2 Basics of Digital Communications

Exercise 2.1 **Equivalent baseband model**

A stream of random data bits $d[i]$ is mapped to a sequence of symbols $x[k]$ with power $\sigma_x^2 = 1$. For the

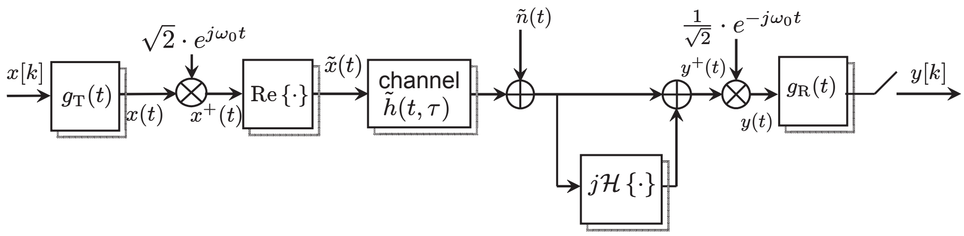

Fig. 2: Equivalent baseband model

transmit side impulse shaping, a square root-cosine filter with frequency response

$$G_{\mathrm{T}}(j\omega) = \begin{cases} \cos(\frac{\omega T}{4}) & |\omega| \leq \frac{2\pi}{T} \\ 0 & \text{else} \end{cases}$$

is applied.

a) Draw the power spectrum density of the signal $x(t)$ at the output of the transmit filter. Calculate the power of the signal $x(t)$ by integrating the power spectrum density over ω.

b) Sketch the power spectrum density of the signal $x^+(t)$ after upconversion (and scaling by $\sqrt{2}$).

c) Sketch the power spectrum density of the signal $\tilde{x}(t) = \mathrm{Re}\{x^+(t)\}$. Calculate the power of this signal. Compare this power to the power of the baseband signal $x(t)$.

d) Assume additive white gaussian noise on the channel with a spectral density of N_0. How large is the noise power σ_n^2 before the symbol decision if receive filter matched to the transmit filter is applied?

Exercise 2.2 **Rayleigh fading**

At the transmitter, symbols with an energy of E_{S} are being transmitted over a flat Rayleigh fading channel with channel coefficient h. How large is the probability that the energy $E_{\mathrm{S,R}}$ of the received symbol is larger than the energy E_{S}?
Hint: The squared magnitude of the channel coefficient follows the χ^2 distribution with 2 degrees of freedom:

$$p_{|H|^2}(\chi) = \begin{cases} \mathrm{e}^{-\chi} & \text{for} \chi \leq 0 \\ 0 & \text{else} \end{cases}$$