Low Complexity Finite-Alphabet based Blind Channel Estimation in OFDM Receivers

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Abstract—***In this paper, we present two novel blind channel estimation algorithms for OFDM systems based upon the knowledge that the modulated and transmitted data are confined to a finite alphabet set. The Minimum Impulse Length (MIL) method exploits the correlations between adjacent subcarrier coefficients caused by time limited impulse responses and shows a very good estimation performance. Unfortunately, MIL and most other Finite-Alphabet based blind channel estimation approaches are known to be extremely complex due to an exhaustive search to be performed over a tremendous number of channel coefficient combinations. Our Clustered SubCarriers (CSC) method, however, dramatically reduces this number of coefficient combinations to be checked without a significant deterioration in estimation quality. Using bit error rates (BERs), both algorithm are tested with simulations and compared to other blind and nonblind channel estimators.***

Keywords— ***OFDM, Blind Channel Estimation, Finite Alphabet.***

I. INTRODUCTION

In recent years, Orthogonal Frequency Division Multiplexing (OFDM) has become one of the most important techniques for high-rate wireless data transmission. Especially, it has been proposed for the European HIPERLAN/2 standard and the American equivalent IEEE802.11a, which are two similar concepts for broadband wireless local area networks (WLAN) in the 5 GHz band.

Since these standards include coherent data demodulation, the transmission channel has to be estimated. Generally, this is achieved by nonblind channel estimators exploiting additionally transmitted training data. Moreover, the training sequences have to be transmitted periodically, since the channel in wireless applications normally is time variant. In order to increase bandwidth efficiency, blind channel estimation, on the other hand, is well motivated since it avoids the need of any training data. The so-called Minimum Distance (MD) algorithm [1] is a blind estimator which is based on the knowledge that the modulated and transmitted data are confined to a finite alphabet set. In this paper, we present the new Minimum Impulse Length (MIL) approach, which shows a slightly better estimation performance than MD. However, both MIL and MD require an enormous computational effort. In contrast, our Clustered SubCarriers (CSC) scheme dramatically reduces this effort without any significant deterioration in estimation quality.

The paper is organized as follows: Section II gives an overview of the OFDM system. In section III, we present our new blind channel estimation algorithms. After showing some simulation results in section IV, the paper is concluded in section V.

This research was supported by the German NSF (DFG contr. #Ka 841/5-1).

II. DATA TRANSMISSION IN OFDM SYSTEMS

Figure 1 shows the conventional OFDM system with Cyclic Prefix (CP). The CP of length N_{cp} larger than the channel order q avoids the received data from being disturbed by inter-symbol (ISI) or inter-carrier interference (ICI). In the transmitter, the chan-

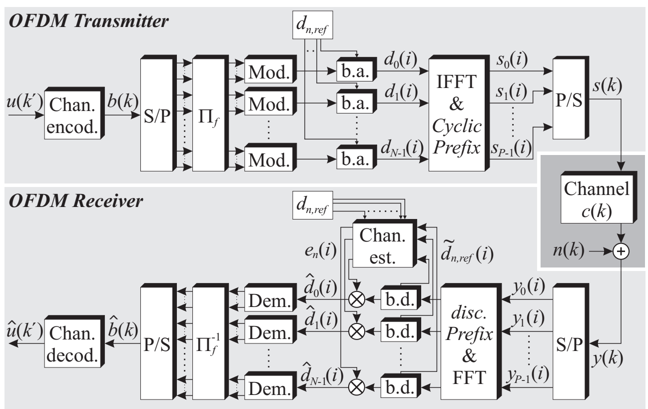

Fig. 1. Conventional OFDM system with Cyclic Prefix

nel encoded information stream $b(k)$ is serial-to-parallel converted (S/P), interleaved (Π_f), modulated, and assembled into so-called OFDM symbols $\mathbf{d}(\mathbf{i}) := [\mathbf{d_0}(\mathbf{i}), \mathbf{d_1}(\mathbf{i}), \ldots, \mathbf{d_{N-1}}(\mathbf{i})]^{\mathbf{T}}$ of length N.

After P/S conversion, the OFDM sequence[1] $s(k)$ is transmitted over the time discrete channel $c(k) = (g_c * c_c * h_c)(t)|_{t=kT}$, where $*$ denotes convolution, T is the chip period, and $g_c(t)$, $h_c(t)$, and $c_c(t)$ are the time continuous transmit and receive filters and the physical channel, respectively. Under assumption that ICI and ISI have been prevented from occurring ($N_{cp} > q$), the ith received OFDM symbol $\tilde{\mathbf{d}}(\mathbf{i})$ after S/P conversion is calculated by

$$\tilde{\mathbf{d}}(\mathbf{i}) = \mathbf{D_C}\mathbf{d}(\mathbf{i}) + \tilde{\mathbf{n}}(\mathbf{i}), \tag{1}$$

where $\mathbf{D_C} := \mathrm{diag}[\mathrm{C}(\mathrm{e}^{\mathrm{j}0}), \mathrm{C}(\mathrm{e}^{\mathrm{j}\frac{2\pi}{\mathrm{N}}}), \ldots, \mathrm{C}(\mathrm{e}^{\mathrm{j}\frac{2\pi}{\mathrm{N}}(\mathrm{N}-1)})]$ is a diagonal matrix with diagonal elements $C(z_n) := \sum_{l=0}^{q} c(l) z_n^{-l}$ evaluated at the subcarriers $z_n = e^{j\frac{2\pi}{N}n}$ for each $n \in [0, N-1]$ and $\tilde{\mathbf{n}}(\mathbf{i})$ represents Additive Gaussian Noise (AGN) without CP and colored by the receive filter $h_c(t)$. From (1) it is obvious that the channel influence is reduced to one complex Rayleigh fading factor (channel coefficient) on each subcarrier.

Since each OFDM symbol has to be demodulated coherently, the channel coefficients $C(\rho_n) := C(e^{j\frac{2\pi}{N}n})$ have to be estimated. Therefore, let $\hat{C}(\rho_n)$ denote the estimate of each subcarrier n which will be used to equalize $\tilde{d}_n(i)$

$$\hat{\mathbf{d}}(\mathbf{i}) = \mathbf{D_e}\tilde{\mathbf{d}}(\mathbf{i}), \tag{2}$$

where $\mathbf{D_e} := \mathrm{diag}[\mathrm{e}_0(\mathrm{i}), \mathrm{e}_1(\mathrm{i}), \ldots, \mathrm{e}_{\mathrm{N}-1}(\mathrm{i})]$ with equalizer coefficients $e_n(i) = 1/\hat{C}(\rho_n)$. Finally, each OFDM symbol is de-interleaved (Π_f^{-1}), P/S converted, and channel decoded into bits $\hat{u}(k')$. If channel decoding is based on soft values, it is important that the demodulated bits of each subcarrier n are multiplied with the *channel state information* $|\hat{C}(\rho_n)|^2$ before deinterleaving (not shown in Fig. 1).

III. BLIND OFDM CHANNEL ESTIMATION

According to the PHY layer of HIPERLAN/2 and IEEE802.11a, several OFDM symbols are combined to *bursts* of different lengths. In case of nonblind channel estimation, each burst is preceded by a preamble consisting of two identical training symbols $d_{n,ref}$ (block *burst assembly* – b.a. in Fig. 1) [2]. If, on the other hand, the channel shall be estimated blindly, a burst only contains information-bearing symbols increasing the bandwidth efficiency.

In [3], Zhou et. al have proven that for any M-ary modulation[2] there can be found a variable $J \leq M$ (and for large signal constellations $J \ll M$) which is sufficient to eliminate the information of the received OFDM symbol $\tilde{\mathbf{d}}(\mathbf{i})$. Concerning the complexity of Finite-Alphabet based blind channel estimators, this fact plays a very important role.

With $\mathrm{E}\{\tilde{d}_n^J(i)\} = C^J(\rho_n)\mathrm{E}\{d_n^J(i)\}$ and the replacement of $\mathrm{E}\{d_n^J(i)\}$ by consistent sample averages (over I OFDM symbols), $C^J(\rho_n)$ is estimated as

$$\hat{C}^J(\rho_n) = -a \cdot \left(\frac{1}{I}\sum_{i=0}^{I-1} \tilde{d}_n^J(i)\right), \quad n \in [0, N-1], \tag{3}$$

where $\mathrm{E}\{\cdot\}$ denotes expectation value and a is a real valued constant whose calculation is described in [3]. Since the colored noise $\tilde{n}_n(i)$ is zero-mean, eq. (3) also holds in the noisy case.

Once we have obtained the estimates $\hat{C}^J(\rho_n)$ from (3), the question arises how to find the correct channel coefficients $\hat{C}(\rho_n)$.

Minimum Impulse Length (MIL) approach

The fundamental idea of our Minimum Impulse Length (MIL) algorithm is that there exists only one estimate $\hat{c}(k)$ out of J^N possible channel impulse responses which is not longer than $q+1$. Hence, with (3), MIL searches for the shortest impulse response over all possible vectors $\hat{\mathbf{C}}_1 := [\lambda_0[\hat{C}^J(\rho_0)]^{1/J}, \ldots, \lambda_{N-1}[\hat{C}^J(\rho_{N-1})]^{1/J}]^T$, where $\lambda_n \in \{e^{j\frac{2\pi}{J}m}\}_{m=0}^{J-1}$ is a scalar ambiguity corresponding to the Jth root, $\forall n$.

After calculating the time domain vectors $\hat{\mathbf{c}}_1$, the correct channel estimate

$$\hat{\mathbf{c}} = \arg\min_{\hat{\mathbf{c}}_1} \sum_{l=q+1}^{N-1} |\hat{c}_1(l)|^2 \tag{4}$$

corresponds to the impulse response with minimum mean power of the coefficients $\hat{c}_1(k)$, $k > q$.

However, the main drawback of MIL and MD [1] is the exhaustive search to be performed over J^N possible channel coefficient combinations. For QPSK modulated signals ($J = 4$) transmitted over a HIPERLAN/2 or IEEE802.11a channel with $N = 52$ active subcarriers this means that $J^N \approx 2 \cdot 10^{31}$. Our new CSC approach is mainly based on the idea of MD, but dramatically reduces the computational effort so that it will be possible to apply a Finite-Alphabet based blind channel estimator even to high-rate modulated OFDM systems.

[1] $k = iN + n, \quad n \in [0, N-1]$ defines the chip index, where n is the subcarrier index in frequency domain and i characterizes the OFDM symbol index in time domain.

[2] For BPSK (Binary Phase Shift Keying), QPSK (Quadrature Phase Shift Keying), and 16-QAM (Quadrature Amplitude Keying) $M = 2, 4,$ and 16, respectively.

Clustered SubCarriers (CSC) algorithm

Figure 2 shows the magnitude and phase of a typical HIPERLAN/2 transfer function, where $N = 64$.

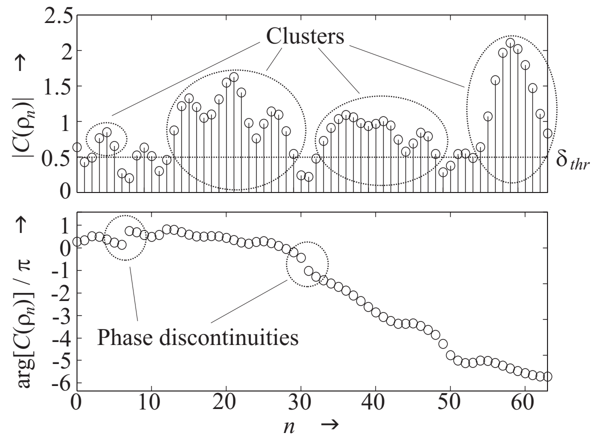

Fig. 2. Magnitude and phase of a channel transfer function

The lower subplot depicts a steady phase course, except from the fading subcarriers $n \in \{6, 7, 30, 31\}$, where phase discontinuities are obvious (dotted circles). Hence, it must be possible to track the scalar ambiguity λ_n of adjacent channel coefficients by choosing their minimum phase distances. This assumption is true as long as no phase discontinuities appear. Figure 3 shows the fading channel coefficients $C(\rho_n)$, $n = 6, 7$ (a) and $n = 30, 31$ (b), and some of their neighbours according to Fig. 2 in the complex z-plane (black circles). If

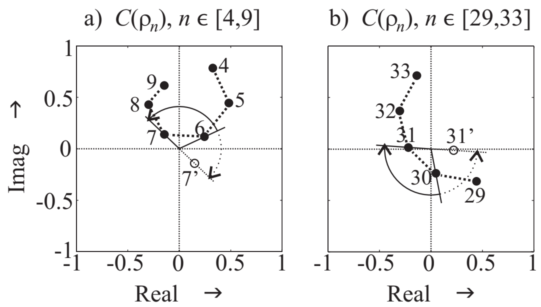

Fig. 3. Phase course of fading channel coefficients

we compare, for instance, the relation between $C(\rho_5)$ and $C(\rho_6)$ on the one hand and $C(\rho_6)$ and $C(\rho_7)$ on the other, it is clear that the closer the coefficients come to the origin, the larger their phase differences can be, even if the Euclidean distance remains constant. Under assumption of BPSK modulated signals and starting from $C(\rho_6)$, the choice of the minimum phase difference would lead to the wrong coefficient $C(\rho_{7'})$ (empty circle) instead of $C(\rho_7)$. Since this error influences all further decisions, a correct estimation of $C(\rho_n)$ will be impossible. Fig. 3b indicates the same behaviour for $29 \leq n \leq 31$. Therefore, we separated the transfer function into Υ *clusters* consisting of L_υ, $\upsilon \in [0, \Upsilon - 1]$, adjacent strong channel coefficients, whose magnitudes are above a certain threshold δ_{thr} (see Fig. 2). Within these clusters, phase discontinuities are very unlikely. By exploiting the correlation between the adjacent channel coefficients, their minimum phase distances within each cluster υ are searched and the scalar ambiguity factors

$$\lambda_{\upsilon,\mu} = \arg\min_{\lambda} \left| \hat{C}_{cl}(\rho_{\upsilon,\mu-1}) - \lambda [\hat{C}^J(\rho_{\upsilon,\mu})]^{1/J} \right|, \quad \mu \in [1, L_\upsilon - 1], \quad \lambda_{\upsilon,0} = 1 \tag{5}$$

can be tracked from one coefficient to the other, where $\hat{C}_{cl}(\rho_{\upsilon,\mu}) = \lambda_{\upsilon,\mu} [\hat{C}^J(\rho_{\upsilon,\mu})]^{1/J}$. Finally, we collect $\hat{C}_{cl}(\rho_{\upsilon,\mu})$ in $L_\upsilon \times 1$ cluster vectors $\hat{\mathbf{C}}_{cl,\upsilon} = [\hat{C}_{cl}(\rho_{\upsilon,0}), \ldots, \hat{C}_{cl}(\rho_{\upsilon,L_\upsilon-1})]^T$ each containing only one scalar ambiguity. Thus, the remaining ambiguities can be resolved by searching over $J^\Upsilon \ll J^N$ possible vectors $\hat{\mathbf{C}}_2 = [\lambda_0 \hat{\mathbf{C}}_{cl,0}, \ldots, \lambda_{\Upsilon-1} \hat{\mathbf{C}}_{cl,\Upsilon-1}]^T$. This means that the computational effort of CSC does not depend anymore on the total number of subcarriers. On the contrary, it rather profits from as much as possible subcarriers, since each cluster may contain more correlated coefficients.

It must be mentioned that with a misadjusted threshold δ_{thr}, CSC might not correctly estimate $C(\rho_n)$ when phase discontinuities appear within clusters. Furthermore, all blind estimators come with an inherent remaining overall scalar ambiguity. This problem can only be solved by the aid of pilot carriers [1].

IV. Simulation Results

In this section, we compare the influence of blind and nonblind channel estimators on the equalization of received data through MONTE-CARLO simulations. With regard to section II, 2000 bursts, each consisting of 20 M-ary modulated OFDM symbols of length N, were transmitted over a time invariant[3] Rayleigh fading channel of order q for different signal-to-noise (SNR) ratios ranging from 0 to 20 dB. In the nonblind case, each burst is preceded by 2 identical training symbols, while the blind estimators are based on $I = 20$ information bearing OFDM symbols. By comparing the sequences $\hat{b}(k)$ and $b(k)$ on the one hand and $\hat{u}(k')$ and $u(k')$ on the other, bit error rates (BERs) were calculated before and after channel decoding, respectively (see Fig. 1), where a half-rate convolutional code with constraint length $L_c = 5$ was applied.

[3]The channel coefficients were changed from burst to burst so that the channel is assumed to be time invariant over one burst period only.

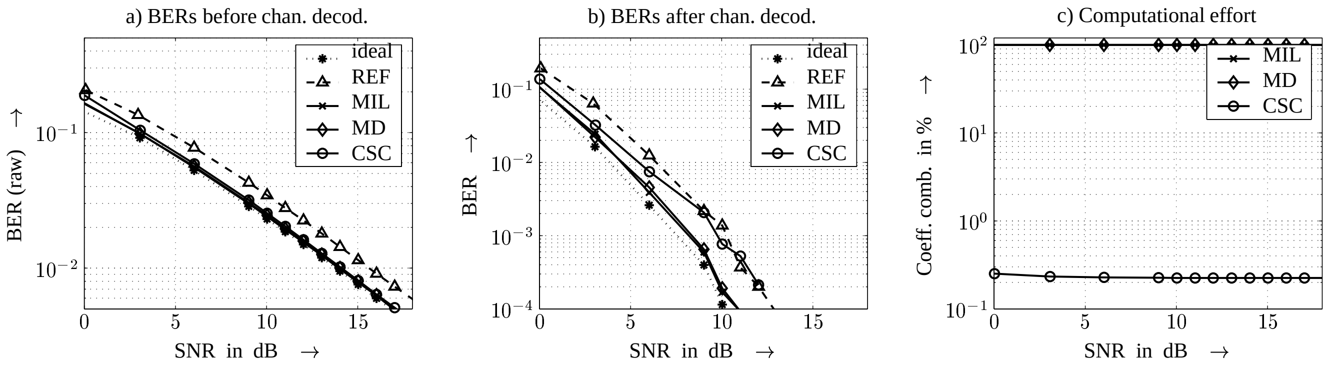

Fig. 4. BERs before (a) and after channel decoding (b) for an "ideal", nonblind ("REF"), and blind ("MIL", "MD", "CSC") channel estimation with $J = 2$ (BPSK), $N = 14$, $N_{cp} = 6$, and $q = 3$. Subplot (c) shows the computational effort of the three blind channel estimators.

Figure 4 shows the BERs before (a) and after channel decoding (b) of BPSK ($M = 2, J = 2$) modulated OFDM symbols with CP length $N_{cp} = 6$ transmitted over a Rayleigh fading channel of order $q = 3$ with $N = 14$ subcarriers for an "ideal", nonblind ("REF"), and blind ("MIL", "MD", "CSC") channel estimation. From subplots (a) and (b) it is obvious that all blind estimators outperform the reference-based channel estimation. While both MIL and MD nearly reach the performance of the "estimator" with ideal channel knowledge, CSC loses app. $2\,\mathrm{dB}$ in SNR after channel decoding. Nevertheless, this is very remarkable, since, according to subplot (c), CSC has to check about 500 times less channel coefficient combinations than MIL or MD.

Figure 5 depicts the BERs for an "ideal", nonblind ("REF"), and blind ("CSC") estimation of a Rayleigh fading channel with order $q = 2$ (a) and $q = 6$ (b) after channel decoding. Since we used a HIPERLAN/2 related system with QPSK ($M = 4, J = 4$) modulated OFDM symbols, $N = 52$ subcarriers, and a CP of length $N_{cp} = 16$, the investigation of MIL and MD was impossible due to the enormous number of channel coefficient combinations (4^{52}) to be checked. The comparison of subplots (a) and (b) shows that in the nonblind case the system benefits from the increased diversity gain caused by the higher channel order. On the other hand, CSC dramatically deteriorates due to the fact that the correlation of the transfer function's coefficients decreases with rising channel order, i.e., there will be more and smaller clusters with a higher risk of appearing phase discontinuities.

V. Conclusions

In this paper, we have presented two novel blind channel estimation approaches for OFDM related systems. While MIL shows an excellent estimation performance with high computational effort, our new CSC approach distinguishes through the fact that it enables the application of Finite-Alphabet based blind channel estimators even to high-rate modulated OFDM signals.

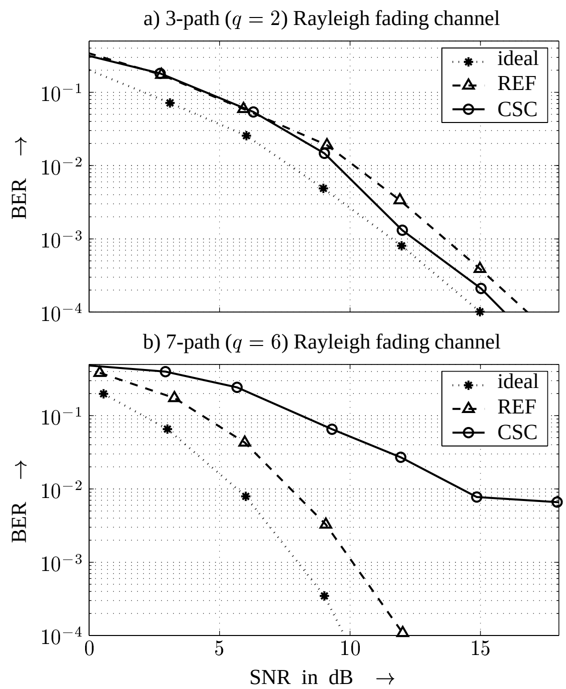

Fig. 5. BERs after chan. decod. for a HIPERLAN/2 system with $J = 4$ (QPSK), $N = 52$, $N_{cp} = 16$ and an "ideal", nonblind ("REF"), and blind ("CSC") estimation of a 3-path (a) and a 7-path Rayleigh fading channel (b).

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