Channel-Optimized Information Bottleneck Design for Signal Forwarding and Discrete Decoding in Cloud-RAN

Tobias Monsees, Dirk Wübben, Armin Dekorsy
Department of Communications Engineering
University of Bremen, 28359 Bremen, Germany
Email: {tmonsees, wuebben, dekorsy}@ant.uni-bremen.de

***Abstract*—We consider the single user uplink of a Cloud Radio Access Network where a single radio access point forwards quantized received signals to the central unit. The focus of our investigation is on the quantization step in the radio access point and the decoding in the central unit. First, we investigate the impact of different quantizer approaches on the performance of the belief propagation decoder applied for low-density party check codes. Second, we investigate the performance of discrete message passing decoders which are optimized via the Information Bottleneck method in order to process quantized samples. The resulting decoder has a low bit representation for each variable and all internal decoder functions are determined by lookup tables. For the investigated scenario the discrete message passing decoder performs close to the floating point implementation of the belief propagation decoder processing real valued log-likelihood ratios.**

I. Introduction

The so-called Cloud Radio Access Networks (Cloud-RANs) are currently investigated for the deployment in 5G [1]. This approach promises many benefits including simplified network management and maintenance along with a more efficient implementation of cooperative processing techniques. The Cloud-RAN uplink relies on the concept of forwarding quantized signals over a rate limited fronthaul channel from Radio Access Points (RAPs) to the Central Unit (CU). Different functional split options between the RAPs and the CU are discussed [2]. Similar to [3], we assume a functional split on the physical layer, where the RAP is forwarding quantized samples. In this case a performance gain due to a joint decoding in the CU is possible. Furthermore, the focus on the quantization step at the RAP is important, since it defines a trade-off between the required data rate on the fronthaul and the decoding performance in the CU. To this end, we utilize the Information Bottleneck (IB) method [8], [11] for channel quantizer design to minimize the information loss at the RAP. The IB method has been successfully utilized in different areas such as the design of channel quantizers [12], relay networks [13] and integer based decoders for Low-Density Parity Check (LDPC) codes [5].

The contributions of this paper are the

- investigation of the quantization step at the RAP on the decoding performance in the CU and
- the comparison between two different decoder structures, i.e. the discrete Message Passing (MP) decoder [4]–[7] and the floating point Belief Propagation (BP) decoder.

The remainder of this paper is organized as follows: In Sec. II, the system model is introduced. In Section III we investigate the influence of the RAP quantizer on the Bit Error Rate (BER) of the BP decoder. Therefore, we utilize the Information Bottleneck (IB) method [8] to minimize the information loss between the UE and the quantizer output. In Sec. IV, we utilize the channel-optimized Information Bottleneck [9] for channel quantizer design to minimize the end-to-end information loss between the UE and the CU. In Sec. V, we compare the BER performance of the discrete message passing decoder and the floating point BP decoder. Sec. VI summarizes the paper[1].

II. System Model

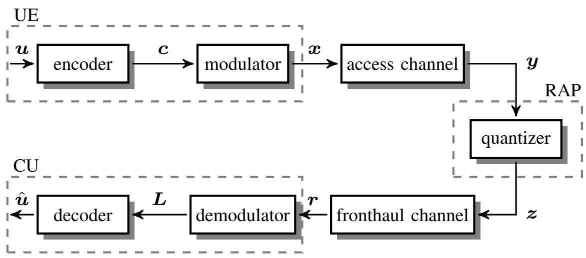

Fig. 1. The system model for the uplink of Cloud-RAN for a single UE

The considered system model is depicted in Fig. 1. The UE encodes the binary information word $\boldsymbol{u} \in \mathbb{F}_2^K$ of length K to the code word $\boldsymbol{c} \in \mathbb{F}_2^N$ of length N, where $\mathbb{F}_2$ is the binary Galois field and $R = \frac{K}{N}$ is the rate of the code. The modulated symbols $\boldsymbol{x} \in \mathcal{X}^M$ with probability mass function (pmf) $p_{\mathsf{x}}(\boldsymbol{x}) = p_{\mathsf{x}_1,...,\mathsf{x}_M}(x_1, ..., x_M)$ are transmitted over the access channel, where $\mathcal{X}$ is the discrete modulation alphabet. Without loss of generality, we model the influence of the access channel and the preprocessing in the RAP as a finely quantized memoryless Additive White Gaussian Noise (AWGN) channel with noise variance σ_{n}^2. The probability mass function (pmf) of the resulting Discrete Memoryless Channel (DMC) is defined as $p_{\mathsf{y}|\mathsf{x}}(\boldsymbol{y}|\boldsymbol{x}) = \prod_{j=1}^{M} p_{\mathsf{y}|\mathsf{x}}(y_j|x_j)$.

[1]*Notation:* Random variables are denoted by sans-serif letters, random vectors by bold sans-serif letters, realizations by serif letters and vector valued realizations by bold serif letters.

At the RAP the received samples $\boldsymbol{y} \in \mathcal{Y}^M$ are mapped into the sequence of quantized samples $\boldsymbol{z} \in \mathcal{Z}^M$ using a scalar quantizer which is defined by the probability mass function (pmf) $p_{\mathsf{z}|\mathsf{y}}(\boldsymbol{z}|\boldsymbol{y}) = \prod_{j=1}^{M} p_{\mathsf{z}|\mathsf{y}}(z_j|y_j)$, where $\mathcal{Z}$ is the set of quantizer outputs. The quantized samples $\boldsymbol{z}$ are transmitted over the digital fronthaul channel. We model the transmission over the fronthaul channel (which would include additional modulation, coding, etc.) as DMC with pmf $p_{\mathsf{r}|\mathsf{z}}(\boldsymbol{r}|\boldsymbol{z}) = \prod_{j=1}^{M} p_{\mathsf{r}|\mathsf{z}}(r_j|z_j)$. The demodulator maps the forwarded quantized samples $\boldsymbol{r} \in \mathcal{R}^M$ into LLRs $\boldsymbol{L} \in \mathbb{R}^N$, where $\mathcal{R}$ is the set of forwarded quantized samples and $\mathbb{R}$ is the set of real numbers. The forwarded quantized samples are used by the BP decoder to obtain the estimated info bit sequence $\hat{\boldsymbol{u}} \in \mathbb{F}_2^K$. As discussed in Sec. V, the IB-based decoder processes discrete values and not LLRs such that the demodulation step can be omitted.

III. Quantizer Design for Ideal Fronthaul Channel

A. Information Bottleneck based Quantizer Design

In this section we focus on the quantizer design at the RAP under the assumption of an ideal fronthaul channel. The quantizer design for the non ideal fronthaul channel is discussed in the next section.

We assume that the modulated symbols are independent and identically distributed (iid) according to $p_{\mathsf{x}}(x)$, i.e. $p_{\mathsf{x}}(\boldsymbol{x}) = \prod_{j=1}^{M} p_{\mathsf{x}}(x_j)$. We consider only scalar quantization. The RAP forwards the quantized samples according to $p_{\mathsf{z}|\mathsf{y}}(z|y)$. The resulting Markov chain is denoted as $\mathsf{x} \leftrightarrow \mathsf{y} \leftrightarrow \mathsf{z}$. Within the RAP, our goal is to obtain a quantized received sample z preserving mutual information[2] (MI) $I(\mathsf{x};\mathsf{z})$ about the transmitted symbol x. The resulting optimization problem is defined via the IB method. The optimal quantizer mapping $p^{\star}_{\mathsf{z}|\mathsf{y}}(z|y)$ is given by

$$p^{\star}_{\mathsf{z}|\mathsf{y}}(z|y) = \underset{p_{\mathsf{z}|\mathsf{y}}(z|y)}{\operatorname{argmin}} \frac{1}{\beta+1}\big(I(\mathsf{y};\mathsf{z}) - \beta I(\mathsf{x};\mathsf{z})\big) \quad \text{s.t. } |\mathcal{Z}| \leq N_{\mathsf{z}}, \tag{1}$$

where $\beta > 0$ is the trade-off parameter between relevant information $I(\mathsf{x};\mathsf{z})$ and compression rate $I(\mathsf{y};\mathsf{z})$ and N_{z} is the upper bound on the number of quantizer representatives. The case of $\beta \to 0$ is not of interest, since no relevant information is kept. For the case $0 < \beta < \infty$, the optimization problem is neither convex nor concave in general [11]. Several heuristics exist [14] to find a locally optimal solution of the optimization problem (1). For the special case of $\beta \to \infty$, the optimization problem reduces to

$$p^{\star}_{\mathsf{z}|\mathsf{y}}(z|y) = \underset{p_{\mathsf{z}|\mathsf{y}}(z|y)}{\operatorname{argmax}}\, I(\mathsf{x};\mathsf{z}) \quad \text{s.t. } |\mathcal{Z}| \leq N_{\mathsf{z}}. \tag{2}$$

In this case, the quantizer mapping maximizes the end-to-end mutual information between UE and CU given $|\mathcal{Z}| \leq N_{\mathsf{z}}$. This optimization problem is a convex maximization problem. In this case, one can show that the optimal solution is of deterministic type [15] (i.e. $p_{\mathsf{z}|\mathsf{y}}(z|y) \in \{0,1\}\, \forall y \in \mathcal{Y}$). Nevertheless, the optimal quantizer can have non-convex quantization regions in general [16]. For the binary input case, an algorithm to find the optimal solution has been developed in [15]. In the following we always assume the asymptotic case $\beta \to \infty$.

Remark 1:

Under the assumed Markov property of the system model $\mathsf{x} \leftrightarrow \mathsf{y} \leftrightarrow \mathsf{z}$, the joint distribution between the discrete modulation symbols and the quantized received samples at the CU $p_{\mathsf{x},\mathsf{z}}(x,z)$ is determined by $p_{\mathsf{x}}(x)$, the access channel $p_{\mathsf{y}|\mathsf{x}}(y|x)$ and the quantizer mapping $p_{\mathsf{z}|\mathsf{y}}(z|y)$, i.e.

$$p_{\mathsf{x},\mathsf{z}}(x,z) = \sum_{y \in \mathcal{Y}} p_{\mathsf{x},\mathsf{y}}(x,y) p_{\mathsf{z}|\mathsf{y}}(z|y), \tag{3}$$

In case of a deterministic quantizer mapping, the set of quantizer boundaries $\mathcal{Q} = \{q_0, ..., q_{N_{\mathsf{z}}}\}$ is identified by convex sets $\mathcal{Y}_z = (q_z, q_{z+1}]$, where it is assumed that $q_0 = -\infty$ and $q_{N_{\mathsf{z}}} = +\infty$.

B. Demodulation

In the CU the quantized samples z are mapped to a-posteriori LLRs for BP decoding. We assume that the statistic is known at the CU, i.e.

$$p_{\mathsf{c}|\mathsf{z}}(c|z) = \sum_{y \in \mathcal{Y}} p_{\mathsf{c}|\mathsf{y}}(x|y) p_{\mathsf{y}|\mathsf{z}}(y|z) \tag{4}$$

and the corresponding LLRs are determined by

$$L(c|z) = \log\left(\frac{p_{\mathsf{c}|\mathsf{z}}(0|z)}{p_{\mathsf{c}|\mathsf{z}}(1|z)}\right). \tag{5}$$

C. Performance Results

In this section we investigate the influence on the BER of a regular LDPC code for 3-bit MMSE, uniform and IB-based quantization. The demodulator calculates the LLRs based on (5), which only considers statistics of the access channel and the quantizer. We used a regular rate $R = \frac{1}{3}$ LDPC code from [17] with row weight $d_c = 6$ and column weight $d_v = 4$ of length $N = 816$. The number of iterations of the floating point BP decoder is limited by $i_{\max} = 50$. We assume BPSK modulated symbols, i.e. $\mathcal{X} = \{-1, 1\}$. For the IB-based quantizer design the continuous AWGN channel is uniformly quantized into 256 clusters. To achieve 99.7 % coverage, the first boundary q_1 and last boundary $q_{N_{\mathsf{z}}-1}$ were set to $\mp(1 + 3\sigma_{\mathsf{n}})$, respectively. We use the algorithm for binary input DMC [15] to obtain the optimal solution for the Information Bottleneck problem in (2). The quantizer labels are predefined as $\mathcal{Z} \subseteq \{0, ..., 7\}$, since the relevant information in (2) is independent of the set of quantizer labels $\mathcal{Z}$.

A bijective function $f : \mathcal{Z} \to \mathbb{F}_2^B$ with $B = \lceil \log_2(N_{\mathsf{z}}) \rceil$ maps each integer to a binary coded bit vector of length B. We model the digital fronthaul as a Binary Symmetric Channel (BSC) with bit-flip probability P_e. Hence, the pmf of the corresponding DMC is given by $p_{\mathsf{r}|\mathsf{z}}(\bar{\boldsymbol{r}}|\bar{\boldsymbol{z}}) = P_e^{d_H(\bar{\boldsymbol{z}},\bar{\boldsymbol{r}})}(1-P_e)^{B-d_H(\bar{\boldsymbol{z}},\bar{\boldsymbol{r}})}$, where $d_H(\bar{\boldsymbol{z}},\bar{\boldsymbol{r}})$ is the Hamming distance between a transmitted bit vector $\bar{\boldsymbol{z}}$ and a received bit

[2]The mutual information between two random variables a and b is given by $I(\mathsf{a};\mathsf{b}) = \sum_{a \in \mathcal{A}} \sum_{b \in \mathcal{B}} p_{\mathsf{a},\mathsf{b}}(a,b) \log \frac{p_{\mathsf{a},\mathsf{b}}(a,b)}{p_{\mathsf{a}}(a) p_{\mathsf{b}}(b)}$ [10]

vector $\bar{\boldsymbol{r}}$, respectively.

We used the Lloyd-Max algorithm [18]–[20] to find the quantizer mapping minimizing the Mean Square Error (MSE) between the received sample y and the quantized output z. Since the Lloyd-Max algorithm converges to a local optimal solution, the algorithm is executed 10^5 times with different initial points and the best result is kept.

The influence of the quantizer design on the BER performance of the floating point BP decoder is shown in Fig. 2. For $P_e = 0$, the difference in the BP decoding performance between uniform, Lloyd-Max and IB-based quantization is within 0.4 dB for a BER of 10^{-3}. Nevertheless, the IB-based quantization performs the best.

In case of a non-ideal fronthaul channel with $P_e = 0.01$, the difference between IB- and Lloyd-Max quantization is approximately 1 dB. IB-based quantization outperforms uniform quantization. In case of $P_e = 0.1$ the BP decoding does not converge for the uniform and the Lloyd-Max quantization methods. In contrast, the gain in BER performance increases for IB-based quantization and the BP decoding converges for $P_e = 0.1$. This observation motivates IB-based quantization for imperfect fronthaul channels. In the next section, the demodulator and quantizer is optimized for an imperfect fronthaul channel.

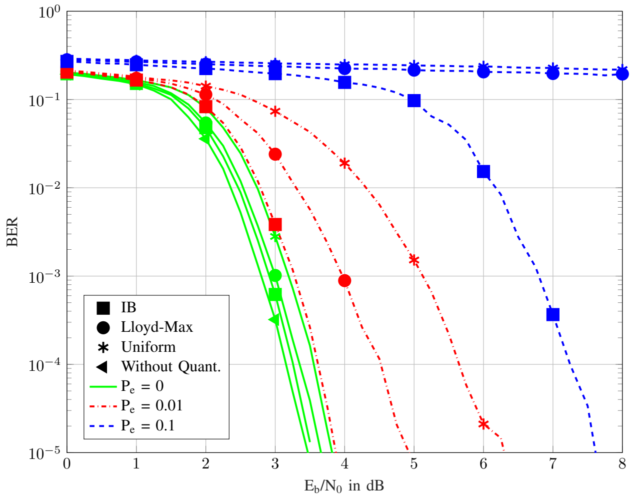

Fig. 2. BER simulations for the (4,6)-regular LDPC code using different 3-bit channel quantizers.

IV. IB-based Quantizer Design for Non-Ideal Fronthaul Channel

A. Channel-Optimized Information Bottleneck (Ch-Opt-IB)

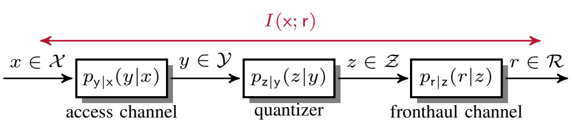

Fig. 3. Channel-optimized IB setup

In [21] the authors extended the IB framework with an additional fronthaul channel. The corresponding Channel-Optimized IB (Ch-Opt-IB) setup is shown in Fig. 3. The variational problem for $\beta \to \infty$ is given by

$$p^\star_{\mathsf{z}|\mathsf{y}}(z|y) = \underset{p_{\mathsf{z}|\mathsf{y}}(z|y)}{\operatorname{argmax}} \; I(\mathsf{x};\mathsf{r}) \;\; \text{s.t.} \; |\mathcal{Z}| \leq N_{\mathsf{z}}, \tag{6}$$

which is again a convex maximization problem [21] and the optimal solution is of deterministic type. The authors of [21] developed an iterative algorithm to find a local optimal deterministic solution. To utilize the algorithm, the access channel must be discretized into a corresponding DMC model. Like in the previous section, we assume a preceding uniform channel quantizer with a sufficient fine grid. We abbreviate this quantizer design approach as Ch-Opt-IB (iterative).

In [3] the authors developed an algorithm to solve (6) on the continuous access channel output under the assumption that the set of output symbols $\mathcal{Y}_z$ are convex. The algorithm requires with the probability density function (pdf) of the AWGN channel, since it utilizes the downhill simplex method [22] using the set of differences between the quantizer boundaries $\Delta \mathcal{Q} = \{\Delta q_0, ..., \Delta q_{N_z - 1}\}$ with $\Delta q_i = q_{i+1} - q_i$ as the difference between two successive boundaries. The corresponding optimization problem can be stated as

$$\Delta \mathcal{Q}^\star = \underset{\Delta \mathcal{Q}}{\operatorname{argmax}} \; I(\mathsf{x};\mathsf{r}) \qquad \text{s.t.} \quad \Delta q_i > 0. \tag{7}$$

This quantizer design approach is abbreviated as Ch-Opt-IB (simplex) in the following.

B. Improved Demodulation

The BP decoding with improved LLR calculation requires the a-posteriori probability which includes the imperfect fronthaul channel, i.e.

$$p_{\mathsf{c}|\mathsf{r}}(c|r) = \sum_{y \in \mathcal{Y}} p_{\mathsf{c}|\mathsf{y}}(c|y) \sum_{z \in \mathcal{Z}} p_{\mathsf{y}|\mathsf{z}}(y|z) p_{\mathsf{z}|\mathsf{r}}(z|r). \tag{8}$$

The corresponding a-posteriori LLRs under the assumption of a imperfect fronthaul channel are determined by

$$L(c|r) = \log \left(\frac{p_{\mathsf{c}|\mathsf{r}}(0|r)}{p_{\mathsf{c}|\mathsf{r}}(1|r)} \right). \tag{9}$$

C. Performance Results

Since the channel-optimized algorithms converge to locally optimal mappings, we initialize the algorithms randomly 10^5 times and keep the best result. Compared to Sec. III-C, the LLR values are calculated by using the a-posteriori distribution of the output of the fronthaul channel Eq. (9).

The BER performance is shown in Fig. 4. The channel-optimized quantization algorithms reveal a performance gain of approximately 1 dB for $P_e = 0.1$ compared to the IB algorithm. Furthermore, we observed that for large values of P_e, the optimal channel quantizer mapping has a reduced alphabet size with increased Hamming distance between neighboring representatives. As also observed in [3], this corresponds to an inherent channel coding of rate $\frac{\log_2(|\mathcal{Z}|)}{\log_2(N_z)}$. For $P_e = 0.01$, the performance gain by considering the fronthaul channel in the LLR calculation is ≈ 1.5 dB for the uniform quantizer and ≈ 0.5 dB for the Lloyd-Max quantizer. The Ch-Opt-IB (iterative) slightly outperforms the Ch-Opt-IB (simplex). In

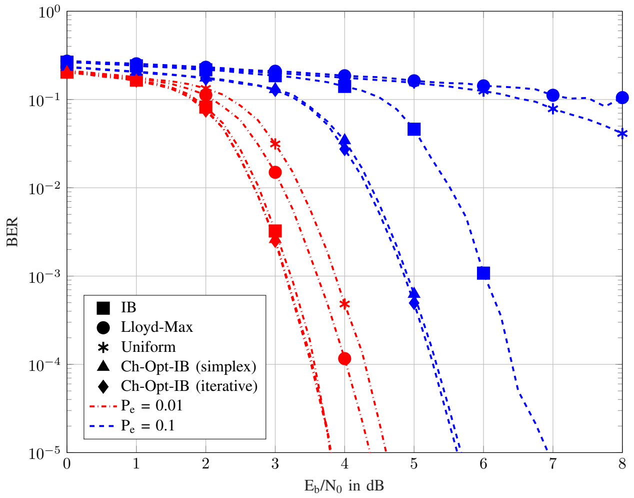

Fig. 4. BER Simulations for the (4,6)-regular LDPC code using different 3-bit channel quantizers with improved demodulation.

the next section we compare the performance between the BP decoder and the discrete message passing decoder.

V. Discrete Decoder Design

A. Discrete Message Passing for LDPC Codes

In message passing decoding of LDPC codes [23], extrinsic information (messages) between the variable and the check nodes is exchanged [24]. In [4], Kurkoski et al. presented a Density Evolution (DE) algorithm for regular LDPC codes which aims to find discrete decoder functions that maximize the mutual information between the code bit and its message. In [5], the DE algorithm is extended by the quantization algorithm for binary input which finds the optimal message mappings [15]. The resulting discrete LDPC decoder processes only unsigned integers by using simple lookup tables and the BER performance of a 4-bit implementation is close BP decoding. As already discussed in [6], the underlying optimization task is closely related to the IB method. In the next subsection we describe this DE algorithm [4], [5] to obtain discrete decoder functions for the channel-optimized IB setup discussed in Sec. IV-A.

B. Density Evolution for the Ch-Opt-IB Setup

For a regular LDPC code, the distribution of check to variable (and vice versa) node messages is the same for all variable and check node and will only change during iterations. Furthermore, it is assumed that the statistical dependencies between the messages can be neglected, which implies a cycle-free graph. The code bit distribution is assumed to be equiprobable in the following. The initial distribution of all variable to check node messages is given by

$$p^{(0)}_{\mathsf{m}|\mathsf{c}} := p^{\star}_{\mathsf{r}|\mathsf{c}}, \qquad (10)$$

where the conditional pmf between the transmitted code bit and the received sample at the CU is defined by

$$p^{\star}_{\mathsf{r}|\mathsf{c}}(r|c) = \sum_{z\in\mathcal{Z}} p_{\mathsf{r}|\mathsf{z}}(r|z) \sum_{y\in\mathcal{Y}} p^{\star}_{\mathsf{z}|\mathsf{y}}(z|y) \sum_{x\in\mathcal{X}} p_{\mathsf{y}|\mathsf{x}}(y|x) p_{\mathsf{x}|\mathsf{c}}(x|c).$$

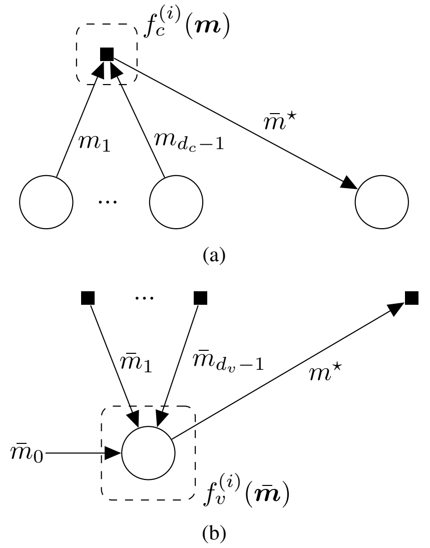

Fig. 5. (a) Check to variable node mapping $f_c^{(i)}(\boldsymbol{m})$. (b) Variable to check node mapping $f_v^{(i)}(\bar{\boldsymbol{m}})$. The quantizer mappings are designed for each decoder iteration i to maximize the (extrinsic) information about the code bits.

The distribution $p^{\star}_{\mathsf{z}|\mathsf{y}}(z|y)$ is determined by the Ch-Opt-IB (iterative) algorithm with $\mathcal{Z} \subseteq \{0,\ldots,N_{\mathsf{z}}-1\}$ for a specific P_e and a sufficient large σ_{n}. The conditional distributions of the variable to check node mappings $p^{(i)}_{\mathsf{m}|\mathsf{c}}$ and check to variable node mappings $p^{(i)}_{\bar{\mathsf{m}}|\mathsf{c}}$ for iteration $i = 1,\ldots,i_{\max}$ are determined by the DE algorithm.

The distribution of the d_c-1 incoming messages of a check node conditioned on the transmitted bit of the target variable node is given by [5], [4]

$$p^{(i)}_{\mathsf{m}|\mathsf{c}}(\boldsymbol{m}|c) = \left(\frac{1}{2}\right)^{(d_c-2)} \sum_{\boldsymbol{b}:\bigoplus \boldsymbol{b}=c} \prod_{j=1}^{d_c-1} p^{(i-1)}_{\mathsf{m}|\mathsf{c}}(m_j|b_j), \qquad (11)$$

where $\boldsymbol{b} = (b_1,\ldots,b_{d_c-1})$ is an auxiliary vector representing the state of the transmitted bits of the incoming variable node messages $\boldsymbol{m} = (m_1,\ldots,m_{d_c-1})$ and $\bigoplus \boldsymbol{b}$ denotes the modulo 2 sum over the elements of the vector $\boldsymbol{b}$. This distribution is used to obtain an IB-based check to variable node mapping

$$p^{(i)}_{\bar{\mathsf{m}}|\mathbf{m}} = \underset{p_{\bar{\mathsf{m}}|\mathbf{m}}}{\operatorname{argmax}}\ I^{(i)}(\bar{\mathsf{m}};\mathsf{c}) \ \text{ s.t. } |\bar{\mathcal{M}}_{(i)}| \le N_{\mathsf{z}}, \qquad (12)$$

where $\bar{\mathcal{M}}_{(i)} \subseteq \{0,\ldots,N_{\mathsf{z}}-1\}$ is the check node message alphabet. This mapping determines the optimized distribution of the check to variable node messages conditioned on the transmitted bit $p^{(i)}_{\bar{\mathsf{m}}|\mathsf{c}}$, which is used to obtain an optimized variable to check node mapping of the next iteration. The distribution of the variable to check node messages is given by

$$p^{(i)}_{\bar{\mathsf{m}}|\mathsf{c}}(\bar{\boldsymbol{m}}|c) = p^{(0)}_{\mathsf{m}|\mathsf{c}}(\bar{m}_0|c) \prod_{j=1}^{d_v-1} p^{(i-1)}_{\bar{\mathsf{m}}|\mathsf{c}}(\bar{m}_j|c), \qquad (13)$$

where $\bar{\boldsymbol{m}} = (\bar{m}_0, ..., \bar{m}_{d_v-1})$ are the incoming check node messages and $\bar{m}_0$ represents the received value r. This distribution is used to obtain an IB-based variable to check node mapping

$$p^{(i)}_{\mathsf{m}|\bar{\mathsf{m}}} = \underset{p_{\mathsf{m}|\bar{\mathsf{m}}}}{\operatorname{argmax}}\ I^{(i)}(\mathsf{m};\mathsf{c}) \ \text{ s.t. } |\mathcal{M}_{(i)}| \leq N_{\mathsf{z}}, \qquad (14)$$

where $\mathcal{M}_{(i)} \subseteq \{0, ..., N_{\mathsf{z}} - 1\}$ is the variable node message alphabet. The mapping $p^{(i)}_{\mathsf{m}|\bar{\mathsf{m}}}$ determines the distribution of the variable to check node messages conditioned on the transmitted bits $p^{(i)}_{\mathsf{m}|\mathsf{c}}$ of the next iteration. The overall DE algorithm starts with the initialization of the variable to check node messages in (10) and iterates over (12), (13) and (14) until the mutual information of the variable to check node messages and the codebits in (14) converges one for a maximum number of iterations $i_{\max}$. If convergence is not possible, the standard deviation of the access channel σ_{n} is slightly reduced and the complete DE algorithm is started again until convergence is obtained. The corresponding convergent initial distribution $p^{\star}_{\mathsf{r}|\mathsf{c}}$ is identified by the *noise threshold* $\sigma^{\star}_{\mathsf{n}}$, since P_e is assumed to be fixed.
For the final decision on $\hat{c}$, an optimized deterministic mapping

$$p^{(i)}_{\hat{\mathsf{c}}|\bar{\mathsf{m}},\bar{\mathsf{m}}_{d_v}} = \underset{p_{\hat{\mathsf{c}}|\bar{\mathsf{m}},\bar{\mathsf{m}}_{d_v}}}{\operatorname{argmax}}\ I^{(i)}(\hat{\mathsf{c}};\mathsf{c}) \ \text{ s.t. } \hat{\mathcal{C}} = \{0, 1\} \qquad (15)$$

is generated by using all incoming check to variable node messages $(\bar{\boldsymbol{m}}, \bar{m}_{d_v})$ in (13). As discussed in Sec. III, for the maximization problems in (12), (14) and (15) the optimal solution is a deterministic mapping, which can be found by the quantization algorithm for binary input DMC [15]. The optimal variable to check node message $m^{\star}$ is determined by the variable node function $f^{(i)}_v : \bar{\mathcal{M}}^{d_v-1}_{(i)} \times \mathcal{Z} \rightarrow \mathcal{M}_{(i)}$ with

$$m^{\star} = f^{(i)}_v(\bar{\boldsymbol{m}}) = \underset{m}{\operatorname{argmax}}\ p^{(i)}_{\mathsf{m}|\bar{\mathsf{m}}}(m|\bar{\boldsymbol{m}}), \qquad (16)$$

for all variable nodes. Likewise, the optimal variable to check node message $\bar{m}^{\star}$ is given by the check node function $f^{(i)}_c : \mathcal{M}^{d_c-1}_{(i-1)} \rightarrow \bar{\mathcal{M}}_{(i)}$ with

$$\bar{m}^{\star} = f^{(i)}_c(\boldsymbol{m}) = \underset{\bar{m}}{\operatorname{argmax}}\ p^{(i)}_{\bar{\mathsf{m}}|\mathsf{m}}(\bar{m}|\boldsymbol{m}), \qquad (17)$$

for all check nodes, respectively. Both mappings are visualized in Fig. 5. For the codebit estimation, an additional function $f^{(i)} : \bar{\mathcal{M}}^{d_v}_{(i)} \times \mathcal{Z} \rightarrow \{0, 1\}$ is generated by using all incoming messages, i.e.

$$\hat{c} = f^{(i)}(\bar{\boldsymbol{m}}, \bar{m}_{d_v}) = \underset{\bar{c}}{\operatorname{argmax}}\ p^{(i)}_{\hat{\mathsf{c}}|\bar{\mathsf{m}},\bar{\mathsf{m}}_{d_v}}(\bar{c}|\bar{\boldsymbol{m}}, \bar{m}_{d_v}), \qquad (18)$$

for all variable nodes. To reduce the memory requirements of the node mappings, we use the node decomposition method as described in [4]. The basic idea of the decomposition of nodes is to split each node into a sequence of nodes of degree 2 and solve the corresponding optimization problem in (12), (14) and (15) for this smaller nodes. Compared to the implementation without node decomposition, which requires (in the worst case) $N_{\mathsf{z}}^{(d-1)}$ memory locations, the required memory size of the decomposed node is $(d-2)N_{\mathsf{z}}^2$, where d is the degree of the corresponding check or variable node.
An extension of DE algorithm to irregular codes is given in [25] by taking the different node degrees into account. For complex modulation alphabets, the idea of message alignment has been proposed [26] in order to utilize the DE algorithm with a codebit-dependent statistic for real and imaginary part.

C. Performance Results

The BER performance of the IB-decoder for both ideal and non-ideal fronthaul channels are shown in Fig. 6. The determined noise thresholds of the access channel are given in Table I. The discrete decoder mappings are optimized for the noise threshold and therefore fixed for all σ_{n}. For the IB-decoder the quantizer at the RAP quantizer is also fixed, which is beneficial since the RAP quantizer and the decoder mappings are optimized once and can be used for all σ_{n} during transmission. Furthermore, as the iteration number increases some quantizer labels are not used and the cardinality of the message alphabet is reduced. This motivates to reduce the upper bound on the cardinality of message mappings N_{z} as the iteration number increases [27], which is not considered here. As shown in Fig. 6, the performance loss of using a fixed channel quantizer and a discrete decoder with 3-bit message mappings is small (≈ 0.3 dB for a BER of 10^{-3}) compared to the floating point BP decoder with quantized input.

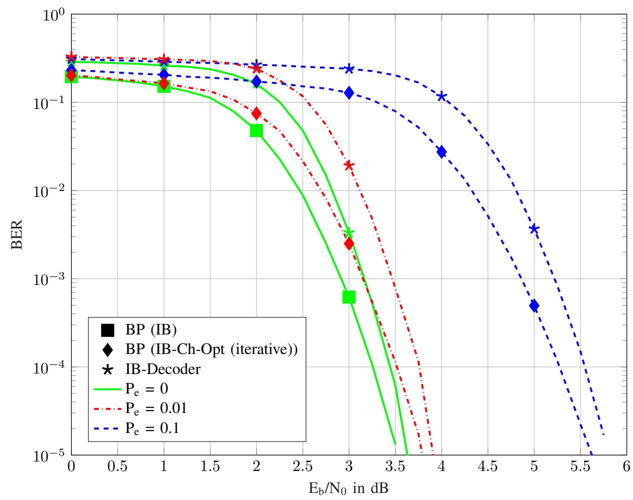

Fig. 6. BER performance for the (4,6)-regular LDPC code using the 3-bit IB-based- vs. BP decoder with 3-bit channel quantizer.

TABLE I
NOISE THRESHOLDS FOR $d_c = 6$, $d_v = 4$, $N_{\mathsf{z}} = 8$ AND $i_{\text{MAX}} = 50$

P_e	Noise threshold $\sigma^{\star}_{\mathsf{n}}$	$\frac{E_b}{N_0}$ in dB
0	0.95451	2.1653
0.01	0.93059	2.3857
0.1	0.76915	4.0407

VI. Summary

We compared the influence on the BER performance of the BP decoding algorithm using uniform, MMSE, IB and channel-optimized IB quantization for the single user uplink model of a Cloud-RAN. The channel-optimized IB quantization performs best since it maximizes the mutual information between the UE and the CU. Furthermore, we utilized channel-optimized IB quantization in the density evolution algorithm for regular LDPC codes to obtain a discrete decoder for the uplink model. The resulting discrete decoder processes only unsigned integers as messages via simple lookup tables. The BER performance of a 3-bit discrete decoder is close to the floating point BP decoder processing real-valued LLRs.

Acknowledgment

This work was partly funded by the German ministry of education and research (BMBF) under grant 16KIS0720 (TACNET 4.0).

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