Advanced Topics in Digital Communications
Spezielle Methoden der digitalen Datenübertragung

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Lecture

Thursday, 10:00 – 12:00 in N3130

Exercise

Wednesday, 14:00 – 16:00 in N1250

Dates for exercises will be announced during lectures.

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Outline

- Part 1: Linear Algebra
  - Eigenvalues and eigenvectors, pseudo inverse
  - Decompositions (QR, unitary matrices, singular value, Cholesky)
- Part 2: Basics and Preliminaries
  - Motivating systems with **M**ultiple **I**nputs and **M**ultiple **O**utputs (multiple access techniques)
  - General classification and description of MIMO systems (SIMO, MISO, MIMO)
  - Mobile Radio Channel
- Part 3: Information Theory for MIMO Systems
  - Repetition of IT basics, channel capacity for SISO AWGN channel
  - Extension to SISO fading channels
  - Generalization for the MIMO case
- Part 4: Multiple Antenna Systems
  - SIMO: diversity gain, beamforming at receiver
  - MISO: space-time coding, beamforming at transmitter
  - MIMO: BLAST with detection strategies
  - Influence of channel (correlation)
- Part 5: Relaying Systems
  - Basic relaying structures
  - Relaying protocols and exemplary configurations

Outline

- Part 6: In Network Processing

- Part 7: Compressive Sensing
  - Motivating Sampling below Nyquist
  - Reconstruction principles and algorithms
  - Applications

Multiple Antenna Systems

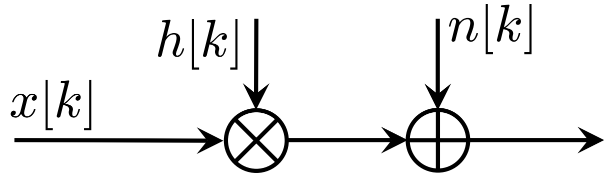
- Exploiting Multiple Antennas for Diversity Enhancement
- SIMO
  - Diversity, Maximum Ratio Combining (beam forming at receiver)
- MISO
  - Beam forming at transmitter
  - Space-Time Coding
    - Orthogonal Space-Time Blockcodes
    - Space-Time Trellis Codes
- MIMO: Layered Space-Time Codes (BLAST) with detection strategies
  - Maximum-Likelihood, Linear Equalization
  - V-BLAST detection algorithm
  - QR-based Successive Interference Cancellation, SQRD
  - Sphere Detection

Exploiting Multiple Antennas

for Diversity Enhancement

Motivation for Antenna Diversity (1)

- Flat Rayleigh fading channel



$$y[k] = h[k] \cdot x[k] + n[k]$$

- Statistic of channel coefficient ($\sigma_h^2 = 1$)
  - Magnitude is Rayleigh distributed
  - Squared magnitude is chi-squared distributed with 2 degrees of freedom

$$p_{|h|}(\xi) = \begin{cases} 2\xi \cdot e^{-\xi^2} & \text{for} \quad \xi \geq 0 \\ 0 & \text{else} \end{cases}$$

$$p_{|h|^2}(\xi) = \begin{cases} e^{-\xi} & \text{for} \quad \xi \geq 0 \\ 0 & \text{else} \end{cases}$$

- Bit Error Rate
  - BER is random variable depending on $|h|^2$

$$P_\mathrm{b}(|h|^2) = \frac{1}{2}\mathrm{erfc}\left(\sqrt{|h|^2 \frac{\overline{E}_\mathrm{b}}{N_0}}\right)$$

  - Average (ergodic) BER

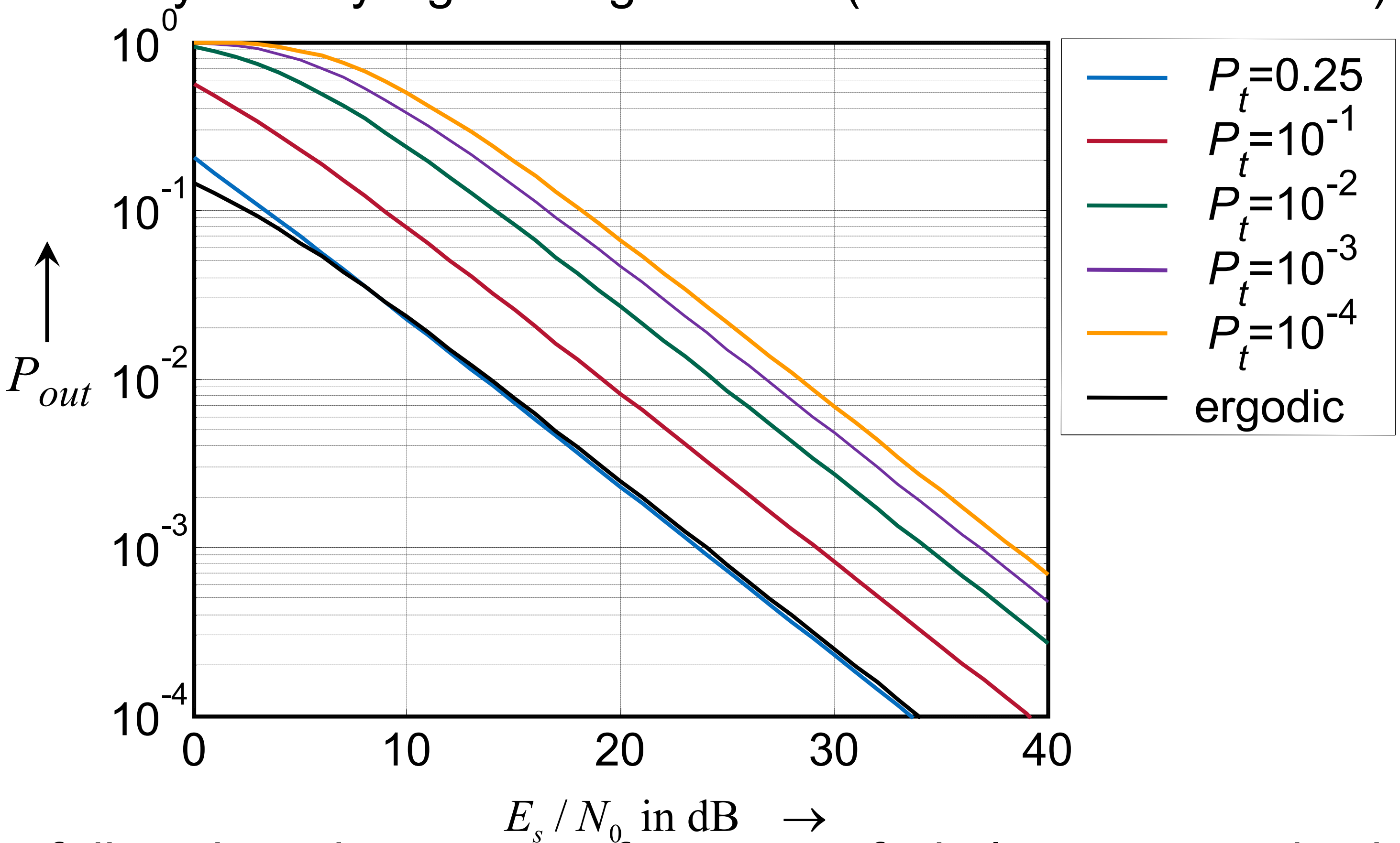
$$\overline{P}_\mathrm{b} = \mathrm{E}\left\{P_\mathrm{b}(|h|^2)\right\} = \int_0^\infty P_\mathrm{b}(|h|^2 = \xi) p_{|h|^2}(\xi) d\xi = \frac{1}{2}\left[1 - \sqrt{1 - \frac{1}{1 + E_\mathrm{b}/N_0}}\right]$$

  - Outage probability for a certain target error rate

$$P_\mathrm{out}(P_\mathrm{b,target}) = \mathrm{P_r}\left\{P_\mathrm{b} > P_\mathrm{b,target}\right\} = 1 - \exp\left(-\frac{[E_\mathrm{b}/N_0]_\mathrm{target}}{\overline{E}_\mathrm{b}/N_0}\right)$$

Motivation for Antenna Diversity (2)

- Outage probability for Rayleigh fading channel (for BPSK transmission)



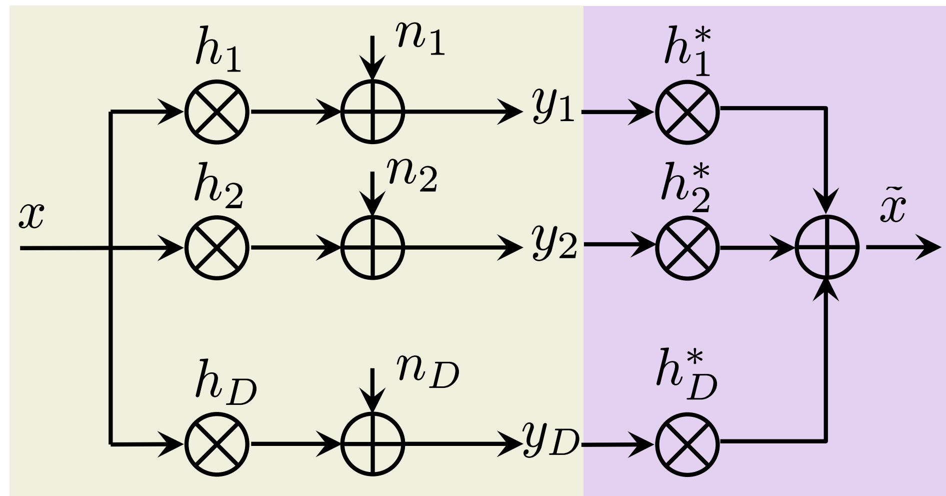
- Utilization of diversity to increase performance of wireless communication

What is Diversity ?

- Different sources of diversity: Frequency, Time, Polarization, Code, Space
- General: receive D statistically independent replicas of same signal
  - **Maximum Ratio Combining** (**MRC**) represents maximum likelihood estimation

$$\tilde{x} = \sum_{j=1}^{D} h_j^* \cdot y_j = \sum_{j=1}^{D} h_j^* \cdot (h_j x + n_j)$$

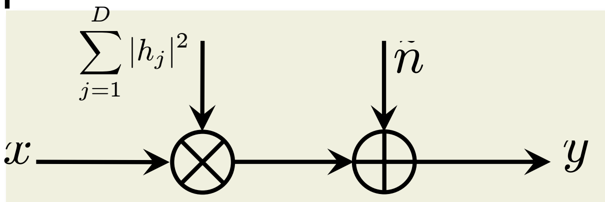
$$= x \cdot \sum_{j=1}^{D} |h_j|^2 + \sum_{j=1}^{D} h_j^* n_j$$



- BER analysis
  - Receive power at each branch $E_s |h_j|^4$
  - Noise term contains sum of D i.i.d. Gaussian processes, weighted by h_j^*
    $\rightarrow$ zero-mean Gaussian process with variance $\sigma_n^2 \sum_{j=1}^{D} |h_j|^2$
  - Average receive power per Bit after MRC: $E_s \left(\sum_{j=1}^{D} |h_j|^2 \right)^2$

SNR Distribution for Maximum Ratio Combining

- MRC: constructive superposition of independent signal parts
- Equivalent SISO channel



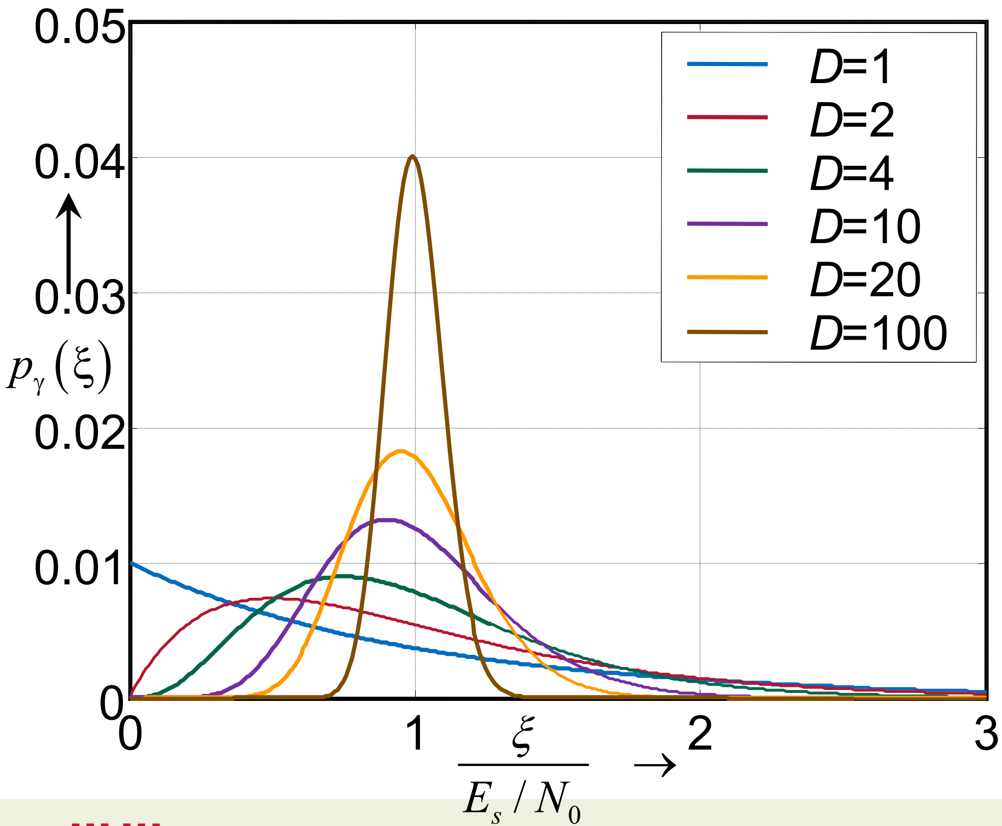
$$P_b = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\frac{E_s\left(\sum_{j=1}^{D}|h_j|^2\right)^2}{N_0\cdot\sum_{j=1}^{D}|h_j|^2}}\right) = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\sum_{j=1}^{D}|h_j|^2\frac{E_s}{N_0}}\right) = \frac{1}{2}\mathrm{erfc}\left(\sqrt{\sum_{j=1}^{D}\gamma_j}\right)$$

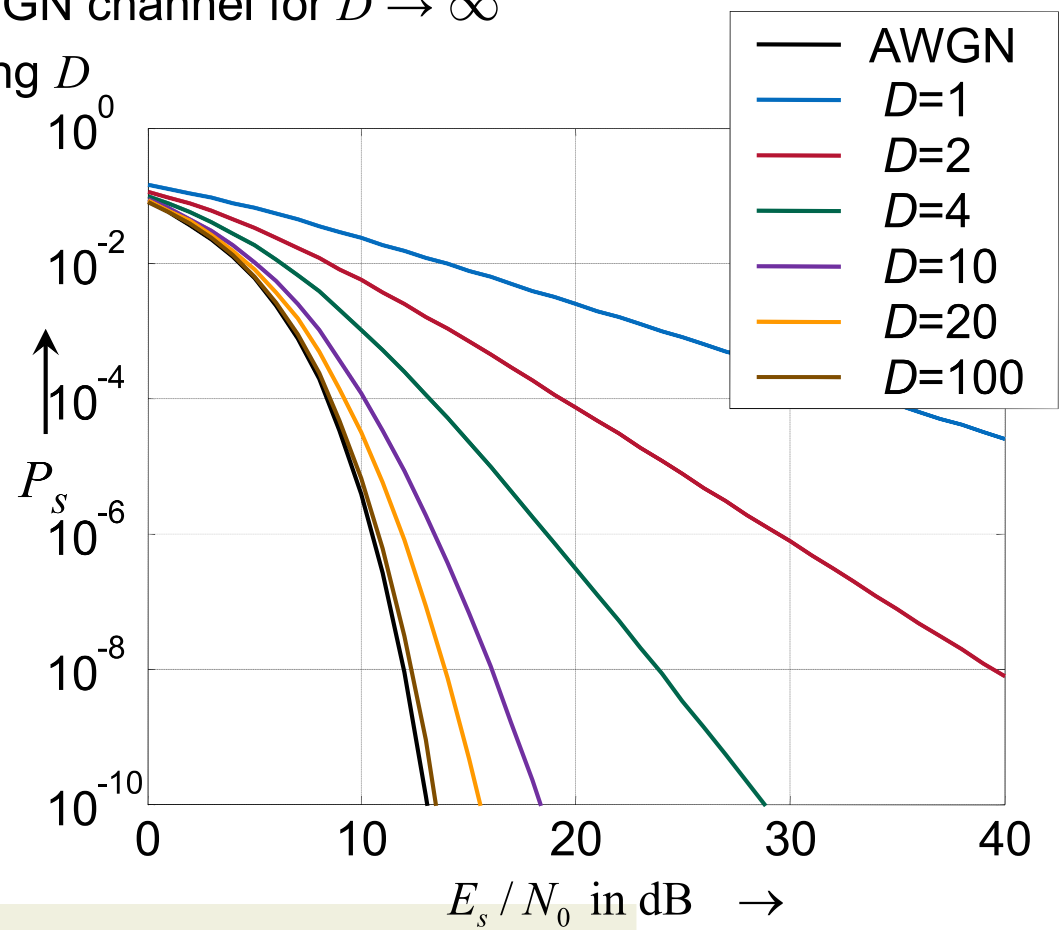
- Distribution of signal to noise ratio after maximum ratio combining $\gamma = \sum_{j=1}^{D}\gamma_j$
  - Chi-squared distribution with $2D$ degrees of freedom

$$p_\gamma(\xi) = \frac{\xi^{D-1}}{(D-1)!\cdot(E_s/N_0)^D}\cdot\exp\left(-\frac{\xi}{E_s/N_0}\right)$$

SNR distribution and BER for Maximum Ratio Combining

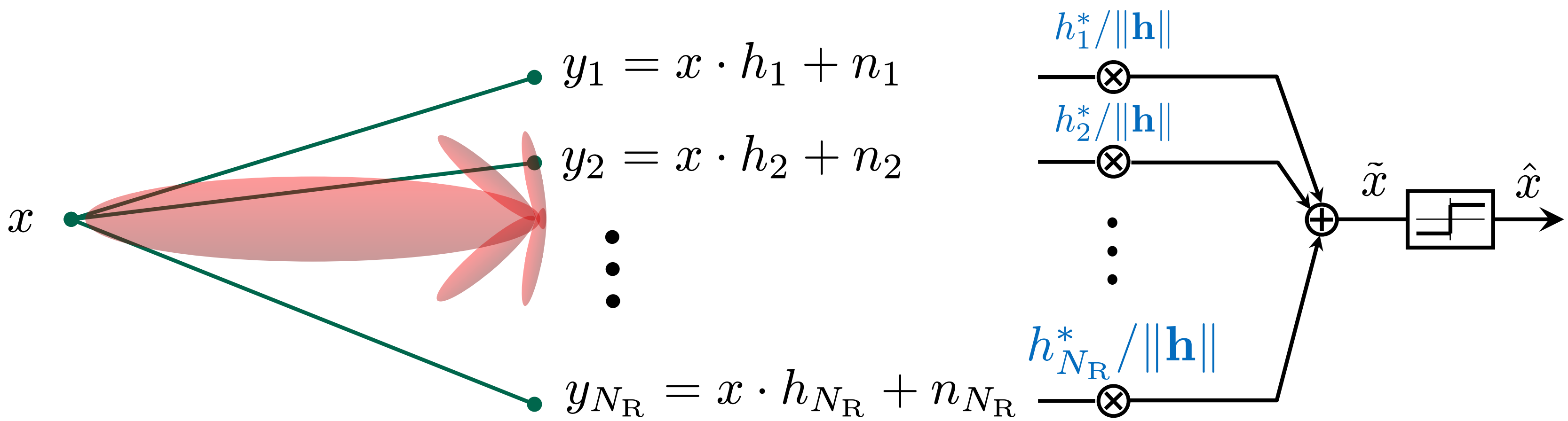
- Density approaches Dirac impulse for $D \to \infty \to$ AWGN
  - Error rate performance reaches AWGN channel for $D \to \infty$
  - **Slope of curve** increases for growing D





Single-Input Multiple-Output Systems (SIMO)

- Multiple antennas only at receiver $\mathbf{y} = \mathbf{h} \cdot x + \mathbf{n}$

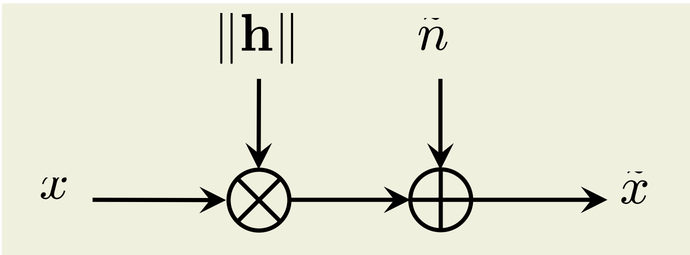


- Optimal receiver performs spatial matched filtering (**Rx-beamforming**)
  - **Matched filter** maximizes SNR by maximum ratio combining (**MRC**)

$$\tilde{x} = \frac{\mathbf{h}^H}{\|\mathbf{h}\|} \cdot \mathbf{y} = x \cdot \frac{1}{\|\mathbf{h}\|} \cdot \sum_{j=1}^{N_R} |h_j|^2 + \tilde{n} = x \cdot \|\mathbf{h}\| + \tilde{n}$$

Gain after Maximum Ratio Combining

- MRC transforms SIMO model into a SISO channel with maximized SNR

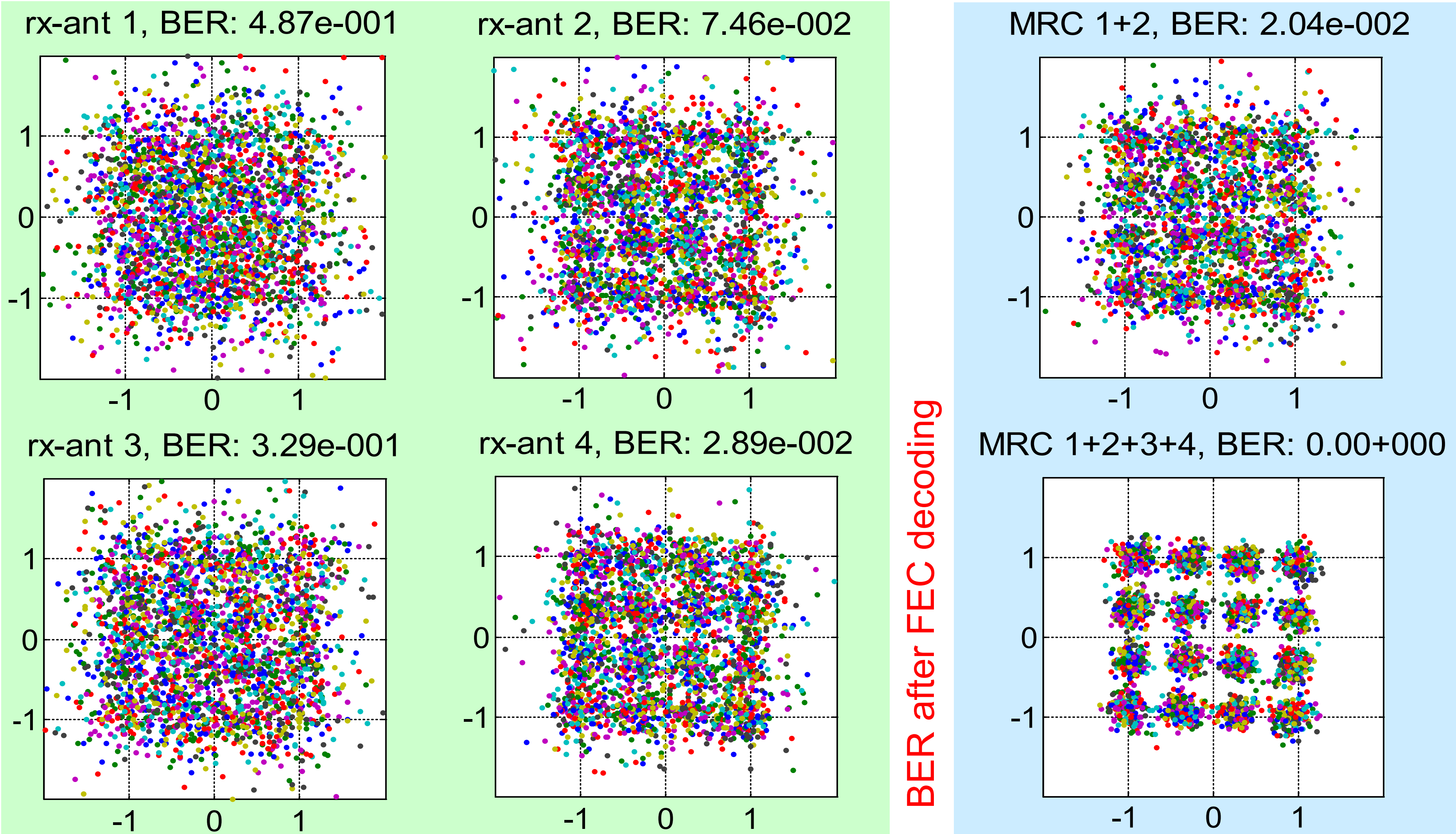


$$\mathrm{SNR} = \sum_{j=1}^{N_\mathrm{R}} |h_j|^2 \cdot \frac{E_s}{N_0}$$

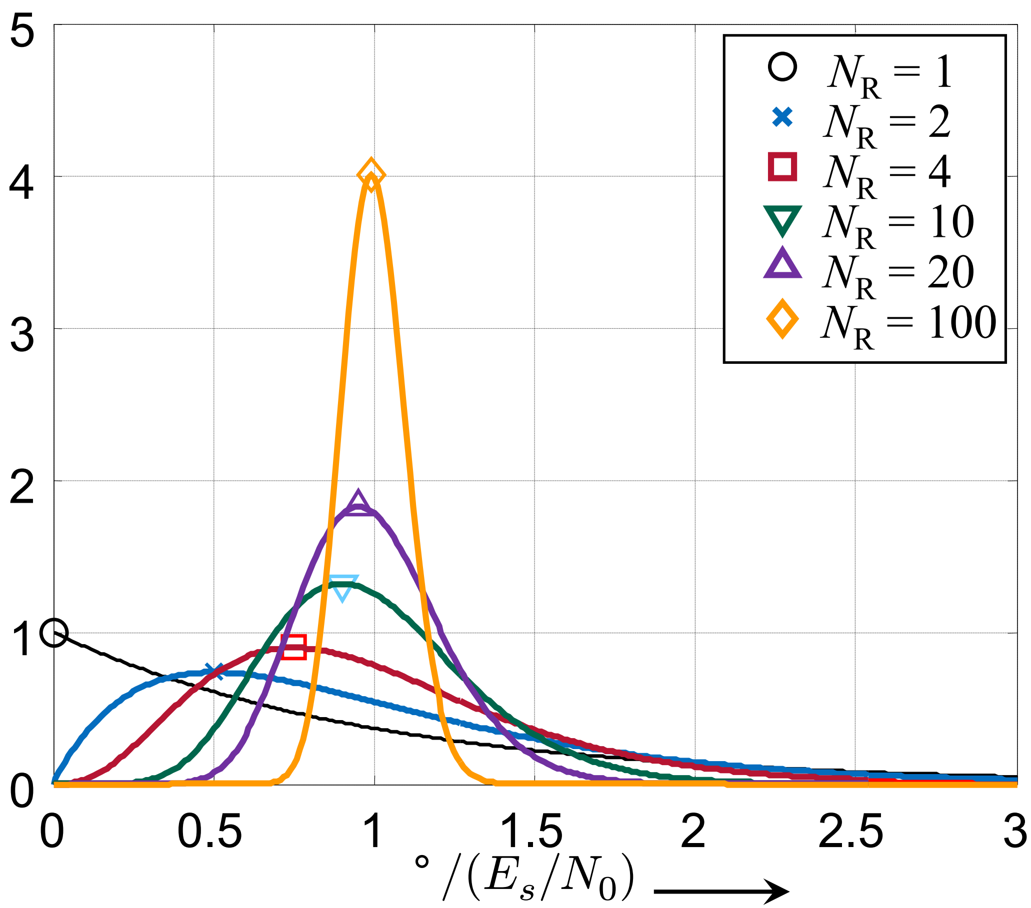
- Two different gains:
  - **Antenna gain** in dB: $10\log_{10}(N_\mathrm{R})$
  - **Diversity gain** due to averaging statistically independent channels
  - Normalizing signal to noise ratio after MRC hides antenna gain for illustration of diversity effect

$$\gamma = \frac{\mathrm{SNR}}{N_\mathrm{R}} = \frac{1}{N_\mathrm{R}} \cdot \sum_{j=1}^{N_\mathrm{R}} |h_j|^2 \cdot \frac{E_s}{N_0}$$

MASI Measurement for IEEE802.11a (36 Mbit/s-Mode)



SNR Distribution after Maximum Ratio Combining



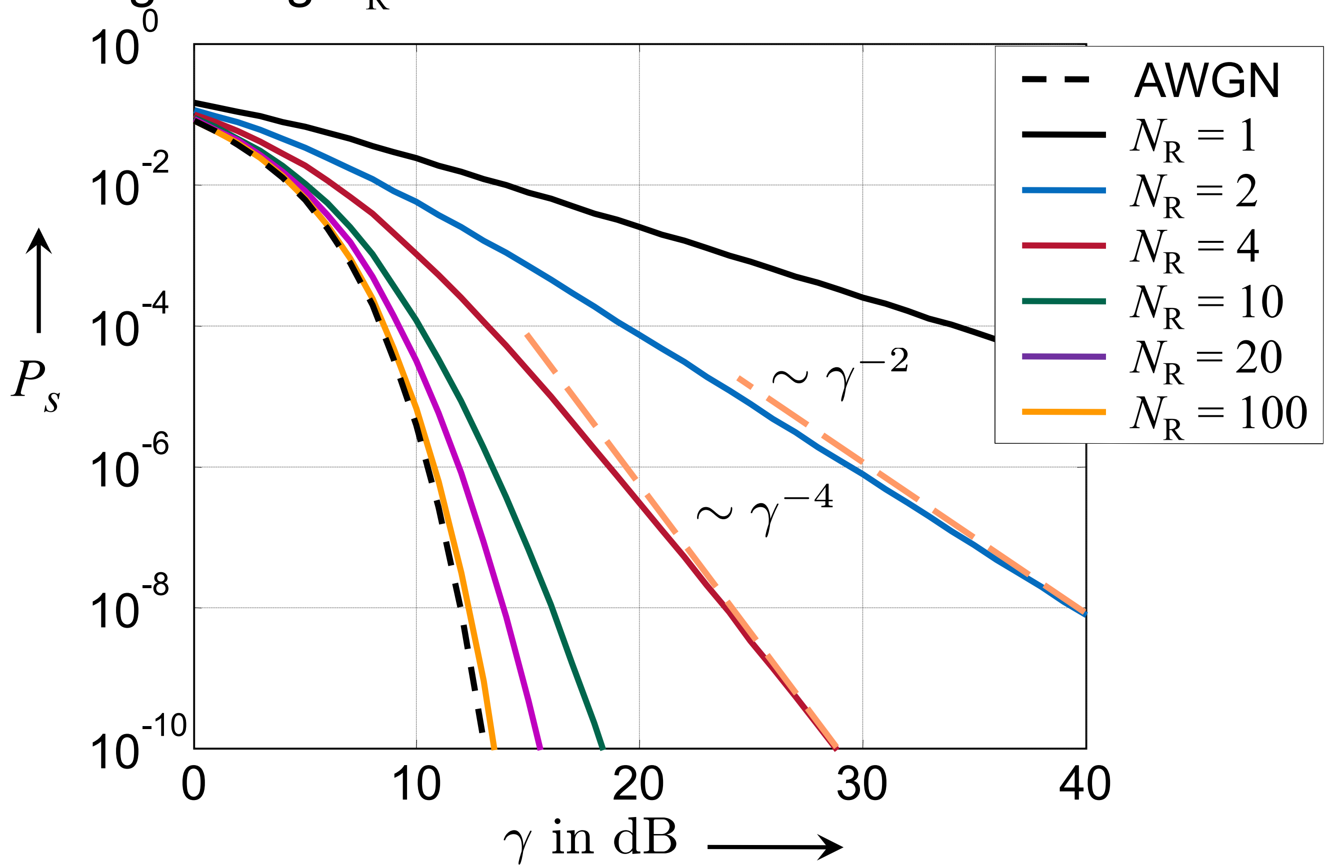
- Rayleigh fading channels
- i.i.d. coefficients
  - Sufficient antenna spacing required
  - Chi-squared distribution with $2N_R$ degrees of freedom

$$p_\gamma(\xi) = \frac{\xi^{N_R - 1}}{(N_R - 1)!} \cdot e^{-\xi}$$

Error Rate Performance for Diversity

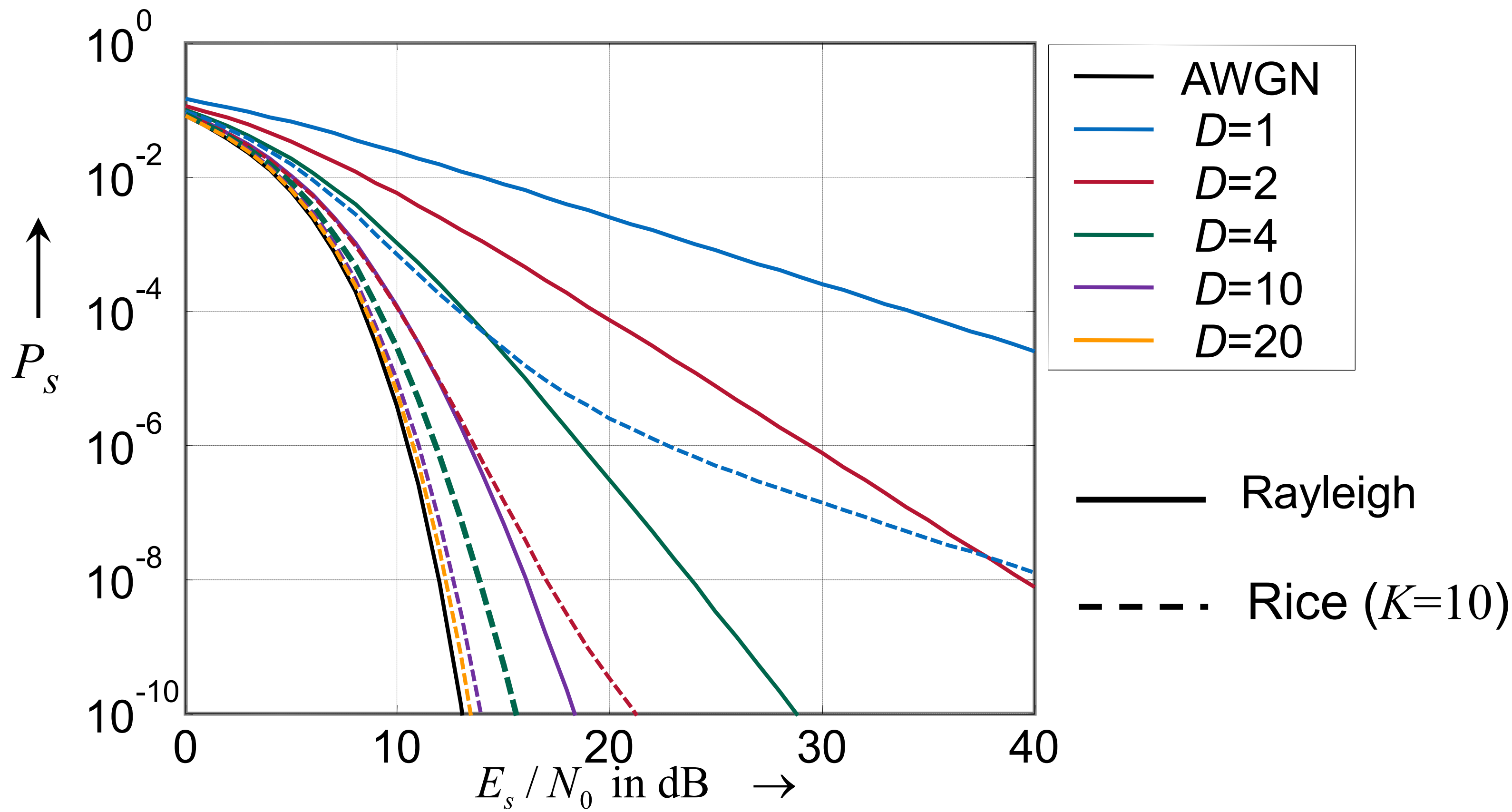
- Error rate performance reaches AWGN channel for $N_\mathrm{R} \rightarrow \infty$
- Slope of curve increases for growing N_R
- **Diversity gain**

$$g_\mathrm{D} \triangleq -\lim_{\gamma\to\infty} \frac{\log(P_s)}{\log(\gamma)}$$



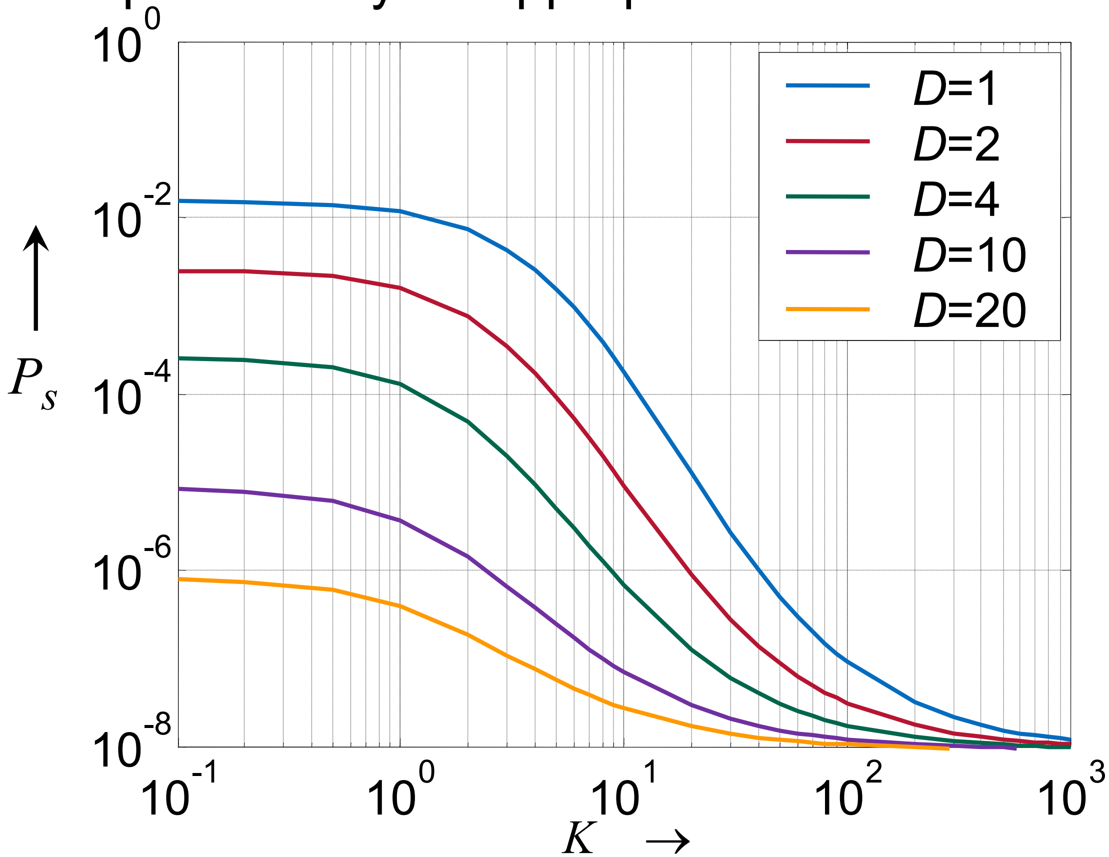
Comparison of Rayleigh and Rice Fading

- Rice suffers less from fading due to line-of-sight path
- Rice channel reaches the AWGN channel with less diversity



Comparison of Rayleigh and Rice Fading

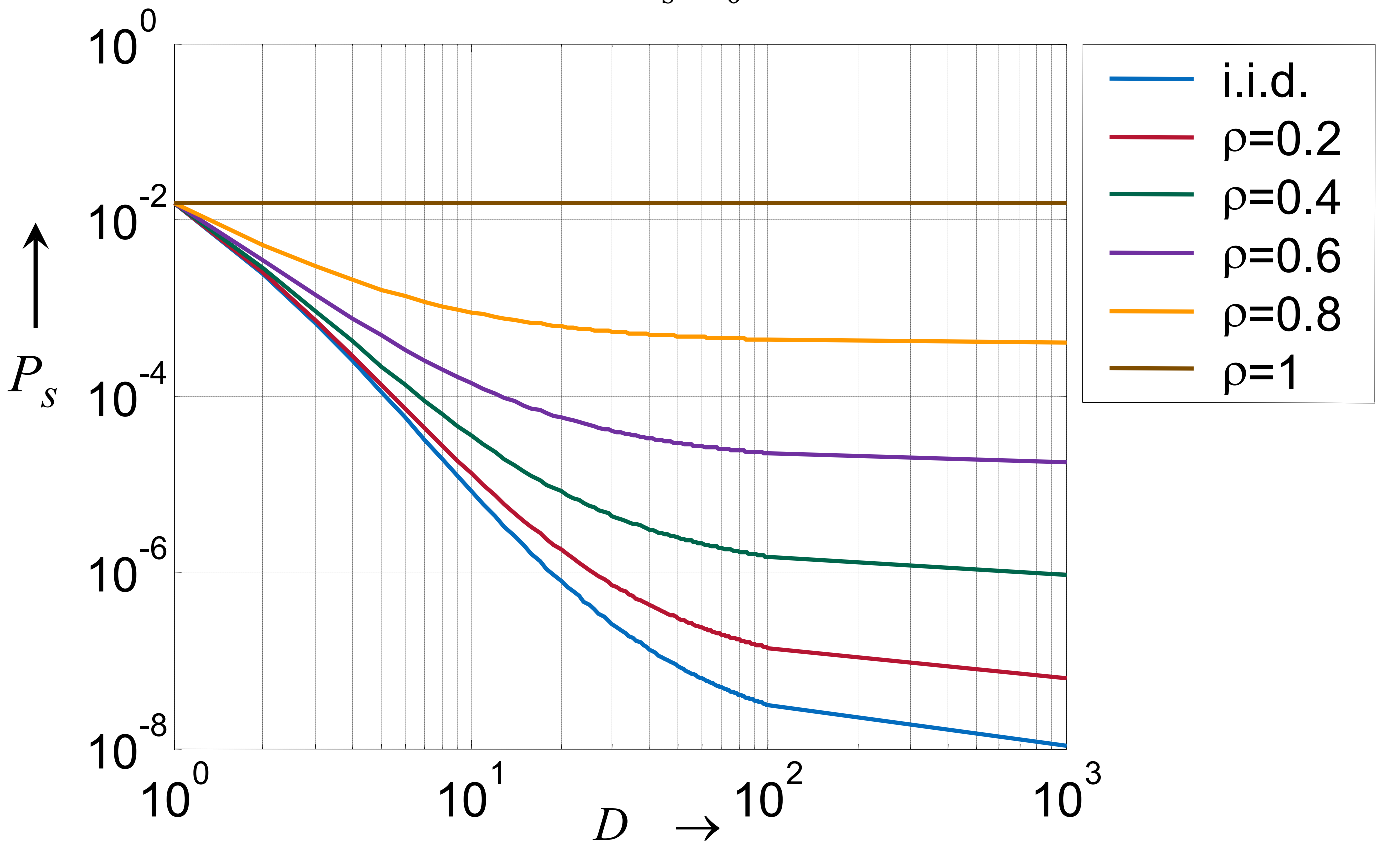
- No diversity gain for Rice factor $K \to \infty$ due to normalization and no fading
- Diversity concepts are only an appropriate means in severe fading conditions



$E_s/N_0 = 12$ dB

Influence of Correlation between Diversity Paths

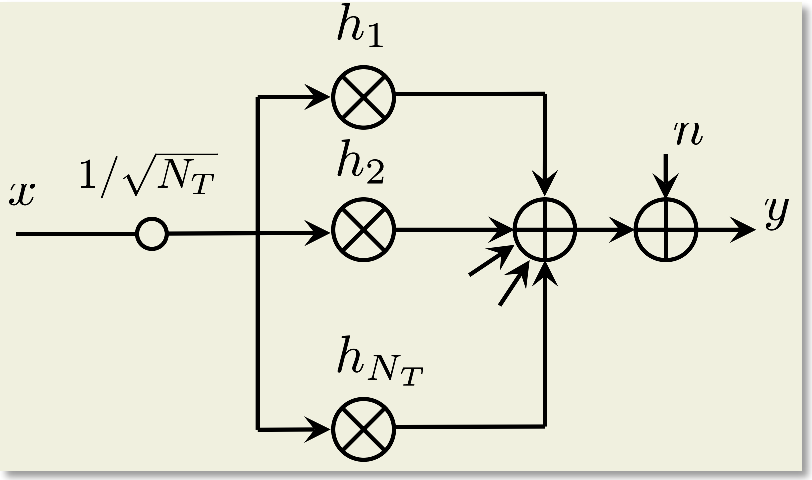
- Diversity gain vanishes for increasing correlation ($\rho \to 1$), here for BPSK with identical distributed channels at $E_s/N_0 = 12$ dB



Transmit Diversity ?

- Receive diversity can be achieved with multiple receive antennas
- Can transmit diversity be obtained by transmitting same signal with N_T antennas?

$$y = \frac{x}{\sqrt{N_T}} \cdot \sum_{i=1}^{N_T} h_i + n$$



- Average receive power per symbol

$$\frac{E_s}{N_T}\left(\sum_{i=1}^{N_T} h_i\right)^2$$

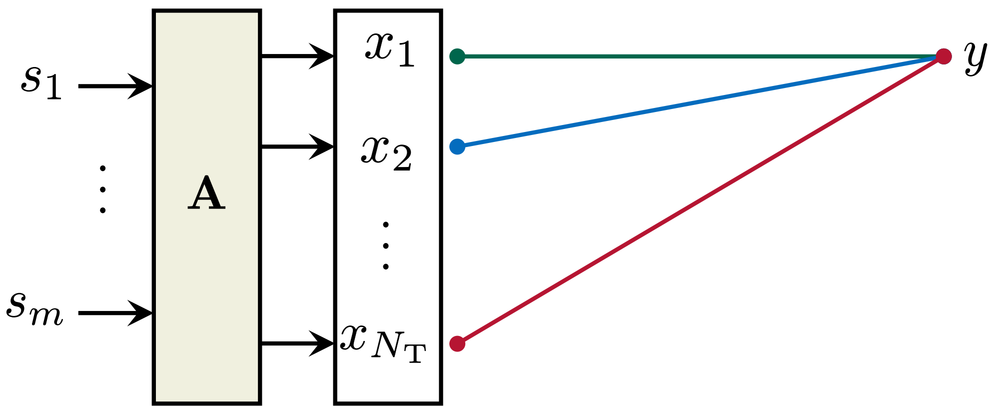
  - Coefficients h_i are i.i.d. with variance $\sigma_h^2 = 1$
  - New Rayleigh distributed coefficient with $\sigma_{\tilde{h}}^2 = N_T$

incoherent superposition
→ constructive and destructive addition of paths

- Error probability $P_b = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E_s\left(\sum_{i=1}^{N_T} h_i\right)^2}{N_T \cdot N_0}}\right) = \frac{1}{2}\operatorname{erfc}\left(\sum_{i=1}^{N_T} h_i \sqrt{\frac{E_s}{N_T \cdot N_0}}\right)$ **No diversity gain!!!**

Multiple-Input Single-Output Systems (MISO)

- Multiple antennas only at transmitter
  - Appropriate pre-processing required



$$y = \underline{\mathbf{h}} \cdot \mathbf{x} + n = \mathbf{h} \cdot \mathbf{A}\mathbf{s} + n$$

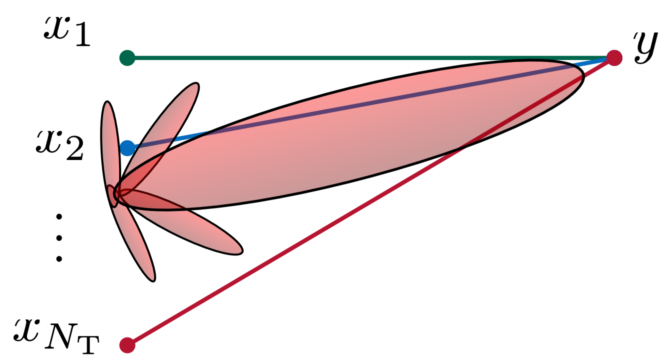
$$\mathbf{h} = \begin{bmatrix} h_1 & h_2 & \cdots & h_{N_\mathrm{T}} \end{bmatrix}$$

- Different levels of channel knowledge (**channel state information**, **CSI**) at transmitter
  - Perfect channel knowledge → beam forming
  - No channel knowledge → space-time coding
- Total transmit energy normalized to $E_s = \mathrm{E}\{\|\mathbf{x}\|^2\}$

MISO Systems: Perfect Channel Knowledge at Transmitter

- Optimum transmitter maximizes SNR at receiver by using matched filter with normalized transmit power: $\mathbf{A} = \underline{\mathbf{h}}^H / \|\underline{\mathbf{h}}\|$

$$\tilde{s} = \underline{\mathbf{h}} \cdot \underbrace{\frac{\underline{\mathbf{h}}^H}{\|\underline{\mathbf{h}}\|} s}_{\mathbf{x}} + n = s \cdot \frac{1}{\|\underline{\mathbf{h}}\|} \cdot \sum_{i=1}^{N_\mathrm{T}} |h_i|^2 + n$$
$$= \|\underline{\mathbf{h}}\| \cdot s + n$$



- **Tx-Beamforming** by maximum ratio combining

- Two different gains
  - Antenna gain: $10 \log_{10}(N_\mathrm{T})$ in dB
  - Diversity gain

- MISO system with perfect CSI at transmitter equivalent to SIMO system

Extension to MIMO-Systems: Multilayer-Transmission

- Assuming instantaneous knowledge at transmitter and receiver
- Singular value decomposition of channel matrix $\mathbf{H} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^H$

- Exploiting all eigenmodes of channel supports multiple data streams
  - Transmitter $\mathbf{x} = \mathbf{V} \cdot \mathbf{s}$
  - Receiver

$$\begin{aligned}\tilde{\mathbf{s}} &= \mathbf{U}^H \cdot \mathbf{y} = \mathbf{U}^H \cdot (\mathbf{H}\mathbf{x} + \mathbf{n}) \\ &= \underbrace{\mathbf{U}^H \cdot \mathbf{U}}_{\mathbf{I}_{N_\mathrm{R}}} \mathbf{\Sigma} \underbrace{\mathbf{V}^H \cdot \mathbf{V}}_{\mathbf{I}_{N_\mathrm{T}}} \mathbf{s} + \mathbf{U}^H \cdot \mathbf{n} = \mathbf{\Sigma} \cdot \mathbf{s} + \tilde{\mathbf{n}}\end{aligned}$$

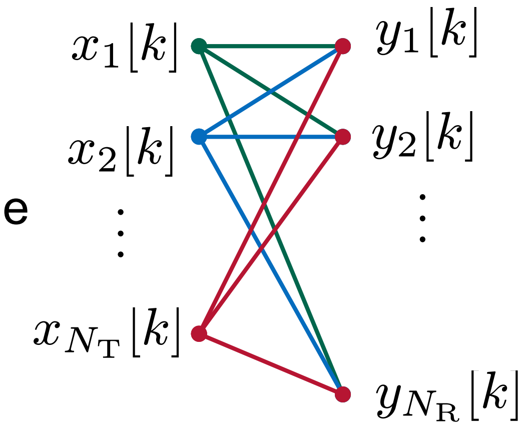
$$\tilde{s}_i = \Sigma_i \cdot s_i + \tilde{n}_i$$

- Transforming MIMO system into parallel SISO systems by singular value decomposition
- Number of parallel layers depend on rank of $\mathbf{H}$
- Adaptation of modulation/coding per parallel layer by water-filling

Space-Time Codes

- General principle of STC
- Error Rate Analysis of MIMO Systems
- Space-Time Blockcodes
- Space-Time Trelliscodes

Space-Time Codes (STC)



- **Space-Time Codes** (**STC**)
  - Achieve transmit diversity without requiring CSI@Tx
  - Coding = arranging the transmitted symbols in space and time
  - **Orthogonal Space-Time Block Codes** (**STBC**)
  - **Space-Time Trellis Codes** (**STTC**) also provide coding gain
  - …
  - Transmit diversity schemes can be combined with multiple receive antennas!
- Transmission of block of length $L \rightarrow$ code matrix $\mathbf{X}$

$$\mathbf{X} = \begin{bmatrix} \mathbf{x}[1] & \mathbf{x}[2] & \ldots & \mathbf{x}[L] \end{bmatrix} = \begin{bmatrix} x_1[1] & x_1[2] & \ldots & x_1[L] \\ \vdots & & & \\ x_{N_\mathrm{T}}[1] & x_{N_\mathrm{T}}[2] & \ldots & x_{N_\mathrm{T}}[L] \end{bmatrix}$$

- Space-Time Code specifies how the code matrix $\mathbf{X}$ is generated
  - Mapping of information symbols s_1, …, s_m onto transmit symbols $x_i[k]$
  - Appropriate design criteria for STC are required!

Instantaneous Pairwise Error Probability

- Probability to decide in favor of code matrix $\mathbf{E}$, when $\mathbf{X}$ was transmitted

$$\mathrm{P}\{\mathbf{X} \to \mathbf{E}|\mathbf{H}\} \square \exp\left(-\frac{E_s}{4N_o} d^2\left(\mathbf{X},\mathbf{E}|\mathbf{H}\right)\right)$$

  - Squared Euclidian distance of corresponding received sequences

$$d^2\left(\mathbf{X},\mathbf{E}|\mathbf{H}\right) = \sum_{k=1}^{L} \left\|\mathbf{H}\cdot\left(\mathbf{x}[k]-\mathbf{e}[k]\right)\right\|^2 = \sum_{j=1}^{N_\mathrm{R}} \underline{\mathbf{h}}_j \cdot \Delta\left(\mathbf{X},\mathbf{E}\right)\cdot \underline{\mathbf{h}}_j^H$$

  - With squared distance matrix and eigenvalue decomposition

$$\Delta\left(\mathbf{X},\mathbf{E}\right) = \left(\mathbf{X}-\mathbf{E}\right)\cdot\left(\mathbf{X}-\mathbf{E}\right)^H = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^H$$

$$r = \mathrm{rank}\{\Delta(\mathbf{X},\mathbf{E})\}$$

$$\lambda_1,\ldots,\lambda_r > 0$$

$$\lambda_{r+1},\ldots,\lambda_{N_\mathrm{T}} = 0$$

  the squared Euclidian distance becomes

$$d^2(\mathbf{X},\mathbf{E}|\mathbf{H}) = \sum_{j=1}^{N_\mathrm{R}} \underline{\mathbf{h}}_j \cdot \mathbf{U}\Lambda\mathbf{U}^H \cdot \underline{\mathbf{h}}_j^H = \sum_{j=1}^{N_\mathrm{R}} \underline{\mathbf{b}}_j \Lambda \underline{\mathbf{b}}_j^H = \sum_{j=1}^{N_\mathrm{R}} \sum_{i=1}^{N_\mathrm{T}} |b_{j,i}|^2 \lambda_i$$

  with $\underline{\mathbf{b}}_j = \underline{\mathbf{h}}_j \cdot \mathbf{U}$ elements $b_{j,i}$ of $\underline{\mathbf{b}}_j$ are still Rayleigh distributed with same variance

$$\Rightarrow \quad \mathrm{P}\{\mathbf{X} \to \mathbf{E}|\mathbf{H}\} \square \exp\left(-\frac{E_s}{4N_0}\sum_{j=1}^{N_\mathrm{R}}\sum_{i=1}^{N_\mathrm{T}} |b_{j,i}|^2\lambda_i\right) = \prod_{j=1}^{N_\mathrm{R}}\prod_{i=1}^{N_\mathrm{T}} \exp\left(-\frac{E_s}{4N_0}|b_{j,i}|^2\lambda_i\right)$$

Average Pairwise Error Probability

- Average pairwise error probability

$$\mathrm{P}\{\mathbf{X} \to \mathbf{E}\} = \mathrm{E}_{\mathbf{H}}\{\mathrm{P}\{\mathbf{X} \to \mathbf{E}|\mathbf{H}\}\} = \mathrm{E}_{\beta}\left\{\prod_{j=1}^{N_\mathrm{R}}\prod_{i=1}^{N_\mathrm{T}} \exp\left(-\frac{E_s}{4N_0}|b_{j,i}|^2\lambda_i\right)\right\}$$

- Calculation of expected value w.r.t to β yields

$$\mathrm{P}\{\mathbf{X} \to \mathbf{E}\} \square \prod_{i=1}^{r}\left(1+\frac{E_s}{4N_0}\lambda_i\right)^{-N_\mathrm{R}} \square \left(\left(\prod_{i=1}^{r}\lambda_i\right)^{1/r} \cdot \frac{E_s}{4N_0}\right)^{-r\cdot N_\mathrm{R}}$$

$$\lambda_1,\ldots,\lambda_r > 0$$
$$\lambda_{r+1},\ldots,\lambda_{N_\mathrm{T}} = 0$$

- **Diversity gain** determines the slope of BER curve in log-scale $g_\mathrm{D} = r \cdot N_\mathrm{R}$

- **Coding gain** determines horizontal shift $g_\mathrm{C} = \min_{(\mathbf{X},\mathbf{E})}\left(\prod_{i=1}^{r}\lambda_i\right)^{1/r}$

  - For difference matrix of full rank ($r = N_\mathrm{T}$) $g_\mathrm{C} = \min_{(\mathbf{X},\mathbf{E})}\left(\det\mathbf{\Delta}(\mathbf{X},\mathbf{E})\right)^{1/N_\mathrm{T}}$

Design Criteria for Space-Time Codes

- **Rank Criterion**:
  In order to achieve the maximum diversity $N_T \cdot N_R$, the difference matrix $(\mathbf{X}\text{-}\mathbf{E})$ has to be full rank for any codeword matrices $\mathbf{X}$ and $\mathbf{E}$.
  If $(\mathbf{X}\text{-}\mathbf{E})$ has a minimum rank r over the set of pairs of distinct words, a diversity of $r \cdot N_R$ is achieved

$$\min_{(\mathbf{X},\mathbf{E})} \operatorname{rank}\{\Delta(\mathbf{X},\mathbf{E})\} = N_\mathrm{T} \quad \Leftrightarrow \quad g_D = \min_{(\mathbf{X},\mathbf{E})} \operatorname{rank}\{\Delta(\mathbf{X},\mathbf{E})\} \cdot N_\mathrm{R} = N_\mathrm{T} \cdot N_\mathrm{R}$$

- **Determinant Criterion**:
  In order to achieve the maximum coding gain for a given diversity gain of $N_T \cdot N_R$, maximize the minimum product of eigenvalues for any two codeword matrices $\mathbf{X}$ and $\mathbf{E}$.

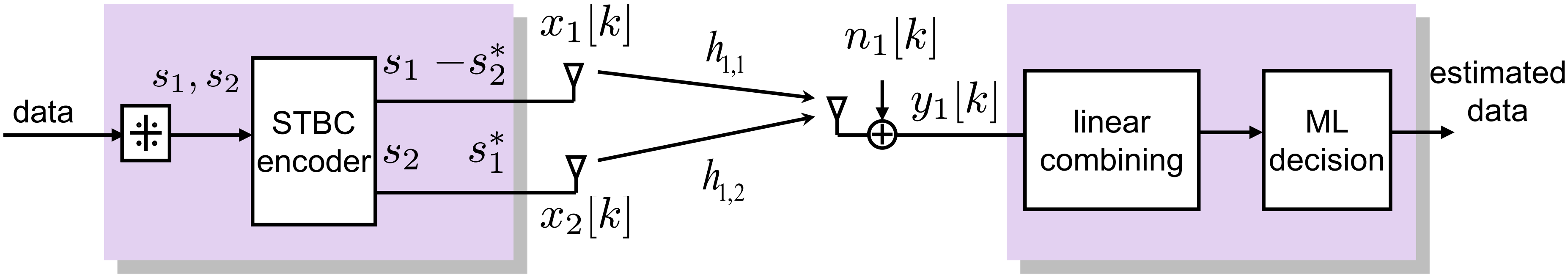
$$g_C = \min_{(\mathbf{X},\mathbf{E})} \left(\prod_{i=1}^{r=N_\mathrm{T}} \right)^{1/N_\mathrm{T}} = \min_{(\mathbf{X},\mathbf{E})} \left(\det \Delta(\mathbf{X},\mathbf{E}) \right)^{1/N_\mathrm{T}}$$

Orthogonal Space-Time Blockcodes (OSTBC)

Orthogonal Space-Time Blockcodes

- Alamouti's scheme
  - Transmission scheme for $N_T = 2$ antennas
  - Equivalent to MRC with 2 antennas at receiver
- Generalization by Tarokh for more than 2 transmit antennas
  - Orthogonal Space-Time Blockcodes
- Simple modulation scheme for limited number of transmit antennas
- Easy detection (demodulation) by linear combination of the received signals
- Transmit diversity schemes can be combined with multiple receive antennas!

Alamouti's Scheme (1)



- Code word matrix of two consecutive time steps

$$\mathbf{X} = \begin{bmatrix} x_1[k] & x_1[k+1] \\ x_2[k] & x_2[k+1] \end{bmatrix} = \begin{bmatrix} s_1 & -s_2^* \\ s_2 & s_1^* \end{bmatrix}$$

- Received signal vector of one block $\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{N}$

$$\begin{bmatrix} y_1[k] & y_1[k+1] \end{bmatrix} = \begin{bmatrix} h_{1,1} & h_{1,2} \end{bmatrix} \cdot \begin{bmatrix} s_1 & -s_2^* \\ s_2 & s_1^* \end{bmatrix} + \begin{bmatrix} n_1[k] & n_1[k+1] \end{bmatrix}$$
$$= \begin{bmatrix} h_{1,1}s_1 + h_{1,2}s_2 & -h_{1,1}s_2^* + h_{1,2}s_1^* \end{bmatrix} + \begin{bmatrix} n_1[k] & n_1[k+1] \end{bmatrix}$$

conjugate second element and rewrite with respect to column vector

$$\Rightarrow \begin{bmatrix} y_1[k] \\ y_1^*[k+1] \end{bmatrix} = \begin{bmatrix} h_{1,1}s_1 + h_{1,2}s_2 \\ -h_{1,1}^*s_2 + h_{1,2}^*s_1 \end{bmatrix} + \begin{bmatrix} n_1[k] \\ n_1^*[k+1] \end{bmatrix} = \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{1,2}^* & -h_{1,1}^* \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_1[k] \\ n_1^*[k+1] \end{bmatrix}$$

Alamouti's Scheme (2)

- Linear combining is matched filtering

$$\begin{bmatrix} \tilde{s}_1 \\ \tilde{s}_2 \end{bmatrix} = \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{1,2}^* & -h_{1,1}^* \end{bmatrix}^H \begin{bmatrix} y_1[k] \\ y_1^*[k+1] \end{bmatrix} = \underbrace{\begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{1,2}^* & -h_{1,1}^* \end{bmatrix}^H \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{1,2}^* & -h_{1,1}^* \end{bmatrix}}_{\left(|h_{1,1}|^2 + |h_{1,2}|^2\right) \cdot \mathbf{I}_2} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \underbrace{\begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{1,2}^* & -h_{1,1}^* \end{bmatrix}^H \begin{bmatrix} n_1[k] \\ n_1^*[k+1] \end{bmatrix}}_{\text{noise term is still white } \begin{bmatrix} \tilde{n}_1 \\ \tilde{n}_2 \end{bmatrix}}$$

- Modified received signal vector after linear combining

$$\begin{bmatrix} \tilde{s}_1 \\ \tilde{s}_2 \end{bmatrix} = \left(|h_{1,1}|^2 + |h_{1,2}|^2\right) \cdot \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} \tilde{n}_1 \\ \tilde{n}_2 \end{bmatrix} \Rightarrow \tilde{\mathbf{s}} = \|\mathbf{H}\|^2 \cdot \mathbf{s} + \tilde{\mathbf{n}}$$

Diversity degree $g_D = 2$!

  - Two independent signals $\tilde{s}_1$ and $\tilde{s}_2$ to represent s_1 and s_2
- Code rate: $R_c^{ST} = 1$ (2 symbols in 2 time slots)
- **Independent detection of signals on basis of linear combining allows very simple receiver structure!**

General Remarks on Orthogonal STBC (1)

- General Results from matrix theory
  - Orthogonal matrices with complex elements for code rate 1 exists only for $N_T = 2$ antennas → **Alamouti**
  - Orthogonal matrices with complex elements for code rate 1/2 exist for any number of transmit antennas
  - Orthogonal matrices with complex elements for code rate 3/4 exist for $N_T = 3$ and $N_T = 4$ antennas
  - Orthogonal quadratic matrices with real valued elements for code rate 1 exist only for $N_T = 2$, $N_T = 4$ and $N_T = 8$

- System with N_T transmit antennas
  - Transmission of m different information symbols s_m
  - Occupation of p time slots for transmission
  - Description by $N_T \times p$ code matrix $\mathcal{G}_{N_T}$

General Remarks on Orthogonal STBC (2)

- Space-Time code rate: m symbols are transmitted in p timeslots $R_c^{ST} = \frac{m}{p}$
- Spectral efficiency $R_c^{ST} \cdot \log_2(M) Bit/s/Hz$

- Elements of $\mathcal{G}_{N_T}$ are given by **linear combinations** of the variables

  $0, s_1, s_1^*, s_2, s_2^*, \cdots, s_m, s_m^*$ → **STBC are linear codes**

  - Only with conjugated elements linear description is possible because conjugation is no linear transformation
  - Code matrix $\mathcal{G}_{N_T}$ consists of orthogonal rows

$$\mathcal{G}_{N_T} \cdot \mathcal{G}_{N_T}^H = \left(|s_1|^2 + |s_2|^2 + \cdots + |s_m|^2\right) \cdot \mathbf{I}_{N_T}$$

- Alternatively, real valued description with twice as large matrices possible

$$\mathcal{G}_{N_T}\left(0, s_1, s_1^*, s_2, s_2^*, \cdots, s_m, s_m^*\right) \rightarrow \tilde{\mathcal{G}}_{N_T}\left(0, s_1', s_1'', s_2', s_2'', \cdots, s_m', s_m''\right)$$

Real and Complex Representation of Alamouti's Scheme

- Alamouti's scheme with $N_T = 2$, $m = 2$, $p = 2$: $R_c^{ST} = \frac{m}{p} = 1$

$$\mathcal{G}_2 = \mathcal{G}_2\left(0, s_1, s_1^*, s_2, s_2^*\right) = \begin{bmatrix} s_1 & -s_2* \\ s_2 & s_1^* \end{bmatrix}$$

$$\mathbf{y} = \begin{bmatrix} y[k] \\ y[k+1] \end{bmatrix} = \begin{bmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{bmatrix} \cdot \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} n[k] \\ n[k+1] \end{bmatrix}$$

- Real representation

$$\tilde{\mathcal{G}}_2 = \tilde{\mathcal{G}}_2\left(s_1', s_1'', s_2', s_2''\right) = \begin{bmatrix} s_1' & s_2' & -s_1'' & -s_2'' \\ -s_2' & s_1' & s_2'' & -s_1'' \\ s_1'' & s_2'' & s_1' & s_2' \\ -s_2'' & s_1'' & -s_2' & s_1' \end{bmatrix} = \begin{bmatrix} \mathrm{Re} & -\mathrm{Im} \\ \mathrm{Im} & \mathrm{Re} \end{bmatrix}$$

$$\mathbf{y} = \begin{bmatrix} y'[k] \\ y'[k+1] \\ y''[k] \\ y''[k+1] \end{bmatrix} = \begin{bmatrix} s_1' & s_2' & -s_1'' & -s_2'' \\ -s_2' & s_1' & s_2'' & -s_1'' \\ s_1'' & s_2'' & s_1' & s_2' \\ -s_2'' & s_1'' & -s_2' & s_1' \end{bmatrix} \cdot \begin{bmatrix} h_1' \\ h_2' \\ h_1'' \\ h_2'' \end{bmatrix} + \begin{bmatrix} n'[k] \\ n'[k+1] \\ n''[k] \\ n''[k+1] \end{bmatrix}$$

Orthogonal STBC for Rate 1/2, $N_T = 3$

- $N_T = 3$ antennas, $m = 4$ information symbols, $p = 8$ time slots

$$\mathcal{G}_3 = \begin{bmatrix} s_1 & -s_2 & -s_3 & -s_4 & s_1^* & -s_2^* & -s_3^* & -s_4^* \\ s_2 & s_1 & s_4 & -s_3 & s_2^* & s_1^* & s_4^* & -s_3^* \\ s_3 & -s_4 & s_1 & s_2 & s_3^* & -s_4^* & s_1^* & s_2^* \end{bmatrix}$$

$\Rightarrow$

$$R_c^{ST} = \frac{m}{p} = \frac{4}{8} = 0.5$$

- Rows of code matrix $\mathcal{G}_3$ are orthogonal $\mathcal{G}_3 \cdot \mathcal{G}_3^H = \left(|s_1|^2 + |s_2|^2 + |s_3|^2 + |s_4|2\right) \cdot \mathbf{I}_3$
- Receive vector $\mathbf{y}$ of dimension 1x8

$$\mathbf{y} = \mathbf{H} \cdot \mathcal{G}_3 + \mathbf{n} = \begin{bmatrix} h_{1,1} & h_{1,2} & h_{1,3} \end{bmatrix} \cdot \begin{bmatrix} s_1 & -s_2 & -s_3 & -s_4 & s_1^* & -s_2^* & -s_3^* & -s_4^* \\ s_2 & s_1 & s_4 & -s_3 & s_2^* & s_1^* & s_4^* & -s_3^* \\ s_3 & -s_4 & s_1 & s_2 & s_3^* & -s_4^* & s_1^* & s_2^* \end{bmatrix} + \mathbf{n}$$

$$\Rightarrow \mathbf{y} = \begin{bmatrix} [h_{1,1}s_1 + h_{1,2}s_2 + h_{1,3}s_3 & -h_{1,1}s_2 + h_{1,2}s_1 - h_{1,3}s_4 & -h_{1,1}s_3 + h_{1,2}s_4 + h_{1,3}s_2] \\ \cdots [-h_{1,1}s_4 - h_{1,2}s_3 + h_{1,3}s_2 & h_{1,1}s_1^* + h_{1,2}s_2^* + h_{1,3}s_3^* & -h_{1,1}s_2^* + h_{1,2}s_1^* - h_{1,3}s_4^*] \\ \cdots [-h_{1,1}s_3^* + h_{1,2}s_4^* + h_{1,3}s_1^* & -h_{1,1}s_4^* - h_{1,2}s_3^* + h_{1,3}s_2^*] & \end{bmatrix} + \mathbf{n}$$

Orthogonal STBC for Rate 1/2, $N_T = 3$

- Generate modified receive vector by conjugation of signals received in time instances 5,…, 8

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5^* \\ y_6^* \\ y_7^* \\ y_8^* \end{bmatrix} = \begin{bmatrix} h_1 & h_2 & h_3 & 0 \\ h_2 & -h_1 & 0 & -h_3 \\ h_3 & 0 & -h_1 & h_2 \\ 0 & h_3 & -h_2 & -h_1 \\ h_1^* & h_2^* & h_3^* & 0 \\ h_2^* & -h_1^* & 0 & -h_3^* \\ h_3^* & 0 & -h_1^* & h_2* \\ 0 & h_3^* & -h_2^* & -h_1^* \end{bmatrix} \cdot \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix} \begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \\ n_5^* \\ n_6^* \\ n_7^* \\ n_8^* \end{bmatrix} \Rightarrow \tilde{\mathbf{y}} = \tilde{\mathbf{H}}\mathbf{s} + \tilde{\mathbf{n}}$$

  - Equivalent channel matrix $\tilde{\mathbf{H}}$ contains orthogonal columns
- Modified received signal vector after linear combining

$$\tilde{\mathbf{s}} = \tilde{\mathbf{H}}^H \cdot \tilde{\mathbf{y}} = \tilde{\mathbf{H}}^H \cdot \tilde{\mathbf{H}} \cdot \mathbf{s} + \tilde{\mathbf{H}}^H \tilde{\mathbf{n}} = \|\tilde{\mathbf{H}}\|^2 \mathbf{s} + \tilde{\tilde{\mathbf{n}}}$$

Orthogonal STBC for Rate 1/2 , $N_T = 4$

- $N_T = 4$ antennas, $m = 4$ information symbols, $p = 8$ time slots

$$\mathcal{G}_4 = \begin{bmatrix} s_1 & -s_2 & -s_3 & -s_4 & s_1^* & -s_2^* & -s_3^* & -s_4^* \\ s_2 & s_1 & s_4 & -s_3 & s_2^* & s_1^* & s_4^* & -s_3^* \\ s_3 & -s_4 & s_1 & s_2 & s_3^* & -s_4^* & s_1^* & s_2^* \\ s_4 & s_3 & -s_2 & s_1 & s_4^* & s_3^* & -s_2^* & s_1^* \end{bmatrix} \Rightarrow R_c^{ST} = \frac{m}{p} = \frac{4}{8} = 0.5$$

  - Modified receive vector → equivalent channel matrix $\tilde{\mathbf{H}}$ with orthogonal columns

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5^* \\ y_6^* \\ y_7^* \\ y_8^* \end{bmatrix} = \begin{bmatrix} h_1 & h_2 & h_3 & h_4 \\ h_2 & -h_1 & h_4 & -h_3 \\ h_3 & -h_4 & -h_1 & h_2 \\ h_4 & h_3 & -h_2 & -h_1 \\ h_1^* & h_2^* & h_3^* & h_4^* \\ h_2^* & -h_1^* & h_4^* & -h_3^* \\ h_3^* & -h_4^* & -h_1^* & h_2* \\ h_4^* & h_3^* & -h_2^* & -h_1^* \end{bmatrix} \cdot \begin{bmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix} \begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \\ n_5^* \\ n_6^* \\ n_7^* \\ n_8^* \end{bmatrix}$$

Orthogonal STBC for Rate 3/4

- $N_T = 3$ antennas, $m = 3$ information symbols, $p = 4$ time slots

$$\mathcal{H}_3 = \begin{bmatrix} s_1 & -s_2^* & \frac{1}{\sqrt{2}}s_3^* & \frac{1}{\sqrt{2}}s_3^* \\ s_2 & s_1^* & \frac{1}{\sqrt{2}}s_3^* & -\frac{1}{\sqrt{2}}s_3^* \\ \frac{1}{\sqrt{2}}s_3 & \frac{1}{\sqrt{2}}s_3 & \frac{1}{2}(-s_1 - s_1^* + s_2 - s_2^*) & \frac{1}{2}(s_1 - s_1^* + s_2 + s_2^*) \end{bmatrix}$$

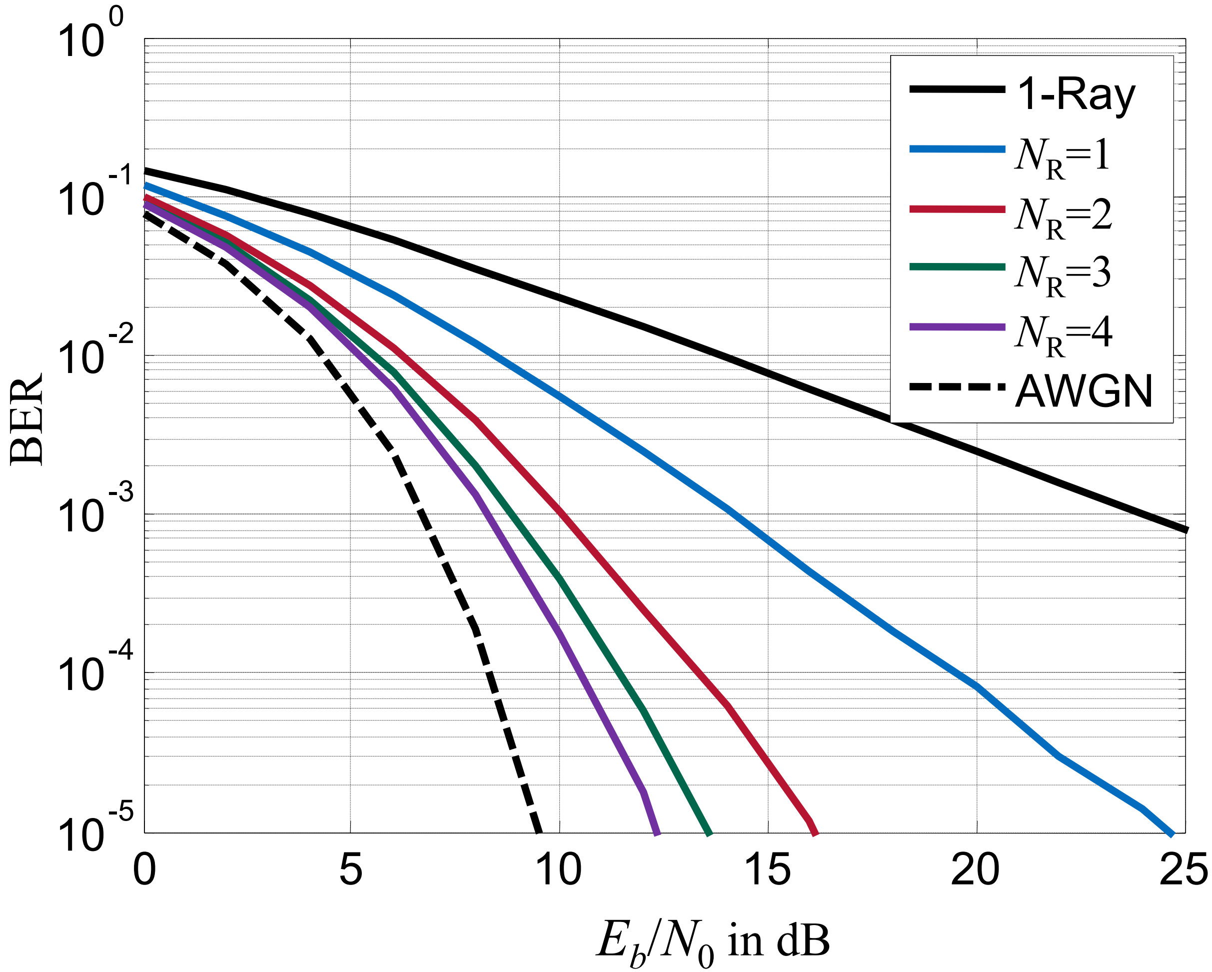
$$R_c^{ST} = \frac{m}{p} = \frac{3}{4}$$

- $N_T = 4$ antennas, $m = 3$ information symbols, $p = 4$ time slots

$$\mathcal{H}_3 = \begin{bmatrix} s_1 & -s_2^* & \frac{1}{\sqrt{2}}s_3^* & \frac{1}{\sqrt{2}}s_3^* \\ s_2 & s_1^* & \frac{1}{\sqrt{2}}s_3^* & -\frac{1}{\sqrt{2}}s_3^* \\ \frac{1}{\sqrt{2}}s_3 & \frac{1}{\sqrt{2}}s_3 & \frac{1}{2}(-s_1 - s_1^* + s_2 - s_2^*) & \frac{1}{2}(s_1 - s_1^* + s_2 + s_2^*) \\ \frac{1}{\sqrt{2}}s_3 & -\frac{1}{\sqrt{2}}s_3 & \frac{1}{2}(s_1 - s_1^* - s_2 - s_2^*) & -\frac{1}{2}(s_1 + s_1^* + s_2 - s_2^*) \end{bmatrix}$$

$$R_c^{ST} = \frac{m}{p} = \frac{3}{4}$$

Simulation Results for STBC (1)

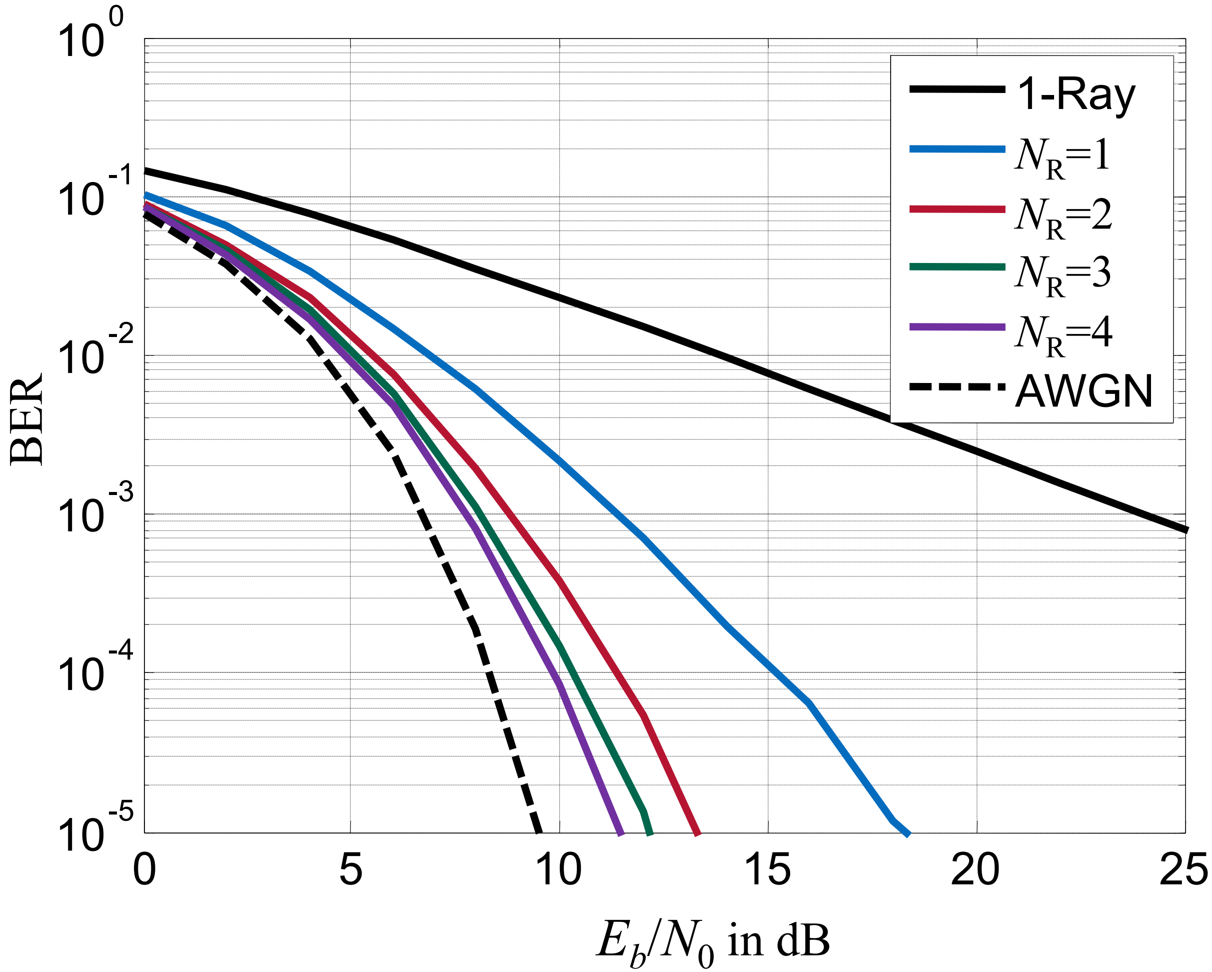


- Simulation parameters
  - Alamouti-STBC, $N_T = 2$,

$$\mathcal{G}_2 = \begin{bmatrix} s_1 & -s_2^* \\ s_2 & s_1^* \end{bmatrix}$$

  - 2 symbols in 2 time slots → $R_c^{ST} = 1$
  - QPSK → 2 bits / time slot
- Results
  - **Diversity gain** determines slope of BER
  - Diversity of $N_T \cdot N_R$ results in strong performance improvement

Simulation Results for STBC (2)



- Simulation parameters
  - STBC for $N_T = 3$,

$$\mathcal{H}_3 = \begin{bmatrix} s_1 & -s_2^* & \frac{1}{\sqrt{2}}s_3^* & \frac{1}{\sqrt{2}}s_3^* \\ s_2 & s_1^* & \frac{1}{\sqrt{2}}s_3^* & -\frac{1}{\sqrt{2}}s_3^* \\ \frac{1}{\sqrt{2}}s_3 & \frac{1}{\sqrt{2}}s_3 & -s_1' + s_2'' & s_1'' + s_2' \end{bmatrix}$$

  - 3 symbols in 4 time slots → $R_c = 3/4$
  - QPSK → 1.5 bits / slot
- Result
  - Increased diversity in comparison to Alamouti due to $N_T \cdot N_R$

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  - V. Tarokh, H. Jafarkhani and A.R. Calderbank: Space-Time Block Codes from Orthogonal Designs, IEEE Trans. on Information Theory, Vol. 45, No. 5, pp. 1456-1467, July 1999
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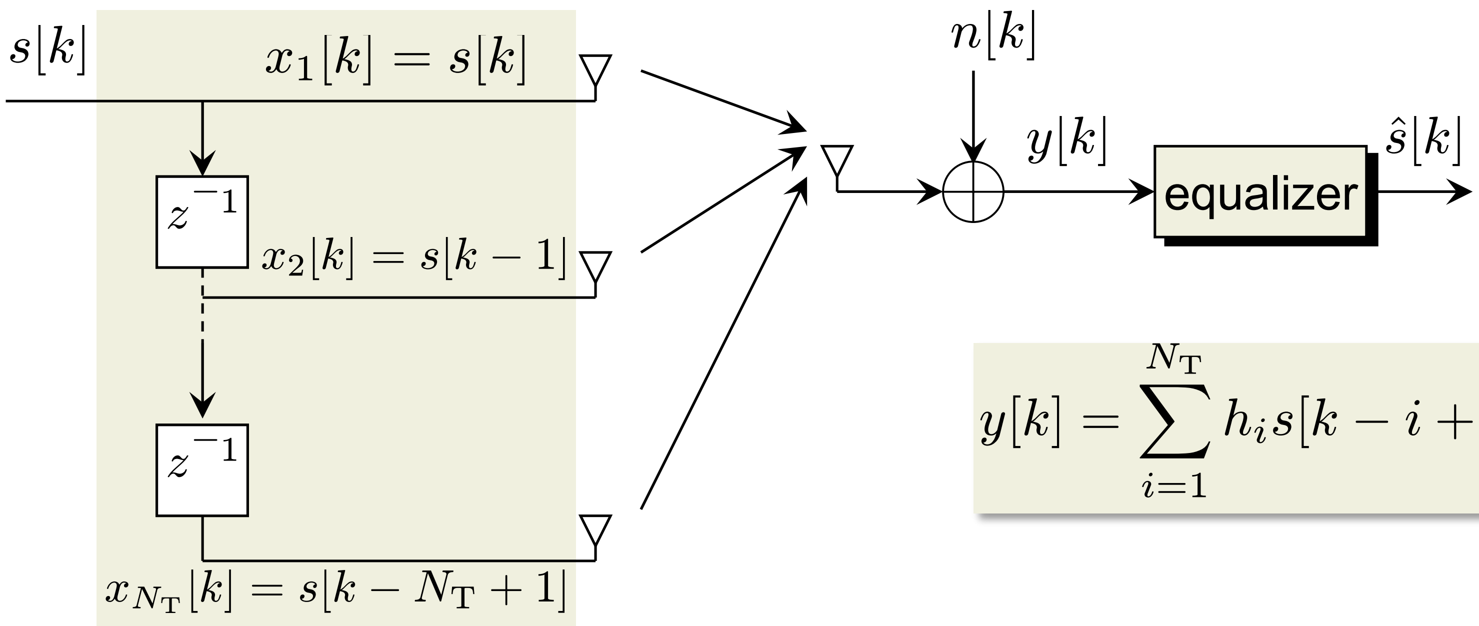
Space-Time Trellis Codes (STTC)

- Orthogonal Space-Time Block Codes do not achieve any coding gain
- Exploit diversity as well as coding gain by applying trellis codes
- Non-orthogonal block codes also possible, but not subject of this course

Space-Time Trellis-Codes: Delay Diversity



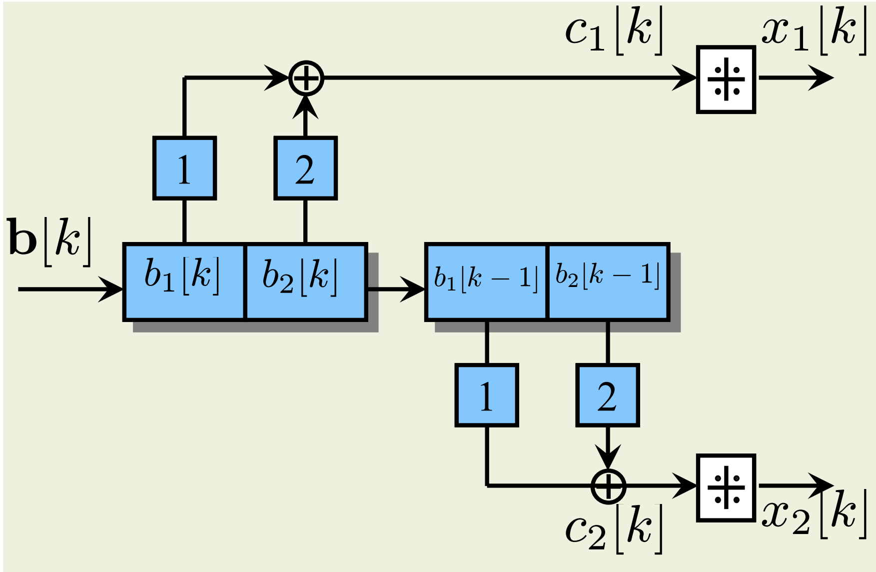
$$y[k] = \sum_{i=1}^{N_\mathrm{T}} h_i s[k-i+1] + n[k]$$

- Properties of delay diversity
  - Transmit delayed replicas of the same signal from different antennas
  - Flat MISO channel transformed into frequency selective SISO channel
  - Equalization of received signal, e.g. by Viterbi equalizer
  - Maximum diversity $N_\mathrm{T} \cdot N_\mathrm{R}$ is achieved
  - Drawback: computational costs grow exponentially with diversity order
- Are there better codes than repetition code?

Encoder Structure for Delay Diversity

- Nonrecursive convolutional encoder realized by binary shift registers
- Example: Delay-Diversity for QPSK, $N_\mathrm{T} = 2$ transmit antennas, 4 states
  - Binary register elements ($b_i[k] \in \{0,1\}$)

$$\mathbf{a}[k] = \begin{bmatrix} \mathbf{b}[k] \\ \mathbf{b}[k-1] \end{bmatrix} = \begin{bmatrix} b_1[k] \\ b_2[k] \\ b_1[k-1] \\ b_2[k-1] \end{bmatrix}$$



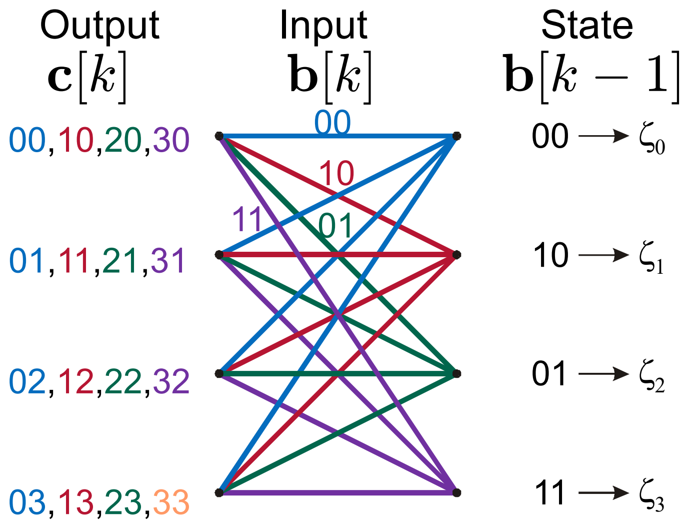
  - Generator matrix $\mathbf{G} = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{bmatrix}$
  - Encoder output: ($c_i[k] \in \{0,1,2,3\}$)

$$\mathbf{c}[k] = \begin{bmatrix} c_1[k] \\ c_2[k] \end{bmatrix} = (\mathbf{G} \cdot \mathbf{a}[k]) \bmod 4$$

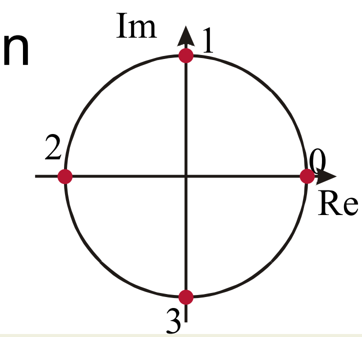
  - Transmit vector (QPSK symbols with natural mapping): $\mathbf{x}[k] = \begin{bmatrix} x_1[k] \\ x_2[k] \end{bmatrix} = \mathcal{M}\{\mathbf{c}[k]\}$

Trellis Representation for Delay Diversity

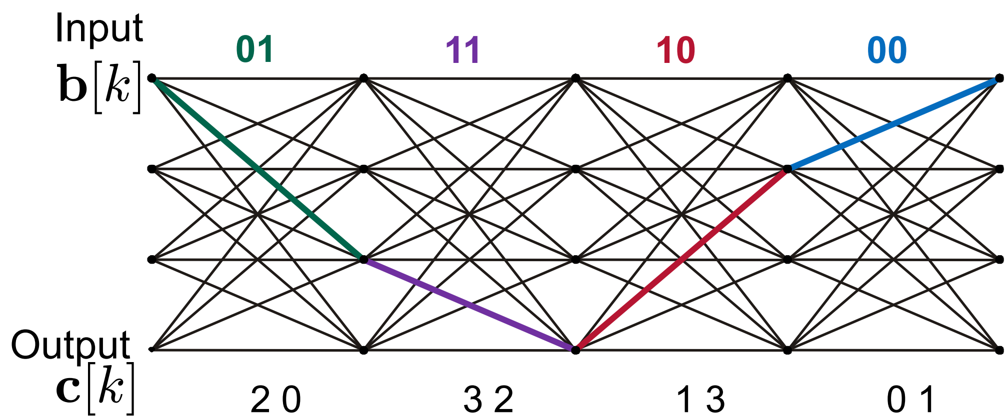
- Trellis structure for delay diversity



- QPSK constellation



- Example
  - Input $\mathbf{b} = \begin{bmatrix} 01 & 11 & 10 & 00 \end{bmatrix}^T$

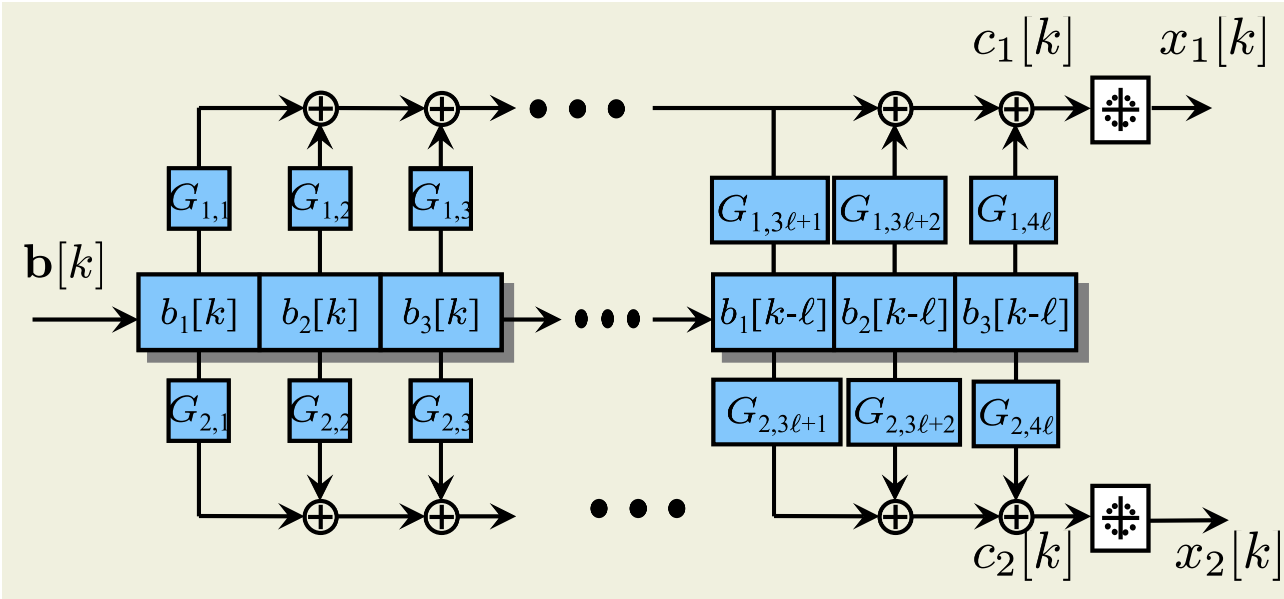


  - Output $\mathbf{C} = \begin{bmatrix} 2 & 3 & 1 & 0 \\ 0 & 2 & 3 & 1 \end{bmatrix}$
  - Code word matrix

$$\mathbf{X} = \begin{bmatrix} -1 & -j & j & 1 \\ 1 & -1 & -j & j \end{bmatrix}$$

General Encoder Structure for STTC

- Example for 8-PSK, $N_\mathrm{T} = 2$ and shift register with memory ℓ



register content:

$$\mathbf{a}[k] = \begin{bmatrix} \mathbf{b}[k] \\ \mathbf{b}[k-1] \\ \vdots \\ \mathbf{b}[k-\ell] \end{bmatrix}$$

Generator matrix

$$\mathbf{G} = \begin{bmatrix} G_{1,1} & G_{1,2} & \dots & G_{1,4\ell} \\ G_{2,1} & G_{2,2} & \dots & G_{2,4\ell} \end{bmatrix}$$

encoder output: $\mathbf{c}[k] = \begin{bmatrix} c_1[k] \\ c_2[k] \end{bmatrix} = (\mathbf{G} \cdot \mathbf{a}[k]) \bmod 8$

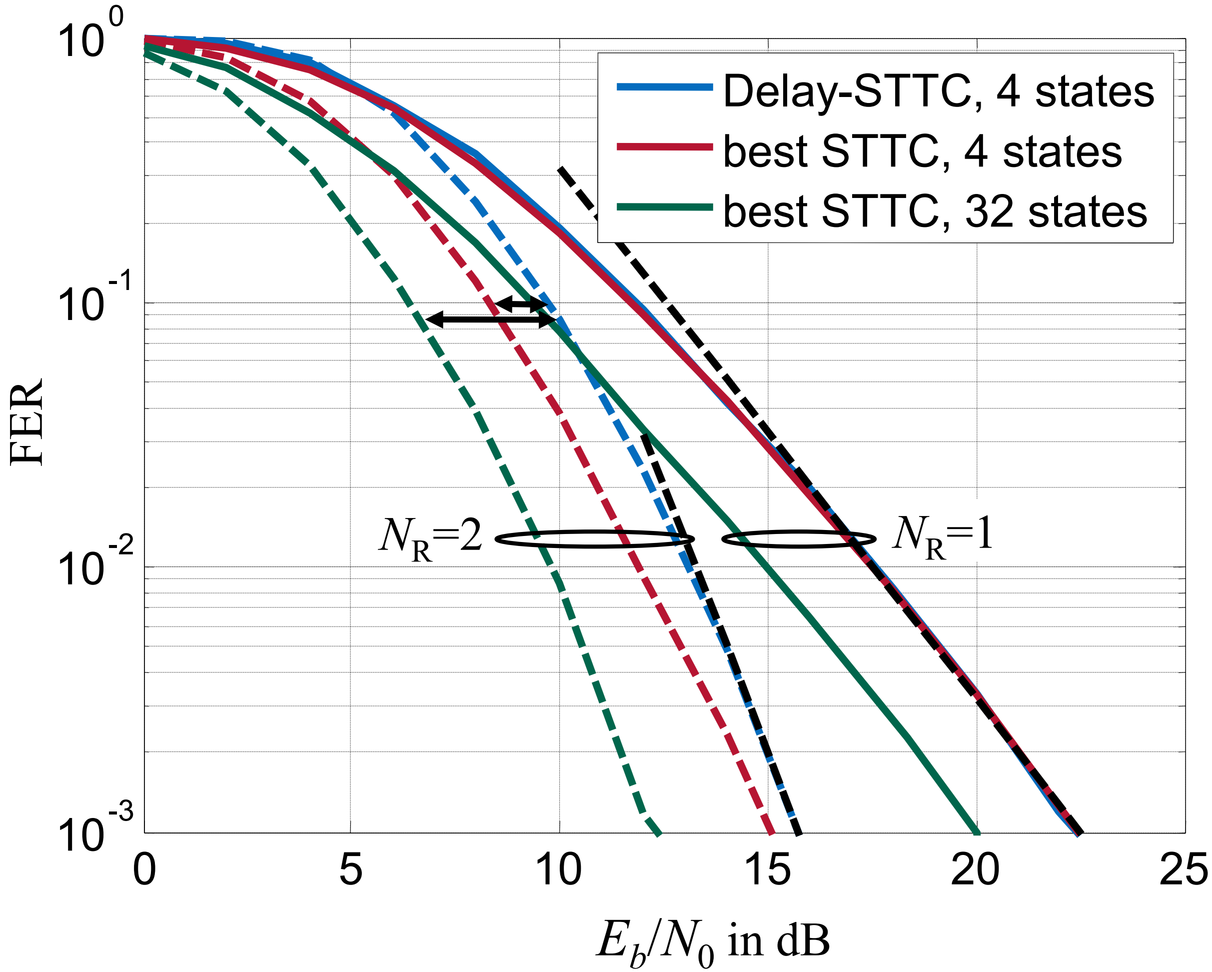
transmit vector: $\mathbf{x}[k] = \begin{bmatrix} x_1[k] \\ x_2[k] \end{bmatrix} = \mathcal{M}\{\mathbf{c}[k]\}$

Code Search for Space-Time Trellis Codes

- Different codes for different configurations
  - Number of transmit antennas
  - Order of modulation
  - Length of register → number of states
- Codes for same configuration differ only in generator coefficients
- Systematic code search by calculating diversity gain and coding gain for all permutations of $\mathbf{G}$
  - Look only for Space-Time Trellis Codes with maximum diversity
  - Choose the code with highest coding gain among those with maximum diversity
  - Best (known) Space-Time Trellis Code for 2 transmit antennas, QPSK, 4 states found by Yan and Blum, Lehigh University

$$\mathbf{G}_{\text{opt}} = \begin{bmatrix} 2 & 0 & 1 & 2 \\ 2 & 2 & 2 & 1 \end{bmatrix}$$

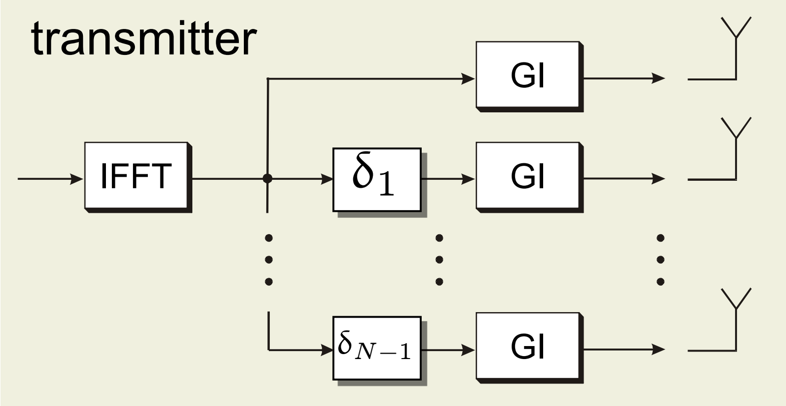
Simulation Results for STTC



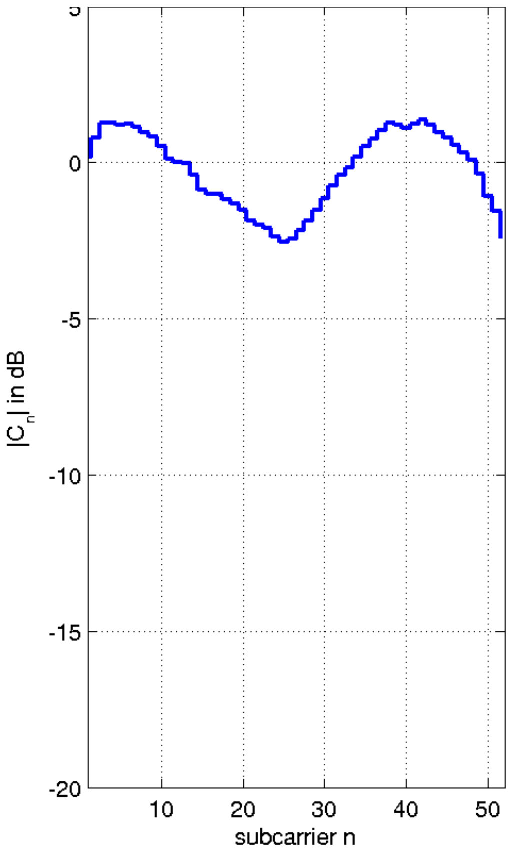
- Simulation parameters
  - $N_T = 2$, 4-PSK, 100 symbols
- Results
  - **Diversity gain** determines slope of FER
  - **Coding gain** affects horizontal shift for codes of same diversity
  - Performance of STTC of same constellation differ only for $N_R > 1$
  - Increased coding gain with larger number of states, but also higher decoding effort

Cyclic Delay Diversity for OFDM

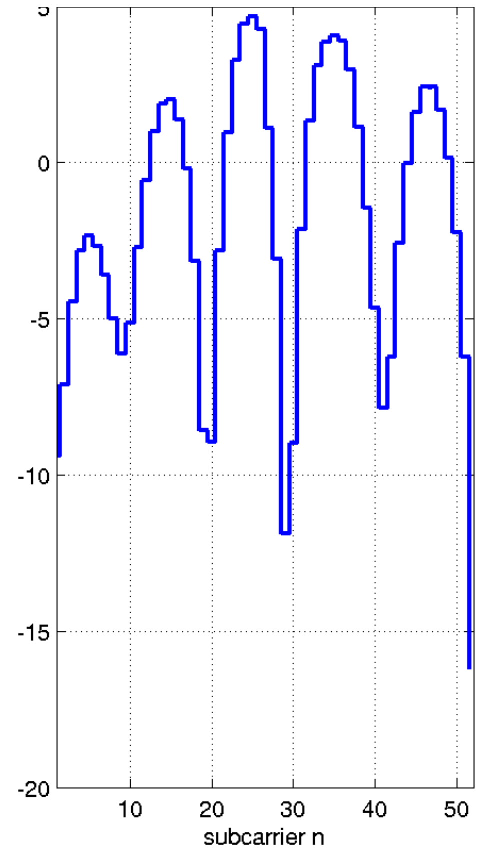
- Transmission of cyclic shifted version of same OFDM symbol
- Frequency selectivity of channel increased (can only be exploited by channel coding)
- Approach consistent with standard:
  → no modification of receiver required



channel transfer function
without CDD



channel transfer function
with CDD



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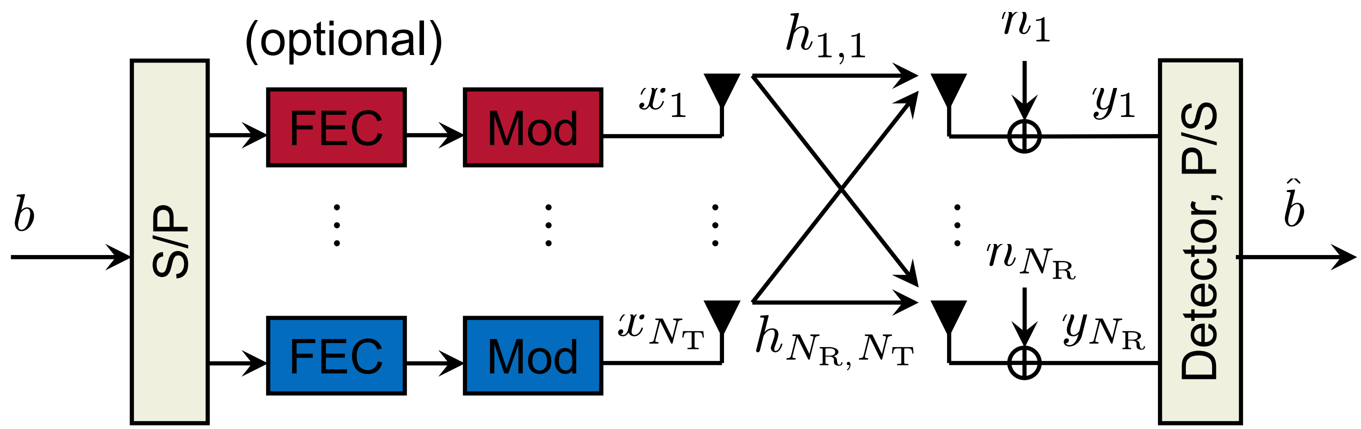
  - E. Biglieri, et al: MIMO Wireless Communications, Cambridge, 2007

Layered Space-Time Codes (BLAST)

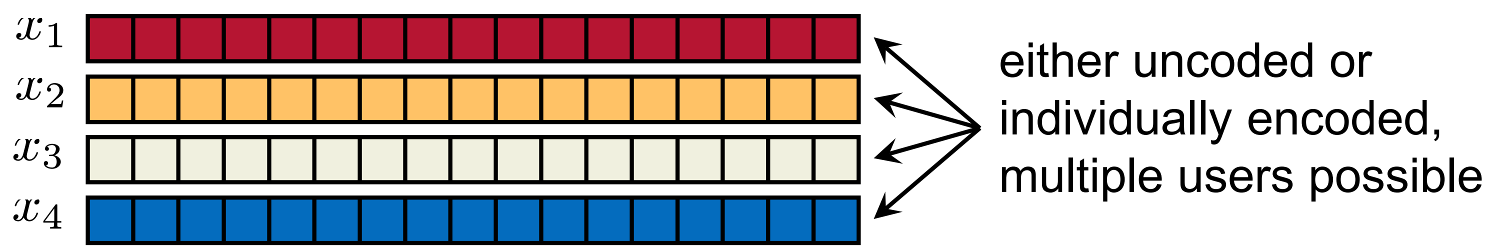
Transmission of multiple (parallel) data streams for higher data rates without increasing bandwidth

V-BLAST Transmitter

- **V-BLAST** → **V**ertical **B**ell-Labs **LA**yered **S**pace-**T**ime Architecture
  - Transmitted code words (layers) are vertically arranged
  - Each layer is transmitted over one particular antenna

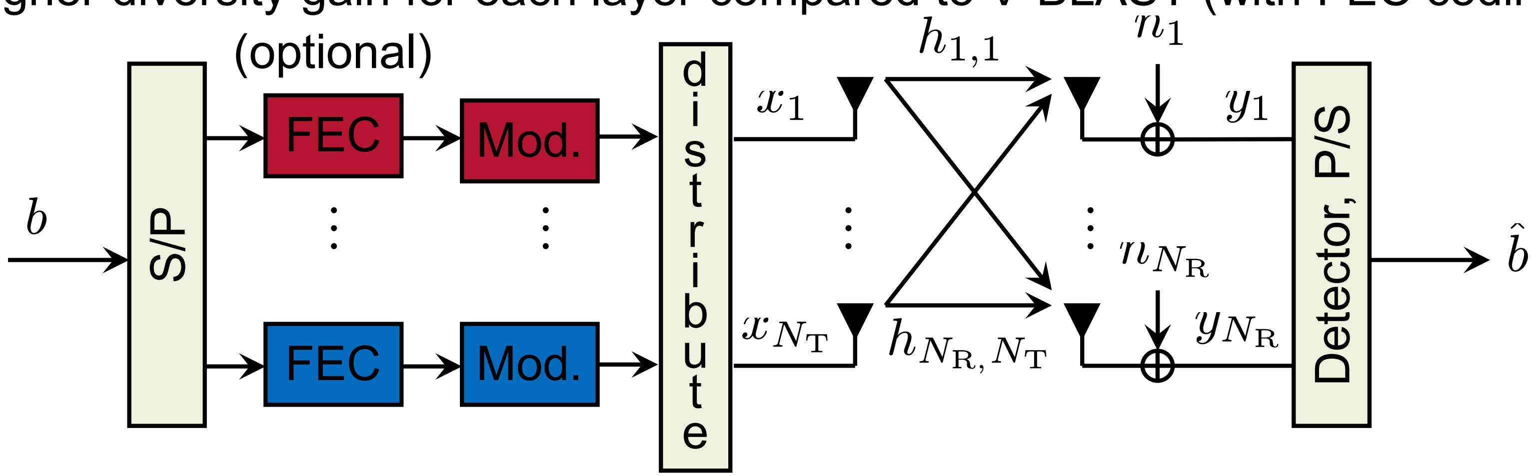


- Structure of transmit signal (example: four transmit antennas)

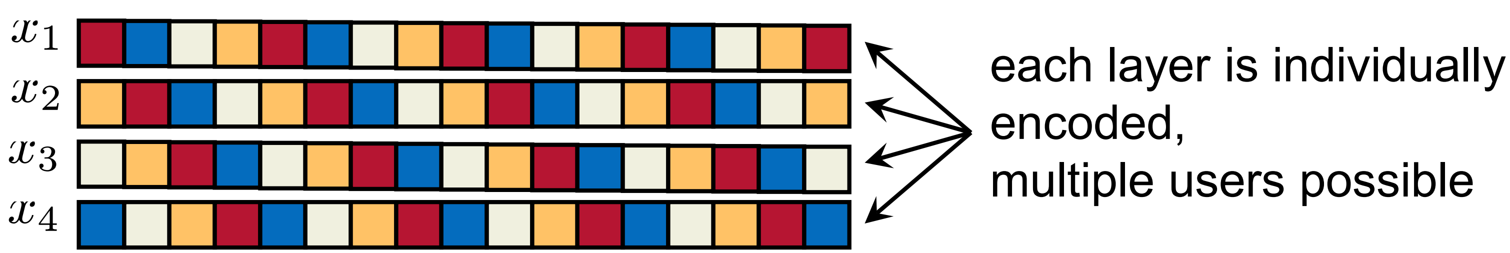


D-BLAST Transmitter

- **D-BLAST** → **D**iagonal **B**ell-Labs **LA**yered **S**pace-**T**ime Architecture
  - Transmitted code words (layers) are distributed over all antennas
  - Higher diversity gain for each layer compared to V-BLAST (with FEC coding)



- Structure of transmit signal (example: four transmit antennas)



Receiver for the V-BLAST Scheme

- Optimal Detection Scheme
  - Maximum-Likelihood Detection
- Linear Equalizer
  - Zero-Forcing Criterion
  - Minimum Mean Square Error Criterion
- Successive Interference Cancellation
  - V-BLAST Detection Algorithm
  - SIC on bases of Sorted QR Decomposition
  - Post Sorting Algorithm
- Sphere Detection

Optimal Detection

- Received signals are superposition of all transmit signals (plus noise)

$$\mathbf{y} = \sum_{i=1}^{N_\mathrm{T}} \mathbf{h}_i x_i + \mathbf{n} = \mathbf{H} \cdot \mathbf{x} + \mathbf{n}$$

- Optimum detector fulfills maximum likelihood (ML) criterion
  - Coded transmission:
    Find set of code words (sequences → MLSE) that was transmitted most likely
    → Extremely high computational complexity
  - Uncoded transmission:
    Find set of symbols that was transmitted most likely
    → Solve linear equation system with respect to the discrete symbol alphabet
    → Still very high computational complexity

Optimal Detection

- Maximum-Likelihood (**ML**)

$$\hat{\mathbf{x}}_{\mathrm{ML}} = \arg \max_{\mathbf{x} \in \mathbb{A}^{N_T}} p(\mathbf{y}|\mathbf{H},\mathbf{x}) = \arg \min_{\mathbf{x} \in \mathbb{A}^{N_T}} \|\mathbf{y} - \mathbf{Hx}\|^2$$

  - Brute Force:
    Find minimum Euclidian distance over all $\mathbf{x} \in \mathbb{A}^{N_\mathrm{T}}$

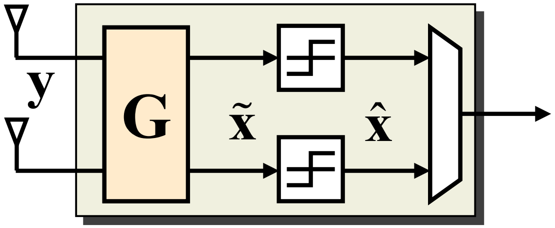
  → Effort grows exponentially with spectral efficiency $\eta = \mathrm{ld}(M) N_\mathrm{T}$

  → Example: N_T=4 and 16-QAM: $M^{N_\mathrm{T}} = 2^{\mathrm{ld}(M) N_\mathrm{T}} = 16^4 = 65536$

  - More efficient implementation by Sphere-Detection (**SD**)
    - Efficient algorithm with low best case complexity
    - Still high worst case complexity

- Less complex detection:
  - Linear processing
  - Suboptimal non-linear processing

Linear Equalizer (Linear Detector, LD)



- Linear filtering of receive signals $\tilde{\mathbf{x}} = \mathbf{G} \cdot \mathbf{y} = \mathbf{G} \cdot (\mathbf{H}\mathbf{x} + \mathbf{n})$
- Quantization per layer $\hat{x}_i = Q\{\tilde{x}_i\}$

- Derivation of the filter matrix $\mathbf{G}$
  - Error vector

$$\mathbf{e} = \tilde{\mathbf{x}} - \mathbf{x} = \mathbf{GHn} + \mathbf{Gn} - \mathbf{x} = (\mathbf{GH} - \mathbf{I}_{N_\mathrm{T}})\mathbf{x} + \mathbf{Gn}$$

  - Error covariance matrix diagonal elements determine layer-specific errors

$$\mathbf{\Phi}_{\mathbf{ee}} = \mathrm{E}\{\mathbf{e}\mathbf{e}^H\} = \mathrm{E}\{(\tilde{\mathbf{x}} - \mathbf{x})(\tilde{\mathbf{x}} - \mathbf{x})^H\} = (\mathbf{GH} - \mathbf{I}_{N_\mathrm{T}})(\mathbf{GH} - \mathbf{I}_{N_\mathrm{T}})^H + \mathbf{G}\mathbf{\Phi}_{\mathbf{nn}}\mathbf{G}^H$$

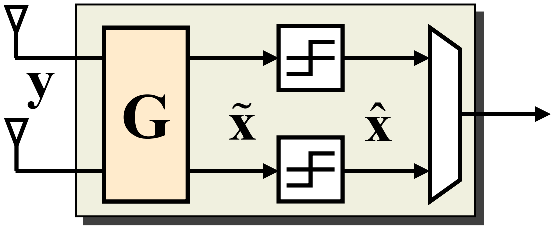
  - Average power of the estimation error is given by

$$\mathrm{E}\{\|\mathbf{e}\|\} = \mathrm{tr}\{\mathbf{\Phi}_{\mathbf{ee}}\}$$

  - General form of the filter output signal

$$\tilde{\mathbf{x}} = \mathrm{dg}\{\mathbf{GH}\} \cdot \mathbf{x} + \overline{\mathrm{dg}}\{\mathbf{GH}\} \cdot \mathbf{x} + \mathbf{Gn}$$

Linear Equalizer (Linear Detector, LD)



- Linear filtering of receive signals $\tilde{\mathbf{x}} = \mathbf{G} \cdot \mathbf{y} = \mathbf{G} \cdot (\mathbf{H}\mathbf{x} + \mathbf{n})$
- Quantization per layer $\hat{x}_i = Q\{\tilde{x}_i\}$

- Derivation of the filter matrix $\mathbf{G}$
  - General form of the filter output signal

$$\tilde{\mathbf{x}} = \mathrm{dg}\{\mathbf{G}\mathbf{H}\} \cdot \mathbf{x} + \overline{\mathrm{dg}}\{\mathbf{G}\mathbf{H}\} \cdot \mathbf{x} + \mathbf{G}\mathbf{n}$$

- Signal-to-Interference-and-Noise-Ratio (SINR)

$$\mathrm{SINR}_i = \frac{P_{S,i}}{P_{I,i} + P_{N,i}} = \frac{P_{S,i}}{P_{T,i} - P_{S,i}} = \frac{P_{S,i}/P_{T,i}}{1 - P_{S,i}/P_{T,i}}$$

$$P_{S,i} = \mathrm{E}\{\left| \left[\mathrm{dg}\{\mathbf{G}\mathbf{H}\} \cdot \mathbf{x}\right]_i \right|^2\}$$

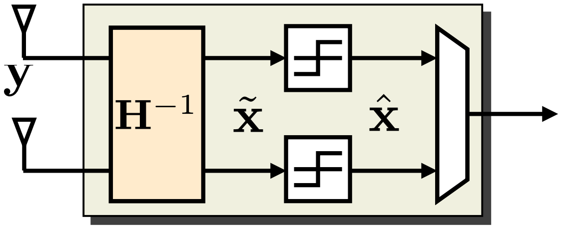
$$P_{I,i} = \mathrm{E}\{\left| \left[\overline{\mathrm{dg}}\{\mathbf{G}\mathbf{H}\} \cdot \mathbf{x}\right]_i \right|^2\}$$

$$P_{N,i} = \mathrm{E}\{\left| \left[\mathbf{G} \cdot \mathbf{n}\right]_i \right|^2\}$$

$\Rightarrow$

$$P_{T,i} = P_{S,i} + P_{I,i} + P_{N,i}$$

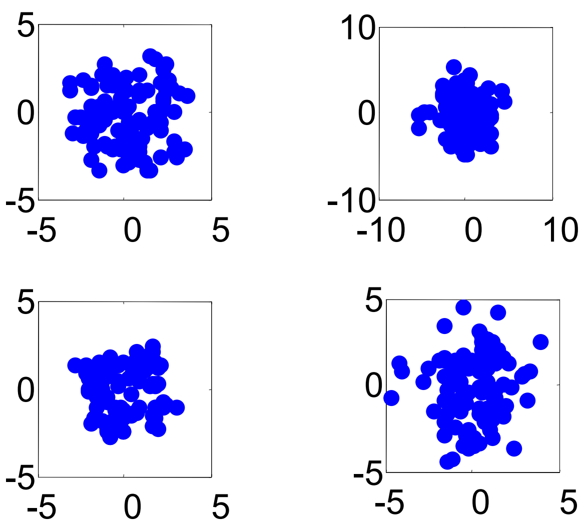
Linear Equalizer by Inversion



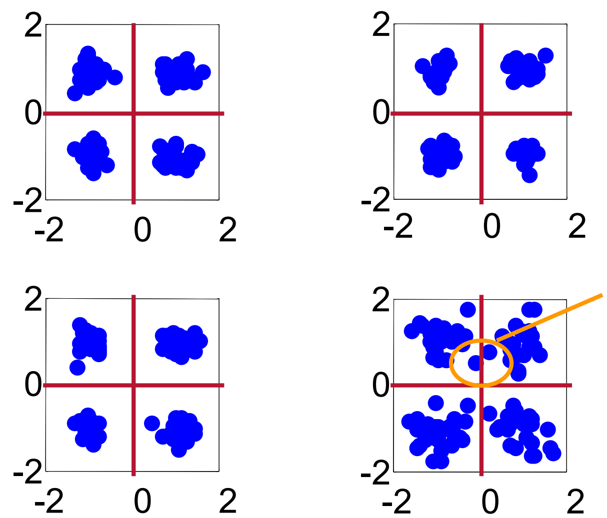
- Inversion of receive signal $\tilde{\mathbf{x}} = \mathbf{H}^{-1} \cdot \mathbf{y}$
- Estimation of transmitted symbol $\hat{x}_i = Q\{\tilde{x}_i\}$

- Signal space diagrams

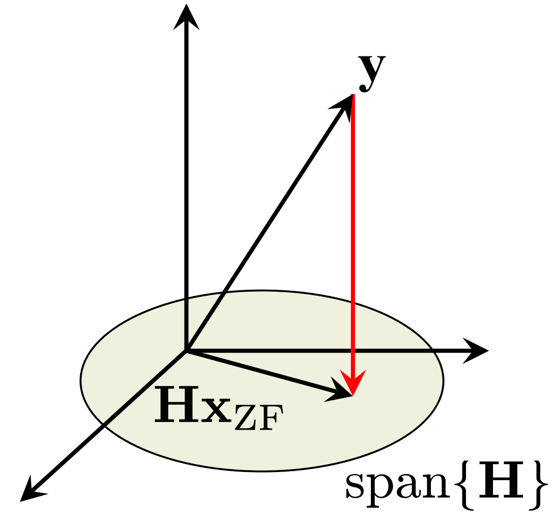
Receive signal $\mathbf{y}$



Filter output signal $\tilde{\mathbf{x}}$



Linear Zero-Forcing Equalizer



- Zero-Forcing Criterion → "Least Square Solution"
  - Minimize the Euclidian distance $\|\mathbf{y}-\mathbf{H}\tilde{\mathbf{x}}\|^2$
  - $\mathbf{G}$ is given by Pseudo-Inverse of channel matrix

    $$\mathbf{G}_{\mathrm{ZF}} = \mathbf{H}^{+} = (\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H$$
  - Projection of the received signal $\mathbf{y}$ onto the N_T-dim. subspace spanned by $\mathbf{H}$ within the N_R-dimensional receive space
  - Perfectly suppresses mutual interference → Problem: Noise enhancement!

    $$\tilde{\mathbf{x}} = \mathbf{G}_{\mathrm{ZF}} \cdot \mathbf{y} = \mathbf{H}^{+}\mathbf{H}\mathbf{x} + \mathbf{H}^{+}\mathbf{n} = \mathbf{x} + \tilde{\mathbf{n}}$$
  - Error covariance matrix

    $$\mathbf{\Phi}_{\mathbf{ee},\mathrm{ZF}} = \mathrm{E}\{\tilde{\mathbf{n}}\tilde{\mathbf{n}}^H\} = \sigma_n^2 \mathbf{G}_{\mathrm{ZF}}\mathbf{G}_{\mathrm{ZF}}^H = \sigma_n^2 \mathbf{H}^{+}\mathbf{H}^{+H} = \sigma_n^2 (\mathbf{H}^H\mathbf{H})^{-1}$$
- SNR

  $$\mathrm{SNR}_{\mathrm{ZF},i} = \frac{P_{S,i}}{P_{N,i}} = \frac{1}{[\mathbf{\Phi}_{\mathbf{ee},\mathrm{ZF}}]_{i,i}} = \frac{1}{\sigma_n^2 \|\mathbf{g}_{\mathrm{ZF}}^{(i)}\|^2}$$

Linear MMSE Equalizer (1)

- Minimum-Mean-Square-Error Criterion (**MMSE**)
  - Minimization of the mean error at the filter output
  - By introducing the receive covariance matrix $\mathbf{\Phi}_{\mathbf{yy}} = \mathrm{E}\{\mathbf{y}\mathbf{y}^H\} = \mathbf{H}\mathbf{H}^H + \mathbf{\Phi}_{\mathbf{nn}}$ the error covariance matrix can be rewritten in quadratic form

$$\begin{aligned}\mathbf{\Phi}_{\mathbf{ee}} &= \mathrm{E}\{(\mathbf{G}\mathbf{y}-\mathbf{x})(\mathbf{G}\mathbf{y}-\mathbf{x})^H\} = \mathrm{E}\{\mathbf{G}\mathbf{y}\mathbf{y}^H\mathbf{G}^H - \mathbf{G}\mathbf{y}\mathbf{x}^H - \mathbf{x}\mathbf{y}^H\mathbf{G}^H + \mathbf{x}\mathbf{x})^H\} \\ &= \mathbf{G}\mathbf{\Phi}_{\mathbf{yy}}\mathbf{G}^H - \mathbf{G}\mathbf{H} - \mathbf{H}^H\mathbf{G}^H + \mathbf{I}_{N_\mathrm{T}} \\ &= \left(\mathbf{G}\mathbf{\Phi}_{\mathbf{yy}} - \mathbf{H}^H\right)\mathbf{\Phi}_{\mathbf{yy}}^{-1}\left(\mathbf{G}\mathbf{\Phi}_{\mathbf{yy}} - \mathbf{H}^H\right)^H - \mathbf{H}^H\mathbf{\Phi}_{\mathbf{yy}}^{-1}\mathbf{H} + \mathbf{I}_{N_\mathrm{T}}\end{aligned}$$

  - $\mathbf{\Phi}_{\mathbf{yy}}$ is non-negative definite → trace of first term can not be negative
  - Minimum for $\mathbf{G}\mathbf{\Phi}_{\mathbf{yy}} - \mathbf{H}^H = \mathbf{0}_{N_\mathrm{T},N_\mathrm{T}}$ $\quad\quad$ $\mathbf{\Phi}_{\mathbf{nn}} = \sigma_n^2\mathbf{I}_{N_\mathrm{R}}$
  - Solution for the filter matrix

$$\mathbf{G} = \mathbf{H}^H \cdot \mathbf{\Phi}_{\mathbf{yy}}^{-1} = \mathbf{H}^H(\mathbf{H}\mathbf{H}^H + \sigma_n^2\mathbf{I}_{N_\mathrm{R}})^{-1} = (\mathbf{H}^H\mathbf{H} + \sigma_n^2\mathbf{I}_{N_\mathrm{T}})^{-1}\mathbf{H}^H$$

  - Error covariance matrix $\mathbf{\Phi}_{\mathbf{ee},\mathrm{MMSE}} = \mathbf{I}_{N_\mathrm{T}} - \mathbf{H}^H\mathbf{\Phi}_{\mathbf{yy}}^{-1}\mathbf{H} = \sigma_n^2(\mathbf{H}^H\mathbf{H} + \sigma_n^2\mathbf{I}_{N_\mathrm{T}})^{-1}$

Linear MMSE Equalizer (2)

- MMSE Filter output signal (biased estimator)

$$\tilde{\mathbf{x}}_{\mathrm{MMSE}} = \mathbf{H}^H \mathbf{\Phi}_{\mathbf{yy}}^{-1} \mathbf{y} = \mathbf{H}^H (\mathbf{H}\mathbf{H}^H + \sigma_n^2 \mathbf{I}_{N_{\mathrm{R}}})^{-1} \mathbf{y}$$

$$\mathbf{H}_{\bar{i}} = \begin{bmatrix} \mathbf{h}_1 & \cdots & \mathbf{h}_{i-1} & \mathbf{h}_{i+1} & \cdots & \mathbf{h}_{N_{\mathrm{T}}} \end{bmatrix}$$

$$\mathbf{x}_{\bar{i}} = \begin{bmatrix} x_1 & \cdots & x_{i-1} & x_{i+1} & \cdots & x_{N_{\mathrm{T}}} \end{bmatrix}^T$$

- i-th filter output signal

$$\tilde{\mathbf{x}}_i = \mathbf{h}_i^H \mathbf{\Phi}_{\mathbf{yy}}^{-1} \mathbf{y} = \mathbf{h}_i^H \mathbf{\Phi}_{\mathbf{yy}}^{-1} (\mathbf{H}\mathbf{x} + \mathbf{n}) = \mathbf{h}_i^H \mathbf{\Phi}_{\mathbf{yy}}^{-1} \mathbf{h}_i \mathbf{x}_i + \mathbf{h}_i^H \mathbf{\Phi}_{\mathbf{yy}}^{-1} \mathbf{H}_{\bar{i}} \mathbf{x}_{\bar{i}} + \mathbf{h}_i^H \mathbf{\Phi}_{\mathbf{yy}}^{-1} \mathbf{n}$$

- SINR

$$\mathrm{SINR}_{\mathrm{MMSE},i} = \frac{P_{S,i}}{P_{I,i} + P_{N,i}} = \frac{\mathbf{h}_i^H \mathbf{\Phi}_{\mathbf{yy}}^{-1} \mathbf{h}_i}{1 - \mathbf{h}_i^H \mathbf{\Phi}_{\mathbf{yy}}^{-1} \mathbf{h}_i} = \frac{1 - [\mathbf{\Phi}_{\mathbf{ee},\mathrm{MMSE}}]_{i,i}}{[\mathbf{\Phi}_{\mathbf{ee},\mathrm{MMSE}}]_{i,i}} = \frac{1}{[\mathbf{\Phi}_{\mathbf{ee},\mathrm{MMSE}}]_{i,i}} - 1$$

- **Unbiased estimator**
  - Assume channel matrix with orthogonal columns $\tilde{\mathbf{x}} = (1 + \sigma_n^2)^{-1} \mathbf{x} + \tilde{\mathbf{n}}$ → biased
  - Bias leads to amplitude scaling → important for QAM
  - Solutions:
    - Adopt filter $\mathbf{G}_{\text{UB-MMSE}} = (\mathrm{dg}\{\mathbf{G}_{\mathrm{MMSE}} \mathbf{H}\})^{-1} \mathbf{G}_{\mathrm{MMSE}}$
    - Consider scaling within the demodulator

Linear MMSE Equalizer (3)

- Relation of MMSE to zero-forcing
  - Definition of extended channel matrix and extended receive vector

$$\underline{\mathbf{H}} = \begin{bmatrix} \mathbf{H} \\ \sigma_n \mathbf{I}_{N_\mathrm{T}} \end{bmatrix} \Rightarrow \underline{\mathbf{H}}^H \underline{\mathbf{H}} = \mathbf{H}^H \mathbf{H} + \sigma_n^2 \mathbf{I}_{N_\mathrm{T}} \qquad \underline{\mathbf{y}} = \begin{bmatrix} \mathbf{y} \\ \mathbf{0}_{N_\mathrm{T} \times 1} \end{bmatrix}$$

- Applying zero-forcing approach to $\underline{\mathbf{H}}$ leads to MMSE solution with $\mathbf{H}$
  - Filter output expressed with $\underline{\mathbf{H}}$ and $\underline{\mathbf{y}}$

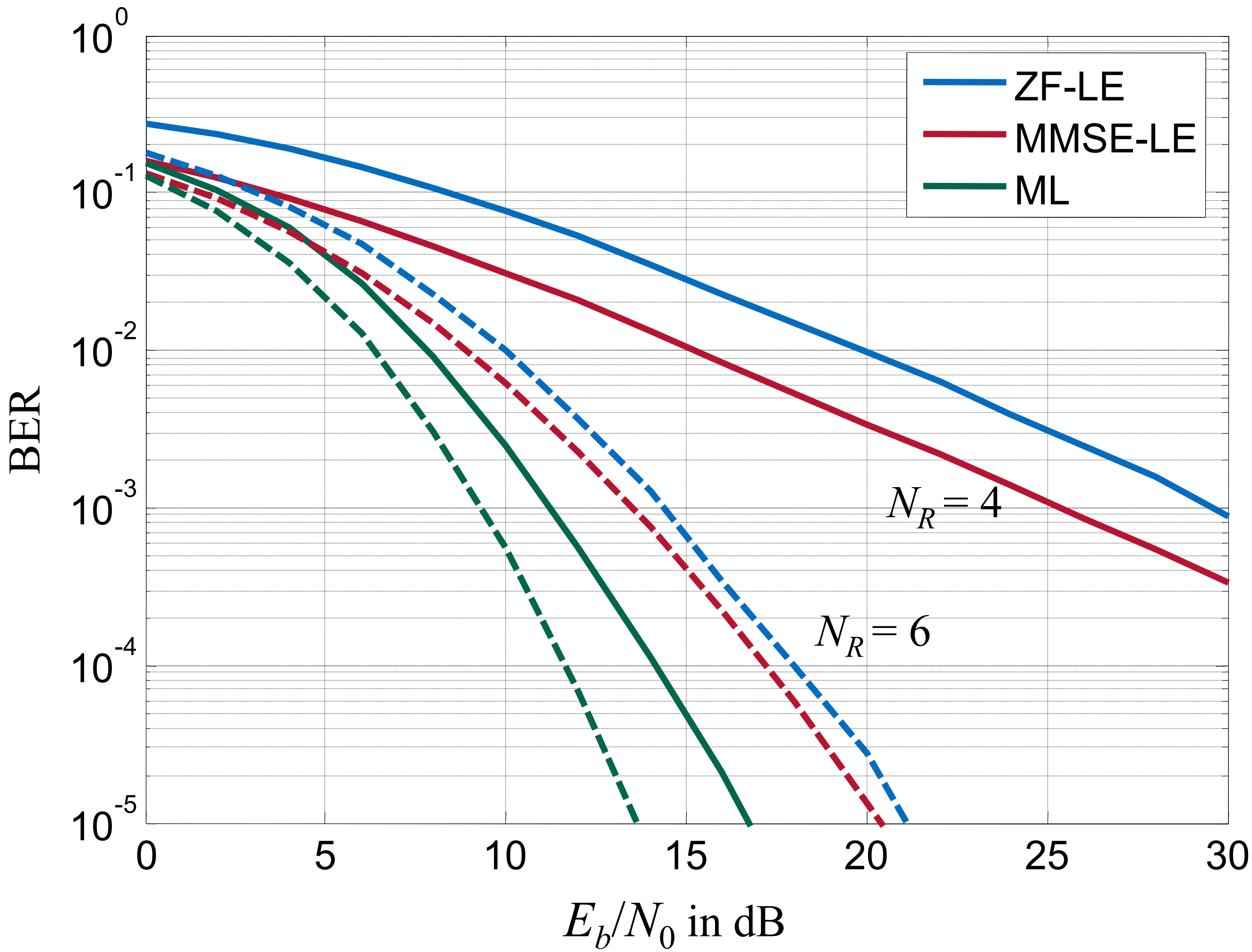
$$\begin{aligned} \tilde{\mathbf{x}}_{\mathrm{MMSE}} &= \left(\mathbf{H}^H \mathbf{H} + \sigma_n^2 \mathbf{I}_{N_\mathrm{T}}\right) \mathbf{H}^H \mathbf{y} = \left(\begin{bmatrix} \mathbf{H}^H & \sigma_n \mathbf{I}_{N_\mathrm{T}} \end{bmatrix} \begin{bmatrix} \mathbf{H} \\ \sigma_n \mathbf{I}_{N_\mathrm{T}} \end{bmatrix} \right)^{-1} \begin{bmatrix} \mathbf{H}^H & \sigma_n \mathbf{I}_{N_\mathrm{T}} \end{bmatrix} \begin{bmatrix} \mathbf{y} \\ \mathbf{0}_{N_\mathrm{T},1} \end{bmatrix} \\ &= (\underline{\mathbf{H}}^H \underline{\mathbf{H}})^{-1} \underline{\mathbf{H}}^H \underline{\mathbf{y}} = \underline{\mathbf{H}}^+ \underline{\mathbf{y}} \end{aligned}$$

  - Error covariance matrix expressed with $\underline{\mathbf{H}}$

$$\mathbf{\Phi}_{\mathrm{MMSE}} = \sigma_n^2 (\underline{\mathbf{H}}^H \underline{\mathbf{H}})^{-1} = \sigma_n^2 \cdot \underline{\mathbf{H}}^+ \underline{\mathbf{H}}^{+H}$$

→ MMSE solution corresponds to zero-forcing for extended system

Bit Error Rate for Linear Equalization

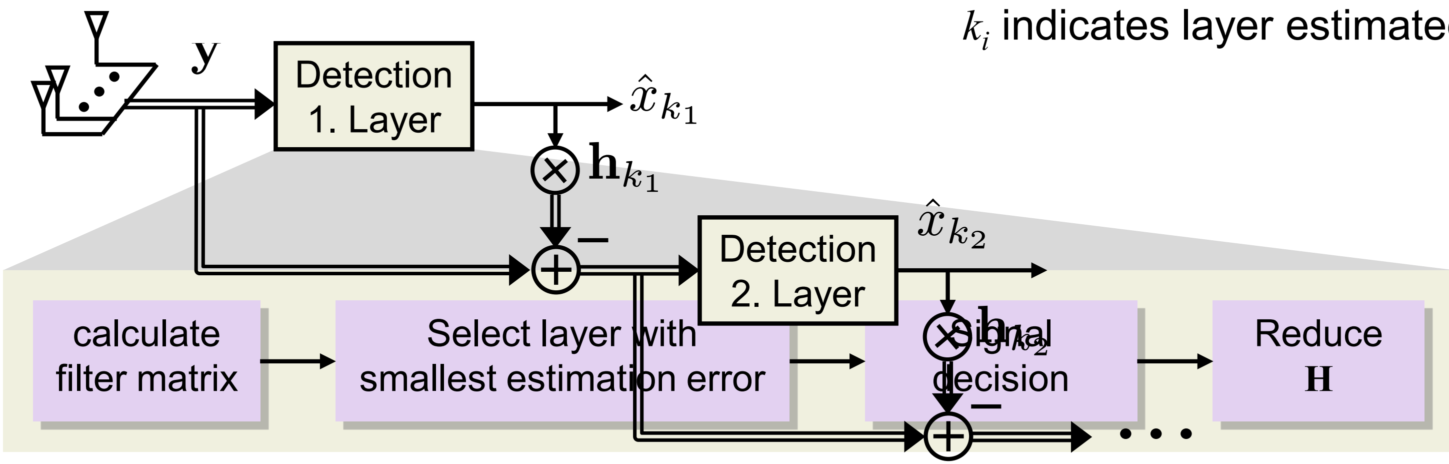


- Simulation parameters
  - $N_\mathrm{T} = 4$, 4-QAM
- Results
  - Linear Equalization leads to strong performance drawback in comparison to ML
  - MMSE outperforms ZF
  - With increased N_R the slope of BER increases (receive diversity) and gap between linear and ML becomes smaller

Successive Interference Cancellation

- Basic principle of Successive Interference Cancellation (**SIC**)
  - Cancel **estimated** interference of already **detected** layer and linearly suppress the interference of **remaining** layer
  - Optimization of detection sequence to reduce **error propagation**
- **V-BLAST Algorithm**
  - Based on linear ZF or MMSE equalization
  - In each step **only the layer with maximum SNR / SINR** is detected

k_i indicates layer estimated in detected step i



V-BLAST Detection Algorithm (2)

- General procedure
  - Apply zero forcing only for one layer (nulling interfering users)
  - Detect best layer and subtract estimated interference
  - Continue with next layer until all layers have been processed
- Order of detection is crucial → sorting criterion is necessary
  - Error covariance matrix: diagonal elements determine layer-specific errors

$$\begin{aligned}\mathbf{\Phi}_{\mathrm{ZF}} &= \mathrm{E}\{(\tilde{\mathbf{x}}_{\mathrm{ZF}} - \mathbf{x})(\tilde{\mathbf{x}}_{\mathrm{ZF}} - \mathbf{x})^H\} \\ &= \mathrm{E}\{(\mathbf{x} + \mathbf{G}_{\mathrm{ZF}}\mathbf{n} - \mathbf{x})(\mathbf{x} + \mathbf{G}_{\mathrm{ZF}}\mathbf{n} - \mathbf{x})^H\} \\ &= \sigma_n^2 \cdot \mathbf{G}_{\mathrm{ZF}} \cdot \mathbf{G}_{\mathrm{ZF}}^H = \sigma_n^2 \cdot (\mathbf{H}^H\mathbf{H})^{-1}\end{aligned}$$

$$\Rightarrow \quad [\mathbf{\Phi}_{\mathrm{ZF}}]_{i,i} = \sigma_n^2 \cdot \|\underline{\mathbf{g}}_{\mathrm{ZF}}^{(i)}\|^2$$

  - Layer corresponding to smallest diagonal element in $\mathbf{\Phi}_{\mathrm{ZF}}$ has smallest error

→ Row $\underline{\mathbf{g}}_{\mathrm{ZF}}^{(i)}$ of $\mathbf{G}_{\mathrm{ZF}}$ with smallest squared norm corresponds to minimum diagonal element in $\mathbf{\Phi}_{\mathrm{ZF}}$

→ Smallest noise amplification → best SNR

V-BLAST Detection Algorithm (3)

- Detailed procedure:
  - Determine layer with smallest noise amplification (best SNR)
  - Apply linear filtering to layer k_i

$$\tilde{x}_{k_i} = \underline{\mathbf{g}}_{\mathrm{ZF}}^{(k_i)} \cdot \mathbf{y} = x_{k_i} + \underline{\mathbf{g}}_{\mathrm{ZF}}^{(k_i)} \mathbf{n} = x_{k_i} + \left[(\mathbf{H}^H \mathbf{H})^{-1} \mathbf{H}^H \right]_{k_i} \mathbf{n}$$

  - Detect layer after filtering, i.e. find estimate $\hat{x}_{k_i}$ for x_{k_i} by quantization of $\tilde{x}_{k_i}$
  - Subtract estimated interference from receive signal

$$\mathbf{y} \leftarrow \mathbf{y} - \mathbf{h}_{k_i} \cdot \hat{x}_{k_i}$$

  - Remove i-th column from channel matrix

$$\mathbf{H} \leftarrow \left[\, \mathbf{h}_1 \quad \cdots \quad \mathbf{h}_{k_i - 1} \quad \mathbf{h}_{k_i + 1} \cdots \mathbf{h}_{N_{\mathrm{T}}} \, \right]$$

- Continue with next layer of reduced system until all layers have been detected

➡ Expensive calculation of pseudo-inverse in each iteration

SIC with QR Decomposition (1)

- Costly calculation of pseudo inverse should be avoided
- Applying QR decomposition of $\mathbf{H}$

$$\mathbf{H} = \mathbf{QR}$$

  - $\mathbf{Q}$ is a $N_R \times N_T$ matrix with orthogonal columns of unit length
  - $\mathbf{R}$ is a $N_T \times N_T$ upper triangular matrix

- Multiplication of $\mathbf{y}$ with $\mathbf{Q}^H$ delivers starting point for successive interference cancellation without any further linear filtering

$$\tilde{\mathbf{x}} = \mathbf{Q}^H \mathbf{y} = \mathbf{Q}^H \mathbf{QRx} + \mathbf{Q}^H \mathbf{n} = \mathbf{Rx} + \eta$$

- Only 1 QR decomposition instead of calculating $N_T - 1$ pseudo inverses

- However, sorting is still an open problem

SIC with QR Decomposition (2)

- Linear filtering of $\mathbf{y}$ with $\mathbf{Q}^H$ yields

$$\begin{pmatrix} \tilde{x}_1 \\ \vdots \\ \tilde{x}_k \\ \tilde{x}_{k+1} \\ \vdots \\ \tilde{x}_{N_\mathrm{T}} \end{pmatrix} = \begin{pmatrix} r_{1,1} & \cdots & r_{1,k} & r_{1,k+1} & \cdots & r_{1,N_\mathrm{T}} \\ \vdots & \ddots & \vdots & \vdots & & \vdots \\ 0 & \cdots & r_{k,k} & r_{k,k+1} & \cdots & r_{k,N_\mathrm{T}} \\ 0 & \cdots & 0 & r_{k+1,k+1} & \cdots & r_{k+1,N_\mathrm{T}} \\ \vdots & & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & 0 & \cdots & r_{N_\mathrm{T},N_\mathrm{T}} \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ \vdots \\ x_k \\ x_{k+1} \\ \vdots \\ x_{N_\mathrm{T}} \end{pmatrix} + \begin{pmatrix} \eta_1 \\ \vdots \\ \eta_k \\ \eta_{k+1} \\ \vdots \\ \eta_{N_\mathrm{T}} \end{pmatrix}$$

- Layer k experiences only interference from layers $k+1, \ldots, N_\mathrm{T}$

$$\tilde{x} = r_{k,k} \cdot x_k + \sum_{i=k+1}^{N_\mathrm{T}} r_{k,i} \cdot x_i + \eta_k$$

  - Signal $\tilde{x}_{N_\mathrm{T}}$ is free of interference and can be directly decided
  - Subtract estimated interference from other layers and continue detection with $\tilde{x}_{N_\mathrm{T}-1}$ until first layer $\tilde{x}_1$ has been decided
  - SNR in layer N_T : $\mathrm{SNR}_{N_\mathrm{T}} = \sigma_n^{-2} |r_{N_\mathrm{T},N_\mathrm{T}}|^2$

SIC with QR Decomposition (3)

- Interference cancellation

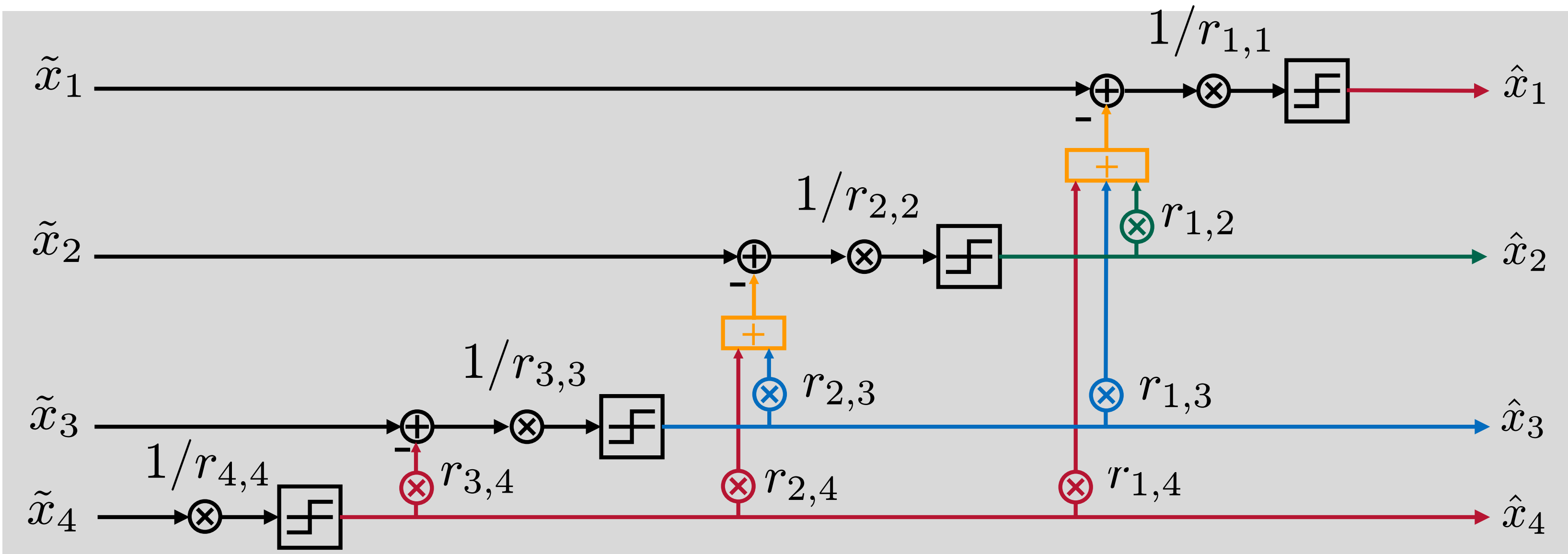
$$\tilde{x}_k^{IC} = \tilde{x}_k - \sum_{i=k+1}^{N_\mathrm{T}} r_{k,i} \cdot \hat{x}_i = r_{k,k} \cdot x_k + \eta_k$$

- Detection

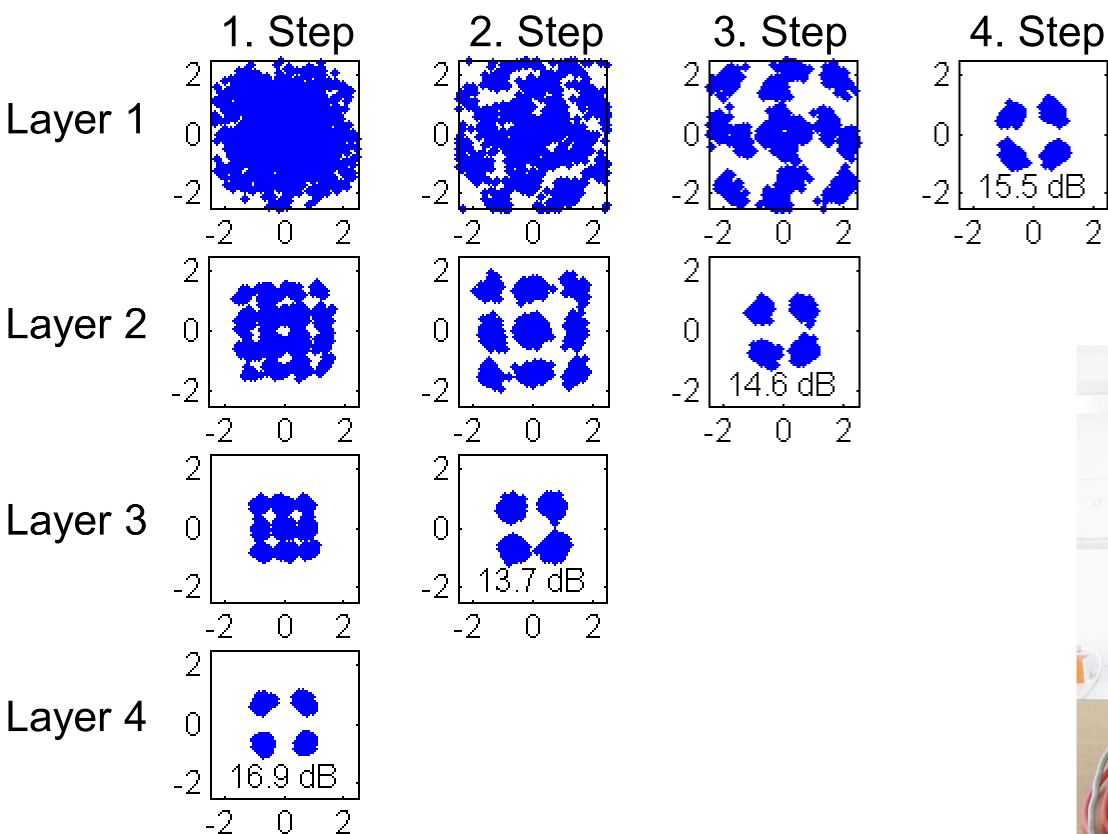
$$\hat{x}_k = \mathcal{Q}\{\tilde{x}_k^{IC} / r_{k,k}\} \Rightarrow \mathrm{SNR}_i = \sigma_n^{-2} |r_{i,i}|^2$$

➢ Example for $N_\mathrm{T} = 4$

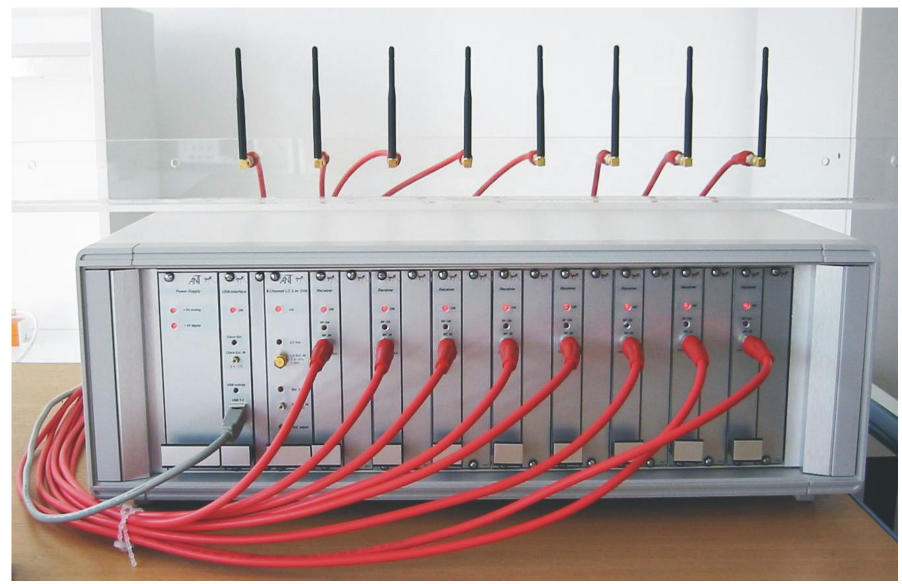
$$\begin{pmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \tilde{x}_3 \\ \tilde{x}_4 \end{pmatrix} = \begin{pmatrix} r_{1,1} & r_{1,2} & r_{1,3} & r_{1,4} \\ 0 & r_{2,2} & r_{2,3} & r_{2,4} \\ 0 & 0 & r_{3,3} & r_{3,4} \\ 0 & 0 & 0 & r_{4,4} \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} + \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \end{pmatrix}$$



SIC-Detection with real MIMO-Transmission



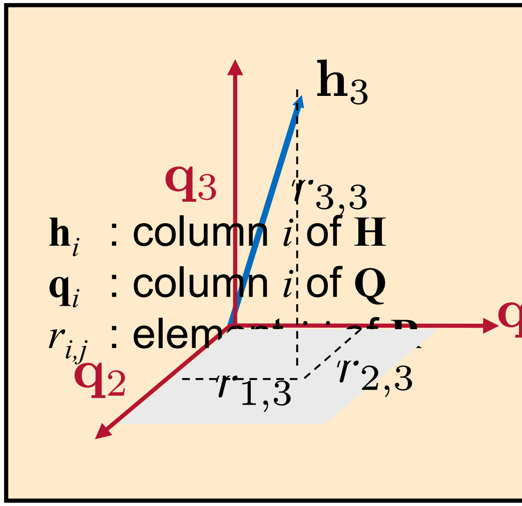
- Parameter
  - **MASI**: Multiple Antenna System for ISM-Band Transmission
  - Transmission between two offices in 2.4 GHz ISM-Band
  - $N_T = N_R = 4$, 4-QAM, $\lambda/2$-ULA



QR Decomposition with Modified Gram-Schmidt Algorithm

- QR decomposition of channel matrix $\mathbf{H} = \mathbf{QR}$
- Gram-Schmidt

$$\begin{bmatrix} \mathbf{h}_1 & \mathbf{h}_2 & \mathbf{h}_3 & \cdots & \mathbf{h}_{N_T} \end{bmatrix} = \begin{bmatrix} \mathbf{q}_1 & \mathbf{q}_2 & \mathbf{q}_3 & \cdots & \mathbf{q}_{N_T} \end{bmatrix} \cdot \begin{bmatrix} r_{1,1} & r_{1,2} & r_{1,3} & \cdots & r_{1,n_T} \\ & r_{2,2} & r_{2,3} & \cdots & r_{2,n_T} \\ & & r_{3,3} & \cdots & r_{3,n_T} \\ & \mathbf{0} & & \ddots & \vdots \\ & & & & r_{n_T,n_T} \end{bmatrix}$$



- Example: Decomposition of $\mathbf{h}_3$
  - Columns ($\mathbf{q}_1$,$\mathbf{q}_2$) form an orthonormal basis of the vector space ($\mathbf{h}_1$,$\mathbf{h}_2$)
  - $r_{1,3}$ and $r_{2,3}$ describe the component of $\mathbf{h}_3$ in the direction of $\mathbf{q}_1$ and $\mathbf{q}_2$
  - $\mathbf{q}_3$ denotes the direction of $\mathbf{h}_3$ perpendicular to the base ($\mathbf{q}_1$,$\mathbf{q}_2$)
  - $r_{3,3}$ describes the component of $\mathbf{h}_3$ in the direction of $\mathbf{q}_3$ $\rightarrow \mathbf{h}_3 = \mathbf{q}_1 \cdot r_{1,3} + \mathbf{q}_2 \cdot r_{2,3} + \mathbf{q}_3 \cdot r_{3,3}$

→ Diagonal element $r_{k,k}$ denotes component of $\mathbf{h}_k$ perpendicular to base ($\mathbf{q}_1$,…,$\mathbf{q}_{k-1}$)

QR Decompositions of Permutated Channel Matrices

- Given channel matrix

$$\mathbf{H} = \begin{bmatrix} 0.0828 & 0.3269 & 0.5548 \\ 0.7662 & 0.8633 & 1.0016 \\ 2.2368 & 0.6794 & 1.2594 \end{bmatrix}$$

- QR decomposition of permutated channel matrices $\mathbf{H}(\mathbf{p})$ (permutat. vector $\mathbf{p}$)

$$\mathbf{R}_{[123]} = \begin{bmatrix} 2.3659 & 0.9334 & 1.5345 \\ 0 & 0.6653 & 0.7056 \\ 0 & 0 & 0.2108 \end{bmatrix} \qquad \mathbf{R}_{[312]} = \begin{bmatrix} 1.7021 & 2.1329 & 1.1173 \\ 0 & 1.0237 & -0.1708 \\ 0 & 0 & 0.1905 \end{bmatrix}$$

$$\mathbf{R}_{[213]} = \begin{bmatrix} 1.1462 & 1.9266 & 1.6591 \\ 0 & 1.3732 & 0.3160 \\ 0 & 0 & 0.2108 \end{bmatrix} \qquad \mathbf{R}_{[231]} = \begin{bmatrix} 1.1462 & 1.6591 & 1.9266 \\ 0 & 0.3799 & 1.1423 \\ 0 & 0 & 0.7622 \end{bmatrix}$$

$$\mathbf{R}_{[132]} = \begin{bmatrix} 2.3659 & 1.5345 & 0.9334 \\ 0 & 0.7365 & 0.6374 \\ 0 & 0 & 0.1905 \end{bmatrix} \qquad \mathbf{R}_{[321]} = \begin{bmatrix} 1.7021 & 1.1173 & 2.1329 \\ 0 & 0.2558 & -0.6834 \\ 0 & 0 & 0.7622 \end{bmatrix}$$

- Which permutation leads to best SNR in each detection step?

Adaptation of the Detection Order

- Optimization of the detection order to reduce problem of error propagation
  - Adaptation by exchanging the columns of $\mathbf{H}$ → different QR decompositions
  - SNR_i is given by diagonal element $r_{i,i}$
    → Exchange the columns of $\mathbf{H}$ in order to maximize the elements $r_{i,i}$ with respect to the detection sequence

$$\begin{bmatrix} r_{1,1} & r_{1,2} & r_{1,3} & r_{1,4} \\ & r_{2,2} & r_{2,3} & r_{2,4} \\ & & r_{3,3} & r_{3,4} \\ & & & r_{4,4} \end{bmatrix}$$

- Sorted QR Decomposition (**SQRD**)

$$\sqrt{\det\left(\mathbf{H}^T\mathbf{H}\right)} = \prod_{i=1}^{N_\mathrm{T}} |r_{i,i}|$$

  - Optimizes the sequence **within one QR Decomposition**
  - Lattice determinant is independent of column sorting
  - Product of diagonal elements is constant

→ Exchange columns within the QR decomposition of $\mathbf{H}$ so that the diagonal elements $r_{i,i}$ are **minimized** in the sequence $r_{1,1}, r_{2,2}, \ldots$!

  - Small elements $r_{1,1}, r_{2,2}, \cdots$ lead to large elements $\cdots, r_{N_{2\mathrm{T}}-1,N_{2\mathrm{T}}-1}, r_{N_{2\mathrm{T}},N_{2\mathrm{T}}}$
  - Only very small computational effort in contrast to unsorted QRD, but does not always lead to the optimal detection sequence → Post-Sorting-Algorithm

Sorted QR Decomposition (SQRD)

- Modification of Gram-Schmidt algorithm by inserting a reordering in each decomposition step
  → Permutation vector $\mathbf{p}$
- Decomposition step i
  - First i-1 elements of $\mathbf{p}$ and $\mathbf{q}_1, \ldots, \mathbf{q}_{i-1}$ are fixed, but order of remaining columns is variable
  - Sorting rule selects column $\mathbf{q}_{k_i}$ of remaining columns with minimum norm
  - Exchange columns of $\mathbf{Q}$, $\mathbf{R}$ and $\mathbf{p}$
  - Proceed with Gram-Schmidt decomposition

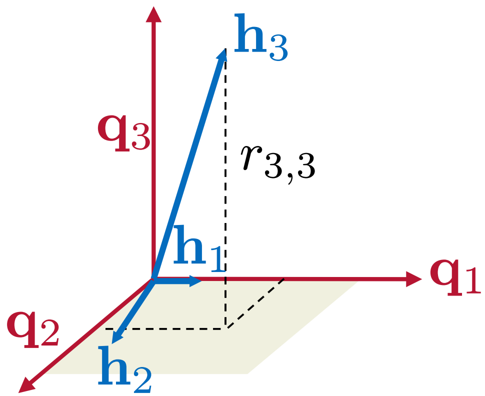
$\mathbf{R} = \mathbf{0}, \mathbf{Q} = \mathbf{H}, \mathbf{p} = (1, \cdots, N_\mathrm{T})$
for $i = 1, \cdots, N_\mathrm{T}$
    $k_i = \arg \min_{l=i,\cdots,N_\mathrm{T}} \|\mathbf{q}_l\|^2$
    exchange col. i and k_i in $\mathbf{Q}, \mathbf{R}$ and $\mathbf{p}$
    $r_{i,i} = \|\mathbf{q}_i\|$
    $\mathbf{q}_i = \mathbf{q}_i / r_{i,i}$
    for $l = i+1, \cdots, N_\mathrm{T}$
        $r_{i,l} = \mathbf{q}_i^H \cdot \mathbf{q}_l$
        $\mathbf{q}_l = \mathbf{q}_l - r_{i,l} \cdot \mathbf{q}_i$
    end
end

➡ Only one decomposition, but *optimum* sorting is not assured!

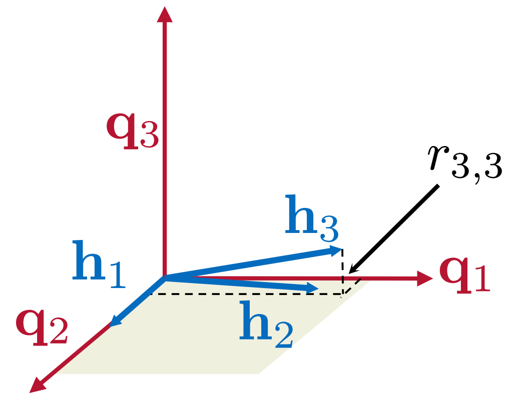
Why is heuristic sorting rule not optimal?

- Example with $N_T = 3$

  - Optimal order



  - Suboptimal order



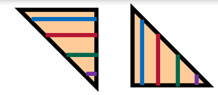
  - Large part of $\mathbf{h}_3$ perpendicular to $\mathbf{h}_1$ and $\mathbf{h}_2$ leads to large coefficient $r_{3,3}$
  - $\mathbf{h}_2$ and $\mathbf{h}_3$ have a large norm but similar direction, which leads to a small perpendicular component $r_{3,3} \rightarrow$ small SNR_3

Post Sorting Algorithm

- Relation between error covariance matrix and QR decomposition

$$\mathbf{\Phi}_{\mathrm{ZF}} = \sigma_n^2 \cdot (\mathbf{H}^H\mathbf{H})^{-1} = \sigma_n^2 \cdot (\mathbf{R}^H\mathbf{Q}^H\mathbf{Q}\mathbf{R})^{-1} = \sigma_n^2 \cdot (\mathbf{R}^H\mathbf{R})^{-1} = \sigma_n^2 \cdot \mathbf{R}^{-1}\mathbf{R}^{-H}$$

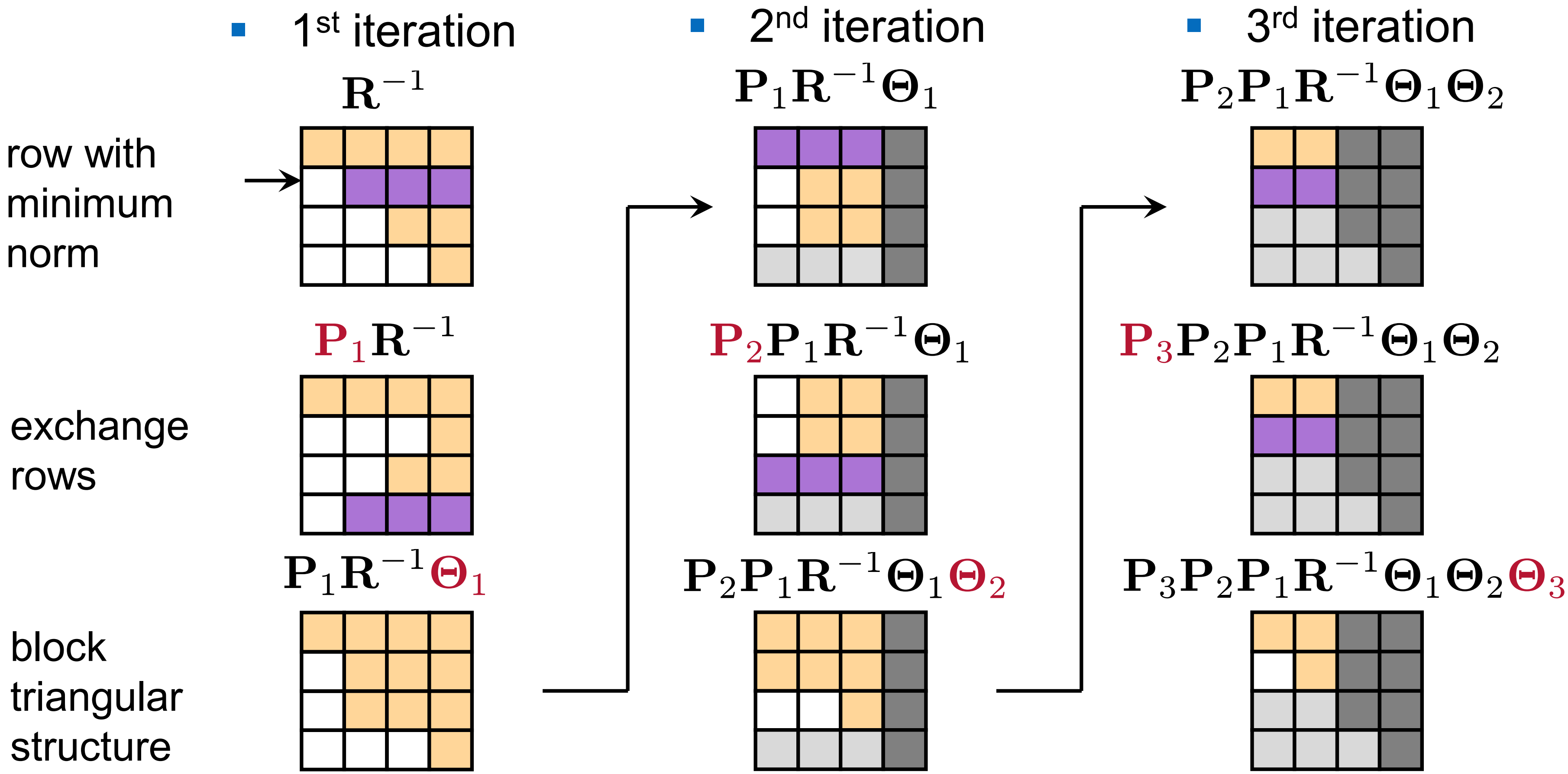
  - $[\mathbf{\Phi}_{\mathrm{ZF}}]_{i,i}$ is proportional to the norm of the i-th row of $\mathbf{R}^{-1}$



  - Due to detection order, last row of $\mathbf{R}^{-1}$ must have minimum norm of all rows
  - If this condition is fulfilled, the last row of the upper left $(N_T\text{-}1)$x$(N_T\text{-}1)$ submatrix of $\mathbf{R}^{-1}$ must have minimum norm of all rows in this submatrix, ...

- Now assume, that this condition is not fulfilled for $\mathbf{R}^{-1}$
  - Exchange row with minimum norm & last row → left multiplication with $\mathbf{P}$
    → destroys triangular structure
  - Block triangular structure is achieved by multiplication with unitary Housholder matrix Θ
    → $\mathbf{R}^{-1} := \mathbf{R}^{-1}\Theta$ and $\mathbf{Q} := \mathbf{Q}\,\Theta$
  - Iterate this ordering and reflection steps for upper left $(N_T\text{-}1)$x$(N_T\text{-}1)$ submatrix of $\mathbf{R}^{-1}$, …

Example: Efficient Sorting Algorithm for 4 Layers



$$\mathbf{R}_{\text{opt}}^{-1} = \mathbf{P}_3\mathbf{P}_2\mathbf{P}_1\mathbf{R}^{-1}\mathbf{\Theta}_1\mathbf{\Theta}_2\mathbf{\Theta}_3 \quad \mathbf{R}_{\text{opt}} = \mathbf{\Theta}_3^H\mathbf{\Theta}_2^H\mathbf{\Theta}_1^H\mathbf{R}\mathbf{P}_1^H\mathbf{P}_2^H\mathbf{P}_3^H \quad \mathbf{Q}_{\text{opt}} = \mathbf{Q}\mathbf{\Theta}_1\mathbf{\Theta}_2\mathbf{\Theta}_3$$

Extension of V-BLAST to MMSE Detection (1)

- MMSE filter matrix

$$\mathbf{G}_{\text{MMSE}} = (\mathbf{H}^H\mathbf{H} + \sigma_n^2\mathbf{I}_{N_\text{T}})^{-1}\mathbf{H}^H$$

  - Output of the MMSE filter

$$\tilde{\mathbf{x}}_{\text{MMSE}} = \mathbf{G}_{\text{MMSE}}\mathbf{y} = (\mathbf{H}^H\mathbf{H} + \sigma_n^2\mathbf{I}_{N_\text{T}})^{-1}\mathbf{H}^H\mathbf{y}$$

  - Error covariance matrix

$$\mathbf{\Phi}_{\text{MMSE}} = \sigma_n^2(\mathbf{H}^H\mathbf{H} + \sigma_n^2\mathbf{I}_{N_\text{T}})^{-1}$$

- Note: row norm of $\mathbf{G}_{\text{MMSE}}$ does not lead to optimum sorting criterion!
- Reason: diagonal elements of $\mathbf{\Phi}_{\text{MMSE}}$ are not squared row norms of $\mathbf{G}_{\text{MMSE}}$

$$\begin{aligned}\mathbf{\Phi}_{\text{MMSE}i,i} &= \sigma_n^2\{(\mathbf{H}^H\mathbf{H}\sigma_n^2\mathbf{I}_{N_\text{T}})^{-1}\}_{i,i}\\ &\neq \sigma_n^2\{(\mathbf{H}^H\mathbf{H} + \sigma_n^2\mathbf{I}_{N_\text{T}})^{-1}\mathbf{H}^H\mathbf{H}(\mathbf{H}^H\mathbf{H} + \sigma_n^2\mathbf{I}_{N_\text{T}})^{-1}\}_{i,i}\end{aligned}$$

- Compare ZF: $\mathbf{\Phi}_{\text{ZF}i,i} = \{(\mathbf{H}^H\mathbf{H})^{-1}\}_{i,i} \overset{!}{=} \{(\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H\mathbf{H}(\mathbf{H}^H\mathbf{H})^{-1}\}_{i,i} = \{(\mathbf{H}^H\mathbf{H})^{-1}\}_{i,i}$

Extension of V-BLAST to MMSE Detection (2)

- Relation of MMSE to zero-forcing
  - Definition of extended channel matrix and extended receive vector

$$\underline{\mathbf{H}} = \begin{bmatrix} \mathbf{H} \\ \sigma_n \mathbf{I}_{N_\mathrm{T}} \end{bmatrix} \Rightarrow \underline{\mathbf{H}}^H \underline{\mathbf{H}} = \mathbf{H}^H \mathbf{H} + \sigma_n^2 \mathbf{I}_{N_\mathrm{T}}$$

$$\underline{\mathbf{y}} = \begin{bmatrix} \mathbf{y} \\ \mathbf{0}_{N_\mathrm{T} \times 1} \end{bmatrix}$$

- Applying zero-forcing approach to $\underline{\mathbf{H}}$ leads to MMSE solution w.r.t. $\mathbf{H}$
  - Filter output expressed with $\underline{\mathbf{H}}$ and $\underline{\mathbf{y}}$

$$\tilde{\mathbf{x}}_\mathrm{MMSE} = (\underline{\mathbf{H}}^H \underline{\mathbf{H}})^{-1} \underline{\mathbf{H}}^H \underline{\mathbf{y}} = \underline{\mathbf{H}}^+ \underline{\mathbf{y}}$$

  - Error covariance matrix expressed with $\underline{\mathbf{H}}$ and $\underline{\mathbf{y}}$

$$\mathbf{\Phi}_\mathrm{MMSE} = \sigma_n^2 (\underline{\mathbf{H}}^H \underline{\mathbf{H}})^{-1} = \sigma_n^2 \cdot \underline{\mathbf{H}}^+ \underline{\mathbf{H}}^{+H}$$

➡ MMSE solution corresponds to zero-forcing for extended system

MMSE-BLAST with QR Decomposition (1)

- Algorithms for ZF V-BLAST can be readily applied to MMSE V-BLAST
  - QR decomposition of extended channel matrix

$$\underline{\mathbf{H}} = \begin{bmatrix} \mathbf{H} \\ \sigma_n \mathbf{I}_{N_\mathrm{T}} \end{bmatrix} = \underline{\mathbf{Q}}\underline{\mathbf{R}} = \begin{bmatrix} \mathbf{Q}_1 \\ \mathbf{Q}_2 \end{bmatrix} \underline{\mathbf{R}} = \begin{bmatrix} \mathbf{Q}_1\underline{\mathbf{R}} \\ \mathbf{Q}_2\underline{\mathbf{R}} \end{bmatrix}$$

with $\underline{\mathbf{Q}} = \begin{bmatrix} \mathbf{Q}_1 \\ \mathbf{Q}_2 \end{bmatrix}$ — $\mathbf{Q}_1$: N_R x N_T, $\mathbf{Q}_2$: N_T x N_T

  - Attention: $\mathbf{Q}_1$ and $\mathbf{Q}_2$ are not unitary since they contain only column parts of $\mathbf{Q}$!
- No matrix inversion for efficient optimum sorting algorithm required

$$\sigma_n \mathbf{I}_{N_\mathrm{T}} = \mathbf{Q}_2 \underline{\mathbf{R}} \Rightarrow \underline{\mathbf{R}}^{-1} = \sigma_n^{-1} \mathbf{Q}_2$$

  - Compensates for higher computational effort of QR decomposition
- Filtered receive signal

$$\tilde{\mathbf{x}} = \underline{\mathbf{Q}}^H \underline{\mathbf{y}} = \begin{bmatrix} \mathbf{Q}_1^H & \mathbf{Q}_2^H \end{bmatrix} \begin{bmatrix} \mathbf{y} \\ \mathbf{0} \end{bmatrix} = \mathbf{Q}_1^H \mathbf{y} = \mathbf{Q}_1^H (\mathbf{H}\mathbf{x} + \mathbf{n}) = \mathbf{Q}_1^H \mathbf{H}\mathbf{x} + \mathbf{Q}_1^H \mathbf{n}$$

- Analyzing $\mathbf{Q}_1^H \mathbf{H}$

$$\underline{\mathbf{Q}}^H \underline{\mathbf{H}} = \underline{\mathbf{R}} = \begin{bmatrix} \mathbf{Q}_1^H & \mathbf{Q}_2^H \end{bmatrix} \underline{\mathbf{H}} = \mathbf{Q}_1^H \mathbf{H} + \sigma_n \mathbf{Q}_2^H$$

MMSE-BLAST with QR Decomposition

- Extracting $\underline{\mathbf{R}}$

$$\underline{\mathbf{Q}}^H \underline{\mathbf{H}} = \underline{\mathbf{R}} = \left[\begin{array}{cc} \mathbf{Q}_1^H & \mathbf{Q}_2^H \end{array} \right] \underline{\mathbf{H}} = \mathbf{Q}_1^H \mathbf{H} + \sigma_n \mathbf{Q}_2^H$$

$$\mathbf{Q}_1^H \mathbf{H} = \underline{\mathbf{R}} - \sigma_n \mathbf{Q}_2^H = \underline{\mathbf{R}} - \sigma_n (\sigma_n \underline{\mathbf{R}}^{-1})^H = \underline{\mathbf{R}} - \sigma_n^2 \underline{\mathbf{R}}^{-H}$$

- Inserting above result into filtered receive signal

$$\tilde{\mathbf{x}} = \mathbf{Q}_1^H \mathbf{H} \mathbf{x} + \mathbf{Q}_1^H \mathbf{n} = \underline{\mathbf{R}} \mathbf{x} - \sigma_n^2 \underline{\mathbf{R}}^{-H} \mathbf{x} + \mathbf{Q}_1^H \mathbf{n}$$

perform successive interference cancellation like before

second term represents remaining interference

Extension for Quadrature Amplitude Modulation

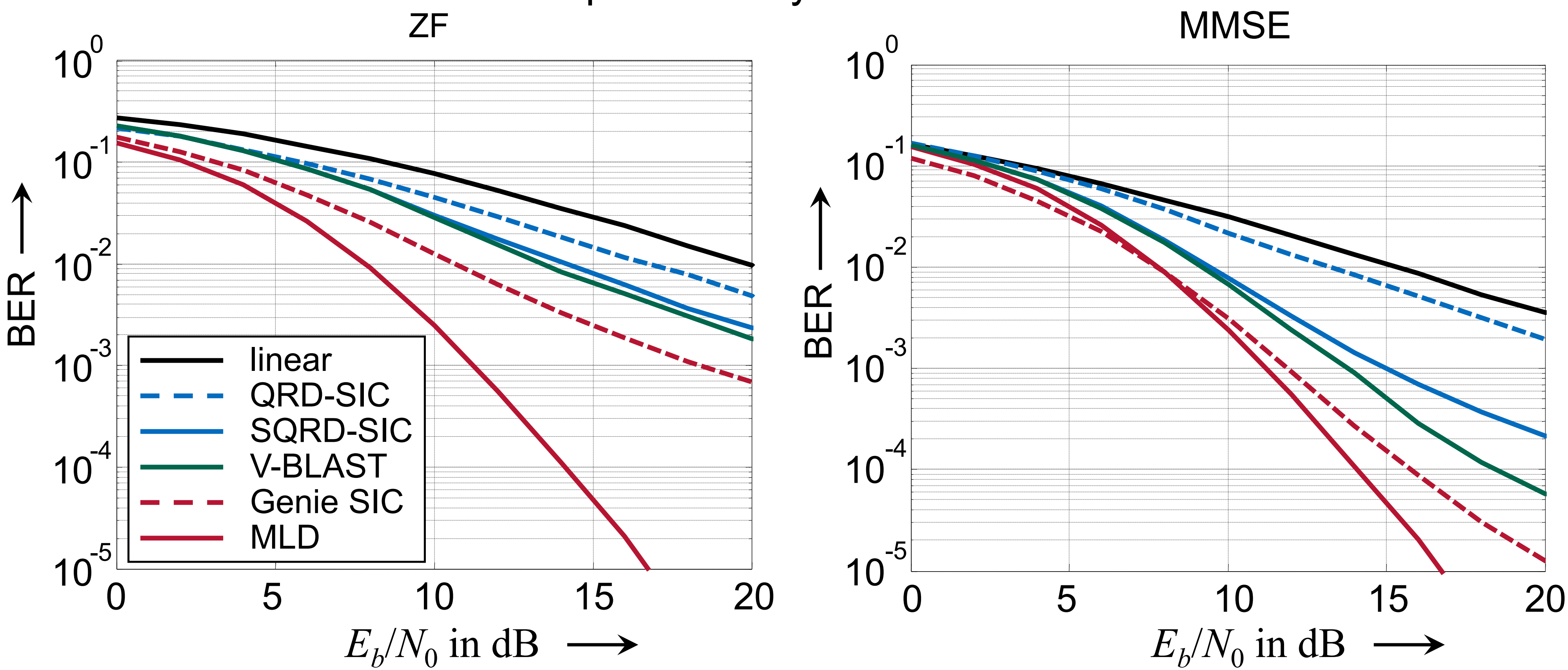
- QAM symbols → real and imaginary parts are independent of each other
  - Real-valued system model

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \quad \Rightarrow \quad \begin{bmatrix} \mathbf{y}_r \\ \mathbf{y}_i \end{bmatrix} = \begin{bmatrix} \mathbf{H}_r & -\mathbf{H}_i \\ \mathbf{H}_i & \mathbf{H}_r \end{bmatrix} \begin{bmatrix} \mathbf{x}_r \\ \mathbf{x}_i \end{bmatrix} + \begin{bmatrix} \mathbf{n}_r \\ \mathbf{n}_i \end{bmatrix}$$

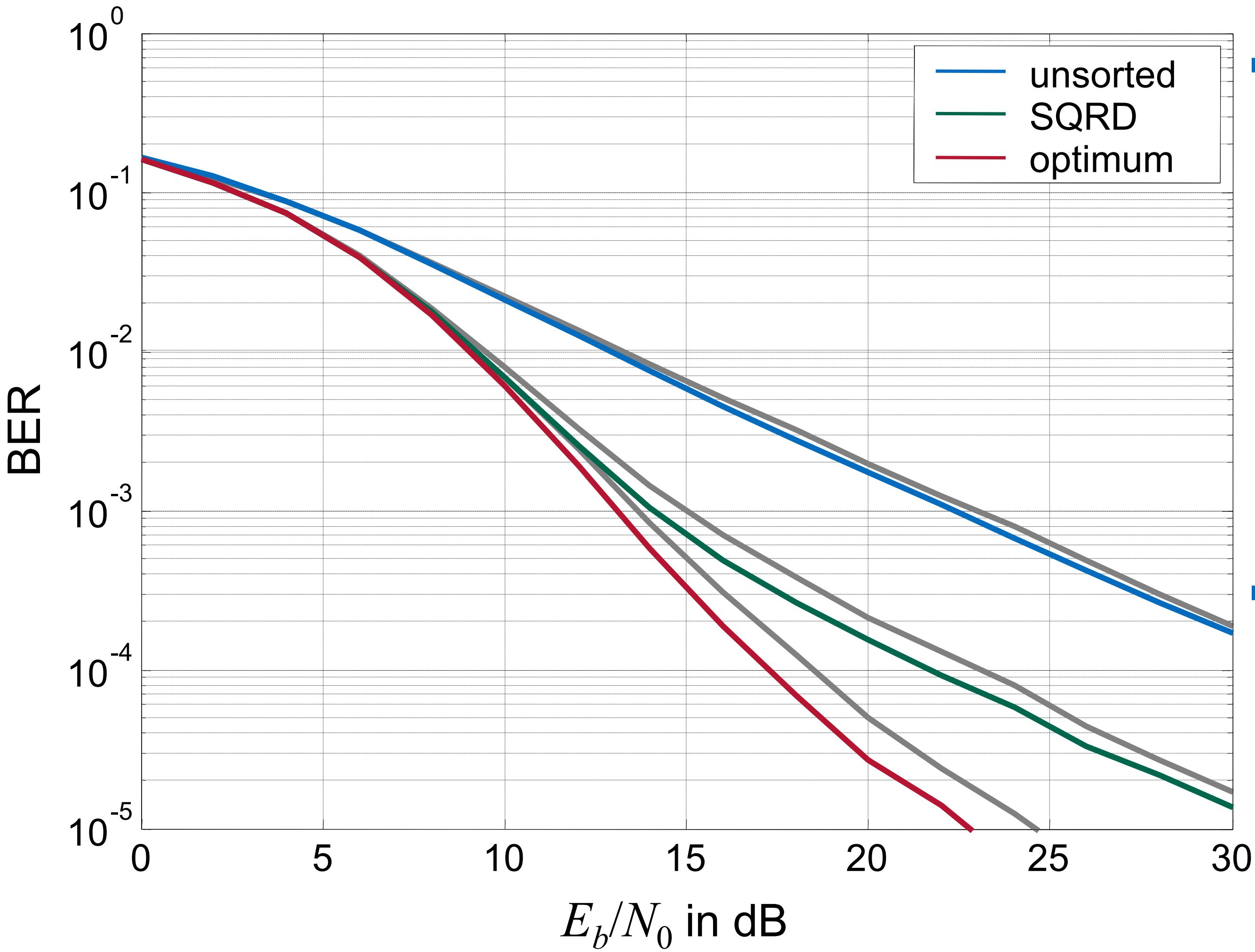
- Real system with doubled number of transmit and receive antennas
  - One QAM symbol does not need to be detected completely
  - Increased degrees of freedom for finding the optimum detection order
  - Leads to additional performance gain
- All algorithms described before can be used without modification
  - Only real-valued operations necessary
  - Nevertheless, slightly increased computational complexity due to larger matrices

Bit Error Rates for V-BLAST Systems

- Simulation parameters: $N_T = N_R = 4$, QPSK → 8 bits per time instant
- Enormous performance gain by sorting → SQRD close to optimum for ZF
  For MMSE SQRD is near optimum only for low SNR



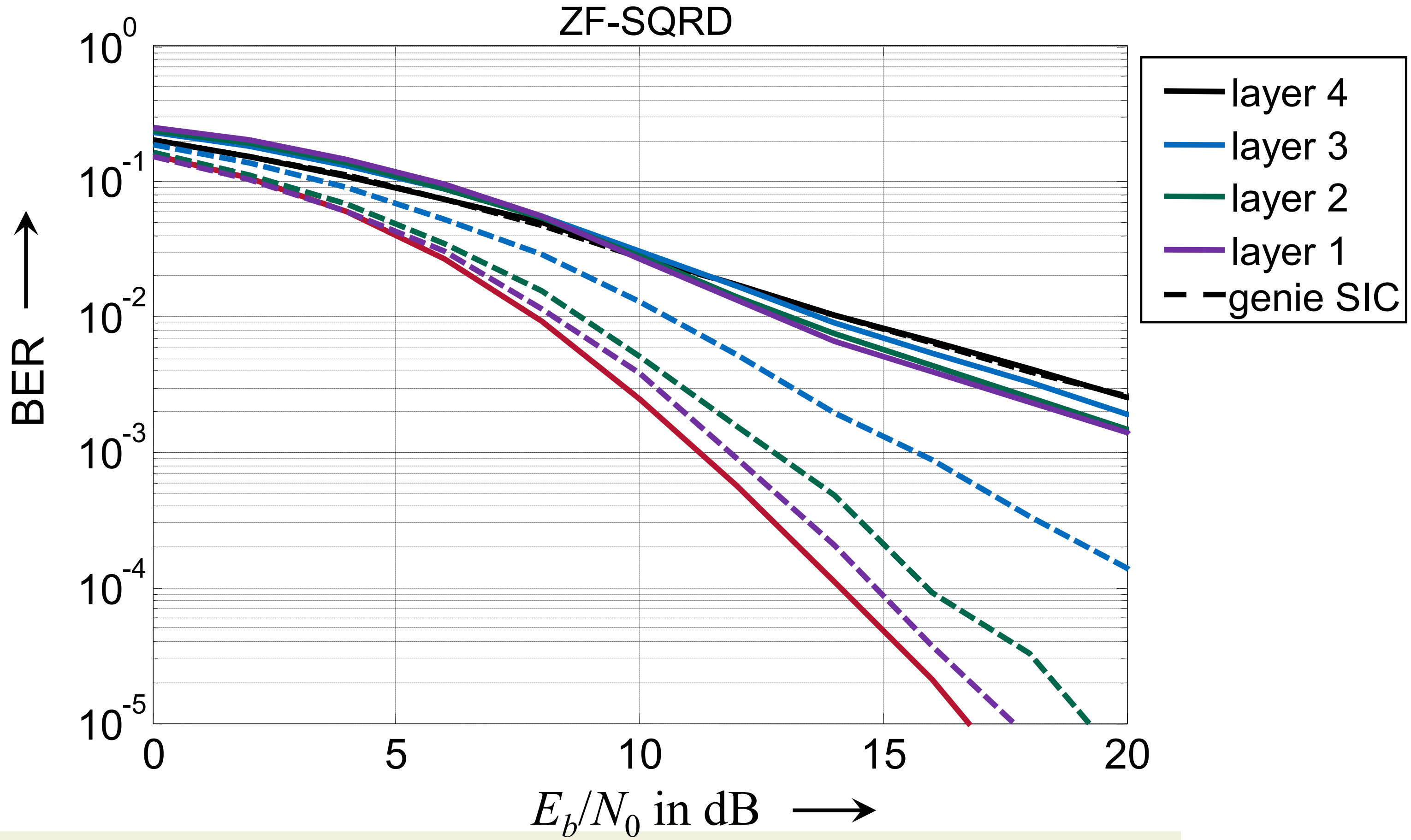
Bit Error Rates for QAM Extension of MMSE V-BLAST



- Simulation parameters
  - 4 transmit antennas
  - 4 receive antennas
  - Flat Rayleigh fading
  - Uncorrelated channels
  - QPSK modulation
  - Uncoded data streams
- Result
  - Small performance gain without sorting
  - Up to 2dB gain with optimum ordering

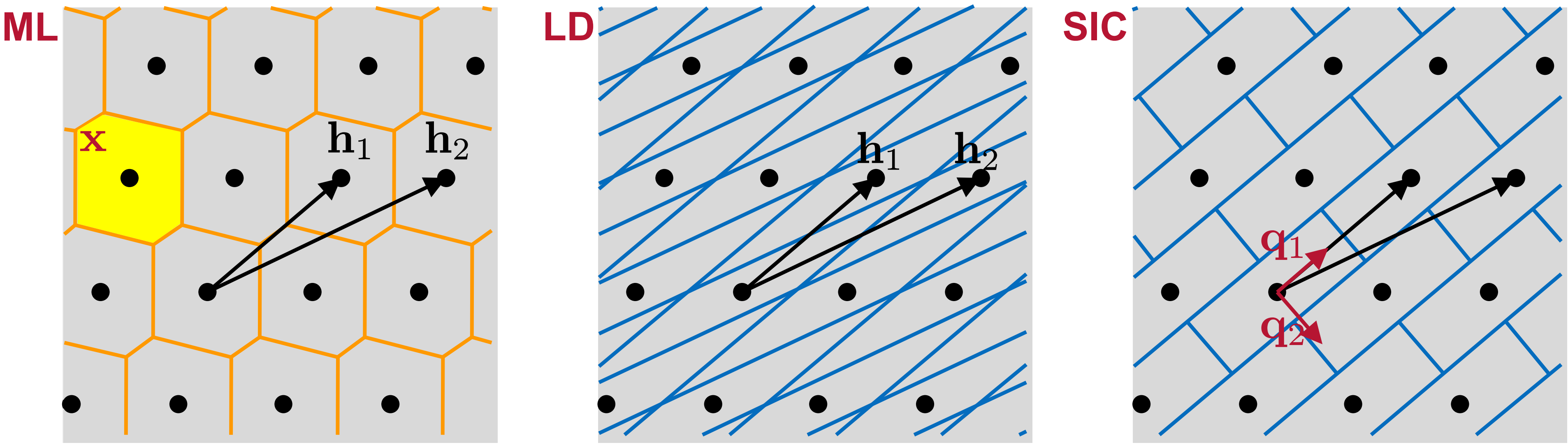
Analysis of Error Propagation

- Uncoded system with $N_\mathrm{T} = N_\mathrm{R} = 4$ antennas
- For genie detection the diversity of layer i is given by $N_\mathrm{R}-i+1$



Decision Regions of the Detection Schemes ($N_T = 2$)

- Maximum-Likelihood (**ML**): Voronoi regions (nearest neighbor)
- Linear Detection (**LD**): parallelogram in direction of $\mathbf{h}_1$ and $\mathbf{h}_2$
- Successive Interference Cancellation (**SIC**): rectangle in direction of $\mathbf{q}_2$ and $\mathbf{q}_1$
  - QR decomposition $\begin{bmatrix} \mathbf{h}_1 & \mathbf{h}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{q}_1 & \mathbf{q}_2 \end{bmatrix} \cdot \begin{bmatrix} r_{1,1} & r_{1,2} \\ & r_{2,2} \end{bmatrix}$

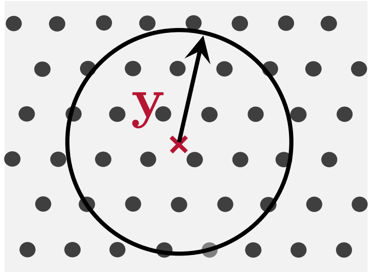


Basic Principle of Sphere Detection (1)

- Maximum-Likelihood Criterion:

$$\hat{\mathbf{x}}_{\mathrm{ML}} = \underset{\mathbf{x}' \in \mathbb{S}}{\operatorname{argmin}} \|\mathbf{y} - \mathbf{H}\mathbf{x}'\|^2$$

Equivalent real-valued system model is assumed in the sequel!



- Basic idea of Sphere Detection (**SD**):
  - Restrict the search to hypothesis $\mathbf{x}'$ within ball of radius $d_{r'}^2$ around $\mathbf{y}$ → easy?

$$d_{r'}^2 \geq \|\mathbf{y} - \mathbf{H}\mathbf{x}'\|^2 = \|\mathbf{y} - \overline{\mathbf{Q}}\,\overline{\mathbf{R}}\mathbf{x}'\|^2 \qquad \text{with} \qquad \mathbf{H} = \overline{\mathbf{Q}}\,\overline{\mathbf{R}} = [\mathbf{Q} \;\; \mathbf{Q}_\perp] \begin{bmatrix} \mathbf{R} \\ \mathbf{O} \end{bmatrix}$$

  - Multiplication with $\overline{\mathbf{Q}}^T$ (orthogonal matrix) does not change distance

$$d_{r'}^2 \geq \|\overline{\mathbf{Q}}^T\mathbf{y} - \overline{\mathbf{Q}}^T\overline{\mathbf{Q}}\overline{\mathbf{R}}\mathbf{x}'\|^2 = \|\mathbf{Q}^T\mathbf{y} - \mathbf{R}\mathbf{x}'\|^2 + \|\mathbf{Q}_\perp\mathbf{y}\|^2 = \|\tilde{\mathbf{x}} - \mathbf{R}\mathbf{x}'\|^2 + \|\mathbf{Q}_\perp\mathbf{y}\|^2$$

  - Last term is independent of the hypothesis → define radius $d_r^2 = d_{r'}^2 - \|\mathbf{Q}_\perp\mathbf{y}\|^2$

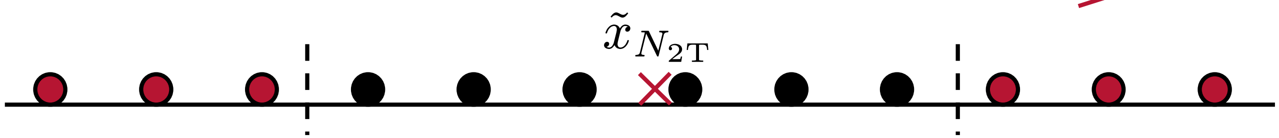
$$d_r^2 \geq \|\tilde{\mathbf{x}} - \mathbf{R}\mathbf{x}'\|^2 = \sum_{i=1}^{N_{2T}} \left(\tilde{x}_i - \sum_{\nu=i}^{N_{2T}} r_{i,\nu} x'_\nu \right)^2$$

All terms in the sum are non-negative!

  - Upper triangular form of $\mathbf{R}$ → successive testing of hypothesis (compare SIC)

Basic Principle of Sphere Detection (2)

- 1\. Step
  - Simplify constraint $d_r^2 \geq \left(\tilde{x}_{N_{2\mathrm{T}}} - r_{N_{2\mathrm{T}},N_{2\mathrm{T}}} x'_{N_{2\mathrm{T}}}\right)^2 + \sum_{i=1}^{N_{2\mathrm{T}}-1} \left(\tilde{x}_i - \sum_{\nu=i}^{N_{2\mathrm{T}}} r_{i,\nu} x'_\nu \right)^2$



  - Choose hypothesis $\hat{x}'_{N_{2\mathrm{T}}}$ that fulfills $d_r^2 = \Delta^2_{N_{2\mathrm{T}}} \geq \left(\tilde{x}_{N_{2\mathrm{T}}} - r_{N_{2\mathrm{T}},N_{2\mathrm{T}}} \hat{x}'_{N_{2\mathrm{T}}}\right)^2$
  - Update the constraint for the remaining layers

$$\Delta^2_{N_{2\mathrm{T}}-1} = \Delta_{N_{2\mathrm{T}}} - \delta^2_{N_{2\mathrm{T}}} \quad \text{with} \quad \delta^2_{N_{2\mathrm{T}}} = \left(\tilde{x}_{N_{2\mathrm{T}}} - r_{N_{2\mathrm{T}},N_{2\mathrm{T}}} \hat{x}'_{N_{2\mathrm{T}}}\right)^2$$

- 2\. Step

Partial Euclidian Distance (PED)

  - Choose hypothesis $\hat{x}'_{N_{2\mathrm{T}}-1}$ that fulfills

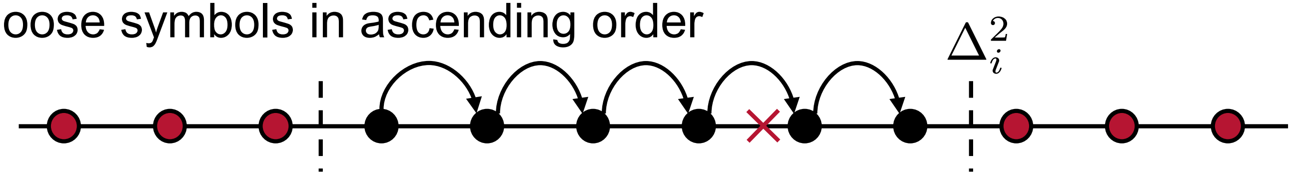
$$\Delta^2_{N_{2\mathrm{T}}-1} \geq \left(\tilde{x}_{N_{2\mathrm{T}}-1} - r_{N_{2\mathrm{T}}-1,N_{2\mathrm{T}}-1} \hat{x}'_{N_{2\mathrm{T}}-1} + r_{N_{2\mathrm{T}}-1,N_{2\mathrm{T}}} \hat{x}'_{N_{2\mathrm{T}}}\right)^2$$

  - Update the constraint for the remaining layers
- …

Search Strategies within each layer

- Fincke-Pohst (**FP-SD**)
  - Determine the range of allowed values for
  - Choose symbols in ascending order

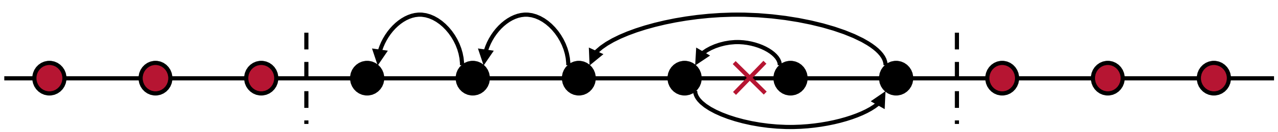
Originally used to count all points within a distinct radius



  - Radius d_r must be initialized appropriately
  - After a valid estimation is found, d_r is reduced and new search is started from root
- Schnorr-Euchner (**SE-SD**)
  - Consider symbols close to the interference reduced signal first

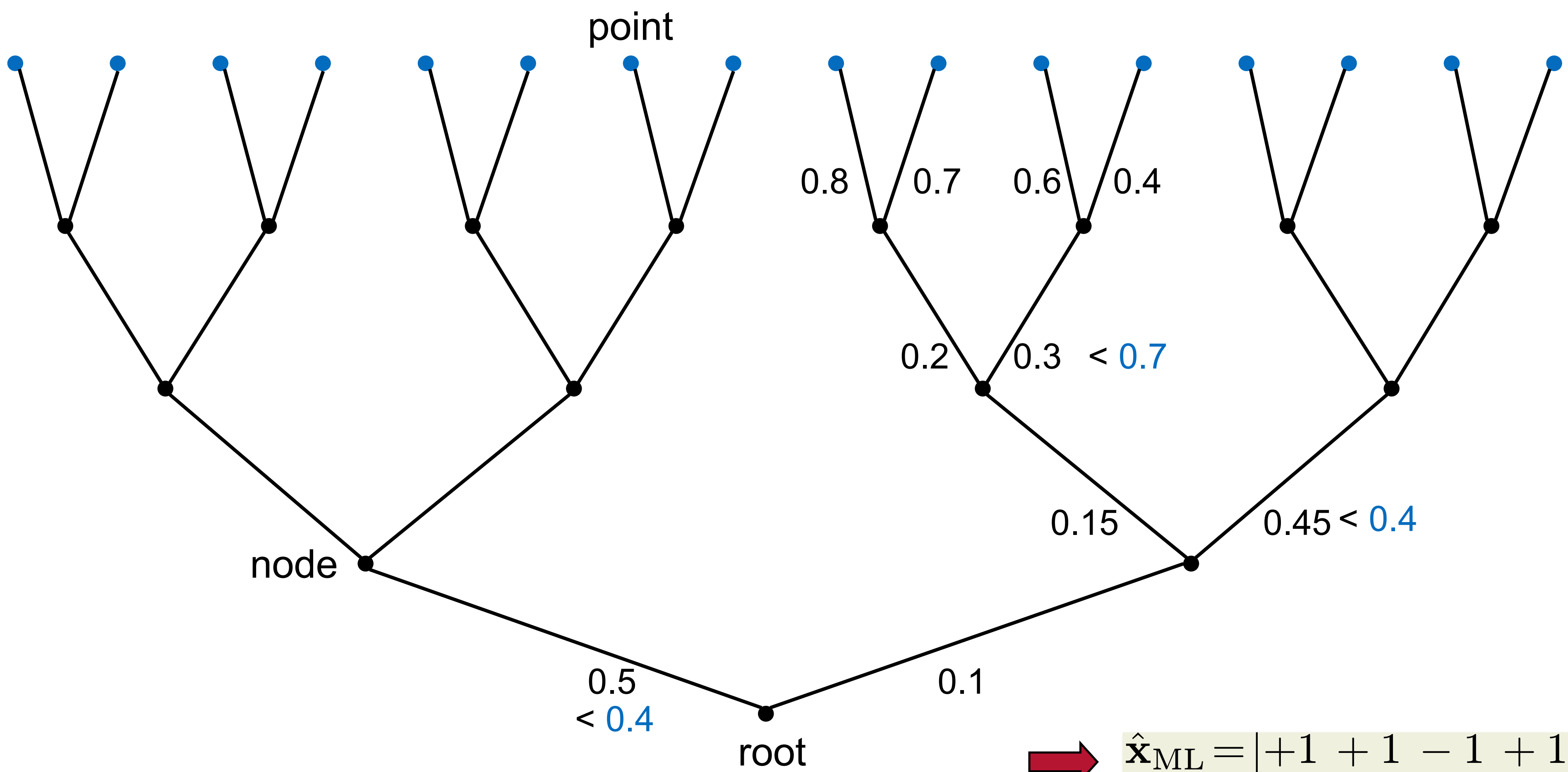
Shortest Vector Problem

$$\frac{1}{r_{i,i}}\left(\tilde{x}_i - \sum_{\nu=i+1}^{N_{2T}} r_{i,\nu}\hat{x}'_\nu\right)$$



  - Initialization $d_r = \infty$ → first point found corresponds to SIC result (Babai-Point)!
  - Update radius if a new point is found and continue search in layer 2, 3, …

Searching Tree of Schnorr-Euchner for N_{2T}=4, 2-ASK per real layer



$$\hat{\mathbf{x}}_{\mathrm{ML}} = [+1\ +1\ -1\ +1]$$

Some Aspects of Implementation

- Computational complexity is determined by number of visited nodes
- Optimization of the detection order
  - Due to the tree structure, a good estimation of hypothesis in first steps is desired → efficient search if the first point is as close to ML as possible
  - By application of SQRD an optimized sorting is achieved
- Choice of initial radius
  - SE-SD selects in each layer the nearest hypothesis → for $d_r=\infty$ → $\hat{\mathbf{x}} = \hat{\mathbf{x}}_{\text{SIC}}$ Nevertheless, an adequate choice of d_r leads to an advance of speeding
  - FP-SD requires a suitable choice of d_r → arrange d_r due to noise, e.g. Hassibi:

$$\|\mathbf{y} - \mathbf{H}\hat{\mathbf{x}}_{\text{ML}}\|^2 = \|\mathbf{n}\|^2 \sim \chi^2_{N_{2\text{R}}}$$

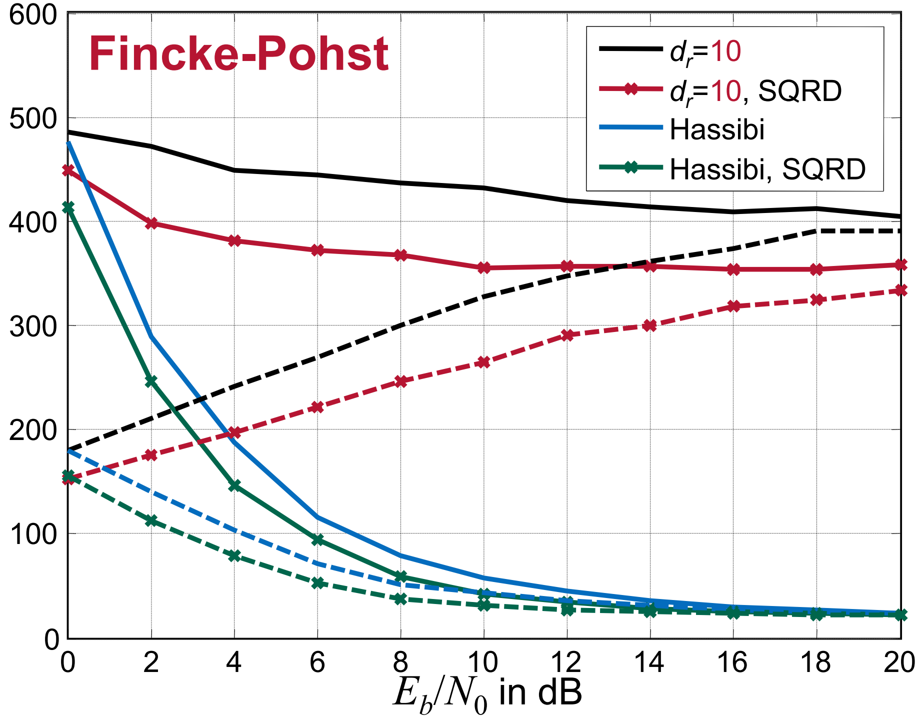
$$\Rightarrow d_{r'} = \alpha \text{E}\{\|\mathbf{n}\|^2\} = \alpha \sigma_n^2 N_{\text{R}}$$

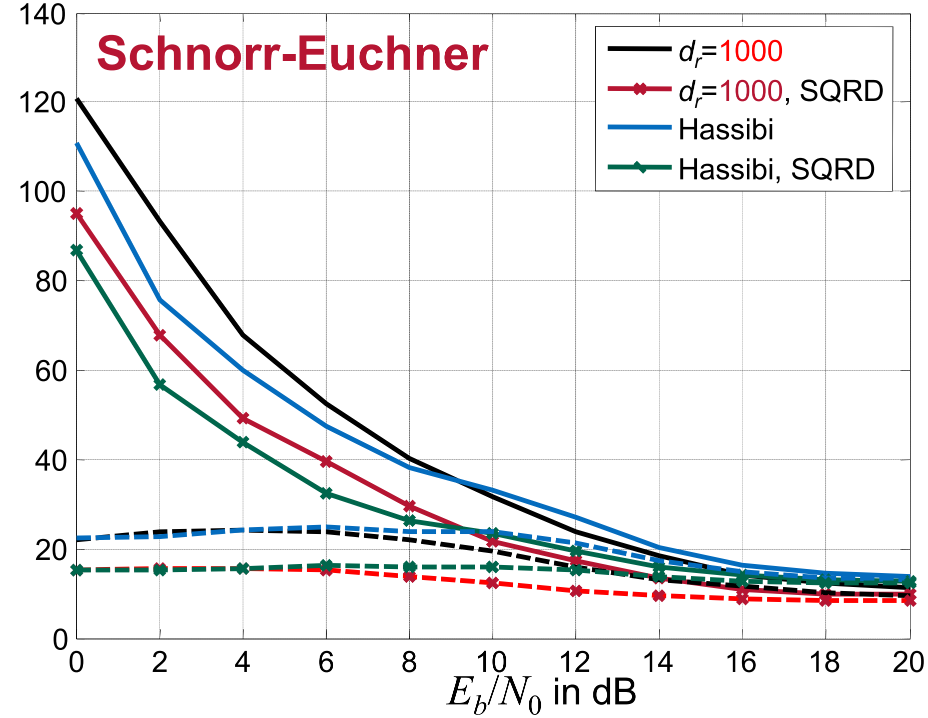
$$\Rightarrow P_{\text{FP}} = \frac{1}{\Gamma(N_{\text{R}})} \gamma(N_{\text{R}}, \alpha N_{\text{R}}) = 1 - \epsilon$$

- Extension to MMSE criterion
  - For SIC the MMSE-extension leads to an improved estimate → speeding up
  - Solution is not necessary ML-solution due to the ignored interference

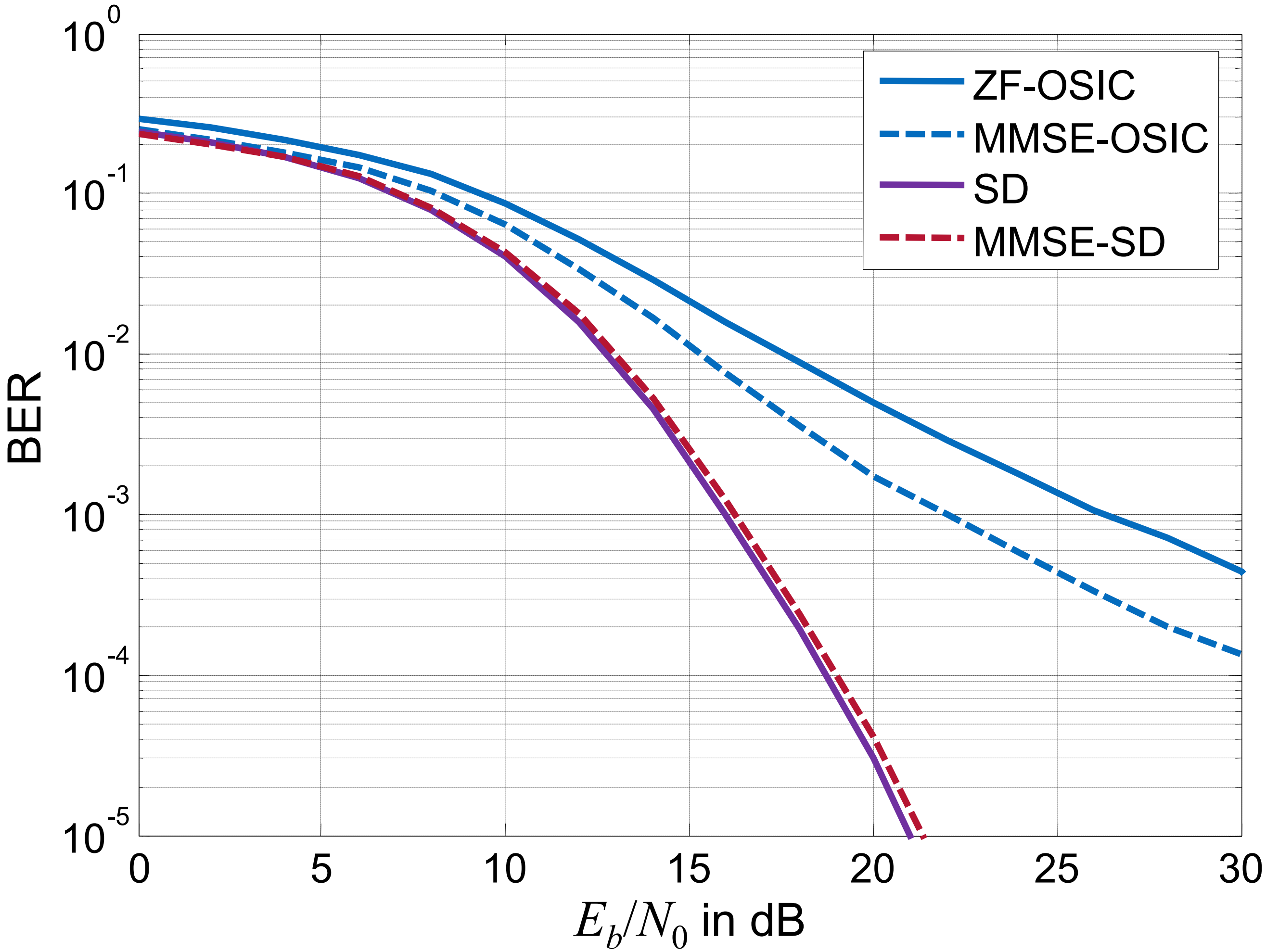
Complexity Evaluation

- Impact of initial radius and sorting for $N_T = N_R = 4$, 16-QAM (→ 65536 points) to the average number of visited nodes (Hassibi with P_{FP}=0.99)
- SQRD and MMSE lead to strong decrease in complexity for both schemes
- SE-SD is first choice of ML-Implementation (notice $d_r = 1000$!)



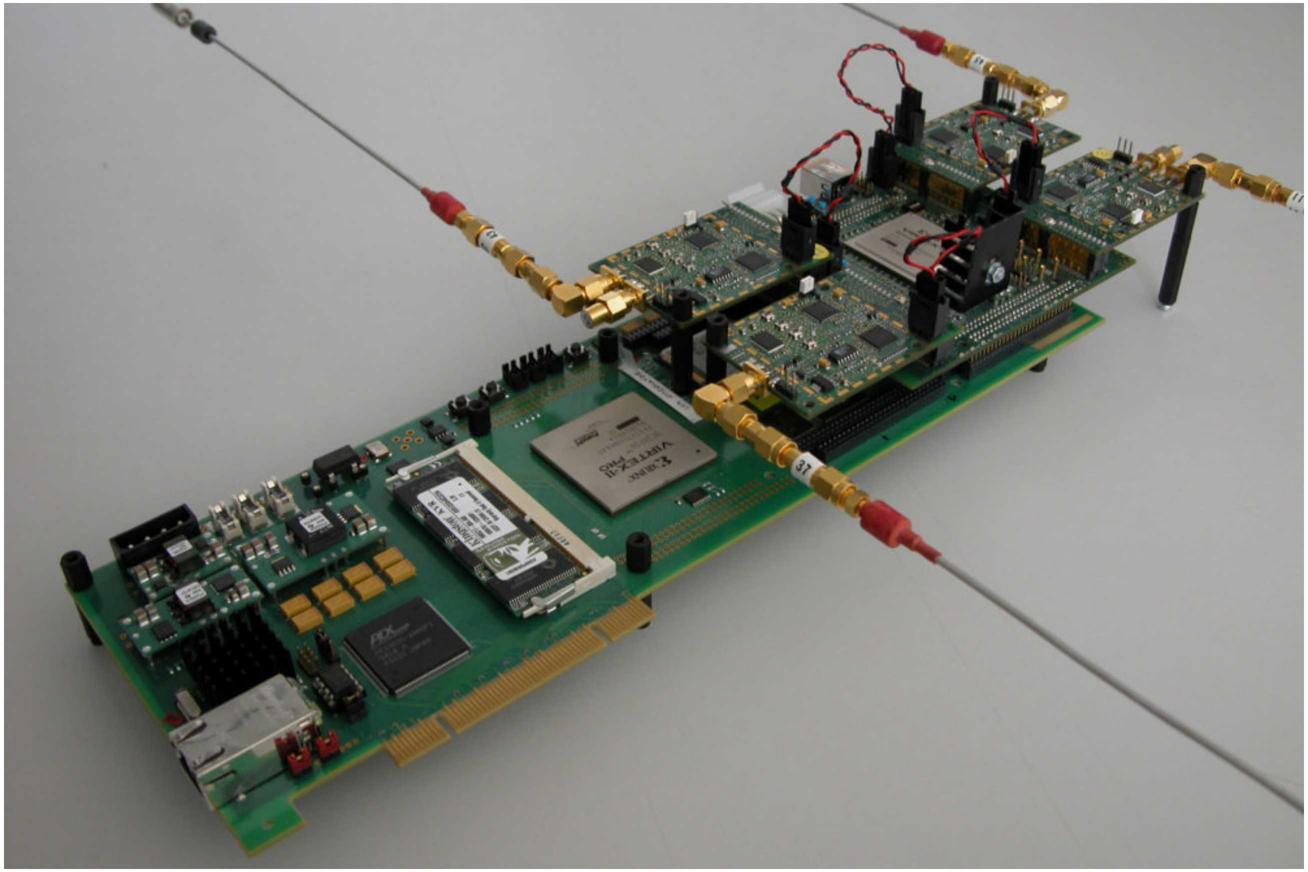


Performance of MMSE Sphere Detection



- Simulation parameters
  - $N_T = N_R = 4$, 16-QAM
- Results
  - Small performance loss of MMSE-extension due to ignored interference term
  - Complexity is significantly reduced by MMSE-extension
    → first choice of implementation

Hardware Implementation



- Real-time implementation of SQRD & Co. at ETH Zürich:
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