Multicarrier Transmission for Scenarios with High Doppler Influence

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***Abstract*— OFDM is an efficient transmission technique for frequency selective channels, but can severely suffer from the effects of Doppler, if the transmitter and/or receiver move with high velocities. In this paper, we present a multicarrier system with soft impulse shaping and tailbiting Max-Log-MAP equalization in order to suppress the intercarrier interference caused by Doppler. With simulation results we demonstrate the achievable performance improvements compared to a conventional OFDM system.**

***Index Terms*— OFDM transmission, intercarrier interference suppression, Doppler spread, tailbiting Max-Log-MAP decoder, multicarrier system with soft impulse shaping**

I. INTRODUCTION

In transmission scenarios where the transmitter and/or receiver of an OFDM system move with high velocities the Doppler effect can severely degrade the system performance. The subcarrier spectra are widened by the Doppler spread leading to intercarrier interference (ICI) and thus the loss of orthogonality. In [1] we presented some more or less complex frequency domain equalization techniques in order to suppress the ICI at the receiver side. Thereby, the strong diagonal structure of the channel matrix offered the applicability of different low-complexity approaches resulting in a strong reduction of the computational effort. The main limiting factor for the simplification was the number of relevant off-diagonals of the channel matrix. According to the leakage effect of the discrete Fourier transform (DFT), under the influence of Doppler spread all subcarriers directly interfere with each other due to the Dirichlet property of the subcarrier spectra. As a result numerous off-diagonals can contain relevant elements and thus have to be regarded in the equalization procedure in order to achieve a sufficient ICI suppression.

Our approach presented in this paper is a modification of the transmitter in such a way that the ICI occurring after the transmission over a mobile channel with strong Doppler influence is focused on a very small number of off-diagonals of the channel matrix only. In [2] a multicarrier system with soft impulse shaping was proposed as a more robust alternative to a conventional OFDM system with rectangular pulse shaping at the cost of a more complex equalization procedure. Based on that proposal, we substitute the rectangular shaped impulses in the OFDM transmitter by Gaussian impulses which are well known for their minimal time-bandwidth product [3]. In fact, with the proper dimensioned Gaussian impulses, it is possible to concentrate the ICI on very few relevant off-diagonals of the channel matrix only, even under severe Doppler influence.

The downside of the soft impulse shaping with Gaussian impulses is the higher overall ICI power compared to OFDM, especially if the Doppler influence is low or not present. With linear equalization methods the ICI suppression can be unsatisfying leading to a weak system performance, as we will demonstrate by simulation results. However, the fact that only a small number of off-diagonals has to be regarded allows the application of maximum-likelihood detection with justifiable computational effort. Motivated by the periodic property of the DFT as well as the strong diagonal structure of the channel matrix we therefore propose the application of a time-variant tailbiting Max-Log-MAP decoder, based on [5], for the combined ICI suppression, channel equalization, and soft demapping of the received symbols.

The paper is organized as follows: Section II describes the considered multicarrier system with soft impulse shaping and section III deals with the application of the time-variant tailbiting Max-Log-MAP decoder as equalizer. In section IV we present our simulation results comparing the soft impulse shaping to a conventional OFDM system, followed by a conclusion of the paper in section V.

II. Multicarrier System with Soft Pulse Shaping

A special property of the Gaussian impulse

$$g(t) = \exp(-at^2)\,;\ a > 0 \tag{1}$$

is that its Fourier transformed is also Gaussian shaped. Moreover, Gaussian impulses show a strong power concentration in time as well as in frequency domain leading to a minimal time-bandwidth product [3], in contrast to the subcarriers of OFDM with their extremely wide Dirichlet spectra. This motivates the application of Gaussian impulses as an alternative to the rectangular impulse shaping filters of OFDM, in order to concentrate the ICI caused by Doppler spread and thereby to avoid the leakage effect. Defining

$$\alpha = \sqrt{\frac{2}{\ln 2}}\,\pi f_{3\mathrm{dB}} T_s \tag{2}$$

with T_s as the symbol duration, the transmit and receive filter impulse responses can be written normalized to the $f_{3\mathrm{dB}}$-bandwidth[1] as

$$g_{tx}(t) = g_{rx}(t) = e^{-\alpha^2 (t/T_s)^2}\ . \tag{3}$$

The corresponding frequency domain expressions are

$$G_{tx}(j\omega) = G_{rx}(j\omega) = \frac{\sqrt{\pi}T_s}{\alpha}\, e^{-(\omega T_s)^2/4\alpha^2}\ . \tag{4}$$

The degree of the overall interference power, caused by intercarrier as well as intersymbol interference (ISI), depends on the symbol spacing T_s in time domain as well as the subcarrier spacing Δf in frequency domain. For a given bandwidth efficiency of $\Delta f \cdot T_s = 1$ the Gaussian impulses overlap in time and/or in frequency domain. Here, their $f_{3\mathrm{dB}}$-bandwidth offers a tradeoff between the produced ISI and ICI power.

In this paper, our intention is to completely avoid ISI in order to combat all occurring interference jointly in frequency domain. For that reason, we truncate the Gaussian impulses in time domain to the duration T_s:

$$g_{tr}(t) = \mathrm{rect}(t/T_s) \cdot e^{-\alpha^2 (t/T_s)^2} \tag{5}$$

Consequently, we obtain an impulse that combines the properties of the pure Gaussian and rectangular impulses: As in the case of OFDM, for a symbol spacing of T_s and neglecting the influence of the multipath channel for the time being no ISI appears. However, in frequency domain, the ripples of the subcarrier spectra are distinctly attenuated due to the convolution with the Gaussian impulse:

$$G_{tr}(j\omega) = \frac{\sqrt{\pi}T_s^2}{\alpha} \int_{-\infty}^{\infty} \mathrm{sinc}\left(\frac{\vartheta T_s}{2}\right) \cdot e^{\frac{-((\omega-\vartheta)T_s)^2}{4\alpha^2}} d\vartheta \tag{6}$$

Fig. 1 shows an example for a truncated Gaussian impulse with $f_{3\mathrm{dB}}T_s = 0.3$.

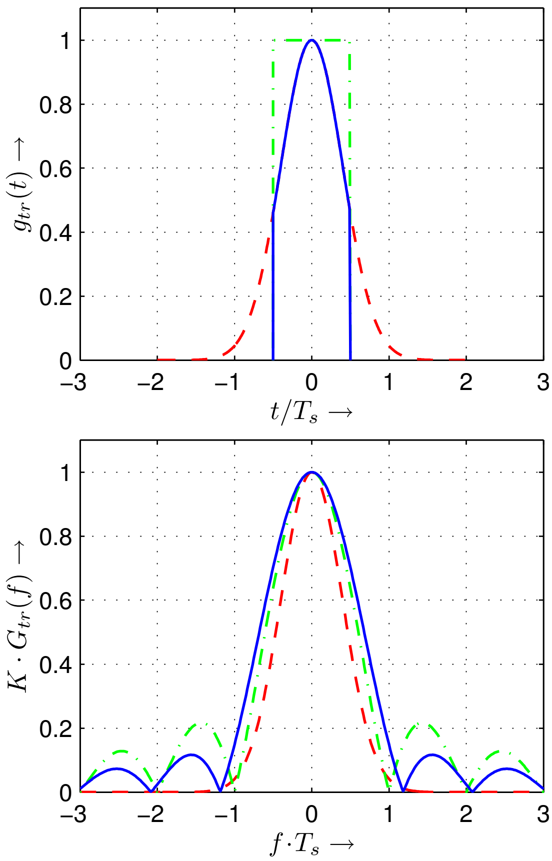

Fig. 1. Normalized Gaussian (– –), rectangular (– · –), and truncated Gaussian impulse (——) in time and frequency domain with $f_{3\mathrm{dB}}T_s = 0.3$

In frequency domain, the zero crossings of the proposed impulse do not appear at multiples of $1/T_s$ anymore, thus the orthogonality of the subcarriers is violated and an ICI reduction becomes necessary. But, under the assumption of a scenario with high Doppler spreads this requirement applies for OFDM also.

In the following, we consider a multicarrier system with N subcarriers, a sampling frequency of $f_s = N\Delta f$ and a cyclic prefix of length N_{cp}/f_s, which is longer than the maximum channel delay $\tau_{\max}$ in order to avoid ISI. Defining a vector $\mathbf{d} := [d_0, ..., d_{N-1}]^T$

[1] The product $f_{3\mathrm{dB}}T_s$ represents the frequency normalized to the symbol rate $1/T_s$, where the spectral power of the impulse reaches just half of the maximum

comprising the transmit symbols of one core symbol in frequency domain, i.e. the symbols on all subcarriers at a particular time, as well as a vector $\mathbf{s} := [s(N-N_{cp}), ..., s(N-1), s(0), ..., s(N-1)]^T$, we can formulate the discrete-time signal at the transmitter output as

$$\mathbf{s} = \mathbf{T}_i \mathcal{D}\{\mathbf{g}\}\mathbf{F}^H \mathbf{d} \ , \tag{7}$$

with $\mathbf{F}$ as the $(N \times N)$ DFT matrix containing the elements[2] $\mathbf{F}(\mu, \nu) := 1/\sqrt{N} \cdot \exp(-j2\pi\mu\nu/N)$, and $\mathcal{D}\{\mathbf{g}\}$ as the $(N \times N)$ diagonal matrix containing the elements of $\mathbf{g}$ on its main diagonal. The vector $\mathbf{g} := [g_{tr}(-T_s/2), ..., g_{tr}(T_s/2)]^T$ contains the values of the truncated Gaussian impulse response, sampled with the frequency f_s. The matrix $\mathbf{T}_i = [\mathbf{I}_{cp}^T, \mathbf{I}_N]^T$, with $\mathbf{I}_{cp}$ as the last N_{cp} rows of the $(N \times N)$ identity matrix $\mathbf{I}_N$, accomplishes the insertion of the cyclic prefix. For practical realizations, the multiplication with $\mathcal{D}\{\mathbf{g}\}\mathbf{F}^H$ can be performed by a polyphase filter bank in order to reduce the computational effort.

The channel is assumed to have a wide-sense-stationary uncorrelated scattering (WSSUS) characteristic. The time-variant convolution matrix $\mathbf{C}$ contains the time domain channel coefficients $h(k, \kappa)$, with the time indices $0 \leq k \leq N-1|_{k=tf_s}$ as well as the delay indices $0 \leq \kappa \leq \tau_{\max} f_s|_{\kappa=\tau f_s}$. The maximum Doppler frequency can be calculated by $f_{D,\max} = v_0 f_c / c_0$, where v_0 denotes the relative velocity between transmitter and receiver, f_c the carrier frequency, and c_0 the speed of light. For the following considerations we define the relative Doppler spread $\gamma = f_{D,\max}/\Delta f$ as the ratio between the maximum Doppler frequency and the subcarrier spacing.

The receive signal after removing the cyclic prefix can be written as

$$\mathbf{r} = \mathbf{T}_r \mathbf{C}\mathbf{s} + \mathbf{n} \ , \tag{8}$$

with the $(N \times 1)$ noise vector $\mathbf{n}$ (AWGN) and $\mathbf{T}_r = [\mathbf{0}_{N,N_{cp}}, \mathbf{I}_N]$ where the matrix $\mathbf{0}_{N,N_{cp}}$ contains N rows and N_{cp} columns of zeros. The transformation into frequency domain as well as the convolution with the Gaussian receive filter leads to the expression

$$\mathbf{x} = \mathbf{F}\mathcal{D}\{\mathbf{g}\}\mathbf{r} = \mathbf{H}\mathbf{d} + \tilde{\mathbf{n}} \tag{9}$$

for the vector of the received symbols. Consequently, the overall channel matrix can be formulated as

$$\mathbf{H} = \mathbf{F}\,\mathcal{D}\{\mathbf{g}\}\,\mathbf{T}_r \mathbf{C} \mathbf{T}_i\,\mathcal{D}\{\mathbf{g}\}\,\mathbf{F}^H \tag{10}$$

[2] Applied notation: $\mathbf{X}(a\!:\!b, c\!:\!d)$ means a submatrix containing the rows a to b and the columns c to d of the matrix $\mathbf{X}$.

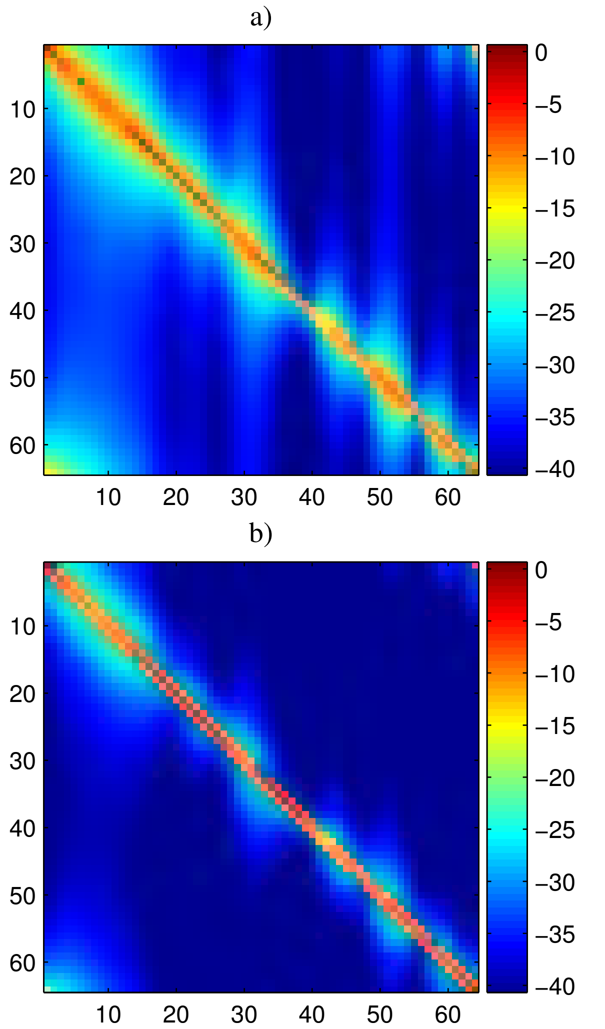

Fig. 2. Example of a (64×64) channel matrix for a) OFDM and b) the multicarrier system with truncated Gaussian impulses ($f_{3\text{dB}}T_s = 0.3$). The color shows the power of an element in dB.

An example of $\mathbf{H}$ for a 10-path Raleigh-fading channel and a relative Doppler spread of $\gamma = 0.4$ is shown in Fig. 2b. Compared to the channel matrix of the corresponding OFDM system presented in Fig. 2a, it is easy to see how the overall ICI power is concentrated around the main diagonal due to the application of the truncated Gaussian impulse in the transmit and receive filter. This power concentration allows for a maximum-likelihood symbol detection with an acceptable computational complexity, which is presented in the following.

III. Time-Variant Tailbiting Max-Log-MAP

In [4], a symbol-by-symbol MAP (maximum a posteriori probability) decoder was presented, which can not only be used for decoding purposes but also for channel equalization. This so called BCJR algorithm requires, analogous to the well known Viterbi algorithm, a trellis structure in order to calculate the probabilities of the states and transitions of a Markov source. In our case, the trellis diagram representing the possible states and transitions, in each stage comprises the possible channel outputs on a specific subcarrier. It is build by the linear combination of the symbols according to a specific state with the coefficients of that column of $\mathbf{H}$ which belongs to the currently considered subcarrier. Due to the frequency selectivity of the channel, the weighting coefficients change from subcarrier to subcarrier, i.e. in frequency direction, we refer to the resulting structure as a "time-variant" trellis.

Unlike a terminated trellis for channel codes, in our case there are no initial or ending sequences forcing the trellis path to specific branches in order to reduce the decision error probability. Thus, there are no absolute starting or ending states known to the receiver. However, due to the circular structure of $\mathbf{H}$ we know that the state following that of the last subcarrier must be identical to the state belonging to the first subcarrier. In other words, we have to deal with a circular tailbiting trellis. In [5] it is shown that after iterating the forward recursion of the BCJR algorithm enough times the resulting output sequence converges to the output of the foregoing iteration. We utilize this fact by initializing the forward as well as the backward recursions of the BCJR algorithm with all possible states considered as equiprobable. When reaching the last subcarrier in the forward recursion sequence or the first subcarrier in the backward recursion sequence, L additional recursions are performed in the same direction in the sense of a cyclic continuation. Here, the previously calculated probabilities of the foregoing iteration are overwritten by the new values. As we will show in our simulation results, only a small number L of additional recursions are necessary in order to compensate the provisional initialization.

Regarding the implementation of the BCJR algorithm it is advantageous to calculate with logarithmic values, because by utilizing the approximation

$$\ln(e^{x_1} + e^{x_2}) \approx \max[x_1, x_2] \qquad (11)$$

multiplications can be substituted by simple maximum functions. This complexity reduction leads to the so called Max-Log-MAP algorithm which produces exactly the same hard decision output as the Viterbi decoder [6].

The computational effort of the time-variant tailbiting Max-Log-MAP equalization directly depends on the number of possible states, which is the number of signal constellation points to the power of the constraint length, in our case the number of considered interfering subcarriers. Therefore, the ICI power concentration to a small number of diagonals in $\mathbf{H}$, as shown in Fig. 2, can greatly reduce the computational complexity of the equalization process.

IV. Simulation Results

For the comparison of the conventional OFDM system and the multicarrier carrier system with truncated Gaussian impulse shaping filters we defined a transmission scenario consisting of $N = 64$ subcarriers with QPSK-modulated symbols, $N_{cp} = 16$ taps for the cyclic prefix, a frame size of 10 OFDM symbols, and a 10-path Raleigh-fading channel with additive white Gaussian noise as well as a relative Doppler spread of $\gamma = 0.4$. In order to combat the ICI influence a time-variant tailbiting Max-Log-MAP equalizer with perfect channel knowledge is applied for the symbol detection. The presented error rates were determined after performing the channel decoding, based on a convolutional code of rate $R_c = 1/2$ and a constraint length $L_c = 5$.

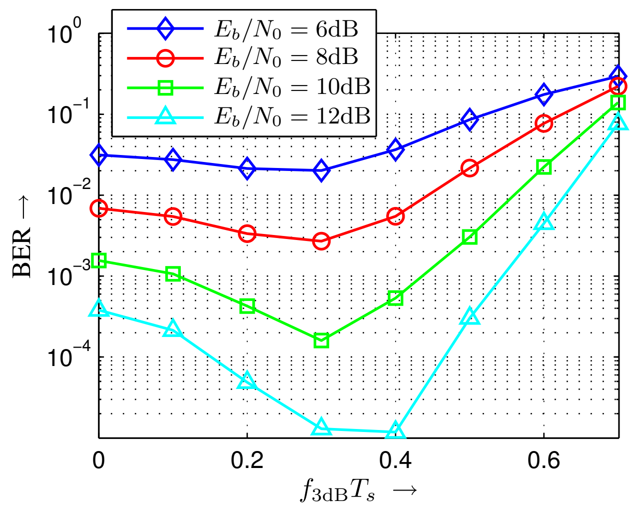

Fig. 3. Performance comparison of different $f_{3\mathrm{dB}}$-bandwidths for the multicarrier system with truncated Gaussian impulses

First, we simulated the influence of the truncated Gaussian impulses applied in the transmitter and receiver on the system performance. Thereby, we assumed the Max-Log-MAP equalizer to have perfect knowledge about the starting state. The resulting bit error rates (BER) for different $f_{3\mathrm{dB}}$-bandwidths and E_b/N_0-ratios are shown in Fig. 3, where the values at $f_{3\mathrm{dB}}T_s = 0$ represent the case of a rectangular pulse shaping and, thus, the performance of the OFDM system. The simulation results demonstrate that for the given scenario the best BER can be achieved with $f_{3\mathrm{dB}}T_s \approx 0.3$.

In Fig. 4 the bit and frame error rates (FER) for the conventional OFDM system, a multicarrier system with truncated Gaussian impulses ($f_{3\text{dB}}T_s = 0.4$) in the transmitter only as well as in the transmitter and receiver ($f_{3\text{dB}}T_s = 0.3$) are compared, all with symbol detection by the Max-Log-MAP equalizer. The higher the E_b/N_0 ratio, the greater become the improvements of the BER due to the soft impulse shaping. The performances of the latter two systems differ by approximately 0.25 dB in the E_b/N_0 ratio, only. Therefore, the truncated Gaussian impulse filter in the transmitter seems to reduce the ICI sufficiently and it is not mandatory to implement another one in the receiver also.

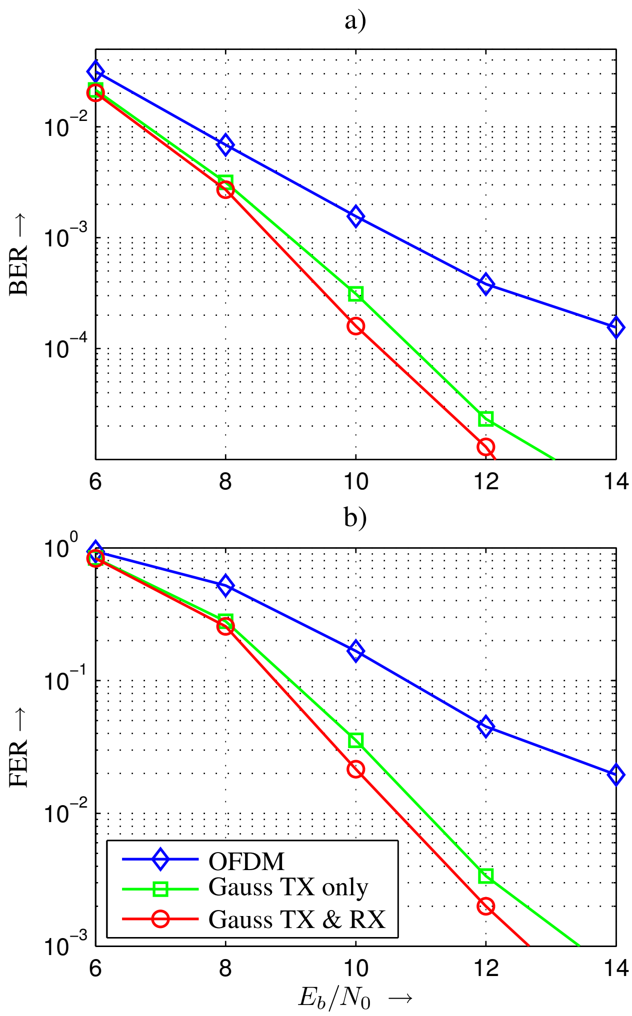

Fig. 4. Comparison of a) bit error rates and b) frame error rates for OFDM and the multicarrier system with truncated Gaussian impulses applied in the transmitter only as well as in the transmitter and receiver

In order to demonstrate the performance gain achieved by additional recursions for exploiting the knowledge of the cyclic property of the channel matrix $\mathbf{H}$, the BERs for the Max-Log-MAP equalization with perfect knowledge of the first and last state, with no as well as with 4 additional recursions in both directions are presented in Fig. 5. From that, it can be concluded that 4 additional recursions are sufficient to closely reach the best achievable performance.

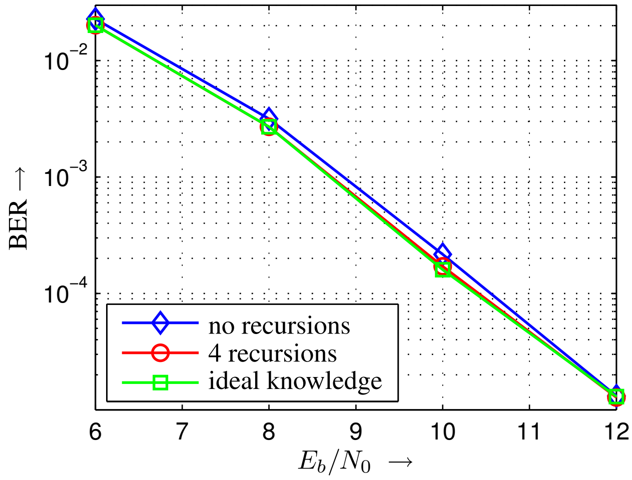

Fig. 5. Bit error rates for the tailbiting Max-Log-MAP equalization with ideal knowledge of the first and last channel state, with no recursions, and with 4 recursions

V. Conclusions

In the presented paper, we propose a multicarrier system with soft impulse shaping as an alternative to OFDM for scenarios with high Doppler spreads. The applied truncated Gaussian impulses concentrate the ICI power on a few diagonals of the channel matrix only, which enables the application of a time-variant tailbiting Max-Log-MAP decoder for the equalization. As we demonstrated in our simulation results, this combination could clearly outperform OFDM in the given scenario.

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