Mutual Information Based Analysis for Physical-Layer Network Coding with Optimal Phase Control

Meng Wu, Dirk Wübben, Armin Dekorsy
Department of Communications Engineering
Otto-Hahn-Allee 1, University of Bremen, 28359 Bremen, Germany
Email: {wu, wuebben, dekorsy}@ant.uni-bremen.de

***Abstract*—In this paper two-way relaying networks are considered using physical-layer network coding in a two-phase protocol. In the multiple access phase, both sources transmit their messages to the relay simultaneously. Subsequently, the relay estimates and broadcasts the XOR-based network coded message back to the sources in the broadcast phase. We concentrate on the critical multiple access phase, where several detection and decoding schemes at the relay are studied and compared with respect to mutual information and system throughput. It is shown, that the system performance of the detection and decoding schemes under investigation is highly dependent on the phase difference of the two incoming messages at the relay, motivating a phase control strategy at the sources before transmission. Simulation results confirm our analysis and the superior performance achieved by phase control in practical systems.**

I. Introduction

In two-way relaying communications two sources exchange messages with each other helped by a relay using network coding [1], [2]. In such a 3-phase protocol, the sources transmit in successive time slots and the relay broadcasts a network coded signal in the third time slot. Thus one time slot can be saved compared to orthogonal transmission requiring 4 time slots. In order to further increase the spectral efficiency, physical-layer network coding (PLNC) can be applied in a 2-phase protocol. In this case, both sources transmit simultaneously, resulting in a superimposed received signal at the relay. To handle such a multiple access (MA) problem, one possible solution is that the relay estimates the bitwise modulo-2 sum (XOR) of the two source messages and sends it back to the sources in the broadcast (BC) phase. It has been recognized by Zhang *et al.* [3] that the relay needs not to decode the individual messages from the sources explicitly, but it can directly estimate the network coded signal from the received message. An identical concept to PLNC was proposed by Popovski *et al.* in [4], [5] termed denoise-forward (DNF). Depending on instantaneous channel knowledges in the MA phase, the received signal at the relay is mapped to the network coded signal adaptively using general exclusive laws that minimizes denoising errors. While the previously mentioned works focused only on uncoded systems, channel decoding has been jointly considered with PLNC for repeat accumulate (RA) codes in [6]. Such a work was extended to low-density parity-check (LDPC) codes using a modified sum-product algorithm over Galois field $\mathbb{F}_4$ [7] for binary phase shift keying (BPSK) modulation in [8]. This has been further extended to quadrature phase shift keying (QPSK) modulation by decoding over Galois field $\mathbb{F}_{16}$ in [9]. The information-theoretical bound for two-way relaying networks using the 2-phase protocol has been elaborated in [10], [11]. More practical analysis based on mutual information (MI) of the MA phase was conducted in [12] for alphabet constrained input.

In this paper we show that the MI is highly dependent on the phase difference of the two incoming messages at the relay for different detection and decoding schemes in the MA phase. Therefore, a phase control strategy is presented that properly adjusts the phase information of the transmitted signal at the sources, which leads to higher MI. It is also demonstrated that only partial channel knowledges at the transmitter side are required for phase control in orthogonal frequency division multiplexing (OFDM) systems.

The remainder of this paper is organized as follows. The system model is introduced in Section II. Focusing on the critical MA phase, several detection and decoding schemes at the relay are demonstrated and compared with respect to their MI in Section III. The phase control strategy with feedback reduction is presented in Section IV. Link level simulations are presented in Section V, which verify the theoretical analysis. Finally, Section VI concludes the paper.

II. System Description

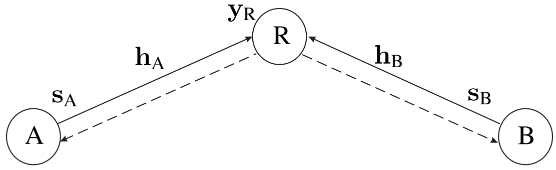

Fig. 1. A two-way relaying network using a 2-phase protocol. Source A and B transmit simultaneously to relay R in the MA phase (solid lines). R broadcasts a network coded signal in the BC phase (dashed lines).

We consider a two-way relaying network shown in Fig. 1, where two sources A and B exchange messages with each other helped by a relay R. All nodes are equipped with a single antenna. A 2-phase protocol is adopted using PLNC. In the MA phase, A and B encode their binary information words $\mathbf{b}_\mathrm{A}$ and $\mathbf{b}_\mathrm{B}$ with the same linear code of rate R_C, resulting in the codewords $\mathbf{c}_\mathrm{A}$ and $\mathbf{c}_\mathrm{B}$. Afterwards, the codewords are mapped to symbol-level vectors $\mathbf{s}_\mathrm{A}$ and $\mathbf{s}_\mathrm{B}$ using a finite alphabet $\mathcal{A}$ with cardinality $|\mathcal{A}| = M$, which are transmitted to the relay simultaneously. The superimposed received signal $y_{\mathrm{R},k}$ for the kth symbol at the relay yields

$$y_{\mathrm{R},k} = h_{\mathrm{A},k} s_{\mathrm{A},k} + h_{\mathrm{B},k} s_{\mathrm{B},k} + n_{\mathrm{R},k} \; . \tag{1}$$

Here $h_{\mathrm{A},k}$ and $h_{\mathrm{B},k}$ describe the channel influence of the two uplinks. When additive white Gaussian noise (AWGN) channels are assumed, $h_{\mathrm{A},k}$ and $h_{\mathrm{B},k}$ are set to 1. In case of multi-path fading scenarios with N_H channel taps, OFDM is applied with N_C subcarriers. Correspondingly, $h_{\mathrm{A},k}$ and $h_{\mathrm{B},k}$ denote the channel coefficients in frequency domain. Furthermore, the complex Gaussian noise term $n_{\mathrm{R},k}$ has zero mean and variance σ_n^2.

Upon receiving $\mathbf{y}_\mathrm{R}$ that collects all symbols containing the whole codeword, relay R estimates the XORed packet $\mathbf{c}_\mathrm{R}$ from $\mathbf{y}_\mathrm{R}$, either by first decoding the individual messages separately or directly estimating the XORed message. Subsequently, $\mathbf{c}_\mathrm{R}$ is modulated and broadcast to the sources in the BC phase. Since A and B know what they transmitted in the MA phase, such self-interference can be removed by XOR operation with the estimate $\hat{\mathbf{c}}_\mathrm{R}$ to recover the desired message.

In the following illustrations the index k will be omitted for the sake of simplicity. Defining $s_\mathrm{AB} = (s_\mathrm{A}, s_\mathrm{B})$ as the pair of transmitted symbols from A and B, the a-posteriori probability (APP) that s_AB is transmitted conditioned on receiving y_R is given by [8]

$$\Pr\{s_\mathrm{AB}|y_\mathrm{R}\} = \frac{\Pr\{y_\mathrm{R}|s_\mathrm{AB}\}\Pr\{s_\mathrm{AB}\}}{\Pr\{y_\mathrm{R}\}} = \frac{\Pr\{y_\mathrm{R}|s_\mathrm{AB}\}}{\sum_{\forall s_\mathrm{AB}} \Pr\{y_\mathrm{R}|s_\mathrm{AB}\}} \; , \tag{2}$$

where Bayes' rule is applied and the following equation holds assuming complex Gaussian noise

$$\Pr\{y_\mathrm{R}|s_\mathrm{AB}\} = \frac{1}{\pi\sigma_n^2} \exp\left\{ -\frac{|y_\mathrm{R} - h_\mathrm{A}s_\mathrm{A} - h_\mathrm{B}s_\mathrm{B}|^2}{\sigma_n^2} \right\} \; . \tag{3}$$

Additionally, equal a-priori probabilities are assumed at the sources, leading to $\Pr\{s_\mathrm{AB}\} = \frac{1}{M^2}$. For BPSK and QPSK, each quaternary signal s_AB appears with probability $\frac{1}{4}$ and $\frac{1}{16}$, respectively.

III. Mutual Information

In this paper we focus on the critical MA phase since it dominates the overall system performance due to error propagation. We initiate a signal analysis for several detection and decoding approaches. The corresponding mutual information between different transmitted signals and the received signal is computed in this section based on instantaneous channel knowledges in the MA phase.

A. Individual Signal

For a MA channel the individual messages from source A and B can be decoded separately, which is termed separate decoding (SDC) [8]. With the help of the APPs defined in (2), the log-likelihood ratio (LLR) L_A of each code bit c_A from source A can be calculated as

$$L_\mathrm{A} = \ln\frac{\Pr\{c_\mathrm{A} = 0|y_\mathrm{R}\}}{\Pr\{c_\mathrm{A} = 1|y_\mathrm{R}\}} = \ln\frac{\sum_{s_\mathrm{AB}\in\mathcal{D}_\mathrm{S}^0} \Pr\{s_\mathrm{AB}|y_\mathrm{R}\}}{\sum_{s_\mathrm{AB}\in\mathcal{D}_\mathrm{S}^1} \Pr\{s_\mathrm{AB}|y_\mathrm{R}\}} \; , \tag{4}$$

where the sets $\mathcal{D}_\mathrm{S}^0$ and $\mathcal{D}_\mathrm{S}^1$ contain all the symbol pairs $s_\mathrm{AB} = (s_\mathrm{A}, s_\mathrm{B})$ with the involved bit c_A equal to 0 and 1, respectively. The LLR L_B for c_B can be calculated similar to Eq. (4). Subsequently, channel decoding is applied to separately calculate the estimates $\hat{\mathbf{c}}_\mathrm{A}$ and $\hat{\mathbf{c}}_\mathrm{B}$, which are then network coded as $\mathbf{c}_\mathrm{R} = \hat{\mathbf{c}}_\mathrm{A} \oplus \hat{\mathbf{c}}_\mathrm{B}$ to be broadcast in the BC phase. The mutual information between the individual bitwise signal c_A and the received signal y_R is computed as [12]

$$\begin{aligned} C_\mathrm{S,A} &= I(c_\mathrm{A}; y_\mathrm{R}) \\ &= \sum_{c_\mathrm{A}=i} \int_{-\infty}^{\infty} \Pr\{c_\mathrm{A}{=}i, y_\mathrm{R}\} \log_2 \frac{\Pr\{c_\mathrm{A}{=}i, y_\mathrm{R}\}}{\Pr\{c_\mathrm{A}{=}i\}\Pr\{y_\mathrm{R}\}} \mathrm{d}y_\mathrm{R} \\ &= \frac{1}{M^2} \sum_{c_\mathrm{A}=i} \int_{-\infty}^{\infty} \sum_{s_\mathrm{AB}\in\mathcal{D}_\mathrm{S}^i} \Pr\{y_\mathrm{R}|s_\mathrm{AB}\} \\ &\quad \cdot \log_2 \frac{\sum_{s_\mathrm{AB}\in\mathcal{D}_\mathrm{S}^i} \Pr\{y_\mathrm{R}|s_\mathrm{AB}\}}{\Pr\{c_\mathrm{A}{=}i\} \sum_{\forall s_\mathrm{AB}} \Pr\{y_\mathrm{R}|s_\mathrm{AB}\}} \mathrm{d}y_\mathrm{R} \end{aligned} \tag{5}$$

with $i = 0, 1$ and $\Pr\{c_\mathrm{A}{=}i\} = \frac{1}{2}$. The signal from source B is treated as non-Gaussian interference due to deterministic channel coefficients. Note that there is no closed-form solution for the integral in (5) and thus this has to be solved numerically. Similarly, the mutual information $C_\mathrm{S,B}$ between c_B and y_R can be calculated, leading to the performance upper-bound for SDC as

$$C_\mathrm{S} = \min\{C_\mathrm{S,A}, C_\mathrm{S,B}\} \; . \tag{6}$$

This is due to the fact that both source messages have to be decoded without errors in order to generate the XOR network coded signal correctly.

B. Network Coded Signal

Motivated by the fact that the relay is not interested in the individual messages from the sources but only broadcasts the network coded signal in the BC phase, joint channel decoding and network coding (JCNC) [8] can be applied assuming to use the same linear channel codes at both sources, which calculates the LLR value $L_{\mathrm{A}\oplus\mathrm{B}}$ for $c_{\mathrm{A}\oplus\mathrm{B}} = c_\mathrm{A} \oplus c_\mathrm{B}$ as

$$L_{\mathrm{A}\oplus\mathrm{B}} = \ln\frac{\Pr\{c_{\mathrm{A}\oplus\mathrm{B}} = 0|y_\mathrm{R}\}}{\Pr\{c_{\mathrm{A}\oplus\mathrm{B}} = 1|y_\mathrm{R}\}} = \ln\frac{\sum_{s_\mathrm{AB}\in\mathcal{D}_\mathrm{J}^0} \Pr\{s_\mathrm{AB}|y_\mathrm{R}\}}{\sum_{s_\mathrm{AB}\in\mathcal{D}_\mathrm{J}^1} \Pr\{s_\mathrm{AB}|y_\mathrm{R}\}} \; . \tag{7}$$

Here $\mathcal{D}_\mathrm{J}^0$ and $\mathcal{D}_\mathrm{J}^1$ contain all the symbol pairs with $c_{\mathrm{A}\oplus\mathrm{B}} = 0$ and $c_{\mathrm{A}\oplus\mathrm{B}} = 1$, respectively. Subsequently, the XORed signal can be decoded directly using $L_{\mathrm{A}\oplus\mathrm{B}}$. The mutual information between the XORed signal $c_{A\oplus B}$ and the received signal y_R is given by [12]

$$
\begin{aligned}
C_J &= I\left(c_{A\oplus B}; y_R\right) \\
&= \sum_{c_{A\oplus B}=i} \int_{-\infty}^{\infty} \Pr\{c_{A\oplus B}=i, y_R\} \log_2 \frac{\Pr\{c_{A\oplus B}=i, y_R\}}{\Pr\{c_{A\oplus B}=i\}\Pr\{y_R\}} \mathrm{d}y_R \\
&= \frac{1}{M^2} \sum_{c_{A\oplus B}=i} \int_{-\infty}^{\infty} \sum_{s_{AB}\in\mathcal{D}_J^i} \Pr\left\{y_R | s_{AB}\right\} \\
&\quad \cdot \log_2 \frac{\sum_{s_{AB}\in\mathcal{D}_J^i} \Pr\left\{y_R|s_{AB}\right\}}{\Pr\left\{c_{A\oplus B}=i\right\} \sum_{\forall s_{AB}} \Pr\left\{y_R|s_{AB}\right\}} \mathrm{d}y_R
\end{aligned}
\tag{8}
$$

with $\Pr\left\{c_{A\oplus B}=i\right\}=\frac{1}{2}$. Note that (8) can be interpreted as the MI of transmitting the network coded signal $c_{A\oplus B}$ to the relay via a virtual channel. For example, this implies that $c_{A\oplus B}=0$ is mapped to the noise-free received signal $\pm(h_A+h_B)$ and $c_{A\oplus B}=1$ is mapped to $\pm(h_A-h_B)$ for BPSK.

C. Tuple Signal

As indicated by Zhang *et al.* in [6], JCNC discussed in the previous sub-section neglects useful information provided by the two channel codes applied at the sources and thus doesn't fully exploit the coding gain. To this end, a generalized sum-product algorithm was proposed in [8], [9] for LDPC codes, where the APPs defined in (2) are fed to a non-binary belief propagation decoder [7]. Such a generalized joint channel decoding and network coding (G-JCNC) strategy motivates the calculation of the mutual information C_G between the tuple signal s_{AB} and y_R [12]

$$
\begin{aligned}
C'_G &= I\left(s_{AB}; y_R\right) \\
&= \sum_{s_{AB}} \int_{-\infty}^{\infty} \Pr\left\{s_{AB}, y_R\right\} \log_2 \frac{\Pr\left\{s_{AB}, y_R\right\}}{\Pr\left\{s_{AB}\right\}\Pr\left\{y_R\right\}} \mathrm{d}y_R \\
&= \frac{1}{M^2} \sum_{s_{AB}} \int_{-\infty}^{\infty} \Pr\{y_R|s_{AB}\} \log_2 \frac{M^2 \Pr\{y_R|s_{AB}\}}{\sum_{\forall s_{AB}} \Pr\{y_R|s_{AB}\}} \mathrm{d}y_R\ .
\end{aligned}
\tag{9}
$$

Note that C'_G corresponds to the sum-rate in the MA phase. Therefore, the code rate from A and B has to be smaller than $C_G = C'_G/2$ in order to recover the individual messages.

D. Comparison

Subsequently, the mutual information C_S for SDC, C_J for JCNC and C_G for G-JCNC are compared in AWGN and fading channels with BPSK, as shown in Fig. 2 and Fig. 3, respectively. Note that extension to higher modulation alphabets is straightforward using(5), (8) and (9). The 3-phase and 4-phase protocols are also considered as benchmarks [2]. Due to higher number of time slots, the normalized MI per bit amounts to $\frac{2}{3}C_{BPSK}$ and $\frac{1}{2}C_{BPSK}$ for these two scenarios with C_{BPSK} given by

$$
C_{BPSK} = I\left(c_A; y_R | s_B = 0\right) \tag{10}
$$

as the single-user bound since no signal superpositions occur. It can be observed, that in AWGN channels, SDC achieves only half the maximum MI due to the ambiguity of transmitting $s_{AB} = (+1, -1)$ and $s_{AB} = (-1, +1)$, both leading to the noise-free received signal $y_R = 0$. However, since $(+1, -1)$ and $(-1, +1)$ are XORed to the same network coded signal, the maximal MI can be achieved for JCNC. The MI saturates at $\frac{3}{4}$ for G-JCNC because 4 transmitted quaternary symbols s_{AB} lead to only 3 received noise-free constellation points in AWGN channels with BPSK.

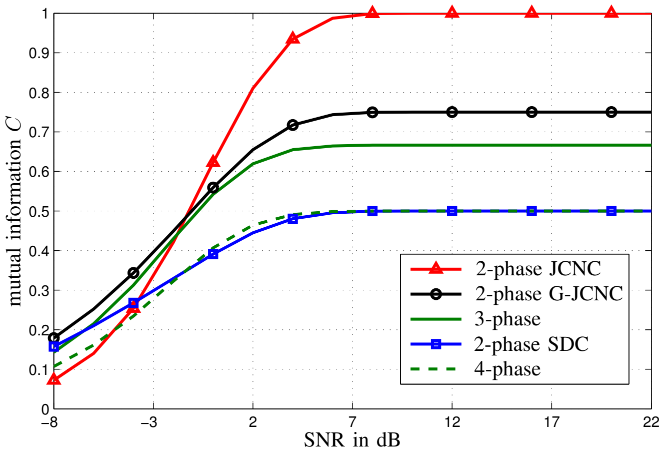

Fig. 2. Mutual information for the 2-phase protocol in AWGN channels. The 4-phase and 3-phase protocols are also considered for comparison. BPSK is adopted for all the involved scenarios.

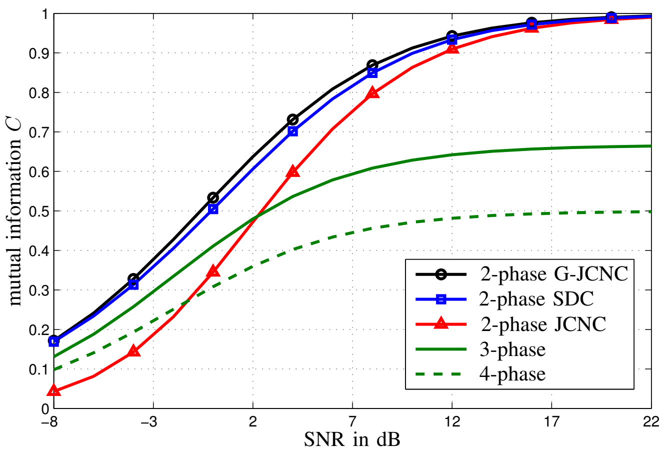

Fig. 3. Mutual information for the 2-phase protocol in fading channels. The 4-phase and 3-phase protocols are also considered for comparison. BPSK is adopted for all the involved scenarios.

For fading channels ergodic MI achieved by averaging 10^6 channel realizations in Monte-Carlo simulations is considered as shown in Fig. 3. Here all three schemes in the 2-phase protocol approach the maximum achievable MI with SDC and G-JCNC outperforming JCNC over the whole SNR regions. This is due to the fact that the randomness of the channel coefficients h_A and h_B helps to avoid the ambiguity of transmitting $s_{AB} = (+1, -1)$ and $s_{AB} = (-1, +1)$ to a large degree. However, it jeopardizes the performance for JCNC because of more separated constellation points that lead to smaller LLRs. Such an impact is also elaborated in [5] based on minimum Euclidean distance analysis.

IV. Phase Control Strategy

A. *Mutual Information with Optimal Phase Control*

As shown in Sub-section III-D, the mutual information C_{S} for SDC saturates at 0.5 with growing signal to noise ratio (SNR) for AWGN channels with BPSK modulation because of the ambiguity of transmitting $s_{\mathrm{AB}} = (+1, -1)$ and $s_{\mathrm{AB}} = (-1, +1)$. However, such ambiguity can be beneficial for JCNC since both $s_{\mathrm{AB}} = (+1, -1)$ and $s_{\mathrm{AB}} = (-1, +1)$ lead to the same XORed message. Inspired by [13] a phase difference can be generated, which distinguishes the superimposed signals by pre-rotating the constellation at, e.g., source B with $\mathrm{e}^{j\Delta\phi}$ before transmission. The corresponding noise-free received signal at R then reads $s_{\mathrm{A}} + \mathrm{e}^{j\Delta\phi} s_{\mathrm{B}}$. In the sequel, the impact of the phase difference $\Delta\phi$ on mutual information is investigated for different decoding schemes over AWGN and fading channels with BPSK and QPSK.

In order to generate a constant phase difference for each channel realization in fading channels, source B, for example, needs to know the phase difference between h_{A} and h_{B} in order to compute $\theta = \Delta\phi + \angle h_{\mathrm{A}} - \angle h_{\mathrm{B}}$ for pre-rotation, which results in extra overhead. The received signal at R yields

$$y_{\mathrm{R}} = h_{\mathrm{A}} s_{\mathrm{A}} + h_{\mathrm{B}} \mathrm{e}^{j\theta} s_{\mathrm{B}} + n_{\mathrm{R}} \; . \tag{11}$$

Note that a 'blind' rotation of $\theta = \Delta\phi$ doesn't help because the channel phase in fading channels is equally distributed. Therefore, the starting point is irrelevant and results in no performance improvement.

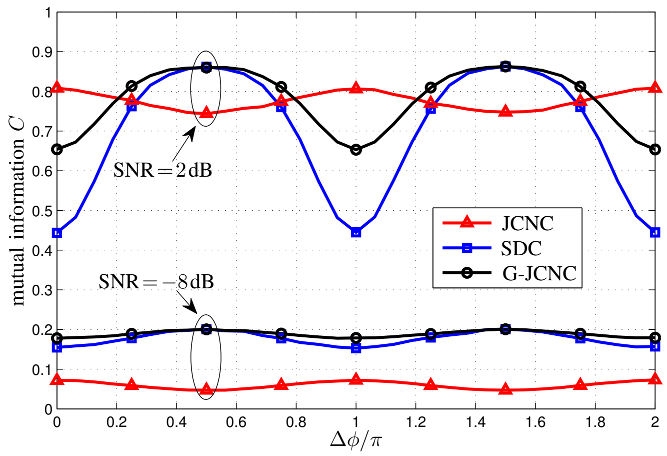

Fig. 4. Impact of phase difference $\Delta\phi$ on the MI for SDC, JCNC and G-JCNC in the high SNR region (SNR = 2dB) and low SNR region (SNR = −8 dB) over AWGN channels with BPSK.

Fig. 4 and Fig. 5 show the dependency of MI on the phase difference $\Delta\phi$ achieved by phase control for different decoding schemes with BPSK over AWGN and fading channels, respectively. It can be observed that the MI for SDC and G-JCNC are highly dependent on $\Delta\phi$ and achieve the maximum with e.g., $\Delta\phi = \frac{\pi}{2}$ which essentially generates a QPSK signal in the complex constellation map. On the other hand, JCNC achieves the maximal MI with e.g., $\Delta\phi = 0$, as illustrated in

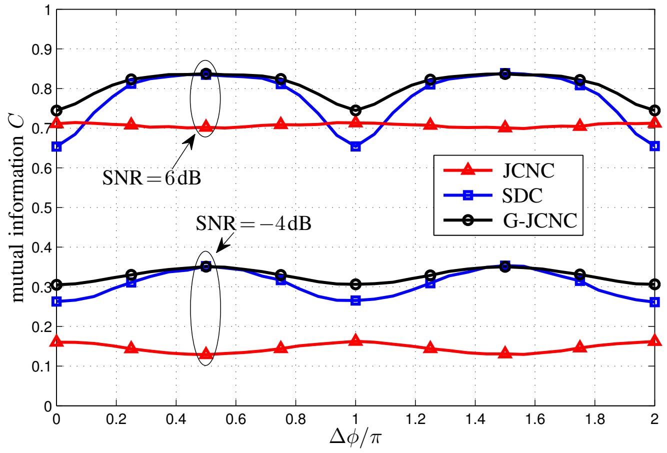

Fig. 5. Impact of phase difference $\Delta\phi$ on the MI for SDC, JCNC and G-JCNC in the high SNR region (SNR = 6dB) and low SNR region (SNR = −4 dB) over fading channels with BPSK.

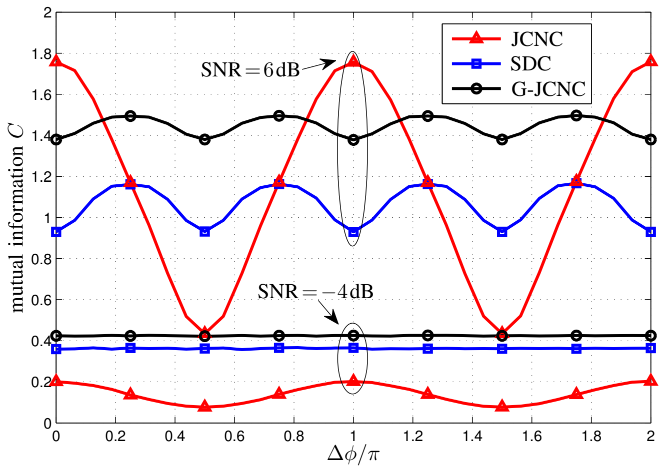

Fig. 6. Impact of phase difference $\Delta\phi$ on the MI for SDC, JCNC and G-JCNC in the high SNR region (SNR = 6dB) and low SNR region (SNR = −4 dB) over AWGN channels with QPSK.

the previous subsection, and the performance is less sensitive to $\Delta\phi$, especially for fading channels.

The results presented in Fig. 4 and Fig. 5 are evaluated for QPSK and shown in Fig. 6 and Fig. 7, respectively. It can be observed that the optimal phase rotation for SDC and G-JCNC is $\Delta\phi = \frac{\pi}{4}$ in AWGN channels and $\Delta\phi = \frac{\pi}{8}$ in fading channels. However, less improvement can be achieved compared to BPSK. This is because a phase rotation $\Delta\phi = \frac{\pi}{2}$ for BPSK makes use of the other dimension in the complex constellation map, whereas the phase rotation for QPSK only tries to generate the most distributed constellation points. Furthermore, the MI for JCNC is improved significantly by phase rotation that achieves $\Delta\phi = 0$, especially in high SNR regions. This corresponds to the observations in [5], which shows that some incoherent channel conditions can be quite catastrophic for XORed denoise-and-forward (DNF) with QPSK, whereas the impact on BPSK is less dramatic.

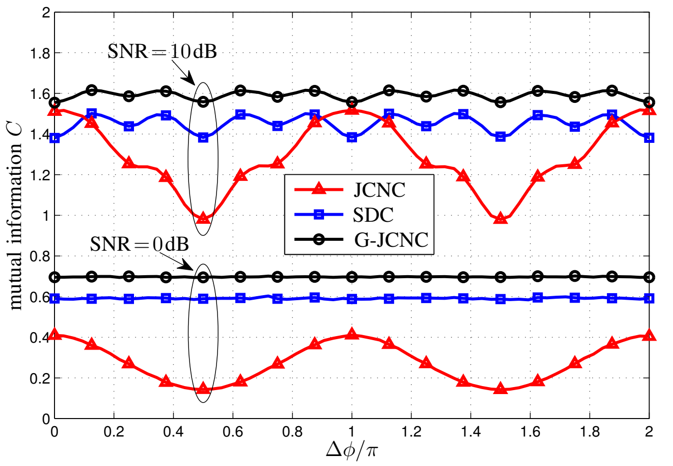

Fig. 7. Impact of phase difference $\Delta\phi$ on the MI for SDC, JCNC and G-JCNC in the high SNR region (SNR = 10 dB) and low SNR region (SNR = 0 dB) over fading channels with QPSK.

B. Phase Approximation in OFDM Systems

For a more realistic scenario multi-path fading channels using OFDM are assumed which yields continuous phases of the channel response in frequency domain. An example is shown in Fig. 8 with $N_{\mathrm{H}} = 5$ equal power Rayleigh faded channel taps and $N_{\mathrm{C}} = 1024$ subcarriers, where $\phi(k)$ denotes the angle of the channel coefficients on subcarrier k, $k = 1, \cdots, N_{\mathrm{C}}$. Since ϕ fluctuates continuously due to the nature of digital Fourier transform (DFT), it can be linearly approximated using only the extreme values $\phi(k) = \pm\pi$ and the boundary value $\phi(1) = \phi(N_{\mathrm{C}})$.

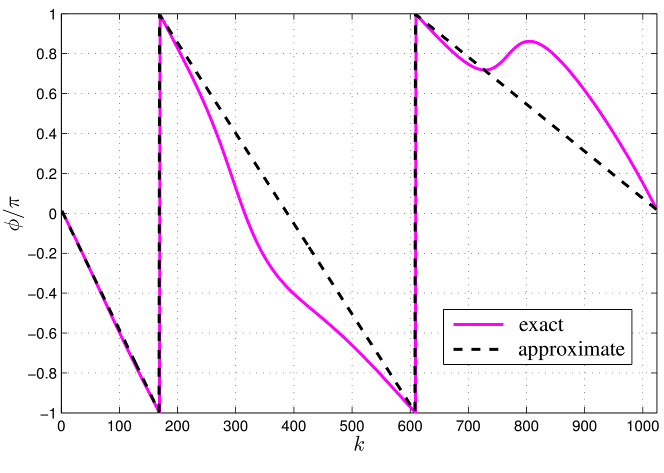

Fig. 8. Exact phase information of the channel frequency response with $N_{\mathrm{H}} = 5$ channel taps and $N_{\mathrm{C}} = 1024$ subcarriers. Its corresponding linear approximation uses the extreme values $\pm\pi$ and the boundary value $\phi(1)$.

Based on such a linear phase approximation strategy, the relay needs only to feedback the boundary value and the indexes of the extreme values $\pm\pi$ per OFDM block to the sources for phase control. Fig. 9 shows the corresponding MI for SDC and G-JCNC with one best phase difference $\Delta\phi = \frac{\pi}{2}$. It can be observed that using the approximated phase loses only 1 dB for SDC and 0.5 dB for G-JCNC, while the feedback overhead is reduced greatly compared to feeding back all the channel knowledges.

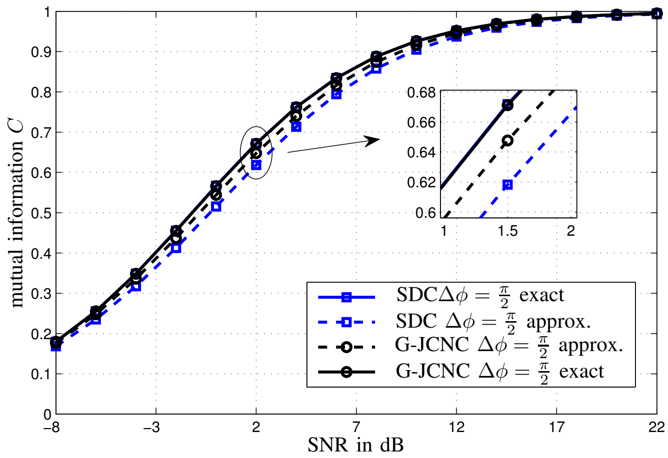

Fig. 9. Mutual information over multi-path MA channel using OFDM with $N_{\mathrm{H}} = 5$ channel taps, $N_{\mathrm{C}} = 1024$ subcarriers and BPSK. The best phase difference $\Delta\phi = \frac{\pi}{2}$ for SDC and G-JCNC is achieved with exact and approximated phase information.

V. Performance Evaluation

A two-way relaying network is considered where relay R is located in the middle of the two sources A and B with the three nodes in a line. We concentrate on the critical MA phase over AWGN channels or multi-path Rayleigh fading channels using OFDM with $N_{\mathrm{H}} = 5$ channel taps and $N_{\mathrm{C}} = 1024$ subcarriers. In the link level simulations, optimized irregular LDPC codes applied in the DVB-S2 standard [14] are used at the sources over a wide range of code rates $R_{\mathrm{C}} = \{\frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \frac{3}{5}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{8}{9}, \frac{9}{10}\}$. The codeword length is set to $n = 16200$ and 100 iterations are employed for both binary and non-binary decoding.

Fig. 10 shows the simulated throughput ξ compared with the analytical MI for different decoding schemes. Note that $\xi = R_{\mathrm{C}}(1 - \mathrm{FER}_{\mathrm{R}})$ with $\mathrm{FER}_{\mathrm{R}}$ denoting the frame error rate (FER) of the XORed packet at relay R. The minimum required SNR γ leading to $\mathrm{FER}_{\mathrm{R}} = 10^{-3}$ is achieved by simulations for different code rates R_{C}. These points (γ, R_{C}) are then connected to compare with the analytical MI. SDC with $\Delta\phi = \frac{\pi}{2}$ and JCNC without phase rotation are considered in AWGN channels for BPSK. In this case, SDC with phase control outperforms JCNC, which corresponds to the behavior shown in Fig. 4. Due to the application of strong LDPC codes, the simulated throughput approaches the analytical MI by approximately 1 dB loss.

In the presence of fading, multi-path block fading using OFDM is considered that leads to a more realistic and implementable scenario. Channel coding is applied to several OFDM frames which are assumed to have varying channel realizations. SDC and G-JCNC with phase control $\Delta\phi = \frac{\pi}{2}$

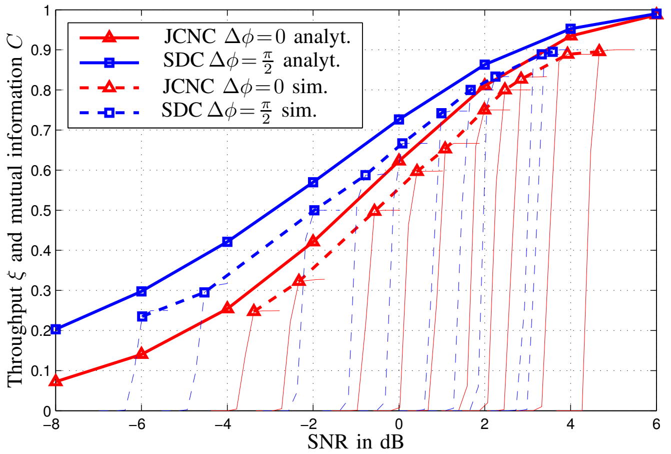

Fig. 10. Throughput performance for SDC and JCNC with BPSK and optimal phase control applying LDPC codes in DVB-S2 over a wide range of code rates in AWGN channels. Codeword length is set to $n = 16200$, 100 iterations.

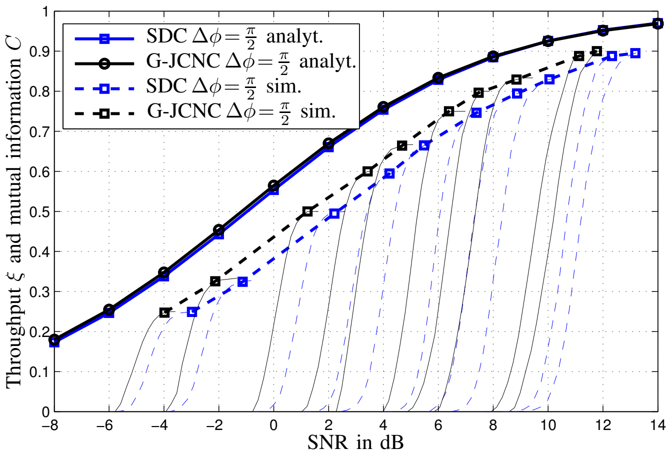

Fig. 11. Throughput performance for SDC and G-JCNC with BPSK and optimal phase control applying LDPC codes in DVB-S2 over a wide range of code rates in fading channels using OFDM with approximate phase feedback. Codeword length is set to $n = 16200$, 100 iterations.

for BPSK are compared in Fig. 11, where linear phase approximation illustrated in Sub-section IV-B is applied. As can be observed, the analytical MI for SDC and G-JCNC are identical with optimal phase control, as shown in Fig. 5. Comparing the throughput achieved from simulations, G-JCNC outperforms SDC because G-JCNC behaves more robustly regarding the phase estimation errors, which corresponds to the conclusion from Fig. 9. Additionally, the gap to the analytical MI for multi-path fading channels using OFDM grows to more than 2dB. Besides the imprecise phase information, this may be due to the lack of frequency selectivity provided by OFDM such that the coding gain is not fully exploited.

VI. Conclusion

We considered a 2-phase two-way relaying network using physical-layer network coding. Focusing on the multiple access channel, different decoding schemes have been investigated and compared with respect to their performance upperbound based on mutual information. A phase control strategy at the sources has been presented that improves the decoding performance by generating a favorable phase difference between the two incoming messages at the relay. The required overhead for phase control can be reasonably reduced in multipath fading channels using OFDM. Furthermore, link level simulations were performed in a practical system setup using strong LDPC codes, which verify the theoretical analysis and achieve capacity-approaching performance.

VII. Acknowledgement

This work was supported in part by the German Research Foundation (DFG) under grant Wu 499/8-1 within the priority program "Communication in Interference Limited Networks (COIN)", SPP 1397.

References

[1] P. Larsson, N. Johansson, and K. E. Sunell, "Coded Bi-directional Relaying," in *63rd Vehicular Technology Conference (VTC'06-Spring)*, Melbourne, Australia, May 2006.

[2] C. Fragouli, J. Y. Boudec, and J. Widmer, "Network Coding: An Instant Primer," *ACM SIGCOMM Computer Communication Review*, vol. 36, no. 1, pp. 63–68, Mar. 2006.

[3] S. Zhang, S .C. Liew, and P. Lam, "Hot Topic: Physical Layer Network Coding," in *International Conference on Mobile Computing and Networking (MobiCom)*, Log Angeles, CA, USA, Mar. 2006.

[4] P. Popovski and H. Yomo, "The Anti-Packets Can Increase the Achievable Throughput of a Wireless Multi-Hop Network," in *International Conference on Communications (ICC'06)*, Istanbul, Turkey, Jun. 2006.

[5] T. Koike-Akino, P. Popovski, and V. Tarokh, "Optimized Constellations for Two-Way Wireless Relaying with Physical Network Coding," *IEEE Journal on Selected Areas in Communications*, vol. 27, no. 5, pp. 773–787, Jun. 2009.

[6] S. Zhang and S. C. Liew, "Channel Coding and Decoding in a Relay System Operated with Physical-Layer Network Coding," *IEEE Journal on Selected Areas in Communications*, vol. 27, no. 5, pp. 788–796, Oct. 2009.

[7] M. C. Davey and D. Mackay, "Low Density Parity Check Codes over GF(q)," *IEEE Communications Letters*, vol. 2, no. 6, pp. 165–167, Jun. 1998.

[8] D. Wübben and Y. Lang, "Generalized Sum-Product Algorithm for Joint Channel Decoding and Physical-Layer Network Coding in Two-Way Relay Systems," in *IEEE Global Communications Conference (GLOBECOM'10)*, Miami, FL, USA, Dec. 2010.

[9] D. Wübben, "Joint Channel Decoding and Physical-Layer Network Coding in Two-Way QPSK Relay Systems by a Generalized Sum-Product Algorithm," in *7th International Symposium on Wireless Communication Systems (ISWCS'10)*, York, United Kingdom, Sept. 2010.

[10] P. Popovski and H. Yomo, "Physical Network Coding in Two-Way Wireless Relay Channels," in *International Conference on Communications (ICC'07)*, Glasgow, Scotland, Jun. 2007.

[11] M. P. Wilson, K. Narayanan, H. D. Pfister, and A. Sprintson, "Joint Physical Layer Coding and Network Coding for BI-Directional Relaying," *IEEE Transactions on Information Theory*, vol. 56, no. 11, pp. 5641–5654, Nov. 2010.

[12] S. Pfletschinger, "A Practical Physical-Layer Network Coding Scheme for the Uplink of the Two-Way Relay Channel," in *45th Asilomar Conference on Signals, Systems and Computers (ASILOMAR'11)*, Monterey, CA, USA, Nov. 2011.

[13] S. Hong, J. Choi, S. U. Hwang, S. Jeon, and J. Seo, "Spatial Diversity Techniques Combined with Rotated Constellation for MIMO-OFDM Systems," in *International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC'09)*, Sydney, Australia, Sept. 2009.

[14] *Digital Video Broadcasting; Second generation framing structure, channel coding and modulation systems for Broadcasting, Interactive Services, News Gathering and other broadband satellite applications (DVB-S2)*, ETSI EN 302 307, Aug. 2009.