Joint Power and Time Allocation for Adaptive Distributed MIMO Multi-hop Networks

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***Abstract*—Distributed MIMO multi-hop relaying can provide cooperative diversity and overcome path losses, hence, boost the end-to-end (e2e) performance. By using a low-complexity adaptive scheme, where one relay stops sending the message if it is in outage and other nodes adapt to a new space-time code, robust communication links can be further achieved. The contribution of this paper is the derivation of near-optimal closed-form solution for joint power and time allocation for such adaptive scheme that minimizes he transmission power while satisfying a given e2e non-ergodic outage probability.**

I. Introduction

Recently, the remarkable capacity potential of multi-hop systems with distributed virtual antenna arrays (VAA) was unveiled [1]. The concept of VAA allows spatially relaying nodes to utilize the capacity-enhancement approaches of multiple-input multiple-output (MIMO) techniques, e.g., distributed space-time codes. They offer significant improvements for the data rate in multi-hop networks. Fig. 1 depicts a distributed MIMO multi-hop network, where one source communicates with one destination via a number of relaying VAAs in multiple hops. Spatially adjacent nodes in a VAA receive data from the previous VAA and relay data to the consecutive VAA until the destination is reached.

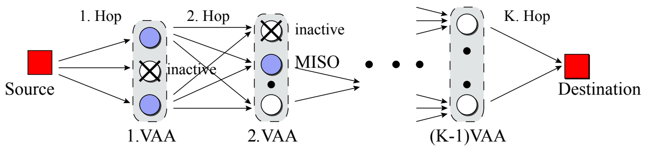

Fig. 1. Topology of adaptive distributed MIMO multi-hop relaying systems.

The general concept of distributed MIMO multi-hop communication systems has been analyzed in [1], where explicit resource allocation strategies were introduced to maximize the e2e data throughput over ergodic fading channels. In contrast, we consider the non-ergodic outage probability which is applicable for the majority of real-world wireless applications [2], [3], [4]. As discussed in [5], the drawback of the fixed decode-and-forward transmission is that it requires full decoding at all relays. Thus, the e2e connection is considered to be in outage if any relay can not decode the message correctly. The e2e performance is then determined by the worst relay link in the network. A similar assumption has also been made in [2], [3], [4] in terms of e2e outage probability and in [6] in terms of ergodic capacity. This strong assumption degrades the e2e performance drastically. Thus, a simple adaptive decode-and-forward scheme for distributed MIMO multi-hop networks will be introduced here.

The aim of this paper is to develop resource-allocation strategies for e2e outage probability constrained adaptive transmissions over slow-fading channels. This is achieved by optimally assigning resources in terms of fractional time and transmission power to each of the hops. The joint power and time allocation problem is formulated as a convex optimization problem, which can be solved by common optimization tools with considerable complexity. To reduce complexity, a sub-optimal but efficient solution will be derived.

The remainder of the paper is organized as follows. In Section II the system model of the adaptive transmission scheme is introduced. The mathematical description of the outage probability will be given in Section III and the optimal power allocation problem is formulated as a convex optimization problem in Section IV. A closed-form solution for an approximated optimization problem will be derived in Section VI. Finally, performance results and conclusions will be given in Section VII and VIII, respectively.

II. System Model

A realization of a distributed MIMO multi-hop network with the utilization of VAAs is depicted in Fig. 1. Here, a source node communicates with a destination node via a number of relaying nodes in K hops. Several relaying nodes are grouped to VAAs performing distributed space-time coding schemes, i.e., *jointly transmission*. For simplicity, each node has only one antenna element and can only operate in half-duplex mode, i.e., the node cannot transmit and receive signal at the same band simultaneously; moreover, the nodes from a VAA do not decode the message jointly due to involved information exchange, i.e., *separately decoding*, which degrades the e2e performance drastically. Thus, a simple adaptive scheme is proposed as a remedy. Here we assume orthogonal decode-and-forward multi-hop relaying scheme, i.e. available network resources are allocated to each hop such that non interference between them occurs. The bandwidth W (FDMA) or time T (TDMA) has to be divided into non-overlapping frequency fractions or time fractions so that they are used by only one

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hop.

The adaptive transmission procedure in TDMA mode is briefly summarized as follows. The source broadcasts the data to the nodes from the first VAA at the first time fraction α_1. Each node decodes the received data separately. If the relay decodes the data successfully (or being not in outage), we denote it as an *active node*, otherwise as an *inactive node*. The inactive node(s) stop(s) transmission at the next time fraction α_2. The active node(s) adapt(s) to transmit the decoded data cooperatively according to a given space-time code with respect to the number of active nodes. If all relays within one VAA fail to decode the data, the e2e connection is considered to be in outage. This adaptive transmission continues at each VAA until the destination is reached. Note that a given fixed network topology is assumed and the task of grouping the VAAs is beyond the scope of this paper.

As mentioned, the nodes from the same VAA decode the data separately. Therefore, the transmission within one hop can be modeled as several multiple-input single-output (MISO) systems. Let k index the hop, t_k, r_k denote the number of transmit nodes and receive nodes within the kth hop, respectively. Each relay transmits signals with the same data rate R but with individual time fraction α_k, of which $\sum_{k=1}^{K} \alpha_k = 1$ holds. All the hops use the total bandwidth W that is available to the network. Define $\mathbf{S}_k \in \mathbb{C}^{t'_k \times L_k}$ as the space-time encoded signal with length L_k from the t'_k active nodes at the kth hop, i.e., $0 \le t'_k \le t_k$. The received signal $\mathbf{y}_{k,j} \in \mathbb{C}^{1\times L_k}$ at the jth node at the kth VAA is given by

$$\mathbf{y}_{k,j} = \sqrt{\theta_k \mathcal{P}_k / t_k}\mathbf{h}_{k,j}\mathbf{S}_k + \mathbf{n}_{k,j} \ , \tag{1}$$

where $\mathbf{n}_{k,j} \sim \mathcal{N}_C(0, N_0) \in \mathbb{C}^{1\times L_k}$ denotes the Gaussian noise vector with power spectral density N_0. Each node from one VAA shares the total transmission power $\mathcal{P}_k$ at the hop k, thus, the active nodes transmit data always with power level $\mathcal{P}_k/t_k$ even if some nodes become inactive. This allows simple power control and hardware implementation at each relaying node which is especially important for relaying nodes with minimal processing functionality. The channel from the t'_k active nodes to the jth receive node within the kth hop is expressed as $\mathbf{h}_{k,j} \in \mathbb{C}^{1\times t'_k}$. The elements of $\mathbf{h}_{k,j}$ obey the same uncorrelated Rayleigh fading statistics with variance 1. The relaying nodes belonging to the same VAA are assumed be spatially sufficiently close as to justify a common path loss θ_k between two VAAs, i.e., the network is symmetric. It can be simply described as $\theta_k = d_k^{-\epsilon}$, where d_k is the distance between the transmit nodes and the receive nodes at the kth hop and ϵ is the path loss exponent within range of 2 to 5 for most wireless channels.

III. Outage Probability at Hop k

Before formulating the outage probability $P_{\text{out},k}$ of hop k, we first consider the outage probability $p_{\text{out},k,j}(t'_k)$ of an active $t'_k \times 1$ MISO system at hop k described in (1). The instantaneous achievable rate of the link is given by

$$C_{k,j}(t'_k) = \alpha_k W \log\left(1 + \frac{\mathcal{P}_k}{\alpha_k W t_k d_k^{\epsilon} N_0}\|\mathbf{h}_{k,j}\|^2\right) \ , \tag{2}$$

with $\|\mathbf{h}_{k,j}\|^2 = \sum_{i=1}^{t'_k} |h_{k,j,i}|^2$. Note that here t'_k is the number of active nodes at hop k, i.e., $0 \le t'_k \le t_k$.

The outage probability $p_{\text{out},k,j}(t'_k)$ can be expressed as the probability that the channel can not support an error-free transmission at rate R,

$$\begin{aligned} p_{\text{out},k,j}(t'_k) &= \Pr(R > C_{k,j}(t'_k)) \\ &= \Pr\left(\|\mathbf{h}_{k,j}\|^2 < \frac{\left(2^{\frac{R}{\alpha_k W}} - 1\right)\alpha_k W N_0 d_k^{\epsilon} t_k}{\mathcal{P}_k}\right) . \end{aligned} \tag{3}$$

Clearly, any analytical optimization on (3) in terms of the fractional time α_k and power $\mathcal{P}_k$ is intractable due to the fairly evolved expression. To overcome that problem, the approximation $\log(1+x) \approx \sqrt{x}$ to the achievable rate in (2) has been suggested and assessed in [1]. Thus, (2) can be simplified by

$$C_{k,j}(t'_k) \approx \sqrt{\frac{\alpha_k W \mathcal{P}_k}{d_k^{\epsilon} N_0 t_k}\|\mathbf{h}_{k,j}\|^2} \ . \tag{4}$$

Therefore, the outage probability (3) becomes

$$p_{\text{out},k,j}(t'_k) \approx \Pr\left(\|\mathbf{h}_{k,j}\|^2 < \frac{R^2 N_0 d_k^{\epsilon} t_k}{\alpha_k W \mathcal{P}_k}\right) = \Pr\left(\|\mathbf{h}_{k,j}\|^2 < x_k\right) . \tag{5}$$

To simplify the notation $x_k = Q_k/\alpha_k\mathcal{P}_k$ is used with variable $Q_k = R^2 N_0 d_k^{\epsilon} t_k / W$. In (5), $\|\mathbf{h}_{k,j}\|^2$ obeys a Gamma distribution [7], therefore its CDF can be described by an incomplete Gamma function $\gamma(t'_k, x_k) = \int_0^{x_k} e^{-u}u^{t'_k - 1}\,\mathrm{d}u$ normalized by Gamma function $\Gamma(t'_k)$. Clearly, the outage probability $p_{\text{out},k,j}(t'_k) = \frac{\gamma(t'_k, x_k)}{\Gamma(t'_k)}$ depends on the number of active t'_k at hop k. In addition, the probability of t'_k active nodes depends on the outage probability of the nodes at the previous hop $k-1$.

Furthermore, the outage probability of receiving node j at hop k is denoted by $P_{\text{out},k,j}$. Under the assumption of symmetric networks the outage probability at each node within one VAA is equal, i.e., $P_{\text{out},k,1} = \cdots = P_{\text{out},k,r_k} = P_{\text{out},k,j'}$ where j' indexes an arbitrary $j \in [1, \cdots, r_k]$. Hence, the number of active relaying nodes t'_k at hop k follows the binomial distribution $\mathcal{B}$ with parameters t_k and $P_{\text{out},k-1,j'}$ [7], i.e.,

$$t'_k \sim \mathcal{B}(t_k, 1 - P_{\text{out},k-1,j'}) \ . \tag{6}$$

The probability of i nodes being active at hop k is expressed by the probability mass function as

$$\Pr(t'_k = i) = \binom{t_k}{i}(1 - P_{\text{out},k-1,j'})^i P_{\text{out},k-1,j'}^{t_k - i} \ , \ \forall\, i \tag{7}$$

where $\binom{t_k}{i} = \frac{t_k!}{i!(t_k-i)!}$. Hence, the outage probability of a $i \times 1$ MISO system is described by $\Pr(t'_k = i)\cdot p_{\text{out},k,j}(i)$. The outage probability $P_{\text{out},k,j'}$ is given by the sum of the outage probabilities over all possible i, namely,

$$P_{\text{out},k,j'} = \sum_{i=1}^{t_k} \Pr(t'_k = i) \cdot p_{\text{out},k,j}(i) \qquad (8)$$
$$= \sum_{i=1}^{t_k} \binom{t_k}{i} (1 - P_{\text{out},k-1,j'})^i P_{\text{out},k-1,j'}^{t_k - i} \frac{\gamma(i, x_k)}{\Gamma(i)} \ .$$

Clearly, an outage occurs in one hop if all the receive nodes within this hop can not decode the message, i.e., the outage probability of hop k is given by

$$P_{\text{out},k} = \prod_{j=1}^{r_k} P_{\text{out},k,j} = P_{\text{out},k,j'}^{r_k} \ . \qquad (9)$$

Consequently the e2e connection is in outage if any hop is broken and the e2e outage probability corresponds to

$$P_{\text{e2e}} = 1 - \prod_{k=1}^{K} (1 - P_{\text{out},k}) = 1 - \prod_{k=1}^{K} \left(1 - P_{\text{out},k,j'}^{r_k}\right) \ . \qquad (10)$$

In the following investigation we use the end-to-end outage probability P_{e2e} as the measurement for the required QoS.

IV. Joint Power and Time Allocation (JPTA)

The joint power and time allocation task for the adaptive scheme is formulated in order to minimize the total power consumption meanwhile supporting a given e2e outage probability requirement e as follows

$$\text{minimize } \mathcal{P}_{\text{total}} = \sum_{k=1}^{K} \alpha_k \mathcal{P}_k (1 - P_{\text{out},k-1,j'}) \qquad (11a)$$

$$\text{subject to } P_{\text{e2e}} \le e, \sum_{k=1}^{K} \alpha_k = 1. \qquad (11b)$$

Here, the calculation of $\mathcal{P}_{\text{total}}$ considers the inactive nodes stopping the transmission to save power and the time fraction α_k. (11) can be proven to be convex for low outage probability requirements by proving the Hessian matrix of $P_{\text{e2e}}(\mathcal{P}_k, \alpha_k, \forall\, k)$ to be positive semi-definite. To this end, the optimal solution $\mathcal{P}_k^\star$, $\alpha_k^\star$ for (11) can be obtained by standard optimization tools leading to considerable complexity [8].

V. Problem Simplification

To simplify the problem, some approximations to the outage probability are invoked in order to derive a near-optimal closed-form power allocation solution. Following the approximation method given in [2], [9], the outage probability $p_{\text{out},k,j}(t'_k)$ in (3) is upper bounded for high SNR as

$$p_{\text{out},k,j}(t'_k) = \frac{\gamma(t'_k, x_k)}{\Gamma(t'_k)} \lessapprox \frac{t_k'^{-1} x_k^{t'_k}}{\Gamma(t'_k)} = \frac{x_k^{t'_k}}{\Gamma(t'_k + 1)} \ . \qquad (12)$$

Hence (8) is approximated by $\tilde{P}_{\text{out},k,j'}$

$$P_{\text{out},k,j'} \lessapprox \sum_{i=1}^{t_k} \Pr(t'_k = i) \frac{x_k^i}{\Gamma(i+1)} \triangleq \tilde{P}_{\text{out},k,j'} \ . \qquad (13)$$

For low outage probabilities, the end-to-end outage probability (10) can be further approximated by [2]

$$P_{\text{e2e}} \le \sum_{k=1}^{K} P_{\text{out},k} = \sum_{k=1}^{K} P_{\text{out},k,j'}^{r_k} \le \sum_{k=1}^{K} \tilde{P}_{\text{out},k,j'}^{r_k} \triangleq \tilde{P}_{\text{e2e}} \ . \qquad (14)$$

For small $P_{\text{out},k-1,j'}$ the objective function of the optimization problem (17) can be rewritten to $\mathcal{P}_{\text{total}} \approx \sum_{k=1}^{K} \alpha_k \mathcal{P}_k$. Thus, the approximated optimization problem is obtained

$$\text{minimize } \mathcal{P}_{\text{total}} \approx \sum_{k=1}^{K} \alpha_k \mathcal{P}_k \qquad (15a)$$

$$\text{subject to } \tilde{P}_{\text{e2e}} = \sum_{k=1}^{K} \tilde{P}_{\text{out},k,j'}^{r_k} \le e, \sum_{k=1}^{K} \alpha_k = 1. \ . \qquad (15b)$$

If we first neglect the time fraction constraint $\sum_{k=1}^{K} \alpha_k = 1$ in (11), the optimization problem depends only on the product $\alpha_k \mathcal{P}_k$. It is therefore approximately symmetric with respect to α_k and $\mathcal{P}_k$. In other word, the optimal power allocation $\mathcal{P}_k^\star$ is proportional to the optimal time fraction $\alpha_k^\star$, i.e., $\mathcal{P}_k^\star \sim \alpha_k^\star$. With consideration of the constraint $\sum_{k=1}^{K} \alpha_k = 1$, we achieve the relation between the optimal power and time fraction

$$\alpha_k^\star = \frac{\mathcal{P}_k^\star}{\sum_{k=1}^{K} \mathcal{P}_k^\star} \ . \qquad (16)$$

Keep this relation in mind, by defining an auxiliary variable $\beta_k = \alpha_k \mathcal{P}_k$ the optimization problem is relaxed to

$$\text{minimize } \mathcal{P}_{\text{total}} = \sum_{k=1}^{K} \beta_k \qquad (17a)$$

$$\text{subject to } P_{\text{e2e}} \le e \ . \qquad (17b)$$

It is similar to the problem in [10], where only power is optimized for the adaptive scheme and the available bandwidth is allocated to each hop equally, i.e., $\alpha_k = \frac{1}{K}$, $\forall k$.

VI. Closed-Form Solution (JPTA-CF)

Clearly, the optimization problem (15) only leads to a near-optimal solution. However, from the complexity point of view, it is attractive to use (15) to derive efficient solutions. To solve the problem, we define the Lagrangian of the approximated optimization problem (15) as

$$L(\beta_k, \lambda) = \sum_{k=1}^{K} \beta_k + \lambda(\tilde{P}_{\text{e2e}} - e) \ , \qquad (18)$$

To obtain the sub-optimal solution that yields minimum total power while meeting the e2e outage constraint e, the derivatives of $L(\beta_k, \lambda)$ w.r.t. β_k has to be zero for all $1 \le k \le K$, i.e.,

$$\frac{\partial L(\beta_k, \lambda)}{\partial \beta_k} = 0, \quad \forall\, k \ . \qquad (19)$$

Furthermore, for the optimum solution of (15) the equality of the constraint function in (15) must be fulfilled, i.e.,

$$\tilde{P}_{\text{e2e}} = \sum_{k=1}^{K} \tilde{P}_{\text{out},k} = e \ . \qquad (20)$$

From (19) and (20), a closed-form solution for β_k can be achieved by several further approximations as outlined in the Appendix.

With consideration of the relation (16), we achieve

$$\beta_k = \alpha_k^\star \mathcal{P}_k^\star = \frac{\mathcal{P}_k^{\star 2}}{\sum_{k=1}^K \mathcal{P}_k^\star} . \tag{21}$$

Rewriting this form yields

$$\sum_{k=1}^K \mathcal{P}_k^\star = \left(\sum_{k=1}^K \sqrt{\beta_k}\right)^2 . \tag{22}$$

Inserting it to (21) and (16), we achieve the following theorem.

Theorem 1: [Joint Power and Time Allocation in Closed Form (JPTA-CF)] The joint power and time (or bandwidth) allocation for outage restricted adaptive distributed MIMO multi-hop networks in closed form is given by

$$\mathcal{P}_k^\star = \sqrt{\beta_k} \sum_{k=1}^K \sqrt{\beta_k} \quad \text{and} \quad \alpha_k^\star = \frac{\sqrt{\beta_k}}{\sum_{k=1}^K \sqrt{\beta_k}} , \tag{23}$$

with outage probability per hop $\tilde{P}_{\mathrm{out},k} = \tilde{P}_{\mathrm{out},k,j'}^{r_k} \approx \frac{\delta_k \cdot e}{\sum_{k=1}^K \delta_k}$, where the parameters δ_k and β_k are given by

$$\delta_k = \frac{\left(2 t_k^{\frac{2}{t_k+1}} \left(\prod_{i=1}^{t_k} \frac{\binom{t_k}{i}(1-e^{\frac{1}{r_{k-1}}})^i e^{\frac{t_k-i}{r_{k-1}}} Q_k^i}{\Gamma(i+1)}\right)^{\frac{2}{t_k+1}}\right)^{\frac{2+(t_k+1)r_k}{(t_k+1)r_k}}}{(r_k(t_k+1))^{\frac{2+(t_k+1)r_k}{(t_k+1)r_k}}} ,$$

$$\beta_k = \frac{t_k^{\frac{2}{t_k+1}}}{\tilde{P}_{\mathrm{out},k}^{\frac{2}{r_k(t_k+1)}}} \left(\prod_{i=1}^{t_k} \frac{\binom{t_k}{i}(1-\tilde{P}_{\mathrm{out},k-1}^{\frac{1}{r_{k-1}}})^i \tilde{P}_{\mathrm{out},k-1}^{\frac{t_k-i}{r_{k-1}}} Q_k^i}{\Gamma(i+1)}\right)^{\frac{2}{t_k(t_k+1)}} .$$

VII. Performance Evaluation

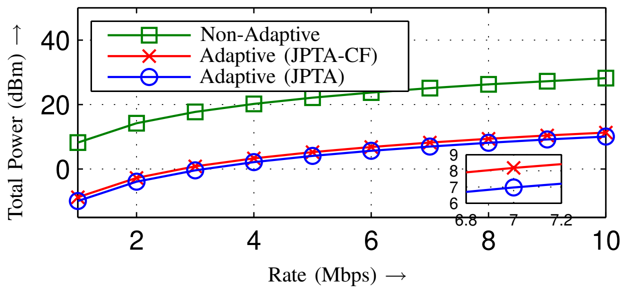

Fig. 2. $\mathcal{P}_{\text{total}}$ in dBm for non-adaptive transmission, closed-form and optimal resource allocation solution.

The performance of the proposed solution for adaptive distributed MIMO multi-hop scheme is assessed here for various network configurations. It is assumed that the e2e communication over $W = 5$ MHz should meet an e2e outage probability constraint of $e = 1\%$ where the path loss exponent $\epsilon = 3$ and $N_0 = -174$ dBm/Hz. Fig. 2 depicts the total power versus the data rate for non-adaptive and adaptive transmissions both with optimized resource allocations for a multi-hop network with 3 hops and 3 nodes per VAA. The distance between the VAAs is $d_k = [3, 1, 1]$km. Note that the optimal solution *JPTA* for adaptive scheme is solved by means of standard optimization tools [8]. The closed-form solution *JPTA-CF* is given by Theorem 1. It is shown that the proposed closed-form solution yields near-optimum total power consumption and almost 20 dBm gain with comparison to the non-adaptive scheme, where the e2e connection is in outage if any node in the network is in outage and no adaptive scheme is applied [2], [4].

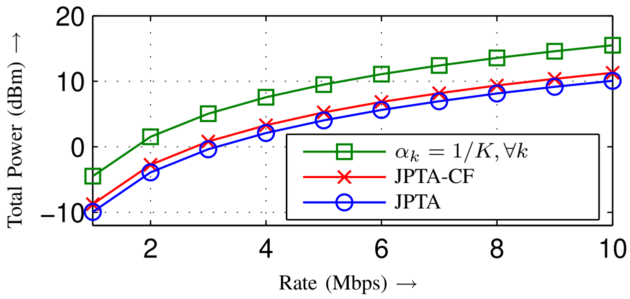

Fig. 3. $\mathcal{P}_{\text{total}}$ in dBm for only power optimized ($\alpha_k = 1/K, \forall k$) and joint power and time allocation for adaptive scheme.

Fig. 3 shows the total transmission power of only power-optimized adaptive scheme [10] and above-mentioned *JPTA* scheme versus data rate. Under this network configuration, joint power and time optimization improves the communication over 5 dBm with comparison to only power optimized communication.

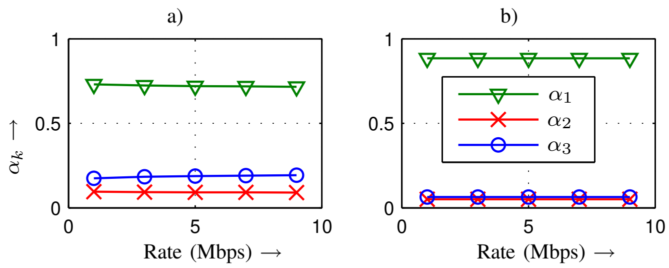

Fig. 4. The time or bandwidth fraction α_k per hop for a) closed-form and b) optimal resource allocation solutions.

Fig. 4 depicts the α_k versus data rate for closed-form and optimal solution. It is shown that the first hop uses over 60% of the time (or bandwidth) due to a large distance 3km. More performance evaluation can be found in [11].

VIII. Conclusion

In this paper, we derived joint power and time allocation for adaptive distributed MIMO multi-hop scheme with optimal as well as closed-form solutions. With comparison the non-adaptive scheme, the adaptive scheme utilizes the nodes from a VAA more efficiently, hence, reduces the power consumption significantly. As shown in simulation results, joint power and time allocation can further improve the e2e performance.

Appendix

Proof of Theorem 1: Referring to (19), the first derivative of $L(\beta_k, \lambda)$ w.r.t. β_k relates not only to $\tilde{P}_{\text{out},k}$ but also to $\tilde{P}_{\text{out},k+1}$, given as follows

$$\frac{\partial L(\beta_k, \lambda)}{\partial \beta_k} = 1 + \lambda \left(\frac{\partial \tilde{P}_{\text{out},k}}{\partial \beta_k} + \frac{\partial \tilde{P}_{\text{out},k+1}}{\partial \beta_k} \right) = 0 \,, \tag{24}$$

which is due to the dependence between $\tilde{P}_{\text{out},k}$ and $\tilde{P}_{\text{out},k+1}$ indicted in (8). This makes the further analysis intricate. Thus, to remove the dependence in (8), $P_{\text{out},k-1,j'}$ is replaced by $e^{\frac{1}{r_{k-1}}}$ which is motivated by the fact that $P_{\text{out},k} < e, \forall\, k$. This relaxes (13) to

$$\tilde{P}_{\text{out},k,j'} \approx \sum_{i=1}^{t_k} \binom{t_k}{i} \left(1 - e^{\frac{1}{r_{k-1}}}\right)^i e^{\frac{t_k - i}{r_{k-1}}} \frac{x_k^i}{\Gamma(i+1)} \,. \tag{25}$$

Furthermore, this sum function can be approximated by its geometric mean

$$\tilde{P}_{\text{out},k,j'} \approx t_k \left(\prod_{i=1}^{t_k} \binom{t_k}{i} \left(1 - e^{\frac{1}{r_{k-1}}}\right)^i e^{\frac{t_k - i}{r_{k-1}}} \frac{x_k^i}{\Gamma(i+1)} \right)^{\frac{1}{t_k}} . \tag{26}$$

With the relation $x_k = Q_k/\beta_k$, (26) the parameter β_k can be represented by $\tilde{P}_{\text{out},k,j'}$ as follows

$$\beta_k = \frac{t_k^{\frac{2}{t_k+1}}}{\tilde{P}_{\text{out},k,j'}^{\frac{2}{t_k+1}}} \left(\prod_{i=1}^{t_k} \frac{\binom{t_k}{i}(1 - e^{\frac{1}{r_{k-1}}})^i e^{\frac{t_k - i}{r_{k-1}}} Q_k^i}{\Gamma(i+1)} \right)^{\frac{2}{t_k(t_k+1)}} . \tag{27}$$

As the dependence between $\tilde{P}_{\text{out},k}$ and $\tilde{P}_{\text{out},k+1}$ has been removed in (25), equation (24) is simplified to

$$\frac{\partial L(\beta_k, \lambda)}{\partial \beta_k} = 1 + \lambda \frac{\partial \tilde{P}_{\text{out},k}}{\partial \beta_k} = 0 \,. \tag{28}$$

Following (28), yields

$$0 = 1 + \lambda r_k \tilde{P}_{\text{out},k}^{\frac{r_k - 1}{r_k}} \frac{\partial \tilde{P}_{\text{out},k,j'}}{\partial \beta_k} \,, \tag{29a}$$

$$= 1 - \lambda r_k \tilde{P}_{\text{out},k}^{\frac{r_k - 1}{r_k}} \sum_{i=1}^{t_k} \frac{\binom{t_k}{i}(1 - e^{\frac{1}{r_{k-1}}})^i e^{\frac{t_k - i}{r_{k-1}}} i x_k^{i-1}}{\Gamma(i+1)} \frac{x_k}{\beta_k} \,, \tag{29b}$$

$$= 1 - \frac{\lambda r_k \tilde{P}_{\text{out},k}^{\frac{r_k - 1}{r_k}}}{\beta_k} \underbrace{\sum_{i=1}^{t_k} \frac{\binom{t_k}{i}(1 - e^{\frac{1}{r_{k-1}}})^i e^{\frac{t_k - i}{r_{k-1}}} x_k^i}{\Gamma(i+1)}}_{\tilde{P}_{\text{out},k,j'}} \cdot i \,. \tag{29c}$$

Due to the weight i in the sum loop it is difficult to represent the sum as a function of $\tilde{P}_{\text{out},k,j'}$. Hence, the approximation $\sum_i^{t_k} z_i \cdot i \approx \frac{1}{t_k} \sum_i^{t_k} i \cdot \sum_i^{t_k} z_i = \frac{t_k+1}{2} \sum_i^{t_k} z_k$ is applied. Thus, (29c) becomes

$$0 = 1 - \frac{\lambda r_k (t_k+1) \tilde{P}_{\text{out},k}^{\frac{r_k - 1}{r_k}}}{2\beta_k} \sum_{i=1}^{t_k} \frac{\binom{t_k}{i}(1 - e^{\frac{1}{r_{k-1}}})^i e^{\frac{t_k - i}{r_{k-1}}} x_k^i}{\Gamma(i+1)} \,, \tag{30a}$$

$$= 1 - \frac{\lambda r_k (t_k+1) \tilde{P}_{\text{out},k}^{\frac{r_k - 1}{r_k}}}{2\beta_k} \cdot \tilde{P}_{\text{out},k}^{\frac{1}{r_k}} \,. \tag{30b}$$

Inserting (27) in (30b), $\tilde{P}_{\text{out},k}$ is expressed as

$$\tilde{P}_{\text{out},k} = \tilde{P}_{\text{out},k,j'}^{r_k} = \lambda^{-\frac{2+(t_k+1)r_k}{(t_k+1)r_k}} \cdot \delta_k \,, \tag{31}$$

where δ_k is introduced to simply the notation

$$\delta_k = \frac{\left(2 t_k^{\frac{2}{t_k+1}} \left(\prod_{i=1}^{t_k} \frac{\binom{t_k}{i}(1 - e^{\frac{1}{r_{k-1}}})^i e^{\frac{t_k - i}{r_{k-1}}} Q_k^i}{\Gamma(i+1)} \right)^{\frac{2}{t_k+1}} \right)^{\frac{2+(t_k+1)r_k}{(t_k+1)r_k}}}{\left(r_k(t_k+1)\right)^{\frac{2+(t_k+1)r_k}{(t_k+1)r_k}}} \,. \tag{32}$$

Since $\lambda^{-\frac{2+(t_k+1)r_k}{(t_k+1)r_k}}$ can be approximated by λ^{-1} for large t_k, inserting (31) in (20)yields

$$\lambda^{-1} \approx \frac{e}{\sum_{k=1}^{K} \delta_k} \,. \tag{33}$$

Hence the sub-optimal outage probability is given by

$$\tilde{P}_{\text{out},k} = \tilde{P}_{\text{out},k,j'}^{r_k} \approx \frac{\delta_k}{\sum_{k=1}^{K} \delta_k} \cdot e \,, \tag{34}$$

which, when inserted into (27), replacing e by $\tilde{P}_{\text{out},k-1}$ we finally achieve β_k given in Theorem 1. This concludes the proof. ■

References

[1] M. Dohler, *Virtual Antenna Arrays*, Ph.D. thesis, King's College London, U.K., November 2003.

[2] Y. Lang, D. Wübben, C. Bockelmann, and K.-D. Kammeyer, "A Closed Power Allocation Solution for Outage Restricted Distributed MIMO Multi-hop Networks," in *Workshop on Resource Allocation in Wireless Networks (RAWNET)*, Berlin, Germany, March 2008.

[3] Y. Lang, D. Wübben, and K.-D. Kammeyer, "Efficient power allocation for outage restricted asymmetric distributed mimo multi-hop networks," in *IEEE International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC)*, Cannes, France, September 2008.

[4] D. Wübben and Y. Lang, "Near-optimum Power Allocation for Outage Restricted Distributed MIMO Multi-hop Networks," in *IEEE Proc. Global Communications Conference (GLOBECOM)*, New Orleans, USA, November 2008.

[5] J.N. Laneman, D. Tse, and G.W. Wornell, "Cooperative Diversity in Wireless Networks: Efficient Protocols and Outage Behavior," *IEEE Trans. on Information Theory*, vol. 50, no. 12, pp. 3062–3080, February 2004.

[6] M. Dohler, A. Gkelias, and H. Aghvami, "A Resource Allocation Strategy for Distributed MIMO Multi-Hop Communication Systems," *IEEE Communications Letters*, vol. 8, no. 2, pp. 98–101, February 2004.

[7] M. Kendall and A. Stuart, *The Advanced Theory of Statistics*, vol. 1, Griffen, London, U.K., 4. edition, 1979.

[8] S. Boyd and L. Vandenberghe, *Convex Optimization*, Cambrigde University Press, 2004.

[9] M. Dohler and M. Arndt, "Inverse Incomplete Gamma Function and Its Application," *IEE Electronics Letters*, vol. 42, no. 1, pp. 35–36, January 2006.

[10] Y. Lang, D. Wübben, and K.-D. Kammeyer, "Power allocations for adaptive distributed mimo multi-hop networks," in *IEEE International Conference on Communications (ICC09)*, Dresden, Germany, June 2009.

[11] Y. Lang and D. Wübben, "Performance evaluation of joint power and time allocations for adaptive distributed mimo multi-hop networks," in *International ITG Workshop on Smart Antennas (WSA 2009)*, Berlin, Germany, February 2009.