BLIND DIRECTION OF ARRIVAL ESTIMATION USING ANTENNA ARRAYS IN MULTIPATH COMMUNICATION ENVIRONMENT

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ABSTRACT

In recent years many digital array processing techniques have been developed for beamforming directed by direction of arrival (DOA) estimation methods. However, the estimates of DOAs using such existing methods are not satisfactory in multipath environment. In this paper, a new DOA estimation algorithm (called SUBMUSIC) is presented, which divides the received wavefronts first into coherent groups and then estimates the DOAs of each coherent group separately with a new modified subspace smoothing technique. Unlike similar existing methods, the SUBMUSIC algorithm can be applied to arbitrary array form and up to $(M-1)^2$ DOAs of direct path and multipath components can be estimated, where M is the number of sensors.

1. INTRODUCTION

In present communication systems the use of antenna arrays is a good option to satisfy the high traffic requirements. The use of multi-sensor front-end terminals allows to improve the efficiency of cellular communication systems regarding the reuse of frequencies and time slots. In recent years a lot of digital array processing techniques have been developed for beamforming directed by direction of arrival (DOA) estimation methods.

However, in cellular communication systems we often work with multipath environment. When a signal wavefront reflects off of a surface, the original wavefronts and the reflected wavefronts will both impinge on the receiving array of sensors, although from different directions. If the delay difference between the two paths is sufficiently small, we can say, that the two wavefronts are coherent. Normally we have to deal with more than two coherent wavefronts. In such an environment the estimation of DOAs of the received wavefronts is often a problem.

In this paper we consider a method which first divides the coherent wavefronts into different coherent groups, and then the DOAs of each coherent group will be estimated separately. We note that such method has also been studied by Gönen *et al.* [1], which algorithm is called FB-VESPA in this paper, and Yuen [2], which algorithm is called SV-DOA. In FBVESPA the steering vectors, which correspond to different coherent groups, are estimated by using the virtual ESPRIT (VESPA) algorithm [3]. Then, the forward and backward spatial smoothing technique [4] in combination with MUSIC algorithm [5] are applied to each coherent group for the DOA estimation. In that paper, the maximal number of DOAs, which can be estimated, is $2(M-1)M/3$, where M is the number of sensors, and uniformly linear array is needed. In SV-DOA, the steering vectors are estimated using fourth order cumulants with the algorithm described in [6]. Then the modified forward-backward linear prediction algorithm proposed by Tufts and Kumaresan [7] is applied to DOA estimation. Using the algorithm SV-DOA uniformly linear array is also needed and up to $(M-1)(M-p/2)$ DOAs can be estimated, where p is the number of coherent wavefronts in each coherent group.

To get more DOAs estimated and no restriction of array form, a new DOA estimation will be presented in this paper. First, the received wavefronts will be divided into coherent groups via estimating the steering vectors. The estimation algorithm used in this step is very similar to that in FBVESPA. But unlike the algorithm there, no doublet sensor [1] is needed in our algorithm. Then, the idea with the subspace smoothing technique by Okamoto *et al.* [8] will be developed further and used for the DOA estimation in each coherent group.

This paper is organized as follows. In section 2 the used data model will be described. In section 3 we repeat briefly the estimation algorithm for the steering vectors and in section 4 the subspace smoothing algorithm will be modified further for DOA estimation in each coherent group. Finally simulation results are presented in section 5 and the conclusion follows in section 6.

2. DATA MODEL

We assume to have an M-element antenna array in a base station. P wavefronts from q independent narrow band sources, which are non-Gaussian, are impinging on the array. The signal vector $\mathbf{x}(t)$ received by the antenna array can be written as:

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) \tag{1}$$

where $\mathbf{A}$ is an $M \times P$ unknown steering matrix, $\mathbf{s}(t)$ is a $P \times 1$ wavefront vector and $\mathbf{n}(t)$ is the independent noise vector, which can be Gaussian as well as non-Gaussian. Since there are only q independent sources among P wavefronts, we can express the received signal vector $\mathbf{x}(t)$ in terms of independent sources:

$$\begin{aligned}\mathbf{x}(t) &= \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) & (2)\\ &= \mathbf{A}\mathbf{Q}\mathbf{u}(t) + \mathbf{n}(t) = \mathbf{B}\mathbf{u}(t) + \mathbf{n}(t) & (3)\end{aligned}$$

where $\mathbf{u}(t)$ is a $q \times 1$ vector with the independent narrow band sources and $\mathbf{Q}$ is a $P \times q$ matrix with the complex attenuation factors of the sources.

$$\mathbf{Q} = \begin{bmatrix} \mathbf{c}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{c}_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{c}_q \end{bmatrix} \tag{4}$$

[1] A doublet sensor is the shifted version of one sensor in the array

here, $\mathbf{c}_i$ is the complex attenuation vector of the ith source ($1 \le i \le q$).

As defined in equation (3), $\mathbf{B}$ is equal $\mathbf{AQ}$, which is called the effective steering matrix. Each column of $\mathbf{B}$ is a linear combination of steering vectors of one coherent group. The combination coefficients are the elements of the complex attenuation vector $\mathbf{c}_i$ for that group. To see that, we can show that each column of $\mathbf{B}$ can be separated as:

$$\mathbf{b}_i = \mathbf{A}_i\mathbf{c}_i, \qquad 1 \le i \le q \tag{5}$$

where $\mathbf{A}_i$ is the $M \times p_i$ steering matrix of the ith coherent group with p_i coherent wavefronts in it ($\sum_{i=1}^{q} p_i = P$). $\mathbf{c}_i$ is the complex attenuation vector of the ith source.

If the effective steering vectors $\mathbf{b}_i$ $(1 \le i \le q)$ can be estimated from the received signals, we can say that the P wavefronts are separated into q coherent groups, since each column of $\mathbf{B}$ is associated with one coherent group. Now, our problem of estimating DOA of the wavefronts is reduced to q subproblems, where we can estimate the DOA of each group separately.

Techniques of estimating the effective steering vectors have been presented by various authors, for example [9] [10] [6] and [1]. In this paper, the method from Gönen *et al.* (FB-VESPA) [1] is used, which will be described briefly in the next section.

3. ESTIMATION OF EFFECTIVE STEERING VECTORS

Let's consider three fourth-order cumulant matrix:

$$\mathbf{R}_1 = \mathrm{cum}(x_1^*(t), x_1(t), \mathbf{x}^H(t), \mathbf{x}(t)) \tag{6}$$
$$\mathbf{R}_2 = \mathrm{cum}(x_1^*(t), x_2(t), \mathbf{x}^H(t), \mathbf{x}(t)) \tag{7}$$
$$\mathbf{R}_3 = \mathrm{cum}(x_2^*(t), x_2(t), \mathbf{x}^H(t), \mathbf{x}(t)) \tag{8}$$

with elements ($1 \le k, l \le M$):

$$\begin{aligned}\mathbf{R}_1(k,l) &= \mathrm{cum}(x_1^*(t), x_1(t), x_k^*(t), x_l(t)) & (9)\\ &= \sum_{i=1}^{q} \gamma_{4,u_i} |B(1,i)|^2 B^*(k,i) B(l,i) & (10)\end{aligned}$$

$$\begin{aligned}
\mathbf{R}_2(k,l) &= \mathrm{cum}(x_1^*(t), x_2(t), x_k^*(t), x_l(t)) && (11)\\
&= \sum_{i=1}^{q} (\gamma_{4,u_i} B^*(1,i) B(2,i) && (12)\\
&\qquad \cdot B^*(k,i) B(l,i)) && (13)\\
\mathbf{R}_3(k,l) &= \mathrm{cum}(x_2^*(t), x_2(t), x_k^*(t), x_l(t)) && (14)\\
&= \sum_{i=1}^{q} \gamma_{4,u_i} |B(2,i)|^2 B^*(k,i) B(l,i) && (15)
\end{aligned}$$

where $x_i(t)$ is the received signal of the ith sensor and $B(m,n)$ denotes the (m,n)th element of **B**. γ_{4,u_i} is the fourth-order cumulant of the ith source.

From equations (10), (12) and (15) we can see that the matrix $\mathbf{R}_1$ $\mathbf{R}_2$ and $\mathbf{R}_3$ can be expressed in terms of **B** and γ_{4,u_i}:

$$\begin{aligned}
\mathbf{R}_1 &= \mathbf{B}\Lambda\mathbf{B}^H && (16)\\
\mathbf{R}_2 &= \mathbf{B}\mathbf{D}\Lambda\mathbf{B}^H && (17)\\
\mathbf{R}_3 &= \mathbf{B}\mathbf{D}\Lambda\mathbf{D}^H\mathbf{B}^H && (18)
\end{aligned}$$

where
$\Lambda = diag\{\gamma_{4,u_1}|B(1,1)|^2, \cdots, \gamma_{4,u_q}|B(1,q)|^2\}$ and $\mathbf{D} = diag\{\frac{B(2,1)}{B(1,1)}, \cdots, \frac{B(2,q)}{B(1,q)}\}$.

Forming a new matrix **R** with:

$$\begin{aligned}
\mathbf{R} &= \begin{bmatrix} \mathbf{R}_1 & \mathbf{R}_2^H \\ \mathbf{R}_2 & \mathbf{R}_3 \end{bmatrix} && (19)\\
&= \begin{bmatrix} \mathbf{B}\Lambda\mathbf{B}^H & \mathbf{B}\Lambda\mathbf{D}^H\mathbf{B}^H \\ \mathbf{B}\mathbf{D}\Lambda\mathbf{B}^H & \mathbf{B}\mathbf{D}\Lambda\mathbf{D}^H\mathbf{B}^H \end{bmatrix} && (20)
\end{aligned}$$

we can see that R has the same form as the covariance matrix in ESPRIT [11]. Then the estimation algorithm there for the steering vectors can be used; hence, **B** can be estimated up to a multiplicative complex scalar. Note that unlike the estimation algorithm in FBVESPA [1] no doublet sensor is needed here, and the maximum number of independent sources that can be resolved is equal $M-1$.

4. MODIFIED SUBSPACE SMOOTHING ALGORITHM

After the effective steering matrix **B** has been estimated, we describe in this section the second step of our DOA estimation algorithm, where DOAs of each coherent group, associated with one column of **B**, will be estimated separately.

As described in [8], due to the relatively large distance between the mobile user and base station, the direction of arrival of both direct path and multipath components do not vary rapidly for a slight move of the mobile user, but the complex attenuation vector $\mathbf{c}_i$ changes more rapidly, especially the phase.

We collect now received signals of M sensors at m time instances, in which time interval the DOAs of all wavefronts can be assumed to be unchanged. At each time instance the received signals can be written as:

$$\begin{aligned}
\mathbf{x}^j(t) &= \mathbf{A}\begin{bmatrix} \mathbf{c}_1^j & \cdots & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{c}_q^j \end{bmatrix}\mathbf{u}(t) + \mathbf{n}^j(t) && (21)\\
&= \mathbf{A}\mathbf{Q}^j\mathbf{u}(t) + \mathbf{n}^j(t) && (22)\\
&= \mathbf{B}^j\mathbf{u}(t) + \mathbf{n}^j(t) \qquad 1 \le j \le m && (23)
\end{aligned}$$

Using the algorithm in section 3 for each time instance, we can get the estimation of $\mathbf{B}^j$ ($1 \le j \le m$). As shown in (5), the column of $\mathbf{B}^j$ can be expressed as:

$$\mathbf{b}_i^j = \mathbf{A}_i\mathbf{c}_i^j \qquad 1 \le i \le q, \quad 1 \le j \le m \quad (24)$$

So we can interpret $\mathbf{b}_i^j$ as received signals of an M element array from a single source $\mathbf{c}_i^j$.

In our paper [12] we have modified the subspace smoothing algorithm from Okomoto *et al.* [8]. Now this modified subspace smoothing algorithm is applied to $\mathbf{b}_i^j$ for $1 \le i \le q$. Using an array, which is not uniformly linear, or we have more than $2/3M$ wavefronts in the ith group, the covariance matrix of $\mathbf{b}_i^j$ is used:

$$\mathbf{R}_{b_i^j} = \mathbf{b}_i^j(\mathbf{b}_i^j)^H = \mathbf{A}_i\mathbf{c}_i^j(\mathbf{c}_i^j)^H\mathbf{A}_i^H \qquad (25)$$

Taking the average of $\mathbf{R}_{b_i^j}$ over all time instances, we can get:

$$\mathbf{R}_{b_i} = \frac{1}{m}\sum_{j=1}^{m}\mathbf{R}_{b_i^j} \qquad (26)$$

$$= \frac{1}{m}\sum_{j=1}^{m}\mathbf{A}_i\mathbf{c}_i^j(\mathbf{c}_i^j)^H\mathbf{A}_i^H \quad (27)$$

$$= \mathbf{A}_i(\frac{1}{m}\sum_{j=1}^{m}\mathbf{c}_i^j(\mathbf{c}_i^j)^H\mathbf{A}_i^H \quad (28)$$

$$= \mathbf{A}_i\mathbf{R}_c\mathbf{A}_i^H \quad (29)$$

Due to the rapid change of the complex attenuation vectors $\mathbf{c}_i^j$ at each time instance, the matrix $\mathbf{R}_c$ would have full rank of the signal subspace of the ith group. Using a subspace based DOA estimation algorithm such as MUSIC [5], maximum $M-1$ DOAs in each group can be estimated.

Under conditions like uniformly linear array and fewer than $2/3M$ wavefronts in each group, the forward and backward spatial smoothing algorithm is used at each time instance. Then the average over all time instances will be the final results for this group under such conditions.

Note that no restriction of uniformly linear array is made in this algorithm and the maximum total number of coherent wavefronts that can be resolved is equal $(M-1)^2$.

5. SIMULATION RESULTS

In our simulations a five-element uniformly linear array is used first, so that the simulation results of our algorithm (SUBMUSIC) can be compared with the algorithm FBVESPA by Gönen [1] and with the SV-DOA algorithm described in [2]. Later a five-element circular array will be used to demonstrate that our algorithm is not restricted to uniformly linear arrays. For each experiment 100 monte-Carlo runs are used.

Experiment 1 (Well Separated Wavefronts): In this experiment we consider two independent QPSK sources impinging on the array at $\{20^o, 40^o\}$ and $\{68^o, -10^o\}$. That means that we have all together 4 wavefronts. Each source has one coherent multipath component. The complex attenuation factors associated with the direct path and the multipath component of the two sources are $\{1,\ 0.7e^{(j1)}\}$ and $\{0.3e^{(-j0.3)}, 1\}$, respectively. White Gaussian noise with $SNR = 10dB$ is considered here.

In Figure 1 the absolute estimation errors of the three algorithms at different data samples are shown. Using the SUBMUSIC algorithm the best estimation results can be obtained. As absolute estimation error in Figure 1 the averaged absolute estimation error of all DOAs over 100 monte-carlo runs is used.

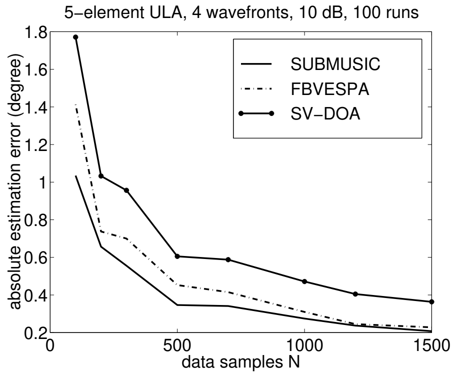

Figure 1: Absolute estimation error of estimated DOAs of well separated wavefronts using uniformly linear array

$$\Delta = \frac{1}{P}\frac{1}{100}\sum_{i=1}^{100}\sum_{j=1}^{P}|\text{DOA}(j) - \hat{\text{DOA}}^i(j)| \quad (30)$$

where Δ is the absolute estimation error, $P = 4$, DOA(j) is the DOA of the jth wavefront and $\hat{\text{DOA}}^i(j)$ is the estimated DOA of the jth wavefront in the ith monte-carlo run. If the absolute estimation error of one monte-carlo run is bigger than 10^0, we say that the using algorithm fails to estimate the DOAs in this run and the corresponding estimation error is not computed in equation (30). We count it to failure rate. In this experiment 2 runs fail at $N = 100$ samples using the SV-DOA algorithm. Using the SUBMUSIC and FBVESPA algorithm no run fails. That means that the SUBMUSIC and FBVESPA algorithm work more robustly than the SV-DOA algorithm.

Experiment 2 (Spatially Close Wavefronts): In this experiment the two independent QPSK sources from experiment 1 are impinging on the array of sensors at $\{20^o, 25^o\}$ and $\{68^o, 20^o\}$. The corresponding attenuation factors are $\{1, 0.7e^{(-j0.3)}\}$ and $\{0.3e^{(1j)}, 1\}$, respectively.

Like experiment 1, the smallest estimation er-

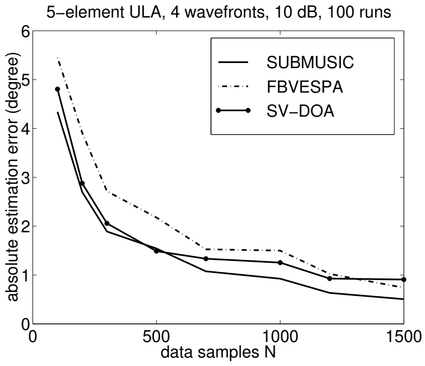

Figure 2: Absolute estimation error of estimated DOAs of spatially close wavefronts using uniformly linear array

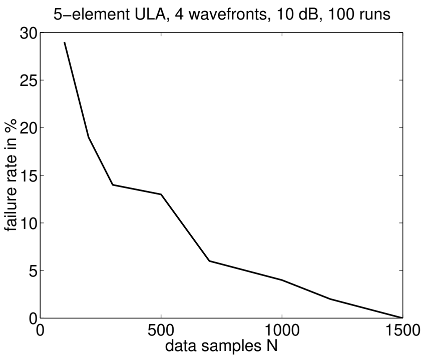

Figure 3: Failure rate of the SV-DOA algorithm with spatially close wavefronts using uniformly linear array

ror in this experiment is also obtained using the SUBMUSIC algorithm (see Figure 2). There is only one difference between the FBVESPA and the SV-DOA algorithm. With more than 1400 data samples using SV-DOA better estimation results than using FBVESPA can be expected.

Let us consider now the stability of the three algorithms in this experiment. Using SUBMUSIC no run fails. Using FBVESPA only 2 runs out of 100 fail at $N = 100$ samples. But using SV-DOA algorithm 28 runs fail at $N = 100$ samples. The failure rate of SV-DOA algorithm is shown in Figure 3.

From Figure 3 we can say that, even more DOAs can be estimated using the SV-DOA algorithm than using the FBVESPA algorithm, the SV-DOA algorithm is not to be recommended because of its weak robustness. Using the new

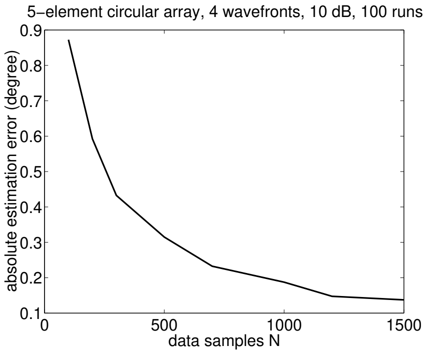

Figure 4: Absolute estimation error of estimated DOAs of well separated wavefronts using circular array

SUBMUSIC algorithm the maximum number of DOAs can be estimated and the best estimation results among the three algorithms can be obtained. The robustness of the SUBMUSIC algorithm is also the strongest. So for estimation of DOAs of the received wavefronts the SUBMUSIC algorithm is the best choice.

Experiment 3 (Using Circular Array): In this experiment the same scenario as in experiment 1 is considered. Instead of the uniformly linear array a five-element circular array is used here.

Figure 4 shows that using the SUBMUSIC algorithm DOAs of coherent wavefronts can be estimated very well even with a non uniformly linear array. In this experiment only 2 runs fail at $N = 100$ samples.

6. CONCLUSION

In this paper a new DOA estimation algorithm in coherent signal environment is presented using both second- and fourth-order statistics. In this new algorithm, a two step approach is applied, in which the received wavefronts is divided first into coherent groups via estimating the steering vectors using fourth-order statistics. Then, the DOAs of each coherent group are estimated using a new modified subspace smoothing algorithm, which uses the second-order statistics of the estimated steering vectors. Unlike similar existing algorithms, the SUBMUSIC algorithm can be applied to arbitrary array form and up to $(M-1)^2$ DOAs can be estimated. Under same conditions where similar existing algorithms also work, the simulations in section 5 have shown that better results can be obtained using the SUBMUSIC algorithm.

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