PERFORMANCE EVALUATION OF JOINT POWER AND TIME ALLOCATIONS FOR ADAPTIVE DISTRIBUTED MIMO MULTI-HOP NETWORKS

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ABSTRACT

Distributed multiple input multiple output (MIMO) multi-hop networks are proven to achieve superior performance in terms of data throughput and communication reliability. In this paper a low-complexity adaptive relaying scheme is considered in order to achieve robust end-to-end (e2e) communications. For this system optimal as well as near-optimal efficient resource allocation strategies that reduce the total transmit power while satisfying a given e2e outage probability are proposed. It will be shown, that notable power savings can be achieved by the adaptive scheme compared to non-adaptive distributed MIMO multi-hop networks. Moreover, the proposed joint power and time allocation in closed form allows simple implementation and achieves near optimal power consumption.

1. INTRODUCTION

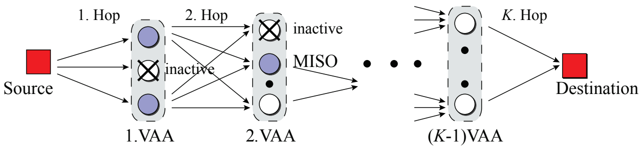

Figure 1: Topology of adaptive distributed MIMO multi-hop relaying systems.

Recently, the remarkable capacity improvement of multi-hop systems by the concept of virtual antenna arrays (VAA) was revealed [1]. The VAA allows the application of traditional MIMO techniques performed on spatially distributed relaying nodes with only one antenna, e.g., distributed space-time codes. A generic realization of a distributed MIMO multi-hop system is depicted in Fig. 1. Here, one source communicates with one destination through several VAAs in multiple hops. Spatially adjacent nodes in a VAA receive data from the previous VAA and relay data to the consecutive VAA until the destination is reached. At each relay, the decode-and-forward protocol [2] and space-time coding will be applied.

In [1], Dohler derived resource allocation strategies to maximize the e2e throughput with respect to the ergodic capacity. In contrast, we consider e2e outage probability due to its higher practical relevance to real wireless communications [3–5]. However, these investigations are based on the assumption that the e2e communication is determined by the weakest link in the network in terms of either ergodic capacity or outage probability. This strong assumption degrades the e2e performance drastically. In order to fully exploit the potential of VAA assisted multi-hop networks, a simple adaptive scheme for distributed MIMO multi-hop networks is introduced here, where one relay stops sending the data when it is in outage and other nodes from the same virtual antenna array (VAA) adapt to a new space-time code. Furthermore, the performance of optimal as well as near-optimal joint power and time allocation strategies will be investigated.

The remainder of the paper is organized as follows. In Section 2 the system model of the adaptive transmission scheme is introduced. The mathematical description of the outage probability will be given in Section 3 and the joint power and time allocation is formulated as a convex optimization problem in Section 4. In order to achieve an efficient solution, a simplified problem is formulated and the corresponding closed-form solution for this approximated optimization problem is derived in Section 5. Finally, the performance is investigated in Section 6 and conclusions will be given in Section 7.

2. SYSTEM DESCRIPTION

We consider a K-hop network with t_k transmit nodes and r_k receive nodes at hop k. Several relays are grouped to a VAA at each hop to apply a distributed space-time code. The data is then transmitted from the source to the destination through $K-1$ VAAs. It is assumed that no interference between the hop occur. Thus, the bandwidth or time has to be divided into non-overlapping parts for each hop such that at any time they are occupied by only one hop, i.e., FDMA or TDMA respectively. Without loss of generality, the TDMA based adaptive scheme will be considered here.

At the first time fraction α_1 the source transmits data to the relays of the 1. VAA. The nodes of the 1. VAA decode the received signals separately in order to avoid enormous information exchanges, i.e., *separately decoding* is performed, which decomposes this hop to several SISO links (or MISO links at the next hops). The t'_k relays successfully decoding the message (or being not in outage) are denoted as *active nodes* and the others failing to decode the message (or being in outage) are denoted as *inactive nodes*, respectively. The inactive nodes will stop transmission at the next time fraction. The t'_k active nodes will *adapt* to transmit the decoded message *cooperatively* according to a space-time code with respect to t'_k, i.e., $0 \leq t'_k \leq t_k$. To this end, each node transmits a spatial fraction of a space-time code. If all relays within one VAA fail to decode the message, the e2e connection is considered to be in outage, denoted by the probability P_{e2e}. Otherwise, the t'_k active nodes send the data to the next VAA at the next time fraction α_{k+1}. This adaptive transmission continues at each VAA until the message reaches the destination. Note that a given fixed network topology is assumed and the task of grouping the VAAs is beyond the scope of this paper.

It is assumed that each relay transmits signals with the same data rate R but with individual time fraction α_k, of which $\sum_{k=1}^{K} \alpha_k = 1$ holds. All the hops use the total bandwidth W that is available to the network. We define $\mathbf{S}_k \in \mathbb{C}^{t'_k \times L_k}$ as the space-time encoded signal with length L_k from the t'_k active nodes at the kth hop, i.e., $0 \leq t'_k \leq t_k$. The received signal $\mathbf{y}_{k,j} \in \mathbb{C}^{1 \times L_k}$ at the jth node at the kth VAA is given by

$$\mathbf{y}_{k,j} = \sqrt{\theta_k \mathcal{P}_k / t_k} \mathbf{h}_{k,j} \mathbf{S}_k + \mathbf{n}_{k,j} \ , \tag{1}$$

where $\mathbf{n}_{k,j} \sim \mathcal{N}_C(0,\, N_0) \in \mathbb{C}^{1 \times L_k}$ is the Gaussian noise with power spectral density N_0. Each active node from one VAA transmits data with power $\mathcal{P}_k / t_k$ equally. This permits simple power control and hardware implementation at each relay which is especially important for relaying nodes with minimal processing functionality. The channel from the t'_k active nodes to the jth receive node within the kth hop is denoted as $\mathbf{h}_{k,j} \in \mathbb{C}^{1 \times t'_k}$, whose elements obey the same uncorrelated Rayleigh fading statistics with unit variance. Note that the relaying nodes belonging to the same VAA are assumed be spatially sufficiently close as to justify a common path loss θ_k between two VAAs, which is known as *symmetric network*. The path loss is simply described as $\theta_k = d_k^{-\epsilon}$, where d_k is the distance between two nodes and ϵ is the path loss exponent within range of 2 to 5 for most wireless channels.

In order to meet a given Quality-of-Services (QoS) requirement, the transmit power $\mathcal{P}_k$ and the time fraction α_k per hop need to be optimized. In the next section the e2e outage probability is introduced as the QoS parameter and optimum as well as near-optimum solutions to the occurring optimization problem are proposed subsequently.

3. END-TO-END OUTAGE PROBABILITY

In order to describe the outage behavior of the adaptive multi-hop network the varying number of active nodes t'_k per hop k has to be considered. To this end, the probability $p_{\text{out},k,j}(t'_k)$ that a *given* MISO link with t'_k active nodes leads to an outage in the jth node in the kth hop is calculated. In combination with the probability $\Pr(t'_k)$ of t'_k active nodes in hop k the effective outage probability $P_{\text{out},k,j}$ of node j in hop k is derived.

The instantaneous achievable rate from t'_k active nodes to the receive node j at hop k is given by

$$C_{k,j}(t'_k) = \alpha_k W \log\left(1 + \frac{\mathcal{P}_k}{\alpha_k W t_k d_k^{\epsilon} N_0} \|\mathbf{h}_{k,j}\|^2\right) \ , \tag{2}$$

with $\|\mathbf{h}_{k,j}\|^2 = \sum_{i=1}^{t'_k} |h_{k,j,i}|^2$. The outage probability $p_{\text{out},k,j}(t'_k)$ is then given by the probability that the channel can not support an error-free transmission at rate R

$$\begin{aligned} p_{\text{out},k,j}(t'_k) &= \Pr(R > C_{k,j}(t'_k)) \\ &= \Pr\left(\|\mathbf{h}_{k,j}\|^2 < \frac{\left(2^{\frac{R}{\alpha_k W}} - 1\right) \alpha_k W N_0 d_k^{\epsilon} t_k}{\mathcal{P}_k}\right) \ . \end{aligned} \tag{3}$$

For simplicity, the approximation $\log(1+x) \approx \sqrt{x}$ is applied to the achievable rate in (2) as assessed in [1]. Thus, (2) can be simplified by

$$C_{k,j}(t'_k) \approx \sqrt{\frac{\alpha_k W \mathcal{P}_k}{d_k^{\epsilon} N_0 t_k} \|\mathbf{h}_{k,j}\|^2} \tag{4}$$

and the outage probability (3) becomes

$$\begin{aligned} p_{\text{out},k,j}(t'_k) &\approx \Pr\left(\|\mathbf{h}_{k,j}\|^2 < \frac{R^2 N_0 d_k^{\epsilon} t_k}{\alpha_k W \mathcal{P}_k}\right) \\ &= \Pr\left(\|\mathbf{h}_{k,j}\|^2 < x_k\right) \ . \end{aligned} \tag{5}$$

For ease notation the variables $x_k = Q_k / (\alpha_k \mathcal{P}_k)$ and $Q_k = R^2 N_0 d_k^{\epsilon} t_k / W$ were introduced, where the parameter x_k is proportional to the inverse signal-to-noise ratio, i.e., $x_k \sim 1/\text{SNR}_k$. In (5), $\|\mathbf{h}_{k,j}\|^2$ obeys a Gamma distribution [6], whose CDF can be described by the lower incomplete Gamma function $\gamma(t'_k, x_k) = \int_0^{x_k} e^{-u} u^{t'_k - 1}\, \mathrm{d}u$ normalized by Gamma function $\Gamma(t'_k)$, i.e.,

$$p_{\text{out},k,j}(t'_k) \approx \frac{\gamma(t'_k, x_k)}{\Gamma(t'_k)} \ . \tag{6}$$

In order to determine the outage probability $P_{\text{out},k,j}$ the probability of active nodes has to be considered. This depends itself on the success of decoding in the previous hop, which is given by the outage probability $P_{\text{out},k-1,j}$. Furthermore, the outage probabilities of all nodes within one VAA are equal under the assumption of symmetric networks, i.e., $P_{\text{out},k,1} = \cdots = P_{\text{out},k,r_k} = P_{\text{out},k,j'}$ where j' indexes an arbitrary $j \in [1, \cdots, r_k]$. Thus, the number of active relaying nodes t'_k follows the binomial distribution $\mathcal{B}$ with parameters t_k and $P_{\text{out},k-1,j'}$ [6], i.e.,

$$t'_k \sim \mathcal{B}(t_k, 1 - P_{\text{out},k-1,j'}) \ . \tag{7}$$

More general, the probability of i nodes being active at hop k is given by the probability mass function as

$$\Pr(t'_k = i) = \binom{t_k}{i}(1 - P_{\text{out},k-1,j'})^i P_{\text{out},k-1,j'}^{t_k-i} \ , \ \forall\, i \tag{8}$$

where $\binom{t_k}{i} = \frac{t_k!}{i!(t_k-i)!}$ is the number of combinations of i active nodes over t_k. Hence, $\Pr(t'_k = i) \cdot p_{\text{out},k,j}(i)$ describes the probability that i nodes are active and lead to an outage event. The outage probability $P_{\text{out},k,j'}$ of node j' in the hop k is given by the sum of the outage probabilities over all possible i, namely,

$$\begin{aligned} P_{\text{out},k,j'} &= \sum_{i=1}^{t_k} \Pr(t'_k = i) \cdot p_{\text{out},k,j}(i) \qquad (9) \\ &= \sum_{i=1}^{t_k} \binom{t_k}{i}(1 - P_{\text{out},k-1,j'})^i P_{\text{out},k-1,j'}^{t_k-i} \frac{\gamma(i, x_k)}{\Gamma(i)} \ . \end{aligned}$$

Clearly, an outage event occurs in one hop if all nodes of one VAA cannot decode the message successfully. Thus, the outage probability of hop k is given by the product of $P_{\text{out},k,j}$, $1 \leq j \leq r_k$,

$$P_{\text{out},k} = \prod_{j=1}^{r_k} P_{\text{out},k,j} = P_{\text{out},k,j'}^{r_k} \ . \tag{10}$$

Consequently the e2e connection is in outage if any hop is broken and the e2e outage probability corresponds to

$$P_{\text{e2e}} = 1 - \prod_{k=1}^{K}(1 - P_{\text{out},k}) = 1 - \prod_{k=1}^{K}\left(1 - P_{\text{out},k,j'}^{r_k}\right) \ . \tag{11}$$

4. OPTIMUM JOINT POWER AND TIME ALLOCATION (JPTA)

In order to minimize the total power consumption $\mathcal{P}_{\text{total}}$ while satisfying a given e2e outage probability constraint e, the optimization problem for joint power and time allocation is formulated as

$$\text{minimize } \mathcal{P}_{\text{total}} = \sum_{k=1}^{K} \alpha_k \mathcal{P}_k (1 - P_{\text{out},k-1,j'}) \tag{12a}$$

$$\text{subject to } P_{\text{e2e}} \leq e, \tag{12b}$$

$$\sum_{k=1}^{K} \alpha_k = 1 \ . \tag{12c}$$

To calculate $\mathcal{P}_{\text{total}}$, the inactive nodes stopping the transmission to save power with the probability $P_{\text{out},k-1,j'}$ is taken into account. Moreover, the duration of each time fraction α_k is also considered. The optimization problem (12) can be shown to be convex for low outage probability requirements by proving the Hessian matrix of $P_{\text{e2e}}(\mathcal{P}_k, \alpha_k, \forall\, k)$ to be positive semi-definite. By using standard optimization methods [7], the optimal solution $\mathcal{P}_k^\star$, $\alpha_k^\star$ for (12) can be achieved leading to considerable complexity. An approximated problem for joint power and time allocation is investigated in the following text.

5. APPROXIMATE JOINT POWER AND TIME ALLOCATION

5.1. Problem Simplification

Following the approximation method given in [3,8], the outage probability $p_{\text{out},k,j}(t'_k)$ in (6) is approximated for high SNR as

$$p_{\text{out},k,j}(t'_k) = \frac{\gamma(t'_k, x_k)}{\Gamma(t'_k)} \lessapprox \frac{t_k'^{-1} x_k^{t'_k}}{\Gamma(t'_k)} = \frac{x_k^{t'_k}}{\Gamma(t'_k + 1)} \ . \tag{13}$$

Hence the outage probability per node (9) and the outage probability per hop (10) are approximated by $\tilde{P}_{\text{out},k,j'}$ and $\tilde{P}_{\text{out},k,j'}$, respectively,

$$P_{\text{out},k,j'} \lessapprox \sum_{i=1}^{t_k} \Pr(t'_k = i) \frac{x_k^i}{\Gamma(i+1)} \triangleq \tilde{P}_{\text{out},k,j'} \tag{14}$$

$$\tilde{P}_{\text{out},k} = \prod_{j=1}^{r_k} \tilde{P}_{\text{out},k,j} = \tilde{P}_{\text{out},k,j'}^{r_k} \ . \tag{15}$$

Furthermore, the e2e outage probability (11) is union bounded by [3]

$$P_{\text{e2e}} \leq \sum_{k=1}^{K} P_{\text{out},k} = \sum_{k=1}^{K} P_{\text{out},k,j'}^{r_k} \leq \sum_{k=1}^{K} \tilde{P}_{\text{out},k,j'}^{r_k} \triangleq \tilde{P}_{\text{e2e}} \ . \tag{16}$$

Finally, for small $P_{\text{out},k-1,j'}$ the objective function of (12) can be relaxed to $\mathcal{P}_{\text{total}} \approx \sum_{k=1}^{K} \alpha_k \mathcal{P}_k$. Thus, the approximated optimization problem is obtained

$$\text{minimize } \mathcal{P}_{\text{total}} \approx \sum_{k=1}^{K} \alpha_k \mathcal{P}_k \tag{17a}$$

$$\text{subject to } \tilde{P}_{\text{e2e}} = \sum_{k=1}^{K} \tilde{P}_{\text{out},k,j'}^{r_k} \leq e, \tag{17b}$$

$$\sum_{k=1}^{K} \alpha_k = 1 \ . \tag{17c}$$

By neglecting the time fraction constraint $\sum_{k=1}^{K} \alpha_k = 1$ in (17), the optimization problem depends only on the product

$\alpha_k \mathcal{P}_k$, which is therefore approximately symmetric with respect to α_k and $\mathcal{P}_k$. In other word, one of the optimal power allocations $\mathcal{P}_k^\star$ is proportional to the optimal time fraction $\alpha_k^\star$, i.e., $\mathcal{P}_k^\star \sim \alpha_k^\star$. With consideration of the constraint $\sum_{k=1}^{K} \alpha_k = 1$, the relation between the optimal power and time fraction can be achieved

$$\alpha_k^\star = \frac{\mathcal{P}_k^\star}{\sum_{k=1}^{K} \mathcal{P}_k^\star} \; . \tag{18}$$

By introducing the auxiliary variable $\beta_k = \alpha_k \mathcal{P}_k$ the optimization problem (17) is relaxed to

$$\text{minimize } \mathcal{P}_{\text{total}} \approx \sum_{k=1}^{K} \beta_k \tag{19a}$$

$$\text{subject to } \tilde{P}_{\text{e2e}} \leq e \; . \tag{19b}$$

5.2. Closed-Form Solution (JPTA-CF)

Similar to the solution introduced in [9], (19) can be used to derive approximative solution by the means of Lagrangian

$$L(\beta_k, \lambda) = \sum_{k=1}^{K} \beta_k + \lambda(\tilde{P}_{\text{e2e}} - e) \; . \tag{20}$$

To obtain the near optimal solution, the derivatives of $L(\beta_k, \lambda)$ with respect to β_k has to be zero for all $1 \leq k \leq K$, i.e.,

$$\frac{\partial L(\beta_k, \lambda)}{\partial \beta_k} = 0, \quad \forall \, k \; . \tag{21}$$

Furthermore, the equality of the constraint function in (19) must be fulfilled,

$$\tilde{P}_{\text{e2e}} = \sum_{k=1}^{K} \tilde{P}_{\text{out},k} = e \; . \tag{22}$$

By several further approximations as outlined in the Appendix, a closed-form solution for β_k can be achieved. Furthermore, according to (18), β_k is given by

$$\beta_k = \alpha_k^\star \mathcal{P}_k^\star = \frac{\mathcal{P}_k^{\star 2}}{\sum_{k=1}^{K} \mathcal{P}_k^\star} \; . \tag{23}$$

Rewriting this form we achieve

$$\mathcal{P}_k^\star = \sqrt{\sum_{k=1}^{K} \mathcal{P}_k^\star} \sqrt{\beta_k} \; , \tag{24}$$

$$\sum_{k=1}^{K} \mathcal{P}_k^\star = \sqrt{\sum_{k=1}^{K} \mathcal{P}_k^\star} \sum_{k=1}^{K} \sqrt{\beta_k} \; , \tag{25}$$

$$\sum_{k=1}^{K} \mathcal{P}_k^\star = \left(\sum_{k=1}^{K} \sqrt{\beta_k} \right)^2 \; . \tag{26}$$

Inserting (26) to (23), the following theorem is obtained.

Theorem 1. [Joint Power and Time Allocation in Closed Form (JPTA-CF)] *The joint power and time (or bandwidth) allocation for outage restricted adaptive distributed MIMO multi-hop networks in closed form is given by*

$$\mathcal{P}_k^\star = \sqrt{\beta_k} \sum_{k=1}^{K} \sqrt{\beta_k} \quad \textit{and} \quad \alpha_k^\star = \frac{\sqrt{\beta_k}}{\sum_{k=1}^{K} \sqrt{\beta_k}} \; , \tag{27}$$

with outage probability per hop $\tilde{P}_{out,k} = \tilde{P}^{r_k}_{out,k,j'} \approx \frac{\delta_k \cdot e}{\sum_{k=1}^{K} \delta_k}$, where the parameters δ_k and β_k are given by

$$\delta_k = \frac{\left(2 t_k^{\frac{2}{t_k+1}} \left(\prod_{i=1}^{t_k} \frac{\binom{t_k}{i} (1 - e^{\frac{1}{r_{k-1}}})^i e^{\frac{t_k - i}{r_{k-1}}} Q_k^i}{\Gamma(i+1)} \right)^{\frac{2}{t_k(t_k+1)}} \right)^{\frac{(t_k+1) r_k}{2 + (t_k+1) r_k}}}{(r_k(t_k+1))^{\frac{(t_k+1) r_k}{2 + (t_k+1) r_k}}} \; ,$$

$$\beta_k = \frac{t_k^{\frac{2}{t_k+1}}}{\tilde{P}_{out,k}^{\frac{2}{r_k(t_k+1)}}} \left(\prod_{i=1}^{t_k} \frac{\binom{t_k}{i} (1 - \tilde{P}_{out,k-1}^{\frac{1}{r_{k-1}}})^i \tilde{P}_{out,k-1}^{\frac{t_k - i}{r_{k-1}}} Q_k^i}{\Gamma(i+1)} \right)^{\frac{2}{t_k(t_k+1)}} \; .$$

6. PERFORMANCE EVALUATION

The performance of joint power and time allocation for adaptive distributed MIMO multi-hop schemes is evaluated here for various network configurations. It is assumed that the e2e communication over $W = 5$ MHz should meet an e2e outage probability constraint of $e = 1\%$ with the path loss exponent $\epsilon = 3$ and $N_0 = -174$ dBm/Hz.

6.1. Total power: Adaptive v.s. non-adaptive.

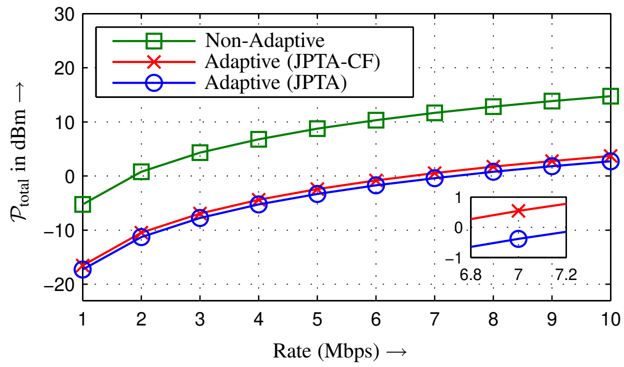

Figure 2: $\mathcal{P}_{\text{total}}$ in dBm for non-adaptive transmission, closed-form and optimal resource allocation solution.

Fig. 2 shows the total power versus the data rate R for non-adaptive and adaptive transmissions both with optimized resource allocations for a 3-hop system with $t_k = 3$ nodes per VAA. The distance between the VAAs is $d_k =$

$[1, 1, 1]$km. Note that the optimal solution *JPTA* is solved by means of standard optimization methods [7] and the closed-form solution *JPTA-CF* is given by Theorem 1. It can be observed that the proposed closed-form solution yields near-optimum total power consumption and almost 15 dBm gain in comparison with the non-adaptive scheme, where the e2e connection is in outage if any node in the network is in outage [3, 5].

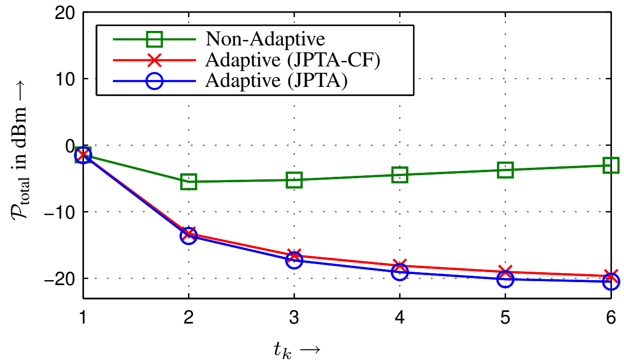

Figure 3: $\mathcal{P}_{\text{total}}$ in dBm versus different number of nodes per VAA from 1 to 6 for non-adaptive transmission, closed-form and optimal resource allocation solution, $R = 1$Mbps.

Fig. 3 shows the total power consumption versus number of nodes per VAA for non-adaptive and adaptive schemes at data rate $R = 1$Mbps. The number of nodes per VAA t_k varies from 1 to 6. It is observed that the performance gap between non-adaptive and adaptive scheme grows for increasing number of nodes t_k. The optimal number of t_k for non-adaptive scheme for this case is 2. Moreover, along with the increasing t_k, the power consumption of the proposed closed-form solution is slightly increased. However, it still achieves near-optimal performance with significantly reduced complexity.

6.2. Total power: Joint power and time allocation (*JPTA*) v.s. only power allocation ($\alpha_k = 1/K$) (*PA*).

In order to reveal the benefits of joint power and time allocation, the optimal power allocation with equal time for each hop is considered for comparison [9], i.e., $\alpha_k = 1/K, \forall k$ which is denoted as *PA* for brevity. Fig. 4 depicts the total transmission power of *PA* and *JPTA* schemes versus data rate. The network configuration is the same as in Section 6.1. *JPTA* improves the communication over 5 dBm in comparison with *PA* for this case.

We consider the impact of different distance d_k on the total power of *JPTA* and *PA* at data rate $R = 1$Mbps. A 3-hop system with 2 VAAs each containing 2 nodes is assumed. At first, the distances $d_2 = d_3 = 1$km are constant, where the distance d_1 from the source to the first VAA varies

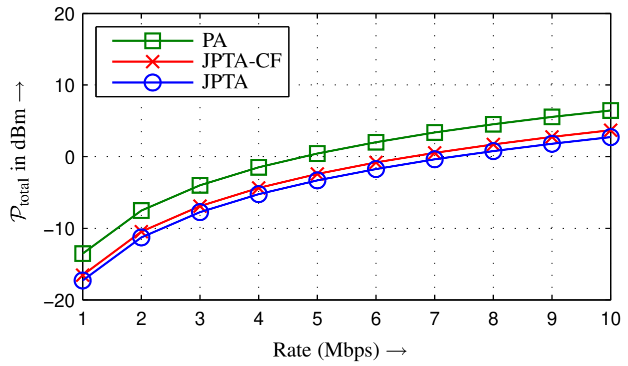

Figure 4: $\mathcal{P}_{\text{total}}$ for only power optimized (equal time $\alpha_k = 1/K, \forall k$) and joint power and time allocation for adaptive scheme.

from 0.5km to 5km. The result is shown in Fig. 5(a). In contrast, for the second case, d_3 varies from 0.5km to 5km and d_1, d_2 are constantly 1km, shown in Fig. 5(b). Clearly, if the distance is increased, the total power of each approach is also enlarged due to the path loss. For the first case, i.e., increasing the distance between the source and the first VAA, the gains due to joint power and time allocation is vanished slowly. This can be explained that in comparison with only power optimization the additional time allocation can not compensate the increased path loss at the first hop, which only has diversity degree 1. In Fig. 5(b) we observe an diametrically opposed phenomenon. When the distance d_3 is increased, the performance gap due to *JPTA* is also increased. Here *JPTA* dominates *PA* by flexible time allocation.

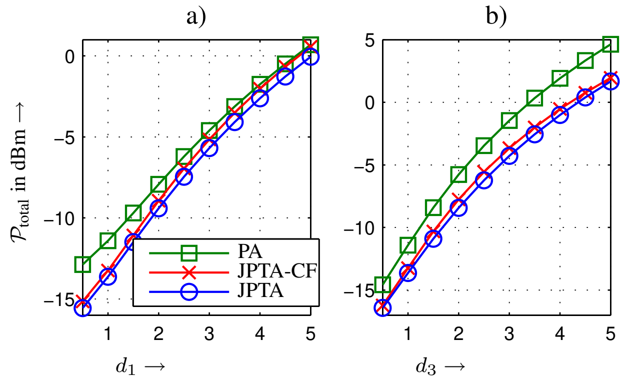

Figure 5: Impact of distance on $\mathcal{P}_{\text{total}}$ in dBm for only power optimized ($\alpha_k = 1/K, \forall k$) and joint power and time allocation for adaptive scheme. a) d_1 varies from 0.5km to 5km; b) d_3 varies from 0.5km to 5km.

6.3. Power of each hop: Adaptive v.s. non-adaptive.

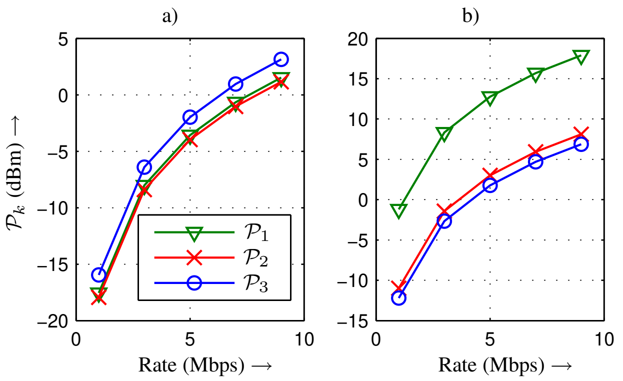

Figure 6: Power $\mathcal{P}_k$ per hop for a) *JPTA* and b) non-adaptive scheme.

Fig. 6 depicts the power $\mathcal{P}_k$ of each hop by *JPTA* and non-adaptive scheme. The network configuration is the same as in Fig. 2, i.e., $t_k = [1, 3, 3, 1]$ and $d_k = [1, 1, 1]$km. It is observed that the power consumed at the third hop is the largest for *JPTA*. The reason for this is that there is only one node at the destination which has to decode the data correctly otherwise an outage event occurs. Similar at the source, there are no nodes to transmit the data cooperatively with high diversity degrees. The source consumes the second most power. Due to the adaptive scheme and space-time coding at the second hop, the second hop uses the least power. In contrast, for non-adaptive scheme the first hop uses the most power, which is due to the lack of diversity degrees at the first hop.

6.4. Time fraction of each hop: *JPTA* v.s. *JPTA-CF*.

As mentioned before, the optimization problem (12) depends only on the product of the variables $\mathcal{P}_k$ and α_k. It can be proven that this problem is not strictly convex with respect to $\mathcal{P}_k$ and α_k [7], i.e., the optimal solution is not unique. There are many optimal combinations of $\mathcal{P}_k$ and α_k to satisfying the optimization problem. This is verified by our simulation results shown in Fig. 7(b)(c). We choose two different starting points for the exact optimization problem (12) which are denoted as *JPTA*$_1$ and *JPTA*$_2$. Different time fractions results can be obtained which do not lead to any increased total power consumption, as the same as that shown in Fig. 2. In other words, these results are also optimal. Fig. 7(a) shows the time fraction derived by *JPTA-CF*, which is near to the optimal solution *JPTA*$_1$.

Fig. 8 shows the relative power allocation per hop $\mathcal{P}'_k = \frac{\mathcal{P}_k}{\sum_{k=1}^{K}\mathcal{P}_k}$ versus data rate for *JPTA* and *JPTA-CF*. Comparing Fig. 8(a)(b) with Fig. 7(a)(b), they are exact the same which verifies the the relation (18), i.e., $\alpha_k^\star = \frac{\mathcal{P}_k^\star}{\sum_{k=1}^{K}\mathcal{P}_k^\star}$.

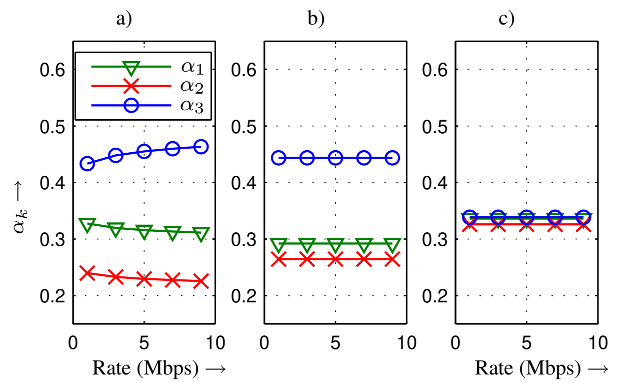

Figure 7: The time fraction α_k per hop for a) *JPTA-CF*, b) *JPTA*$_1$ and c) *JPTA*$_2$.

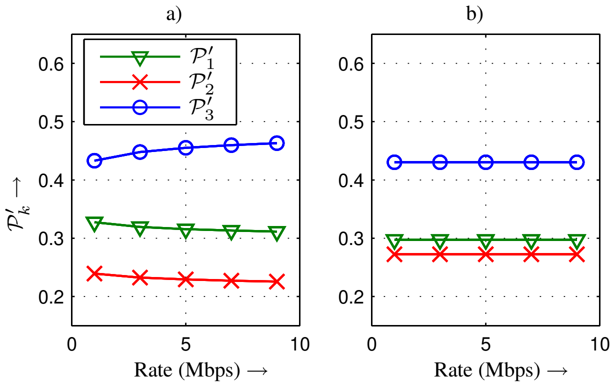

Figure 8: The time fraction α_k per hop for a) *JPTA-CF*, b) *JPTA*.

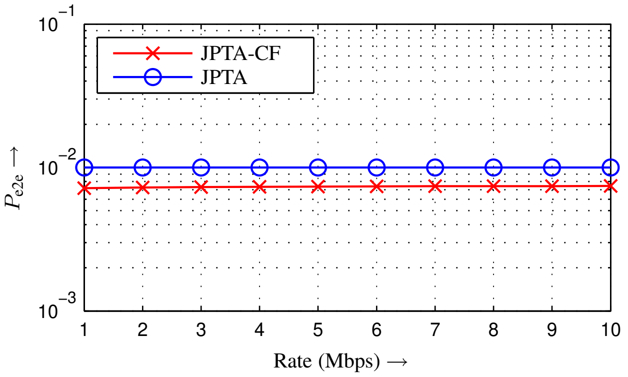

Figure 9: The e2e outage probability with *JPTA-CF* and *JPTA*.

6.5. Outage probability: *JPTA* v.s. *JPTA-CF*.

Fig. 9 shows the e2e outage probability P_{e2e} versus data rate by the proposed *JPTA* and *JPTA-CF* solutions. It is observed that both e2e outage probabilities are independent

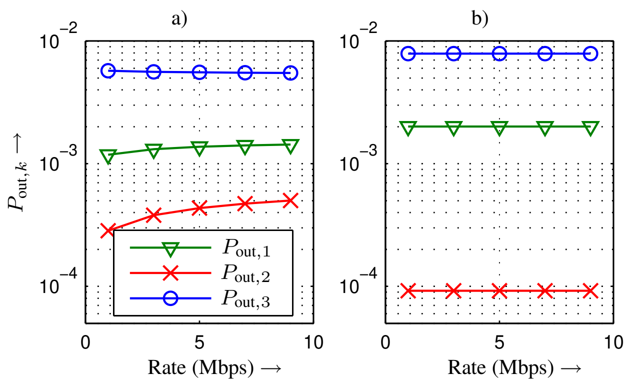

Figure 10: The outage probability per hop with a) *JPTA-CF*, b) *JPTA*.

from the data rate. Moreover, the *JPTA-CF* solution leads to a lower outage level as required which results in slightly higher power consumption as dedicated in Fig. 2. Fig. 10 depicts the outage probability of each hop $P_{\text{out},k}$ versus data rate. The most outage events happen at the third hop where the most power is used as shown in Fig. 8.

7. CONCLUSION

In this paper, we proposed optimal as well as near-optimal joint power and time (or bandwidth) allocation for adaptive distributed MIMO multi-hop networks, which is required to support an given outage probability level with minimized total transmission power. As shown in simulation results, the adaptive scheme outperforms the non-adaptive scheme significantly. Furthermore, the closed form of joint power and time allocation *JPTA-CF* achieves near-optimal performance with lower complexity comparing to the optimal solution *JPTA*.

8. APPENDIX

[Proof of Theorem 1] The first derivative of $L(\beta_k, \lambda)$ in (21) with respect to β_k relates to $\tilde{P}_{\text{out},k}$ as well as $\tilde{P}_{\text{out},k+1}$, expressed as follows

$$\frac{\partial L(\beta_k,\lambda)}{\partial \beta_k} = 1 + \lambda \left(\frac{\partial \tilde{P}_{\text{out},k}}{\partial \beta_k} + \frac{\partial \tilde{P}_{\text{out},k+1}}{\partial \beta_k} \right) = 0 \,, \quad (28)$$

which is due to the dependence between $\tilde{P}_{\text{out},k}$ and $\tilde{P}_{\text{out},k+1}$ indicted in (9). This makes the further analysis involved. To simplify the analysis, we replace $\tilde{P}_{\text{out},k-1,j'}$ by $e^{\frac{1}{r_{k-1}}}$ which is motivated by the fact that $\tilde{P}_{\text{out},k} < e, \forall k$. Thus, (14) becomes

$$\tilde{P}_{\text{out},k,j'} \approx \sum_{i=1}^{t_k} \binom{t_k}{i} \left(1 - e^{\frac{1}{r_{k-1}}}\right)^i e^{\frac{t_k - i}{r_{k-1}}} \frac{x_k^i}{\Gamma(i+1)} \,. \quad (29)$$

Furthermore, (29) can be approximated by its geometric mean

$$\begin{aligned} \tilde{P}_{\text{out},k,j'} &\approx t_k \left(\prod_{i=1}^{t_k} \frac{\binom{t_k}{i}(1-e^{\frac{1}{r_{k-1}}})^i e^{\frac{t_k-i}{r_{k-1}}} x_k^i}{\Gamma(i+1)} \right)^{\frac{1}{t_k}} & (30) \\ &= x_k^{\frac{t_k+1}{2}} t_k \left(\prod_{i=1}^{t_k} \frac{\binom{t_k}{i}(1-e^{\frac{1}{r_{k-1}}})^i e^{\frac{t_k-i}{r_{k-1}}}}{\Gamma(i+1)} \right)^{\frac{1}{t_k}} \\ &= \left(\frac{Q_k}{\beta_k}\right)^{\frac{t_k+1}{2}} t_k \left(\prod_{i=1}^{t_k} \frac{\binom{t_k}{i}(1-e^{\frac{1}{r_{k-1}}})^i e^{\frac{t_k-i}{r_{k-1}}}}{\Gamma(i+1)} \right)^{\frac{1}{t_k}} \end{aligned}$$

Hence, β_k can be expressed by $\tilde{P}_{\text{out},k,j'}$ as

$$\beta_k = \frac{t_k^{\frac{2}{t_k+1}}}{\tilde{P}_{\text{out},k,j'}^{\frac{2}{t_k+1}}} \left(\prod_{i=1}^{t_k} \frac{\binom{t_k}{i}(1-e^{\frac{1}{r_{k-1}}})^i e^{\frac{t_k-i}{r_{k-1}}} Q_k^i}{\Gamma(i+1)} \right)^{\frac{2}{t_k(t_k+1)}} . \quad (31)$$

As the dependence between $\tilde{P}_{\text{out},k}$ and $\tilde{P}_{\text{out},k+1}$ has been removed, equation (28) simplifies to

$$\frac{\partial L(\beta_k,\lambda)}{\partial \beta_k} = 1 + \lambda \frac{\partial \tilde{P}_{\text{out},k}}{\partial \beta_k} = 0 \,. \quad (32)$$

Differentiating (30) along β_k yields

$$\begin{aligned} 0 &= 1 + \lambda r_k \tilde{P}_{\text{out},k}^{\frac{r_k-1}{r_k}} \frac{\partial \tilde{P}_{\text{out},k,j'}}{\partial \beta_k} & (33a) \\ &= 1 - \frac{\lambda r_k (t_k+1) \tilde{P}_{\text{out},k}^{\frac{r_k-1}{r_k}}}{2\beta_k} \tilde{P}_{\text{out},k,j'} & (33b) \\ &= 1 - \frac{\lambda r_k (t_k+1) \tilde{P}_{\text{out},k}^{\frac{r_k-1}{r_k}}}{2\beta_k} \tilde{P}_{\text{out},k}^{\frac{1}{r_k}} & (33c) \\ &= 1 - \frac{\lambda r_k (t_k+1) \tilde{P}_{\text{out},k}}{2\beta_k} \,. & (33d) \end{aligned}$$

Inserting (31) in (33d), $\tilde{P}_{\text{out},k}$ is given by

$$\tilde{P}_{\text{out},k} = \tilde{P}_{\text{out},k,j'}^{r_k} = \lambda^{-\frac{(t_k+1)r_k}{2+(t_k+1)r_k}} \cdot \delta_k \,, \quad (34)$$

where δ_k is introduced to simply the notation

$$\delta_k = \frac{\left(2 t_k^{\frac{2}{t_k+1}} \left(\prod_{i=1}^{t_k} \frac{\binom{t_k}{i}(1-e^{\frac{1}{r_{k-1}}})^i e^{\frac{t_k-i}{r_{k-1}}} Q_k^i}{\Gamma(i+1)} \right)^{\frac{2}{t_k(t_k+1)}} \right)^{\frac{(t_k+1)r_k}{2+(t_k+1)r_k}}}{(r_k(t_k+1))^{\frac{(t_k+1)r_k}{2+(t_k+1)r_k}}} \,. \quad (35)$$

Since $\lambda^{-\frac{(t_k+1)r_k}{2+(t_k+1)r_k}}$ can be approximated by λ^{-1} for large t_k, inserting (34) in (22) yields

$$\lambda^{-1} \approx \frac{e}{\sum_{k=1}^{K} \delta_k} \,. \quad (36)$$

Hence the approximated outage probability is written as

$$\tilde{P}_{\text{out},k} = \tilde{P}^{r_k}_{\text{out},k,j'} \approx \frac{\delta_k}{\sum_{k=1}^{K} \delta_k} \cdot e \ , \quad (37)$$

insert it into (31), replacing e by $\tilde{P}_{\text{out},k-1}$ we finally achieve β_k given in Theorem 1. This concludes the proof.

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