Power Allocation for Adaptive Asymmetric distributed MIMO Networks

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Abstract: Distributed MIMO network topology is one of the most promising technologies in modern communication networks. Compared to classical Single Input Single Output (SISO) links, where information is broadcasted from one sender over huge distances, the distance between two points in multi-hop networks is rather small. By means of such relay networks it becomes possible to guarantee a constant throughput while meeting a given end-to-end (e2e) outage constraint over extensive areas. In each hop several relay nodes form a virtual antenna array (VAA). By using this concept, capacity improving techniques like distributed space time codes can be implemented [Doh03]. The possibility of allocating recourses like transmit power is much more complex in distributed networks than in SISO links. The goal of power optimization in relay networks is to minimize the total transmit power while meeting a given e2e outage constraint. However, in practical scenarios, the structure of a relaying network does not have to follow certain geometrical regularities but can be arranged asymmetrically. As a consequence of that, the closed form expression of power allocation for asymmetric networks is intricate. In this paper the performance of power allocation for an adaptive asymmetric distributed relay networks is analyzed. The significant power saving compared to a non adaptive network is demonstrated by numerical calculations.

1 Introduction

By the concept of relaying networks a classical SISO link can be transformed into a virtual MIMO multi-hop system. A certain amount of relay nodes form a VAA. In each hop, the information is transmitted from one VAA to the next one until the destination is reached. In order to guarantee a give data rate and outage probability for each user, the transmit power in such a network has to be shared reasonable among the transmitting nodes. In the sequel the e2e outage probability is considered as a measure of a given Quality of Service (QoS) requirement that has to be fulfilled. Relays are due to their mobility highly energy restricted. Therefore we consider the transmit power as one of the most important factors that is required to meet a given QoS of a network. Since most of the wireless communication is done over slow fading channels, where the model of ergodic capacity is not applicable, the measure of the e2e outage probability is valid and will be considered in this paper. In order to reduce the overall transmit power in relay networks, several investigations have been made yet. In [LBW$^+$08] the authors derived a closed form expression for a non adaptive network. A symmetric multi-hop network was assumed, where the VAAs are distributed along a line in each hop. Therefore the channels in each hop could be modeled

to be equal with same pathlosses. Power optimization has been done in a non adaptive sense, means that if one relay fails, the whole communication fails. The proposed closed form solution was derived by utilizing common optimization techniques. In such a scenario the whole performance of the network is strongly dependent on the worst link. If one link fails, the whole e2e communication collapses. In order to overcome this drawback, a scheme for an adaptive network was introduced in [LWK09], where nodes that cannot decode the received signal correctly, just stop transmitting in order to save energy. The remaining nodes adapt to the new scenario by a new space time code scheme. The achieved gain by utilizing an adaptive network was for a three hop system with a distance between each VAA of 1km and three nodes per VAA about 15dB. As far as an optimal alignment of the nodes cannot always be presumed, asymmetric networks have been investigated in [LWK08]. A closed form power allocation scheme for asymmetric non adaptive networks was derived with the help of several approximations for the outage probability per hop. Since practical multi-hop networks can be modeled at best by an asymmetric structure, power allocation schemes for such structures have to be investigated. The contribution of this paper is to investigate the gain of power allocation for adaptive asymmetric distributed MIMO networks. The problem is formulated as a convex optimization problem. The performance is investigated later by numerical calculations.
The remainder of this paper is organized as follows. In section 2 we introduce the system model for asymmetric MIMO multi-hop networks as well as the mathematical description. The optimization problem for the overall transmit power is given in section 3. Finally the achieved performance is analyzed and conclusions are drawn from that in section 4 and 5 respectively.

2 System Model

The setup for the asymmetric MIMO multi-hop system is shown in Fig. 1.

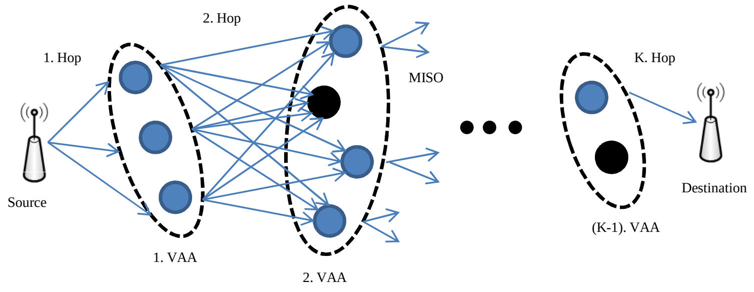

Figure 1: An asymmetric distributed MIMO multi-hop system

The source sends the information to the first VAA in the first time slot. Each node that has received the information successfully is considered to be active. The nodes that have not received or were not able to decode the information properly are considered to be in outage. In the next time slot the active nodes in a VAA adapt to the new setup of the VAA and transmit the data cooperatively according to a space time code, tailored for the number of active nodes. If all nodes within one VAA fail to decode the information, the VAA is considered to be in outage. In this case the whole e2e communication fails. This adaptive scheme is continued until the message reaches its destination. It is assumed that each relay node has only one antenna element, so that it communicates in half duplex mode. Decode and Forward (D&F) is presumed to be implemented in each node [LTW04]. Furthermore it is assumed that there is no exchange of data between the relays in a VAA. Since every node decodes the information separately and re-encodes it by using a fraction of a shared space time code, the communication in one VAA can be modeled as several multiple-input single-output (MISO) links. Data rate R and duration of transmission are assumed to be equal in the whole network. In all hops the total bandwidth denoted by W is used. In the sequel k indexes the current hop, t_k denotes the transmit nodes and r_k the receive nodes. The set of active transmit nodes in hop k is denoted by $\mathbb{T}'_k$ and the set of inactive nodes by $\tilde{\mathbb{T}}_k$ The pathloss between a sending node i and a receiving node j is denoted by $1/d^{\epsilon}_{k,j,i}$. Where $d_{k,j,i}$ denotes the distance between both nodes and ϵ denotes the pathloss exponent within a range of 2 to 5 as used for the most wireless channels. The Signal $\mathbf{S}_k \in \mathbb{C}^{t_k \times T_k}$ denotes the space time encoded signal of length T_k transmitted from the set of active nodes $\mathbb{T}'_k$ at hop k. The signal received by node j is defined by the following equation.

$$\mathbf{y}_{k,j} = \mathbf{h}_{k,j} \cdot \mathbf{\Lambda}_k \cdot \mathbf{S}_k + \mathbf{n}_{k,j} \ , \tag{1}$$

Where $\mathbf{\Lambda}_k$ corresponds to the diagonal matrix given by

$$\mathbf{\Lambda}_k = \mathrm{diag}\left\{ \sqrt{\frac{\mathcal{P}_{k,1}}{d^{\epsilon}_{k,1,j}}}, \cdots, \sqrt{\frac{\mathcal{P}_{k,t_k}}{d^{\epsilon}_{k,t_k,j}}} \right\} . \tag{2}$$

The gaussian noise vector with power spectral density N_0 is expressed by $\mathbf{n}_{k,j} \sim \mathcal{N}_C(0,\, N_0) \in \mathbb{C}^{1 \times T_k}$. The transmit power supplied by the ith node in the kth hop is expressed by $\mathcal{P}_{k,i}$. The channel from the t_k transmit nodes to the jth receive node at the kth hop is expressed as $\mathbf{h}_{k,j} \in \mathbb{C}^{1 \times t_k}$. Its elements $h_{k,i,j}$ obey the same uncorrelated Rayleigh fading statistics, i.e., they are complex zero-mean circular symmetric Gaussian distributed with variance 1. The capacity of an asymmetric MISO channel is given by

$$C_{k,j} = W \log\left(1 + \frac{1}{W N_0} \sum_{i \in \mathbb{T}'_k} g_{k,i,j} |h_{k,i,j}|^2 \right) \ , \tag{3}$$

with the co-factors $g_{k,j,i} = \mathcal{P}_{k,i}/d^{\epsilon}_{k,j,i}$. Note that $g_{k,j,i}$ corresponds to the squared diagonal elements of $\mathbf{\Lambda}_k$ in Eq. 2 [Doh03]. The probability that the link from the set of $\mathbb{T}'_k$ sending nodes to the j_k receiving nodes in hop k is in outage can be interpreted as that this link cannot support the required data rate R.

This can be expressed by $p_{\text{out},k,j}(\mathbb{T}'_k) = \Pr(R > C_{k,j})$. The following closed form expression was derived in [Doh03, p.65]

$$\begin{aligned}
p_{\text{out},k,j}(\mathbb{T}'_k) &= \text{Pr}(R > C_{k,j}) \\
&= \text{Pr}\left(\sum_{i\in\mathbb{T}'_k} g_{k,i,j}|h_{k,i,j}|^2 < \left(2^{\frac{R}{W}}-1\right) W N_0 \right) \\
&= \sum_{i\in\mathbb{T}'_k} K_i \left(1 - e^{-g_{k,i,j}^{-1} Q_k}\right) \\
\text{where} \quad K_i &= \prod_{\substack{i'\in\mathbb{T}'_k \\ i'\neq i}} \left[\frac{g_{k,i,j}}{g_{k,i,j}-g_{k,i',j}}\right] .
\end{aligned} \tag{4}$$

The parameters including data rate, bandwidth and noise are collected in the variable $Q_k = \left(2^{\frac{R}{W}} - 1\right) W N_0$.

3 Optimization Problem

The outage probability of node j in hop k depends on the outage probability $p_{out,k,j}(\mathbb{T}'_k)$, on the set of $\mathbb{T}'_k$ active nodes and on the set of $\tilde{\mathbb{T}}_k$ inactive nodes, which depends itself on the outage probability in hop $k-1$. Note that the intersection of $\mathbb{T}'_k$ and $\tilde{\mathbb{T}}_k$ is zero, so that

$$\mathbb{T}'_k \cap \tilde{\mathbb{T}}_k = 0, \quad \forall k \tag{5}$$

holds. Since the outage probabilities for the nodes in each VAA are not equal, no common distribution function can be applied here. For each VAA there exists 2^{t_k} different sets $\mathbb{T}'_k$ of transmitting nodes. The outage probability of a MISO system with a set of $\mathbb{T}'_k$ active nodes can be described by $\text{Pr}(\mathbb{T}'_k) \cdot p_{\text{out},k,j}(\mathbb{T}'_k)$ that concludes that the outage probability $P_{\text{out},k,j}$ can be expressed by the sum over all possible outage scenarios, i.e. the sum over all possible sets of active nodes $\mathbb{T}'_k$.

$$P_{\text{out},k,j} = \sum_{i=1}^{2^{t_k}} \prod_{a_i\in\mathbb{T}'_{k,i}} \left[1 - P_{\text{out},k-1,a_i}\right] \prod_{b_i\in\tilde{\mathbb{T}}_{k,i}} \left[P_{\text{out},k-1,b_i}\right] \cdot p_{\text{out},k,j}(\mathbb{T}'_{k,i}) \tag{6}$$

An outage occurs in one hop, if all nodes in the active VAA cannot decode the message correctly. Taking into account that the single outage probabilities can be unequal, the outage probability per hop has to be defined as

$$P_{\text{out},k} = \prod_{j=1}^{r_k} P_{\text{out},k,j} . \tag{7}$$

The e2e outage probability for the adaptive relaying transmission can be expressed as

$$P_{\text{e2e}} = \sum_{k=1}^{K} P_{\text{out},k}. \tag{8}$$

For the required total power for this transmission scheme follows

$$\mathcal{P}_{\text{total}} = \sum_{k=1}^{K} \sum_{i=1}^{t_k} \mathcal{P}_{k,i}(1 - P_{\text{out},k-1,i}). \tag{9}$$

Eq. 9 considers the inactive nodes to stop transmission if they are in outage. To optimize the transmit power in the network, we formulate the following optimization problem

$$\begin{aligned} &\text{minimize } \mathcal{P}_{\text{total}} = \sum_{k=1}^{K} \sum_{i=1}^{t_k} \mathcal{P}_{k,i}(1 - P_{\text{out},k-1,i}) \\ &\text{subject to } P_{\text{e2e}} = \sum_{k=1}^{K} P_{\text{out},k} \leq e \ . \end{aligned} \tag{10}$$

Generally the problem stated in Eq. 10 is not convex. It can be shown that for a low outage probability constraint this problem can assumed to be convex [BV04]. We solved this optimization problem numerically with the help of common optimization toolboxes.

4 Performance Evaluation

In this work we investigated the performance of the adaptive transmission scheme for an arbitrary generated multi-hop network. It is assumed that the e2e outage probability should meet a maximum constraint of $1\,\%$. The total bandwidth is $W = 5\,\text{MHz}$, the pathloss exponent ϵ is assumed to be 3 and the noise power spectral density is assumed to be $N_0 = -174\,\text{dBm/Hz}$. We assume a $K = 4$ hop multi-hop system. The distance between source and destination is $4\,\text{km}$. Fig. 2 represents the network setup. Nodes are represented by circles, the solid lines show which nodes are mapped to one VAA. How the mapping of nodes to VAAs is archived is out of the scope of this paper. To show the improvement of the adaptive network compared to the non adaptive one, proposed by [LWK08], Fig. 3 shows the archived curves for the required total power versus data rate in Mbps. As a reference Fig. 3 shows also the required power for SISO links over the given distance of $4\,\text{km}$. It can be observed that the adaptive network with optimized transmit power offers a significant saving of total power compared to the non adaptive structure. This can be interpreted by the capability of the adaptive scheme to adapt to a new network setup if some nodes are in outage. The non adaptive scheme has to rely on the worst link in the network. Only if all nodes in on VAA fail, the adaptive scheme can be considered to be in outage.

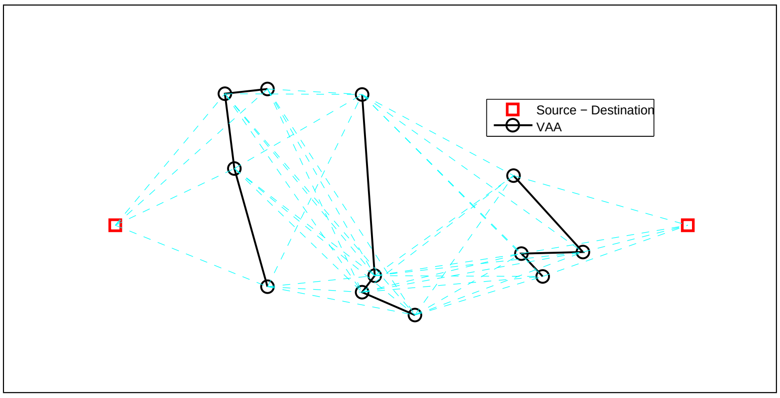

Figure 2: Setup of an asymmetric network with $K = 4$ hops and VAAs consisting of $t_k = 4$ nodes.

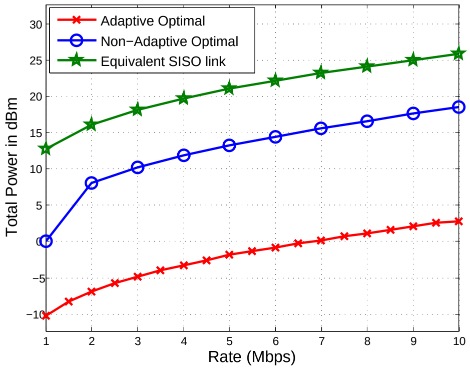

Figure 3: Total power consumption versus data rate for the asymmetric network with $K = 4$ hops and $t_k = 4$ nodes per hop.

5 Conclusions and Outlook

In this paper we introduced and investigated the performance of a power allocation scheme for adaptive asymmetric relay networks. It could be shown that power allocation in adaptive networks provides a huge potential to save overall transmit power. The solution for the optimal transmit power was obtained by common optimization tools and since there has no closed form been found yet, it is not possible to implement this scheme in practical setups. Since the performance of the proposed scheme is much higher than a non adaptive scheme, it is worth to spend further investigations in order to find a closed form solution that can be implemented in practical applications.

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