Outage Capacity based Link Adaptation for BICM-OFDM with Imperfect CSI

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Abstract—Adapting the modulation, power and channel code to the current channel state for OFDM systems is an important topic for future wireless data transmission to fully utilize the given channel conditions. The consideration of practical channel codes in Link Adaptation schemes is still under investigation, especially if imperfect knowledge of the channel conditions is assumed. In this paper, the ramifications of imperfect channel knowledge on capacity based Link Adaptation will be discussed and a solution will be proposed. In particular, robustly adapting the LTE turbo code employed in this paper was found to be extremely important for the overall performance of the system.

I. INTRODUCTION

Link Adaptation for Orthogonal Frequency Division Multiplex (OFDM) systems has become common in literature nowadays. Regarding the problem of power and rate distribution aiming either at power minimization or rate maximization for a known channel, many contributions have been made, of which only the most recent include channel coding into their designs [1], [2], [3]. Previous schemes, e.g., [4][5] either focus on uncoded BER or Gaussian capacity as the figure of merit.

Nonetheless, imperfect channel state information (CSI) at the transmitter either due to estimation or delays is still an open topic in the field of "coded" Link Adaptation. Specifically, the adaptation of the code rate has shown high sensitivity to imperfect CSI, whereas bit and power allocation are generally more robust against small to medium deviations from perfect CSI [6].

In this paper, we will present a robust Link Adaptation scheme for rate maximization based on the observation that the performance of capacity approaching codes can be described by the average mutual information (AMI) of one codeword [2]. To this end, closed form approximations of the bit interleaved coded modulation (BICM) [7] capacity of the OFDM system will be employed to describe the statistics of the capacity due to imperfect CSI in closed form. Building on this the bit and power loading as well as adapting the code rate of an Long Term Evolution (LTE) turbo code to the achieved AMI can be facilitated.

The remainder of this paper is organized as follows. Section II introduces the system model and information on channel state information. The basics of the Link Adaptation scheme are discussed in Section III. Section IV then presents numerical results and Section V finally concludes this paper.





Fig. 1. Adaptive OFDM System Model.

II. OFDM SYSTEM MODEL

The OFDM frequency domain description for subcarriers $k \in \{1, ..., N_C\}$ applied in this contribution is given by

$$y_k = h_k \sqrt{p_k} d_k + n_k \tag{1}$$

with receive signal y_k , transmit symbols $d_k \in \mathcal{A}_k$, power factor p_k with total power $\mathcal{P} = \sum_{k=1}^{N_C} p_k$, AWGN noise $n_k \sim \mathcal{N}_C(0, 1)$ and frequency domain channel coefficients h_k stemming from the FFT of the time domain channel $\tilde{\mathbf{h}}$, which is modeled as L Gaussian distributed channel coefficients of equal power 1/L. The subcarrier power p_k and modulation alphabet \mathcal{A}_k (see Section II-A) on a subcarrier k are adapted to the current channel state by a loading algorithm knowing an estimate \hat{h}_k of the channel that is modeled according to Section II-B.

A. Modulation and Coding

Fig. 1 shows the overall system model including forward error correction (FEC). Following the BICM approach, a channel code followed by a random interleaver protects the information bits of one OFDM symbol. Throughout this paper the standard 3GPP LTE turbo code [8] with random interleaving will be applied, which enables flexible code rate choices. In all cases the codeword length is fixed to the number of bits in one OFDM symbol, leading to longer codewords for higher data rates, which may decrease the performance of the channel code at lower data rates.

The interleaved codebits \mathbf{c}' are mapped to transmit symbols d stemming from Gray-mapped *M*-QAM modulation alphabets \mathcal{A} . Each subcarrier *k* may use an individual alphabet of cardinality $M_k = |\mathcal{A}_k|$ carrying m_k bits. Soft-Demapping via a-posteriori-probability (APP) detection is used to supply soft information to the decoder.

B. Channel State Information

The channel estimate \hat{h}_k known at the transmitter either by feedback or a former transmission in the reverse direction is considered as a typical MMSE error model in frequency domain

$$h_k = \hat{h}_k + \Delta_k \,, \tag{2}$$

where the estimate \hat{h}_k of mean energy $E\{|\hat{h}_k|^2\} = \rho^2$ and the error Δ_k are independent. Furthermore, the error is assumed to be independent over the subcarriers neglecting any correlation in frequency domain, which for instance may result from time domain channel estimation or interpolation in frequency domain. Consequently, Δ_k is modeled as Gaussian noise with power $\sigma_e^2 = 1 - \rho^2$, which leads to an overall complex valued Gaussian distribution $\mathcal{N}_C(\hat{h}_k, \sigma_e^2)$ for the channel h_k on a subcarrier k. At the receiver perfect CSI is assumed throughout this paper.

III. OUTAGE BICM CAPACITY LINK ADAPTATION

In the following we will first discuss the BICM capacity considering the imperfect CSI model from Section II-B and then detail a simple Link Adaptation scheme based on the observations. Finally, we will elaborate on the choice of the code rate and its importance for the overall performance.

A. BICM Capacity and Imperfect CSI

The BICM capacity assuming perfect CSI at the receiver on a subcarrier k is given by [9]

$$C_{\text{BICM},k} = m_k - \sum_{\ell=1}^{m_k} \mathbb{E}_{b,y_k} \left\{ \log_2 \frac{\sum_{\tilde{d} \in \mathcal{A}_k} p\left(y_k | \tilde{d}, \hat{h}_k\right)}{\sum_{\tilde{d} \in \mathcal{A}_k^{\ell,b}} p\left(y_k | \tilde{d}, \hat{h}_k\right)} \right\},$$
(3)

where $b \in \{0, 1\}$ denotes a bit label of a symbol d at position ℓ and $\mathcal{A}_k^{\ell,b}$ is then the set of all symbols with value b at bit position ℓ . The mean $\mathbb{E}_{b,y_k}\{\cdot\}$ is taken over the distribution of y_k and bit values b assuming a fixed channel, thereby only averaging over the noise and hypotheses for \tilde{d} . Due to the fact that the transmitter only possesses imperfect CSI, the true channel-to-noise-ratios (CNRs) $|h_k|^2$ (noise power is normalized to 1) are unknown, but follow non-central χ^2 -distributions with non-centralities $\lambda_k = |\hat{h}_k|^2$ assuming (2) [10]. Specifically, the probability density function (pdf) of $|h_k|^2$ is given by

$$\mathbf{p}_{|h_k|^2}(\xi) = \frac{1}{\sigma_e^2} \mathbf{e}^{-\frac{1}{\sigma_e^2}(\xi + \lambda_k)} I_0\left(\frac{2\sqrt{\lambda_k\xi}}{\sigma_e^2}\right) \,, \tag{4}$$

where $I_0(\cdot)$ denotes the modified Bessel function of first kind and zeroth order.

Consequently, the BICM capacity $C_{\text{BICM},k}$ on a subcarrier is no longer a fixed value, but a random variable. Unfortunately, there is no closed form solution for (3), meaning that the pdf of $C_{\text{BICM},k}$ is also not known generally. To this end, a typical model for approximation is applied, estimating the capacity expression by a sum of exponential terms

$$\tilde{C}_{\text{BICM},k} = \sum_{\ell=1}^{N} \frac{m_k}{N} \left(1 - e^{-b_\ell |h_k|^2 p_k} \right) \,, \tag{5}$$



Fig. 2. Comparison of approximated capacity and true capacity for various modulations

where N denotes the number of approximation terms and the b_{ℓ} are to be determined by least squares curve fitting [11]. Depending on N even high order modulations, e.g., 1024-QAMs, can be well approximated. Fig. 2 shows the BICM capacity for various Gray-mapped modulation alphabets and the approximations with N = 4 using the b_{ℓ} given by Table I as dashed black lines. For small modulation alphabets there is virtually no deviation, whereas for larger ones a slight degradation is to be expected. In principle N could be chosen arbitrarily to enhance the approximation, though, the derivations shown in the following might get intractable. For the numerical evaluation in Section IV, N = 4 will be assumed to minimize errors stemming from the capacity approximation.

One important consequence of this approximation is that the pdf of $\tilde{C}_{\text{BICM},k}$ can be formulated in closed form for small N and, furthermore, the standard deviation $\sigma_{\tilde{C}_{\text{BICM},k}}$ as well as the mean $\mu_{\tilde{C}_{\text{BICM},k}}$ can be determined in closed form. Exemplary, for N = 1, the mean $\mu_{\tilde{C}_{\text{BICM},k}}$ at a subcarrier k using (4) and (5) can be calculated through

$$\mu_{\tilde{C}_{\text{BICM},k}} = \int_{0}^{\infty} m_k \left(1 - e^{-b_1 \xi p_k} \right) \cdot p_{|h_k|^2}(\xi) \, d\xi \,. \tag{6}$$

The main obstacle for all calculations regarding the transformed random variable given by (5) lies in the Bessel function $I_0(\cdot)$. After some steps, the following integral has to be solved

$$c \cdot \int_{0}^{\infty} e^{-\left(\frac{1}{\sigma_e^2} + b_1 p_k\right)\xi} I_0\left(\frac{2\sqrt{\lambda_k\xi}}{\sigma_e^2}\right) d\xi , \qquad (7)$$

where c summarizes all constant terms. Fortunately, (7) can be solved by 6.614-3 on page 689 in [12], which allows for closed form solutions. For N > 1, however, symbolic math tools provide the necessary means to achieve solutions for $\sigma_{\tilde{C}_{\rm BICM,k}}$ and $\mu_{\tilde{C}_{\rm BICM,k}}$. After some manipulations the final result

TABLE I LS FITTED APPROXIMATION VARIABLES

M	b_1	b_2	b_3	b_4
2	0.838575170651701	0.471554002572644	0.999999999598585	0.470145668308432
4	0.735499531731037	0.196059856765561	0.108268657021434	0.103417496414273
6	0.021825133060320	0.036463870847281	0.091909128635594	0.511871171620869
8	0.047713835197178	0.005431599944494	0.386827752042358	0.012712259998329
10	0.026585257392315	0.001417248097012	0.296717215845762	0.004678319957500

can be simplified to

$$\mu_{\tilde{C}_{\text{BICM},k}} = m_k \frac{1 + \sigma_e^2 b_1 p_k - e^{-\frac{\lambda_k b_1 p_k}{1 + \sigma_e^2 b_1 p_k}}}{1 + \sigma_e^2 b_1 p_k} \,. \tag{8}$$

Even for N = 1 the standard deviation cannot be shown due to space constraints, but the solution is obtained in a very similar way.

According to the central limit theorem the summation of many independent random variables will lead to an overall Gaussian distribution, hence the approximated sum BICM capacity for one OFDM symbol

$$\tilde{C}_{\text{BICM}} = \sum_{k=1}^{N_C} \tilde{C}_{\text{BICM},k} , \qquad (9)$$

will be Gaussian distributed for large N_C with variance $\sigma_{\tilde{C}_{\text{BICM}}}^2 = \sum_{k=1}^{N_C} \sigma_{\tilde{C}_{\text{BICM},k}}^2$ and mean $\mu_{\tilde{C}_{\text{BICM}}} = \sum_{k=1}^{N_C} \mu_{\tilde{C}_{\text{BICM},k}}$. Therefore, the closed form approximation for the pdf of the sum capacity of one OFDM symbol $p_{\tilde{C}_{\text{BICM}}}(\xi|\hat{h}_1, \dots, \hat{h}_{N_C}, \sigma_e^2)$ given channel estimates \hat{h}_k and estimation error variance σ_e^2 provides a convenient way for further considerations on how to adapt the system. Note, however, that the subcarrier capacities $\hat{C}_{\text{BICM},k}$ are usually not independent with respect to their mean due to the inherent correlation in frequency domain even though the estimation error is assumed to be independent. Thus, the variance estimate $\sigma^2_{\tilde{C}_{\text{BICM}}}$ may require further knowledge of the channel covariance (or at least the length of the impulse response), which will be neglected here due to the CSI model in (2).

B. Rate Maximization

A simple approach to optimize the data rate for given channels and total power \mathcal{P} usually can be obtained by equal power allocation and choosing the modulation with the highest capacity $C_{\text{BICM},k}$ on a subcarrier. The capacities, however, are now random variables and as previously discussed, the sum capacity, respectively the AMI, of one code word, i.e., one OFDM symbol, will determine the average FER/BER outcome. To this end, we use the LTE turbo code to match the code rate to the sum capacity including a code rate gap ϵ_{R_C} to ensure a defined FER/BER performance, which will be discussed in further detail in Section III-C.

As has been shown in Section III-A the pdf of the sum BICM capacity \tilde{C}_{BICM} is determined not only by the estimates \hat{h}_k and the error variance σ_e^2 , but also by the choice of the M_k and the power distribution. Thus, before the channel code will be adapted to the capacity of the current channel, the bit loading should be determined in such a way that maximizes the achievable rate.

Considering the approximated distribution of the capacity $p_{\tilde{C}_{\text{BICM}}}(\xi|\hat{h}_1,\ldots,\hat{h}_{N_C},\sigma_e^2) \sim \mathcal{N}(\mu_{\tilde{C}_{\text{BICM}}},\sigma_{\tilde{C}_{\text{BICM}}}^2)$, it is natural to devise an outage criterion for the maximization. For a given outage probability $\delta = P(\tilde{C}_{BICM} \leq \hat{C}_{\delta})$, which describes the probability that the capacity will be lower than or equal the outage capacity C_{δ} that we want to maximize. The inverse of the cumulative density function (cdf) - also called quantile function - that defines \hat{C}_{δ} for a given $\mu_{\tilde{C}_{\text{BICM}}}$, $\sigma_{\tilde{C}_{\text{BICM}}}^2$ and δ is

$$\hat{C} = \mu_{\tilde{C}_{\text{BICM}}} + \sigma_{\tilde{C}_{\text{BICM}}} \cdot \sqrt{2} \cdot \operatorname{erf}^{-1}(2\delta - 1), \qquad (10)$$

where $\operatorname{erf}^{-1}(\cdot)$ denotes the inverse error function. Fortunately, the second term of (10) is usually very small for reasonable δ meaning that it is sufficient to maximize the individual means $\mu_{\tilde{C}_{\mathrm{BICM}},k}$ on a per subcarrier basis, thus, greatly simplifying the optimization and eliminating the need to find the optimal outage probability δ . In terms of power loading, the worst (regarding the mean) subcarriers can be switched off to enhance the performance for low SNRs. The overall power is then evenly distributed to active subcarriers without a finer distribution of power as this would require some discontinuous optimization approach which is a topic for future works.

Thus, the bit and power loading algorithm can be summarized as follows:

- 1) Initialize: Set $\ell = 0$. For each subcarrier k, assuming $p_k = \mathcal{P}/N$ find M_k , that maximizes $\mu_{ ilde{C}_{\mathsf{BICM},k}}$ and calculate \hat{C}_{ℓ} with (10).
- Sort the μ_{C̃BICM,k} from lowest to highest.
 Set ℓ = ℓ + 1 and switch off k = ℓ by p_ℓ = 0 (μ_{C̃BICM,ℓ} = 0), and calculate \hat{C}_{ℓ} applying power $p_k = \mathcal{P}/(N-\ell)$ to the active subcarriers.
- 4) If $\hat{C}_{\ell} > \hat{C}_{\ell-1}$ goto 2, else stop.

C. Code Rate Adaptation

Besides adaptation of bit allocation and power the most important aspect to control in a coded system is an appropriate design of the channel code to achieve the optimum performance. In this work, we want to adapt the code rate R_C of an LTE turbo code to the capacity of the channel. Given the capacity of the channel $C_{\text{BICM},k}$ and the allocated modulation alphabets M_k for all subcarriers k, the code rate should be

$$R_{C} = \frac{\sum_{k=1}^{N_{C}} C_{\text{BICM},k}}{\sum_{k=1}^{N_{C}} \log_{2} M_{k}} + \epsilon_{R_{C}}, \qquad (11)$$

where the first part is the minimum required code rate for a perfect capacity approaching code to perform error free and



Fig. 3. Histograms of the errors in code rate estimation $R_C - \hat{R}_C$ and $R_C - \tilde{R}_C$ using robust bit and power loading; SNR = 20dB, $\delta = 0.01$, $N_C = 1024$, L = 10 equal power Rayleigh fading taps.

 ϵ_{R_C} denotes a design margin that can be used to tune the actual channel code to a required FER performance. Simply applying the channel estimates \hat{h}_k to calculate $\hat{C}_{\text{BICM},k}$ and, thus, achieving an estimate of the code rate \hat{R}_C is straightforward but neglects further knowledge about the estimation error. Employing \hat{C} (8) to calculate another estimate \tilde{R}_C , however, does not necessarily lead to enhanced results as can be seen by Fig. 3. The histograms of the deviation from R_C for a fixed parameter choice clearly show that \tilde{R}_C generates an overall worse estimate of the code rate with higher standard deviation σ . The quality of the estimation will of course depend on the chosen system parameters and, especially with increasing number of channel taps, the assumptions of independence in Section III-A should lead to less errors.

Generally, overestimating the code rate will lead to outage events, but underestimating reduces the efficiency of the system due to unnecessary redundancy. Unfortunately, the authors have found that even by MAP estimation of the code rate assuming the pdfs given by the approximation discussed in Section III-A, no gains could be achieved. Thus, the straightforward estimate \hat{R}_C will be used in the following.

IV. NUMERICAL EVALUATION

A. Perfect Knowledge of the Code Rate R_C

At first, the code rate selection for the turbo code shall be based on genie information, thereby choosing the code rate for the actually achieved capacity and not for the estimate \hat{R}_C . The code rate choice is critical for the performance of the code, so that the results in this section solely present the performance of bit and power loading scheme from Section III-B. Fig. 4 shows results for an $N_C = 1024$ subcarrier system with L = 10 equal power channel taps using the bit and power loading from Section III-B termed "Robust Cap Loading" using N = 4 approximation terms, the non-robust Coded Bisection (CB) algorithm [1] that uses a discrete set of code rates to adapt



Fig. 4. Goodput and FER vs. SNR with genie code rate and a design gap $\epsilon_{R_c} = 0.07$; OFDM parameters: $N_C = 1024$, L = 10 equal power Rayleigh fading taps.

the system and the algorithm developed in [11], which aims at maximizing the capacity given a range of valid code rates (here in the range [0.1, 0.9]) for comparison. Furthermore, the ergodic capacity of the OFDM system is shown for comparison. Three error powers $\sigma_e^2 = \{0, -5, -15\}$ dB are employed to model "bad", "medium" and "good" CSI. In the lower half, the so-called Goodput is shown, i.e., erroneous codewords are excluded from the rate. Therefore, the achieved rates are coupled with the FER results. A lower FER might decrease the achieved Goodput. With respect to the choice of ϵ_{R_C} a fixed value of 0.07 has been chosen, which ensures FER results roughly below 10^{-2} at medium to high SNRs, which is necessary for a fair comparison with the CB approach.

From the figure it is clear that the proposed scheme is very robust even for "bad" CSI, i.e., $\sigma_e^2 = 0$ dB, and that the FER performance is virtually unchanged due to the perfect knowledge of R_C . In comparison with the other schemes, the performance is better than [11] for medium to bad CSI qualities, but slightly worse at good CSI quality, which is a consequence of the more conservative robust loading. Overall, the loss through bad CSI quality is rather small, which stresses the robustness of the LTE turbo code and the importance of proper code rate estimation. In contrast to the former two the CB approach adapts the code rate imperfectly specifically aiming at a target FER of $P_{Target} = 10^{-2}$. Overall, the CB performance is worse, and offers less control over the achieved FER with decreasing CSI quality. The higher FER at lower SNR is mostly due to shorter codewords at lower rates, whereas small variations at higher SNRs are caused by the bit and power loading. In particular for the "Robust Cap Loading" the bit and power loading depends on the available power and the error variance, which may lead to small variations in the achieved FER due to the fact that a practical channel code with limited codeword length is still dependent on the realizations of data and noise.



Fig. 5. Goodput and FER vs. SNR for robust and non-robust bit and power loading with LTE turbo code fitted to estimated \hat{R}_C , $\epsilon_{R_c} = \{0.06, 0.04, 0\}$ variable, but fixed $\epsilon_{R_c} = 0.07$ for Sankar et al.; OFDM parameters: $N_C = 1024$, L = 10 equal power Rayleigh fading taps.

B. Overall Adaptive System

After investigation of the bit and power loading performance, we now investigate the results for a system, where an estimate \hat{R}_C of the true code rate is employed to adapt the code rate in case of the capacity maximizing schemes and a variable code rate choice as before in case of CB. Furthermore, ϵ_{R_C} has been adapted to the different CSI cases for the approach presented in Section III-B to enhance the Goodput, which can be easily obtained by a numerical analysis like in Section III-C. Note, that this is unnecessary for the other two schemes because the bit and power loading is not dependent on the quality of CSI.

Fig. 5 shows numerical results for the same system as in Section IV-A, now applying the aforementioned assumptions. Notably, the FER performance is less predictable/stable over different SNRs for both capacity based schemes, but less variable for different σ_e^2 for the presented "Robust Cap Loading". By choice of ϵ_{R_c} a certain amount of control can be exercised, even though coarse at best. Due to the fact, that the $\epsilon_{R_c} = \{0.06, 0.04, 0\}$ have been chosen to achieve the best Goodput at the respective σ_e^2 , the FER performance is lacking with respect to the FER originally targeted. This results hints at a fundamental problem: imperfect knowledge of the true R_C reduces the ability to fulfill both maximum rate and a certain FER target.

In terms of Goodput, the presented scheme performs best at all qualities of CSI knowledge and especially at higher error powers. The absolute gains are, however, rather small at a maximum gain of 0.3 bit/s/Hz for $\sigma_e^2 = 0$ dB in comparison to the non-robust capacity based scheme and slightly greater compared to the CB results. Nonetheless, the presented Link Adaptation scheme provides a robust and relatively simple alternative to existing solutions, which ignore CSI quality completely. The additional complexity added by our robust scheme is very small, because the calculation of the $\mu_{\tilde{C}_{\text{BICM},k}}$ is just requiring basic operations and could also be calculated in advance and used as a look-up table. Most importantly, a graceful transition from typical bit and power loading solutions at low CSI errors to a non-adaptive system with fixed modulations and power at very high CSI errors has been achieved.

V. CONCLUSION

In this paper we have introduced an outage based link adaptation in combination with capacity achieving codes even for imperfect CSI at the transmitter. To this end, the distribution of the BICM capacity, which results from the assumption of imperfect CSI at the transmitter, has been analyzed and approximated by a tractable form that allows closed form expressions for further analysis and optimization. This approximation has then been used to formulate a simple bit and power loading scheme to optimize the estimated capacity of a system. Besides the adaptation of the bit allocation and power, the choice of the correct code rate has been discussed as one of the most important aspects to achieve the best possible performance. Unfortunately, the estimation of the bit wise capacity, which indicates the code rate required to successfully transmit, proved difficult to improve beyond the simplest solution. Overall, the presented bit and power loading scheme provides robustness against imperfect CSI at the transmitter at relatively low additional complexity in comparison to known bit and power loading schemes. For future works, the importance of proper code rate/capacity estimation should be stressed as this is the determining factor with respect to robust adaptive systems.

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