Robust Link Adaptation using SNR-Outage based Loading for BICM-OFDM

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***Abstract*— In order to increase the data rate or power efficiency of OFDM systems several adaptive strategies to select the power, modulation and even code rate on a per subcarrier basis have been discussed in literature. The underlying assumption of perfect channel state information (CSI) at the transmitter, however, is hardly fulfilled in practical systems. Hence, either existing approaches have to be modified to achieve a certain robustness against imperfect CSI or entirely new schemes have to be found. In this paper, we will pursue the former and present an extension of the coded bisection algorithm previously presented by the authors to an outage probability based design for enhanced robustness against erroneous CSI.**

***Index Terms*— OFDM, Link Adaptation, Bit and Power Loading, Imperfect CSI**

I. Introduction

Orthogonal Frequency Division Multiplex (OFDM) is one of the most prominent techniques applied in upcoming communication standards and has been in the focus of research in many areas. The application of resource allocation and/or link adaptation schemes to enhance the overall performance or data rate of OFDM systems has become common in literature. Especially regarding the link adaptation, many contributions have been made, where older works mostly neglect channel coding [1], [2] and more recent publications include channel coding into the design [3], [4], [5].

However, most contributions still consider the presence of perfect channel state information (CSI) at the transmitter to allow adaptation to the current transmission situation, which is often not fulfilled. On the one hand, any channel knowledge has to be gained by estimation leading to errors strongly depending on the spent effort and on the other hand the CSI may be fed back to the transmitter introducing delays. Consequently, imperfect knowledge has to be considered either in the design of adaptation algorithms or existing solutions have to be modified to cope with the uncertainties. In this paper, the latter approach will be used to adapt the coded bisection scheme presented by the authors [3] to imperfect CSI.

Existing contributions in the field of link adaptation for OFDM systems with imperfect CSI often neglect the influence of specific channel codes and simply use capacity maximizing schemes to enhance performance [6]. Accordingly, this paper considers the performance of a specific channel coding scheme under the assumption of imperfect CSI and offers some analysis and advice how to adapt the presented solutions to other channel codes.

The remainder of this paper is organized as follows. Section II introduces the system model and information on channel state information. The basics of the link adaptation scheme are discussed in Section III, which is extended in Section IV to an outage based approach. Section V then presents some code specifics that have to be considered in the overall design followed by numerical results shown in Section VI. Section VII finally concludes this paper.

II. OFDM System model

A typical OFDM frequency domain description with perfect synchronization and sufficiently long cyclic prefix is assumed, where the received signal y_k at subcarrier $k \in \{1, \ldots, N_C\}$ can be described as

$$y_k = h_k \underbrace{\sqrt{p_k} d_k}_{x_k} + n_k \tag{1}$$

with transmit symbols $d_k \in \mathcal{A}_k$, power factor p_k, AWGN noise $n_k \sim \mathcal{N}_C(0, 1)$ and frequency domain channel coefficients h_k stemming from the FFT of the time domain channel $\tilde{\mathbf{h}}$, which is modeled as L Gaussian distributed channel coefficients of equal

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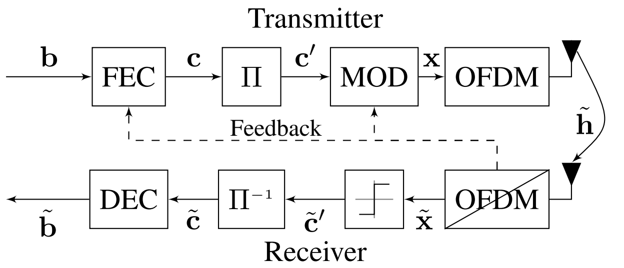

Fig. 1. Adaptive OFDM System Model.

power. The subcarrier power p_k and modulation alphabet $\mathcal{A}_k$ (see Section II-A) on a subcarrier k are adapted to the current channel state by a loading algorithm knowing an estimate $\hat{h}_k$ of the channel that is modeled according to Subsection II-B.

The total power over all subcarriers for a specific OFDM symbol

$$\mathcal{P} = \sum_{k=1}^{N_C} p_k \tag{2}$$

is dependent on the link adaptation. However, for presentation of numerical results, the mean subcarrier Signal-to-Noise-Ratio (SNR) $\mathrm{E}\{\mathcal{P}\}/N_C$ will be applied to scale the link adaptation outcome at a specific target error rate for illustration of the performance in terms of frame error rates (FER) versus SNR.

A. Modulation and Coding

Fig. 1 shows the overall system model including foward error correction (FEC). Following the bit interleaved coded modulation (BICM) approach [7], a single channel code followed by a random interleaver protects the information bits of one OFDM symbol. Throughout this paper rate compatible convolutional codes (RPCP) of rates $R_C \in \{1/3, 4/11, 2/5, 4/9, 1/2, 4/7, 4/6, 4/5, 8/9\}$ with constraint length $L_C = 4$ are considered [8]. In all cases the code word length is fixed to the number of bits in one OFDM symbol, leading to longer code words for higher data rates.

The interleaved code bits $\mathbf{c}'$ are mapped to transmit symbols $\mathbf{d}$ stemming from M-QAM modulation alphabets $\mathcal{A}$. Each subcarrier k may use an individual alphabet of cardinality $M_k = |\mathcal{A}_k|$. Soft-Demapping via a-posteriori-probability (APP) detection is used to supply soft information to the decoder.

B. Channel State Information

The channel estimate $\hat{h}_k$ known at the transmitter either by feedback or a former transmission in the reverse direction is considered as a typical MMSE error model in frequency domain

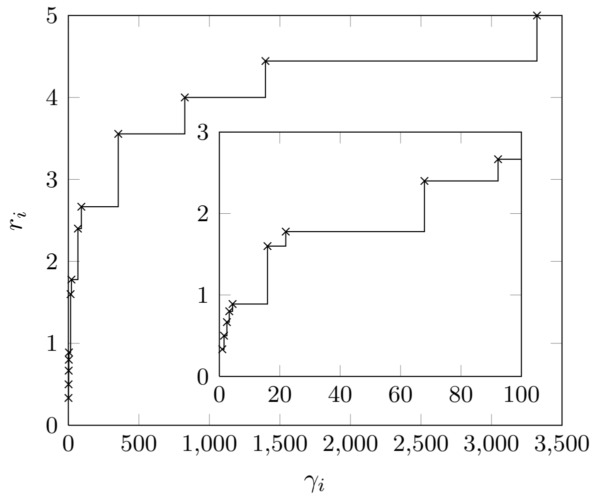

Fig. 2. Exemplary rate-power function $f(\cdot)$ for $N_C = 1024$, $\mathrm{P}_{\text{Target}} = 0.01$, RCPC width $L_C = 4$ and maximum 1024-ASK (real valued constellation).

$$h_k = \hat{h}_k + \Delta_k\,, \tag{3}$$

where the estimate $\hat{h}_k$ of mean energy $\mathrm{E}\{|\hat{h}_k|^2\} = \rho^2$ and the error Δ_k are independent. Furthermore, the error is assumed to be independent over the subcarriers neglecting any correlation in frequency domain, which for instance may result from time domain channel estimation or interpolation in frequency domain. Consequently, Δ_k is modeled as Gaussian noise with power $\sigma_e^2 = 1 - \rho^2$, which leads to an overall Gaussian distribution $\mathcal{N}_c(\hat{h}_k, \sigma_e^2)$ for the channel h_k on a subcarrier k.

III. Coded Bisection Summary

The coded bisection approach discussed in [3] aims at minimizing the transmit power $\mathcal{P}$ given a target rate R_{Total} and target FER $\mathrm{P}_{\text{Target}}$.

$$\text{minimize } \mathcal{P} = \sum_{k}^{N_C} p_k$$
$$\text{subject to } \sum_{k}^{N_C} r_k = R_{\text{Total}} \text{ and } \mathrm{P}_f < \mathrm{P}_{\text{Target}}\,, \tag{4}$$

where r_k defines the bit rate on subcarrier k. To solve this optimization problem approximately, a discrete rate-power function $r_i = f(\gamma_i)$ describing the rate r_i supported by a certain power γ_i while guaranteeing $\mathrm{P}_{\text{Target}}$ is introduced. Fig. 2 shows one example of such a function generated from simulation results of different transmission modes (modulation + coding) over an AWGN channel.

Hence, the Lagrangian of (4) is given by

$$J(\lambda) = \sum_{k=1}^{N_C} p_k + \lambda \left(\sum_{k=1}^{N_C} r_k - R_{\text{Total}} \right) , \quad (5)$$

where the FER constraint is now included in $f(\cdot)$. Considering the unconstrained problem, i.e., neglecting R_{Total}, each λ corresponds to an optimal power and rate distribution, which minimizes the cost function $J(\lambda)$. More specifically, $\lambda/|h_k|^2$ defines the required slope of the rate-power function $f(\cdot)$ on a subcarrier k, which is fulfilled by a unique r_k and p_k only. Due to this simple dependence, a common rate-power function $f(\cdot)$ can be applied for all subcarriers. The solution of the constrained problem (5) is then found by bisection with respect to λ. For further details see [3], [1].

This scheme uses a rate-power function constructed from all possible combinations of code rates and modulations to solve the optimization and thereby leads to a solution with "local" code rates for all subcarriers. Due to the system setup, however, a single code with fixed code rate should be used to encode the information bits for one OFDM symbol. This constraint motivated a solution in two steps, where the second step solves the bit and power loading problem again for a fixed code rate, which is calculated via the mean over all "local" code rates. Note, that the selection of the right code rate is crucial for the overall performance of the scheme, especially at higher spectral efficiencies.

IV. Outage Based Rate-Power Functions

Applying the loading algorithm summarized in Section III under the assumption that the estimate $\hat{h}_k$ equals the actual channel h_k will lead to degradations of the overall performance depending on the error variance σ_e^2. In order to enhance this scheme we consider the discrete rate-power function $r_i = f(\gamma_i)$ describing the supported rate r_i given a SNR γ_i, which is the basis of the coded bisection approach.

Due to the CSI model (3) the true channel-to-noise ratio (CNR) $|h_k|^2$ on subcarrier k is no longer known at the receiver, but can be interpreted as a random variable following a non-central chi-square distribution [9] with 2 degrees of freedom and non-centrality parameter $\lambda_k = 2|\hat{h}_k|^2/\sigma_e^2$ assuming knowledge of σ_e^2. To include this uncertainty into the loading algorithm, we propose an outage based approach. Given the SNR thresholds γ_i for a specific rate r_i defined by the rate-power function, the probability that a threshold γ_i is actually exceeded while applying the power p_k to subcarrier k is given by

$$P(\gamma_i) = 1 - F_{h_k}(\gamma_i/p_k) , \quad (6)$$

where $F_{h_k}(\hat{x}) = P(x \leq \hat{x})$ is the cumulative distribution function (CDF) of the CNR on subcarrier k. Consequently, by choosing an acceptable probability $P(\gamma_i) = \delta$, we can determine the power $p_{k,i}^*$, that is required to support a certain rate r_i with outage probability δ, thereby creating a new rate-power function $r_i = f_k(p_{k,i}^*)$, which depends on the subcarrier k (or more specifically, the estimate $\hat{h}_k$). Hence, the original algorithm is modified by using the subcarrier specific rate-power function $f_k(\cdot)$ instead of $f(\cdot)$ multiplied with the inverse CNR $1/|h_k|^2$.

Besides the design of the rate-power functions the appropriate choice of δ still has to be solved. Ideally, the probability that the threshold γ_i is exceeded by the actual CNR $|h_k|^2$ should be high to ensure proper adaptation. However, choosing δ too high, will strongly increase the required powers $p_{k,i}^*$. A tradeoff between power increase and reliability of the bit and power loading has to be found. Unfortunately, there is no analytical form to determine an optimum δ due to the channel coding, which necessitates numerical simulation to analyze the behavior in dependence of δ.

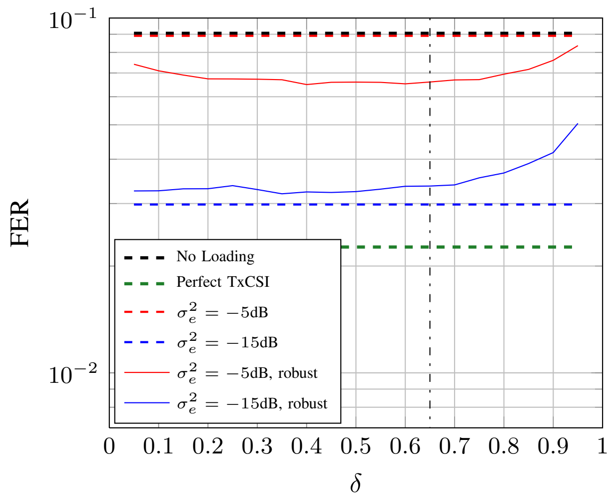

Fig. 3. FER vs. δ for two CSI qualities with and without outage design at a fixed SNR of 15dB, 2 bit/s/Hz using RCPC with code rate $R_C = 1/2$ and $L_C = 4$, maximum modulation 1024-QAM, $\text{P}_{\text{Target}} = 0.01$ and OFDM parameters: $N_C = 1024$, $L = 10$ equal power Rayleigh fading taps.

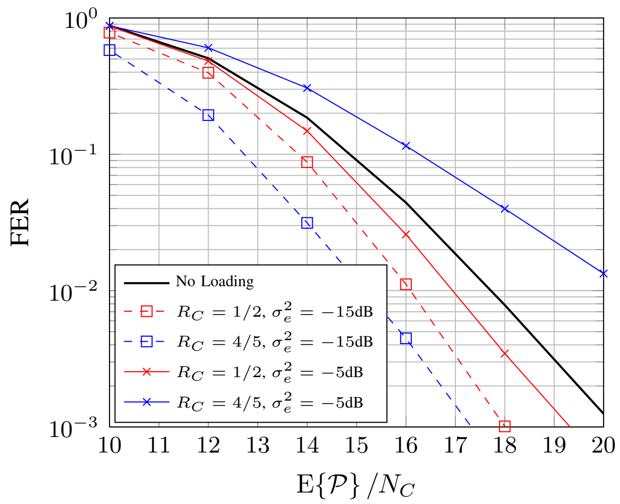

Fig. 4. Results for the robust design using RCPC of fixed $R_C = 1/2$ and $R_C = 4/5$ and $L_C = 4$ at 2 bit/s/Hz with maximum modulation 1024-QAM, $\mathrm{P_{Target}} = 0.01$, $\delta = 0.65$; OFDM parameters: $N_C = 1024$, $L = 10$ equal power Rayleigh fading taps.

Fig. 3 shows the FER performance for a fixed code rate of $R_C = 1/2$ and fixed SNR of 15dB for $\sigma_e^2 = -5$dB (weak CSI) and $\sigma_e^2 = -15$dB (good CSI) using the proposed outage design and the standard approach assuming the estimate to be true. From the numerical analysis it is quite clear that the behavior is rather good-natured over a wide range of δ values. We choose $\delta \approx 0.65$ as a good compromise for the given code for both cases.

Note, that for the chosen code rate nearly no enhancements (or even slight degradation) can be achieved by the proposed outage design for rather good CSI, while the improvement is significant for strongly disturbed CSI. Further details will be discussed in the next Section.

V. Code Specific Design

As already mentioned, the performance of the bit and power loading and the possible enhancements by robust design depends on the quality of the CSI. Furthermore, it strongly depends on the code rate R_C due to heightened noise sensitivity of bit loadings with lower code rates (mean modulation cardinality increases) and the fact, that the decreased redundancy of the channel code at higher code rates offers less protection against wrong CSI (and thereby wrong bit and power allocations). Fig. 4 exemplary shows this for the outage based bit and power loading with fixed $R_C = 1/2$ and $R_C = 4/5$ again at $\sigma_e^2 = -5$dB and $\sigma_e^2 = -15$dB. For "good" CSI at $\sigma_e^2 = -15$dB it is quite clear, that the higher code rate $R_C = 4/5$ can still be exploited to apply modulation schemes of smaller cardinality with reasonable protection against small errors in the bit and power loading. In contrast to that $R_C = 1/2$ offers much better protection against errors in the "bad" CSI case at $\sigma_e^2 = -5$dB. Furthermore, the performance loss of $R_C = 4/5$ with $\sigma_e^2 = -5$dB compared to $\sigma_e^2 = -15$dB is huge and even results in a worse performance than the "static" transmission applying no loading.

The link adaptation summarized in Section III adapts the code rate in addition to bit and power loading to optimize the performance for coded systems. Due to the structure of the algorithm, however, the main criterion for the selection of a certain transmission mode is the power required for that mode at a certain FER $\mathrm{P_{Target}}$, which may still be lowest for "bad" code rates like in the example above if we apply outage based rate-power functions $f_k(\cdot)$ with imperfect CSI. In other words, the true FER performance is no longer indicated by the rate-power functions leading to wrong mode decisions. To solve this problem, and still use the same algorithm robustly for any CSI quality a modification of the code rate adaptation step is required. A simple but effective solution is the application of different δ values for different code rates to balance the FER performance and required power in the outage based $f_k(\cdot)$ during the code rate selection step. The choice of good δ values, however, is heuristic, but can be supported by the numerical analysis presented in Section IV.

VI. Simulation Results

Fig. 5 shows results for a system with two cases of imperfect CSI ($\sigma_e^2 = -15$dB and $\sigma_e^2 = -5$dB), where different δ values according to Table I have been assumed for the available code rates (see Section II-A) to achieve a robust code rate selection. The cases "No Loading" and "Perfect CSI" show results for a fixed modulation of 16-QAM with $R_C = 1/2$ and the non-robust coded bisection two-step scheme outlined in Section III achieving 2 bit/s/Hz with a maximum modulation of 1024-QAM, respectively.

More important, the dashed lines indicate the performance of the non-robust coded bisection scheme under the assumption that the estimated channels $\hat{h}_k$ are correct. Note, that the non-robust scheme copes well with small CSI errors, which is mostly due to a correct code rate choice. For small CSI errors the relative performance of the code rates in comparison to the perfect CSI case is still preserved

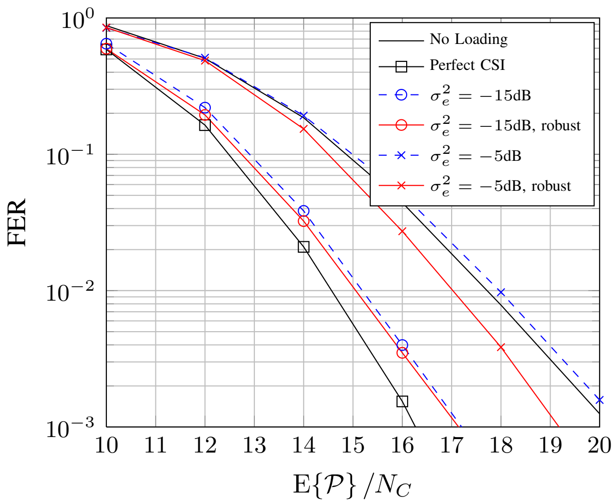

Fig. 5. Results for the non-robust and robust coded bisection using RCPC of variable rates at 2 bit/s/Hz with maximum modulation 1024-QAM, $\mathrm{P}_{\text{Target}} = 0.01$, δ according to Table I; OFDM parameters: $N_C = 1024$, $L = 10$ equal power Rayleigh fading taps.

(i.e., most of the time $R_C = 4/5$ will be chosen, also see Fig. 4), whereas for large CSI errors the relative performances of the code rates may drastically change (e.g., $R_C = 1/2$ should be chosen instead of $R_C = 4/5$). Accordingly, a large performance loss can be observed for $\sigma_e^2 = -5$dB, which coincides with the losses seen in Fig. 4 for $R_C = 4/5$ due to the fact, that these modes seemingly provide the best rate-power efficiency.

The robust outage based scheme on the other hand provides small performance enhancements for small CSI errors by considering the imperfect CSI, but leads to considerable gains for large CSI errors. Note, that the main gain is achieved by the corrected code rate selection. Assuming, that some form of CSI is available disregarding the cost (e.g., due to feedback), the robust approach can now be applied without losing performance in comparison to the non-adaptive transmission even if the CSI quality is rather bad.

VII. Conclusion

In this paper we have shown a modification of the coded bisection method to achieve higher robustness in the presence of imperfect CSI. By considering the CNR distribution an outage based approach has been formulated, which adapts the rate-power functions that are essential for the performance of the adaptation algorithm. Furthermore, we discussed the impact of imperfect CSI on the performance of different code rates of the rate set used in this contribution and provided a simple modification to

TABLE I

Values for δ in dependence of the code rate R_C

R_C	δ
1/3	0.65
4/11	0.65
2/5	0.65
4/9	0.65
1/2	0.65
4/7	0.70
4/6	0.75
4/5	0.77
8/9	0.79

ensure proper code rate selection. The heuristic choice of the probability threshold δ allows flexible tuning of the overall performance, but is of course depending on the specific channel codes that are applied. Overall, the presented numerical results clearly show, that the proposed outage based design enhances the performance in the presence of erroneous CSI, especially enabling a smooth degradation from perfect CSI with the best FER performance to bad or even no CSI with a worst case performance equaling the non-adaptive transmission using a fixed code rate, modulation and equal power distribution.

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