Effects of Downlink Channel Quantization on the Performance of Relative Calibration in OFDM Systems

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Abstract— The application of calibration techniques mitigating the front-end impairments in adaptive multi-antenna TDD systems designed for very high data rates is of paramount importance. Excellent calibration purely based on signal processing can be achieved by means of relative calibration. There, additional estimated DL channel state information is required at the transmitter, which must be sent back in reverse direction during a special calibration phase. This paper deals with the implementation of UL feedback of DL channel state information for online calibration. The fed back DL channel needs to be quantized before retransmission, which results in an inherent quantization error. The effects of this quantization error are evaluated and a recursive calibration algorithm will be presented that achieves good a performance in terms of MSE and BER.

I. INTRODUCTION

T HE application of time division duplex (TDD) schemes will gain more importance in future adaptive wireless transmissions with high data rates since it does not waste additional frequency resources which are necessary when using frequency duplexing. As uplink (UL) and downlink (DL) use the same frequency, channel state information (CSI) at the transmitter can easily be achieved by utilizing the reciprocity principle [1]. Then, CSI based on the UL channel estimate can be used to adapt to the DL channel. Unfortunately, with mismatched real-world front-ends the effective baseband channels in both directions do not fulfill the reciprocity theorem [1].

One possibility to overcome this problem is calibration by means of additional hardware calibration circuitries. If additional hardware costs for calibrating the effective channels should be saved, digital signal processing is sufficient to cope with the transceiver impairments. The idea of using total least squares (TLS) methods for calibration was introduced recently [2] and the application to Orthogonal Frequency Division Multiplexing (OFDM) systems was presented by the authors [3]. This solution was explicitly designed to work with a special calibration phase providing the base station (BS) with UL and DL CSI. Hence, the overall efficiency of the system is degraded due to a discontinuous data transmission. Consequently, this contribution deals with a recursive online calibration algorithm exploiting direct quantized DL channel feedback in the UL to avoid the need for such a calibration phase. The quantization error between the true and the calibrated channel in terms of the mean square error (MSE) performance is analyzed and bit error rate (BER) as well as measurement results are presented to verify the applicability of the algorithm.

The remainder of the paper is organized as follows. In Sec. II the system and the applied extended channel model are described. In addition, the non-reciprocity and mutual coupling models are introduced. Subsequently, the relative calibration principle based on the total least squares methods is stated in Sec. III. The DL channel quantization effects are reflected in Sec. III-B, while an efficient recursive algorithm is given in Sec. III-C. Simulation results of the calibration performance in different transceiver mismatch conditions are shown in Sec. IV-A and measurement results with a hardware demonstrator applying the recursive algorithm are presented in Sec. IV-B, respectively. Finally, a conclusion is given in Sec. V.

II. SYSTEM MODEL

In the investigations an adaptive multi-user multipleinput single-output (MISO) OFDM system with N_B base station antennas and N_M decentralized single-antenna mobile stations (MS) using OFDM with N_C subcarriers is considered, where the relation $N_B \ge N_M$ should hold. The effective DL matrix $\mathbf{H}(k)$ and the effective UL matrix $\mathbf{G}(k)$ in frequency-domain on subcarrier k can be written to

$$\mathbf{H}(k) = \mathbf{A}_{RM} \mathbf{W}_{RM} \mathbf{S}_{MB}(k) \mathbf{W}_{TB} \mathbf{A}_{TB}, \qquad (1)$$

and

$$\mathbf{G}(k) = \mathbf{A}_{TM} \mathbf{W}_{TM}^T \mathbf{S}_{MB}(k) \mathbf{W}_{RB}^T \mathbf{A}_{RB}, \qquad (2)$$

respectively. There, the scattering matrix approach of [1] and [3] is applied. Consequently, in (1) and (2) the matrices

$$\mathbf{W}_{T[B/M]} = \left(\mathbf{I}_{N_{[B/M]}} - \mathbf{\Gamma}_{T[B/M]} \mathbf{S}_{[BB/MM]}\right)^{-1} \quad (3a)$$

$$\mathbf{W}_{R[B/M]} = \left(\mathbf{I}_{N_{[B/M]}} - \mathbf{S}_{[BB/MM]} \mathbf{\Gamma}_{R[B/M]}\right)^{-1} \quad (3b)$$

describe the mutual coupling and the reflection at the transceivers, whereas the matrices $A_{[T/R][M/B]}$ contain

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the antenna gains in the transmit and the receive paths, respectively. It can be shown that the gains at the mobile stations have a minor influence on the adaptive DL transmission [4]. If feedback effects of the BS antennas on the radiation of the MSs are negligible, the remaining matrices $\mathbf{A}_{[T/R]B}$ and $\mathbf{\Gamma}_{[T/R]B}$ with complex gain factors $\alpha_{[T/R]B,i}$ and input/output reflection coefficients $\gamma_{[T/R]B,i}$, respectively, can be modeled as diagonal matrices [1], [3].

Concerning an error model of the complex gain factors that describe the non-reciprocal behavior of the transceiver chains, each front-end is assumed to have an allpass-like characteristic. This motivates the introduction of slightly mismatched gain factors $\alpha_{[T/R]B,i} = 1 + \delta_{[T/R]B,i}$, where the statistically independent error terms $\delta_{[T/R]B,i}$ are zero mean complex Gaussian random variables with variance σ_{δ}^2 [3]. These factors are expected to change very slowly in time compared to the duplex phase and are assumed to be equal per antenna on all subcarriers k.

The modeling of the reflection coefficients $\gamma_{[T/R]B,i}$ is motivated by the fact that the input reflection coefficient is usually around 20 dB below the transmission factor in a frequency range of interest. Hence, the mean value for $\gamma_{[T/R]B,i}$ was set to 0.1 [3]. Then $\gamma_{[T/R]B,i} = 0.1 + \kappa_{[T/R]B,i}$ is used to model the reflection coefficients. Again additional error terms $\kappa_{[T/R]B,i}$ are added, which are zero mean complex Gaussian random variables with variance σ_{κ}^2 .

If the BS applies linear pre-equalization for space division of the users per subcarrier, the receive signal $\mathbf{y}(k) = [y_1(k), \dots, y_{N_M}(k)]^T$ on subcarrier k stacking the signals of all mobile stations reads

$$\mathbf{y}(k) = \beta(k)\mathbf{H}(k)\mathbf{F}(k)\mathbf{d}(k) + \mathbf{n}(k), \qquad (4)$$

where $\mathbf{d}(k) \in \mathbb{C}^{N_M \times 1}$ is the data vector to be transmitted to the N_M MSs. The pre-equalization matrix $\mathbf{F}(k) \in \mathbb{C}^{N_B \times N_M}$ in the minimum mean square error (MMSE) case is determined using the uplink channel matrix $\mathbf{G}(k)$ such that

$$\mathbf{F}(k) = \mathbf{G}^{H}(k) \left(\mathbf{G}(k)\mathbf{G}^{H}(k) + \sigma_{n}^{2}\mathbf{I}_{N_{M}}\right)^{-1}$$
(5)

holds. Here, the same noise power σ_n^2 on all subcarriers and all MSs is assumed. The scalar $\beta(k)$ is chosen such that the total sum power constraint per subcarrier is fulfilled [3]. In terms of MMSE channel estimation in uplink direction, the estimated channel matrix $\hat{\mathbf{G}}(k)$ of one subcarrier can be modeled by [3]

$$\hat{\mathbf{G}}(k) = \sqrt{1 - \sigma_e^2} \,\mathbf{G}(k) + \sqrt{\sigma_e^2 \left(1 - \sigma_e^2\right)} \,\Psi(k) \,, \quad (6)$$

where $\Psi(k)$ is a Gaussian error matrix with an entry variance of one and estimation error variance σ_e^2 . The same holds for $\hat{\mathbf{H}}(k)$ with an independent error matrix but here with identical estimation error variance, which does not need to be the same in general.

III. RELATIVE CALIBRATION BASED ON TOTAL LEAST SQUARES

A. Calibration Method and Optimization Problem

The original idea of relative calibration is based on the assumption that the data transmission is interrupted to enable a time interval exclusively dedicated for calibration purposes [2]. The calibration procedure is based on knowledge of both UL and DL CSI at the base station, which in this regard requires feedback from all the MSs to the BS in selected time intervals. This fact is discussed more profound in Sec. III-B.

Based on the estimates of an UL and DL channel on one subcarrier k, taking (2), solve for $\mathbf{S}_{MB}(k)$ and insert into (1) leads to (7) at the top of this page, which is the initial point of the derivations. For further considerations, we assume estimates of the channels and neglect the $\hat{}$ indication.

Taking (7), we define the auxiliary vectors $\mathbf{c}_B \triangleq \operatorname{vec} \{ \mathbf{C}_B^{-1} \}$ and $\mathbf{c}_M \triangleq \operatorname{vec} \{ \mathbf{C}_M^T \}$, where the vec-operator is defined as $\operatorname{vec} \{ \mathbf{B} \} = \operatorname{vec} \{ [\mathbf{b}_1 \dots, \mathbf{b}_i] \} = [\mathbf{b}_1^T, \dots, \mathbf{b}_i^T]^T$, and $\mathbf{g}_i(k)$ as the *i*-th column of matrix $\mathbf{G}(k)$. Hence, (7) can be reformulated with

$$\boldsymbol{\Theta}(k) = \begin{bmatrix} \mathbf{I}_{N_M} \otimes \mathbf{g}_1^T(k) \\ \vdots \\ \mathbf{I}_{N_M} \otimes \mathbf{g}_{N_B}^T(k) \end{bmatrix}$$
(8a)

and

to

$$\mathbf{\Omega}(k) = \mathbf{I}_{N_B} \otimes \mathbf{H}(k) \tag{8b}$$

$$\mathbf{\Omega}(k)\mathbf{c}_B - \mathbf{\Theta}(k)\mathbf{c}_M = \mathbf{0}_{N_B N_M \times 1} \,. \tag{9}$$

Here, \otimes is the Kronecker product. If we set $\mathbf{c} \triangleq \begin{bmatrix} \mathbf{c}_B^T \ \mathbf{c}_M^T \end{bmatrix}^T \in \mathbb{C}^{N_B^2 + N_M^2} \times 1$ as well as

$$\mathbf{E}_k = \begin{bmatrix} \mathbf{\Omega}(k) & -\mathbf{\Theta}(k) \end{bmatrix}$$
(10a)

and an augmented matrix of \boldsymbol{K} independent measurements

$$\mathbf{E} = \begin{bmatrix} \mathbf{E}_1^T, \dots, \mathbf{E}_K^T \end{bmatrix}^T \in \mathbb{C}^{KN_BN_M \times N_B^2 + N_M^2}, \quad (10b)$$

then (9) can be compactly written to

$$\mathbf{E}\mathbf{c} = \mathbf{0}_{KN_BN_M \times 1} \,. \tag{11}$$

Obviously, matrix **E** depends on estimates of $\mathbf{G}(k)$ and $\mathbf{H}(k)$ (cf. [2]). Here, K defines the number of subcarriers used for calibration, where the k's can be arbitrarily chosen due to the presumed virtually frequency-flat transfer functions of the transceivers. But, as there are $N_M^2 + N_B^2$ number of unknowns and KN_MN_B linear equations, multiple subcarriers or measurements K must be available to obtain a non-zero solution to (11).

With the assumption of inherent estimation errors in (10a), the solution to the overdetermined set of equations (11) can be obtained by solving the total least squares (TLS) optimization problem [2]

$$\underset{\Delta \mathbf{E}}{\text{minimize}} \quad \|\Delta \mathbf{E}\|_F \tag{12a}$$

such that $(\mathbf{E} + \Delta \mathbf{E}) \mathbf{c} = \mathbf{0}_{KN_BN_M \times 1}$. (12b)

The goal is to find a perturbation matrix $\Delta \mathbf{E}$ with minimum Frobenius norm that lowers the rank of \mathbf{E} , where $\Delta \mathbf{E}$ is the correction term of the TLS optimization problem. The solution to (12) \mathbf{c}_{TLS} lies in the right null space of \mathbf{E} and can be computed with the singular value decomposition (SVD) of \mathbf{E} as shown in [3], [4].

In case of an uncoupled system the matrices C_M and C_B in (7) reduce to diagonal matrices, which leads to an overdetermined system with only $N_B + N_M$ unknowns compared to $N_B^2 + N_M^2$ in the coupled case. Consequently, the matrix **E** looks slightly different and multiple measurements are not necessary to obtain a solution, but likewise increase the accuracy of the parameter estimation [4].

B. DL Channel Quantization

The TLS calibration from the previous section is based on CSI of both effective UL and DL channels at the BS. While the effective UL channel is easily obtained with standard UL channel estimation procedures within each duplex phase, the estimated effective DL channel is only existent at the MSs in general. Consequently, quantized DL CSI needs to be fed back from the MSs to the BS for channel calibration in adaptive DL systems, which shows the compliance to standard quantization theory for FDD systems. Hence, at the BS (8b) is now generated with

$$\mathbf{\Omega}(k) = \mathbf{I}_{N_B} \otimes \mathcal{Q}\{\mathbf{H}(k)\}$$
(13)

and $\mathcal{Q}{\mathbf{H}(k)} = \left[\mathcal{Q}{\mathbf{h}^{(1)}(k)}^T, \dots, \mathcal{Q}{\mathbf{h}^{(N_M)}(k)}^T\right]^T$ denotes the stacked version of the quantized DL vector channels of the users on subcarrier k. As the transceiver parameters in (1) and (2) remain constant over a long time period, e.g., several seconds or even minutes, it is sufficient to feed back the DL channels infrequently or to split the amount of feedback into several parts that are retransmitted in multiple phases. The latter allows for a rigorous feedback reduction within one UL packet.

In literature, Marzetta et al. [5] showed that raw analog feedback can be used to transfer reverse link information to the transmitter in frequency duplex systems. In contrast to codebook-based quantization of the whole DL channel matrix or quantization of the user vector channels as described in [6], direct quantization of each channel coefficient is necessary here. This is due to insufficient accuracy of the quantization using codebooks, leading to too large quantization errors or, assuming very large codebooks, resulting in too complex search algorithms at the MSs. Unfortunately, the direct quantization of the coefficients implies a large number of feedback bits. Without loss of generality, in this paper it is assumed that two bits per coefficient are reserved for the digits before the decimal point, while the remaining bits are used for the fractional digits. This choice promises only small quantization errors under the assumption of Gaussian distributed channel coefficients. Accordingly, the total amount of feedback bits per channel matrix in direct quantization is determined by $N_{\text{tot}} = 2 \cdot N_M \cdot N_B \cdot q \cdot K$, where q defines the number of bits per real coefficient. The factor two indicates the separate quantization of real and imaginary part of a single complex channel coefficient and, again, K determines the number of subcarriers or channels used for calibration.

As opposed to the assumption of equal transceiver factors on all subcarriers, even a frequency-selective behavior of the transceivers can be handled by reserving different subcarriers for calibration in each UL phase and perform subsequent interpolation in frequency direction. The subcarrier selection could also be regulated by means of the subcarrier SNR. In the further studies, the parameter q is analyzed, whereas the total number of feedback bits N_{tot} necessary for calibration is determined by considering the remaining specific system parameters N_M , N_B and K.

C. A Recursive TLS Algorithm

It was shown in [3] that the utilization of multiple subcarriers or duplex phases (in single-carrier systems), respectively, leads to more accurate calibration results. But for a continuous transmission according to Sec. III-B this means a huge amount of additional feedback per UL transmission.

Therefore, to enable online calibration with the help of feedback and to decrease the number of feedback bits in each UL phase, efficient QR updating is introduced such that $\mathbf{E}_k = \mathbf{Q}_k \mathbf{R}_k$ with k being the subcarrier/duplex phase number [7]. As upper triangular matrix \mathbf{R}_k and matrix \mathbf{E}_k have the same singular values [8], we can now write

$$\mathbf{R}_k \, \mathbf{c}_{\mathsf{TLS},k} = \mathbf{0}_{N_B N_M \times 1} \tag{14}$$

instead of (11). Hence, the total number of feedback bits per duplex phase is only depending on q and independent of K. This results in less feedback bits in total without sacrificing performance in terms of MSE and BER compared to multiple calibration carriers.

Then, the procedure is as follows. Assuming that in duplex phase k + 1 another DL channel is fed back in UL direction, a matrix α that contains information about DL and UL channels as in (10a) can be built, where the DL information has additional errors due to quantization. Adding this α below matrix \mathbf{R}_k leads to

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$$\mathbf{E}_{k+1} \leftarrow \begin{pmatrix} \mathbf{R}_k \\ \boldsymbol{\alpha} \end{pmatrix}. \tag{15}$$

 \mathbf{E}_{k+1} has no triangular structure as necessary for solving the TLS problem with (14). With a series of Givens rotations represented by matrix $\mathbf{Q}_{k+1} \in \mathbb{C}^{2N_M N_B \times 2N_M N_B}$ in phase k+1, the lower part can be set to zero to restore the upper triangular structure such that [8]

$$\begin{pmatrix} \mathbf{R}_{k+1} \\ \mathbf{0} \end{pmatrix} \leftarrow \mathbf{Q}_{k+1} \begin{pmatrix} \mathbf{R}_k \\ \boldsymbol{\alpha} \end{pmatrix} . \tag{16}$$

For the coupled case $N_M^2 + N_B^2$ Givens rotations are necessary, whereas $N_M + N_B$ rotations are sufficient for the uncoupled case. Then, \mathbf{R}_{k+1} is the new upper triangular matrix that is used for the determination of the calibration vector \mathbf{c}_{TLS} . In [7], the computation of the solution is achieved via an inverse power method, whereas the application of conjugate gradient methods [3] or even direct low-cost implementations of the SVD are possible.

It is worth to mention that the calculation of the solution vector in a special calibration phase does not have a specific delay constraint as the calibration phase is usually much shorter compared to the transmission phase. Now, if the online calibration in combination with instantaneous DL channel feedback is applied, the computing time of c_{TLS} has to be much shorter than half the time of the duplex phase to ensure an instantaneous update of the front-end parameters while providing the same accuracy.

IV. RESULTS

A. Simulation Results

This section presents simulation results obtained by feeding back the quantized effective DL channels and performing relative calibration solving the TLS problem at the BS. Therefore, a system with $N_B = N_M = 4$ BS antennas and single-antenna users as well as $N_C = 256$ subcarriers was selected. The channel estimation error variance according to (6) was fixed to $\sigma_e^2 = 10^{-4}$. The channel has an almost exponentially decaying power delay profile with a length of six samples and never exceeds the cyclic prefix [3].

Then, Fig. 1 a) and b) show the MSE results between the real effective DL channel and the UL channel after calibration depending on the number of duplex phases or calibration carriers K, respectively. As a reference curve the perfect analog feedback assuming no errors in UL direction is given. For the uncoupled system in a) and the coupled system in **b**) it can be seen that at least q = 7 bits per real-valued coefficient (corresponds to $N_{\text{tot}} = 56 \cdot K$ bits feedback per user in this system) are necessary to achieve the same class of accuracy with the TLS calibration compared to the analog case, which also proves the numerical stability if the quantization error is not too large. In contrast, in the coupled system if only q = 5 are applied this stability cannot be guaranteed. Additionally, it can be concluded that the calibration for an uncoupled system is more robust with respect to quantization errors as the degradation in terms of MSE is small. Furthermore, Fig. 2 a) and b) show the BER performance of the system versus q at $E_b/N_0 = 40 \text{ dB}$ and a fixed reciprocity error variance of $\sigma_{\delta}^2 = -30~\mathrm{dB}$ and $\sigma_{\kappa}^2 = -30$ dB for the coupled case. The total number of subcarriers K used for calibration is also varied. As expected, the performance with a larger amount of



Fig. 1. Mean square error (MSE) after calibration versus subcarrier/measurement number k for time-invariant mismatch effects, with $N_B = N_M = 4$ for different amount of UL feeback bits; channel estimation error variance set to $\sigma_e^2 = 10^{-4}$, $\sigma_\delta^2 = -30$ dB, a) without coupling, b) with coupling $\sigma_{\kappa}^2 = -30$ dB



Fig. 2. Bit error rate (BER) for uncoded MU-MISO-OFDM systems with $N_B = N_M = 4$ and $N_C = 256$ versus different amount of UL feeback bits at $E_b/N_0 = 40$ dB; channel estimation error variance set to $\sigma_e^2 = 10^{-4}$, $\sigma_\delta^2 = -30$ dB, **a**) without coupling, **b**) with coupling $\sigma_\kappa^2 = -30$ dB

feedback gets better and saturates at the BER bound of the analog TLS solution. If K is increased a smaller number of bits can be used to achieve the same performance and in accordance to [3] the coupled system needs $K \gg 1$ in frequency-domain in contrast to the uncoupled system, which even works for K = 1. Fig. 3 a) and b) depict BER results for the same system including a punctured half-rate 3GPP Turbo code at $E_b/N_0 = 20$ dB, $\sigma_{\delta}^2 = -20$ dB and $= \sigma_{\kappa}^2 = -20$ dB. As channel coding is applied in almost every communication system, these results indicate that at least five bits in the uncoupled and seven bits per real coefficient in the coupled case are required to perform an accurate calibration of the BS transceivers in terms of 10^{-3} BER.

To conclude, the choice of q and K is a trade-off between accuracy, the amount of feedback and utilized duplex phases, respectively. The latter directly affects the time to wait before the calibrated UL channel can be exploited in the adaptive system. The more bits spent for quantization, the more accurate the results and obviously the more bits must be retransmitted. On the other hand, the total amount of utilized duplex phases can be



Fig. 3. Bit error rate (BER) for encoded MU-MISO-OFDM systems using a half-rate 3GPP Turbo code with $N_B = N_M = 4$ and $N_C = 256$ versus different amount of UL feeback bits at $E_b/N_0 = 20$ dB; channel estimation error variance set to $\sigma_e^2 = 10^{-4}$, $\sigma_\delta^2 = -20$ dB, **a**) without coupling, **b**) with coupling $\sigma_\kappa^2 = -20$ dB

increased to achieve the same BER performance with less feedback bits per uplink.

B. Measurements

To this end, the results from the previous subsection are evaluated by real-world measurements including non-reciprocal transceivers as in [9]. There, the multipleantenna demonstrator MASI-2 allows for a 2×2 TDD transmission.

To account for the slowly time-varying property of the transceiver parameters, the update equation in (16) can be adjusted by introducing a weighting with an exponential forgetting factor λ . Hence, the update

$$\begin{pmatrix} \mathbf{R}_{k+1} \\ \mathbf{0} \end{pmatrix} \leftarrow \mathbf{Q}_{k+1} \begin{pmatrix} \lambda \mathbf{R}_k \\ \boldsymbol{\alpha} \end{pmatrix}$$
(17)

is used in each iteration, where $0 \le \lambda \le 1$. As λ directly affects the number of measurements in the memory, $\lambda = 1$ defines the original algorithm, whereas $\lambda = 0$ implies that just the instantaneous duplex phase measurements are used for calibration. Considering slowly time-varying gain factors suggests $\lambda \to 1$. Thus, $\lambda = 0.98$ was chosen exemplarily in the following and channel sounding measurements with the MASI-2 demonstrator in a flatfading Line-of-Sight scenario as in [9] are conducted. In contrast to the simulation results, the OFDM system is not utilized and solely the MIMO channels are measured in multiple duplex phases. Uncoupled antenna elements are assumed throughout the measurements, leading to the described TLS problem with less unknown coefficients. Fig. 4 shows the MSE results between the true measured DL channel, the measured UL and the calibrated UL channel, respectively. In case of the uncalibrated MASI-2 system, the MSE remains at around 10% in each duplex phase, whereas the possible MSE reduction is from 10%to $2 \cdot 10^{-3}$ on average for perfect (analog) feedback. Assuming 5 bits per real coefficient per DL channel, the MSE performance shows comparably good results. The implementation of only 4 bit feedback per real coefficient leads to MSE results around one order of magnitude



Fig. 4. Channel sounding MSE measurement results with the 2×2 MIMO MASI-2 system in a Line-of-Sight scenario [9]

larger than the perfect feedback case. Consequently, the amount of feedback must be large enough. However, the feedback can be split up into several packets distributed over multiple duplex phases.

V. CONCLUSION

In this contribution the application of a recursive online calibration approach exploiting QR decomposition methods for multi-user TDD MISO-OFDM systems was presented. This approach avoids the need for a special calibration phase by exploiting infrequent DL channel feedback and therefore not only improves the BER performance but also increases the system efficiency in terms of throughput. The amount of DL feedback for calibration has to be chosen carefully as the amount directly relates to the calibration performance. In the future, the application of time-domain calibration is investigated, which leads to a smaller number of coefficients to quantize but, in contrast, requires more complex algorithms to solve structured TLS (STLS) problems.

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