BER-based Power Allocation for Amplify-and-Forward and Decode-and-Forward Relaying Systems

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***Abstract*—In this paper, power allocation schemes are proposed for the total transmit power of the source and the relay in a triplet relaying system based on Bit Error Rate (BER) analysis. Considering Amplify-and-Forward (AF), Decode-and-Forward (DF) for relaying systems and direct transmission (DT) without relay, we perform a fair comparison with respect to their power consumption in order to achieve the same target BER at the destination. For coded systems, the relationship between the input and the output BERs of the decoder for specific channel codes is modeled by polynomials to facilitate the BER analysis. Additionally, the Feasible Relay Region (FRR) is investigated, which determines geographically when AF or DF outperforms DT.**

I. Introduction

Cooperative transmission with the help of relays nowadays attracts increasing interests by providing performance gain compared to the direct transmission (DT). Among the large amount of cooperative strategies, Amplify-and-Forward (AF) and Decode-and-Forward (DF) are most commonly in use. By AF, the received signal at the relay is simply amplified to the available power and forwarded, however the noise from the source-relay link is also amplified at the relay. By DF, the signal is first decoded to re-generate the source message, which is then forwarded to the destination. DF achieves extra coding gain but may suffer from error propagation caused by decoding errors at the relay. Since in a relaying system, both the source and the relay consume power for transmission, the question arises how much power should be allocated to the nodes to achieve a given Quality of Service (QoS). In [1] and [2], power allocation schemes have been proposed to maximize the capacity under a total transmit power constraint for uncoded AF and DF, respectively. In [3], a BER-based power allocation method has been proposed that minimizes the BER at the destination for uncoded AF for Rayleigh fading channels also subject to a total power consumption. The BER-based power allocation was discussed for coded DF in [4], but error-free decoding was considered at the relay due to the assumption of a perfect channel code. Adaptive relaying strategies for uncoded DF were also investigated with respect to BER-based power allocation in [5] and [6], where the relay is activated and transmits to the destination only in case of successful decoding at the relay.

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In this paper, we aim at minimizing the total transmit power subject to a target BER P_e^{tar} at the destination for AF and DF in both uncoded and coded scenarios. To this end, for coded systems the input-output BER behavior of the decoder for the applied channel codes must be modeled. Here we concentrate on AWGN channels with path-loss, which should be extended in the future for fading channels. Additionally, the optimized total power consumption is compared with the power required for DT to achieve the same P_e^{tar}. Since the optimized power varies with the relay position, this leads to the determination of the Feasible Relay Region (FRR) [4] in which AF or DF is superior to DT without relay.

The remainder of this paper is organized as follows. The system model is introduced and described in Section II. Our proposed power allocation scheme based on BER analysis is presented in detail for AF and DF in Section III and IV, respectively. In Section V, the performances of the proposed BER-based power allocation schemes are evaluated and compared for AF, DF and DT with respect to the total transmit power that is required to achieve a target BER P_e^{tar} at the destination. Section VI concludes this paper.

II. System Description

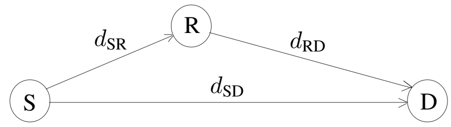

Fig. 1. A triplet relaying system model with source S, relay R and destination D. The distances of the SR, RD and SD links are denoted as d_{SR}, d_{RD} and d_{SD}, respectively.

We consider a triplet relaying system, where the communication from source S to destination D is supported by one relay R, as illustrated in Fig. 1. The system exploits a half-duplex transmission mode, i.e., S transmits in the first time slot to R and D and R transmits to D in the second time slot. The distances for the source-relay (SR), the relay-destination (RD) and the source-destination (SD) links are defined as d_{SR},

d_{RD} and d_{SD}, respectively. By denoting the transmit signals at S and R by x_{S} and x_{R}, the corresponding receive signals at R and D are given by

$$y_{\text{SR}} = d_{\text{SR}}^{-\frac{\alpha}{2}} x_{\text{S}} + n_{\text{SR}} \tag{1a}$$

$$y_{\text{SD}} = d_{\text{SD}}^{-\frac{\alpha}{2}} x_{\text{S}} + n_{\text{SD}} \tag{1b}$$

$$y_{\text{RD}} = d_{\text{RD}}^{-\frac{\alpha}{2}} x_{\text{R}} + n_{\text{RD}} \; . \tag{1c}$$

Here, n_{SR}, n_{SD} and n_{RD} describe the additive Gaussian noise terms on the different links with variance σ_n^2 and α is the path-loss exponent. With $\mathcal{P}_{\text{S}} = \text{E}\{|x_{\text{S}}|^2\}$ and $\mathcal{P}_{\text{R}} = \text{E}\{|x_{\text{R}}|^2\}$ denoting the transmit powers of S and R, the signal to noise ratios (SNRs) of the three AWGN links are

$$\gamma_{\text{SR}} = \frac{\mathcal{P}_{\text{S}} d_{\text{SR}}^{-\alpha}}{\sigma_n^2}, \; \gamma_{\text{RD}} = \frac{\mathcal{P}_{\text{R}} d_{\text{RD}}^{-\alpha}}{\sigma_n^2} \; \text{ and } \; \gamma_{\text{SD}} = \frac{\mathcal{P}_{\text{S}} d_{\text{SD}}^{-\alpha}}{\sigma_n^2} \tag{2}$$

and the total transmit power is given by $\mathcal{P}_{\text{tot}} = \mathcal{P}_{\text{S}} + \mathcal{P}_{\text{R}}$. When the SD link is considered at the destination, Maximum Ratio Combining (MRC) by

$$\tilde{x}_{\text{D,MRC}} = d_{\text{SD}}^{-\frac{\alpha}{2}} y_{\text{SD}} + d_{\text{RD}}^{-\frac{\alpha}{2}} y_{\text{RD}} \tag{3}$$

is used to exploit maximum receive SNR at D, otherwise the transmission degrades to a simple two-hop system. Both cases are investigated and analyzed separately in the sequel. For AWGN channels with receive SNR γ the BER for M-QAM transmission can be approximated as [7]

$$P_e(\gamma, M, R_{\text{C}}) = \frac{2(1-1/\sqrt{M})}{\text{ld}(M)} \text{erfc}\left(\sqrt{\frac{3\gamma R_{\text{C}}}{2(M-1)}}\right). \tag{4}$$

Note that for uncoded systems the code rate is $R_{\text{C}} = 1$.

For direct transmission, i.e., system without relay, the total power equals the transmit power of the source $\mathcal{P}_{\text{S}} = \mathcal{P}_{\text{tot}}$ and the SNR is given by γ_{SD}. In order to achieve a given target BER P_e^{tar} at the destination for uncoded transmission, the required target SNR γ^{tar} can be calculated based on (4)

$$\gamma^{\text{tar}} = \frac{2(M-1)}{3R_{\text{C}}} \left[\text{erfc}^{-1}\left(\frac{\text{ld}(M)}{2(1-1/\sqrt{M})} P_e^{\text{tar}}\right)\right]^2 . \tag{5}$$

Thus, the required transmit power $\mathcal{P}_{\text{S}}$ to achieve this target BER can be determined. For coded systems with code rate $0 < R_{\text{C}} < 1$, the error probability P_e in (4) represents the bit error rate of the code bits at the input of the decoder. This CBER (code bit error rate) is later on used to derive a relation between the receive SNR γ and the BER of the information bits at the output of the corresponding decoder. Although this relation is derived for hard decisions at the input and the output of the decoder by the mapping of CBER onto BER, soft-input decoders are used in general. Based on this mapping, it is again possible to determine the required transmit power to a achieve a given BER at the decoder output. In order to achieve the same target BER for relaying systems with minimum total transmit power, we present the corresponding optimization problems for AF and DF in the subsequent sections.

III. Power Allocation for AF Relaying

A. SNR for AF Relaying

For AF, the received signal at R is simply amplified without decoding or other signal processing and then forwarded to D. In order to meet the power budget $\mathcal{P}_{\text{R}}$ of the relay, the transmit signal is amplified by

$$\beta = \sqrt{\frac{\mathcal{P}_{\text{R}}}{\mathcal{P}_{\text{S}} d_{\text{SR}}^{-\alpha} + \sigma_n^2}} \tag{6}$$

leading to the transmit signal $x_{\text{R}} = \beta y_{\text{SR}}$. Thus, the receive SNR at D for AF without SD link is given by [8]

$$\begin{aligned}\gamma_{\text{D}}^{\text{AF}} =& \frac{\beta^2 \mathcal{P}_{\text{S}} d_{\text{SR}}^{-\alpha} d_{\text{RD}}^{-\alpha}}{\beta^2 \sigma_n^2 d_{\text{RD}}^{-\alpha} + \sigma_n^2} = \frac{\mathcal{P}_{\text{S}} \mathcal{P}_{\text{R}} d_{\text{SR}}^{-\alpha} d_{\text{RD}}^{-\alpha} \sigma_n^{-2}}{\mathcal{P}_{\text{S}} d_{\text{SR}}^{-\alpha} + \mathcal{P}_{\text{R}} d_{\text{RD}}^{-\alpha} + \sigma_n^2} \\ =& \frac{\gamma_{\text{SR}} \gamma_{\text{RD}}}{\gamma_{\text{SR}} + \gamma_{\text{RD}} + 1} \; .\end{aligned} \tag{7}$$

Obviously, this receive SNR $\gamma_{\text{D}}^{\text{AF}}$ depends on the transmit powers $\mathcal{P}_{\text{S}}$ and $\mathcal{P}_{\text{R}}$ on both component channels. Additionally, the SD link and the RD link have an equivalent impact, i.e., $\gamma_{\text{D}}^{\text{AF}}$ is symmetric with respect to the component SNRs γ_{SR} and γ_{RD}.

When the SD link is considered at D, the corresponding receive SNR after MRC is given by the summation of the SNRs from the S-R-D link and the SD link [8]

$$\gamma_{\text{D,MRC}}^{\text{AF}} = \frac{\gamma_{\text{SR}} \gamma_{\text{RD}}}{\gamma_{\text{SR}} + \gamma_{\text{RD}} + 1} + \gamma_{\text{SD}} \; . \tag{8}$$

B. Uncoded AF without SD Link

The power optimization problem that minimizes the total transmit power $\mathcal{P}_{\text{tot}}$ subject to a given target SNR γ^{tar} for uncoded AF relaying without SD link can be formulated as

$$\min \; \mathcal{P}_{\text{tot}} \tag{9a}$$

$$\text{s.t. } \gamma_{\text{D}}^{\text{AF}} \geq \gamma^{\text{tar}} \tag{9b}$$

$$\mathcal{P}_{\text{S}}, \mathcal{P}_{\text{R}} > 0. \tag{9c}$$

Note that $\gamma_{\text{D}}^{\text{AF}}$ is defined in (7) and depends on the power components $\mathcal{P}_{\text{S}}$ and $\mathcal{P}_{\text{R}}$. The constraints (9b) and (9c) guarantee that the target SNR γ_{tar} can be achieved and both power components are positive, respectively.

Using the fact that the minimum total transmit power is required for $\gamma_{\text{D}}^{\text{AF}} = \gamma^{\text{tar}}$ in (7), the relay power $\mathcal{P}_{\text{R}}$ can be represented by $\mathcal{P}_{\text{S}}$ as

$$\mathcal{P}_{\text{R}} = \frac{\mathcal{P}_{\text{S}} d_{\text{RD}}^{\alpha} \sigma_n^2 \gamma^{\text{tar}} + d_{\text{SR}}^{\alpha} d_{\text{RD}}^{\alpha} \sigma_n^4 \gamma^{\text{tar}}}{\mathcal{P}_{\text{S}} - d_{\text{SR}}^{\alpha} \sigma_n^2 \gamma^{\text{tar}}}. \tag{10}$$

Thus the optimization problem (9) reads

$$\min_{\mathcal{P}_{\text{S}}} \; \mathcal{P}_{\text{S}} + \frac{\mathcal{P}_{\text{S}} d_{\text{RD}}^{\alpha} \sigma_n^2 \gamma^{\text{tar}} + d_{\text{SR}}^{\alpha} d_{\text{RD}}^{\alpha} \sigma_n^4 \gamma^{\text{tar}}}{\mathcal{P}_{\text{S}} - d_{\text{SR}}^{\alpha} \sigma_n^2 \gamma^{\text{tar}}} \tag{11a}$$

$$\text{s.t. } \mathcal{P}_{\text{S}} > d_{\text{SR}}^{\alpha} \sigma_n^2 \gamma^{\text{tar}}. \tag{11b}$$

The constraint (11b) guarantees, that both $\mathcal{P}_{\text{S}}$ and $\mathcal{P}_{\text{R}}$ are larger than 0 as in (9c). The optimized value for $\mathcal{P}_{\text{S}}$ is calculated by setting the first derivative of (11a) with respect to $\mathcal{P}_{\text{S}}$ equal to

0, which results in a quadratic equation possessing two roots. The root that fulfills the constraint (11b) is given by

$$\mathcal{P}_{\mathrm{S,opt}} = d_{\mathrm{SR}}^{\alpha}\sigma_n^2\gamma^{\mathrm{tar}} + \sqrt{d_{\mathrm{SR}}^{\alpha}d_{\mathrm{RD}}^{\alpha}\sigma_n^4\gamma^{\mathrm{tar}}(1+\gamma^{\mathrm{tar}})} \tag{12}$$

and denotes the optimum power for S. Using this solution, the corresponding optimum value $\mathcal{P}_{\mathrm{R,opt}}$ is achieved by (10).

C. Uncoded AF with SD Link

When the SD link is considered, the receive SNR after MRC at D is $\gamma_{\mathrm{D,MRC}}^{\mathrm{AF}}$ (8). Beyond that, the optimization problem follows the same manner as for AF without SD link. The corresponding expression of $\mathcal{P}_{\mathrm{R}}$ with respect to $\mathcal{P}_{\mathrm{S}}$ is determined from (8) as

$$\mathcal{P}_{\mathrm{R}} = \frac{d_{\mathrm{RD}}^{\alpha}\left(d_{\mathrm{SD}}^{\alpha}d_{\mathrm{SR}}^{\alpha}\sigma_n^4\gamma^{\mathrm{tar}} + \mathcal{P}_{\mathrm{S}}\sigma_n^2\left(d_{\mathrm{SD}}^{\alpha}\gamma^{\mathrm{tar}} - d_{\mathrm{SR}}^{\alpha}\right) - \mathcal{P}_{\mathrm{S}}^2\right)}{\mathcal{P}_{\mathrm{S}}\left(d_{\mathrm{SD}}^{\alpha}+d_{\mathrm{SR}}^{\alpha}\right) - d_{\mathrm{SD}}^{\alpha}d_{\mathrm{SR}}^{\alpha}\sigma_n^2\gamma^{\mathrm{tar}}} \,. \tag{13}$$

Using this representation of $\mathcal{P}_{\mathrm{R}}$, the optimization problem can be solved as in the previous subsection. The optimum value for $\mathcal{P}_{\mathrm{S}}$ is derived as

$$\begin{aligned}\mathcal{P}_{\mathrm{S,opt}} = &\frac{\gamma^{\mathrm{tar}}\sigma_n^2 d_{\mathrm{SD}}^{\alpha}d_{\mathrm{SR}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}+d_{\mathrm{SR}}^{\alpha}} + \\ &\frac{\sigma_n^2 d_{\mathrm{SD}}^{\alpha}d_{\mathrm{SR}}^{\frac{\alpha}{2}}d_{\mathrm{RD}}^{\frac{\alpha}{2}}}{d_{\mathrm{SD}}^{\alpha}+d_{\mathrm{SR}}^{\alpha}}\sqrt{\frac{\gamma^{\mathrm{tar}}d_{\mathrm{SR}}^{\alpha}+\gamma^{\mathrm{tar}}(1+\gamma^{\mathrm{tar}})d_{\mathrm{SD}}^{\alpha}}{d_{\mathrm{SD}}^{\alpha}+d_{\mathrm{SR}}^{\alpha}-d_{\mathrm{RD}}^{\alpha}}}\end{aligned} \tag{14}$$

and is again given by the root of the quadratic equation achieved by the first derivation of $\mathcal{P}_{\mathrm{tot}}$. This solution guarantees positive power components. The corresponding optimum value $\mathcal{P}_{\mathrm{R,opt}}$ can be calculated using $\mathcal{P}_{\mathrm{S,opt}}$ in (13), which finalizes the optimization.

D. Coded AF with and without SD Link

Considering coded systems with a specific channel code $\mathcal{C}$ of rate R_{C}, a relation between the channel output (SNR or CBER at decoder input) and the BER after decoding is necessary in order to determine the required transmit power for a given target BER. Although soft-input decoders are used in general, the input-output behavior of the decoder is modeled by the relation of the average CBER and the average BER assuming hard decisions. This input-output BER characteristic of the decoder can be described by a function $P_e^{\mathrm{out}} = f_{\mathcal{C}}(P_e^{\mathrm{in}})$, where P_e^{in} and P_e^{out} denote the average input CBER and output BER of the decoder, respectively. $f_{\mathcal{C}}$ essentially maps the channel errors in (4) to errors at the output of the decoder and is treated as an inherent property of $\mathcal{C}$. This function can be determined by simulations and can be properly approximated by a polynomial.

Subsequently, this polynomial-based modeling is illustrated for LDPC codes of rate $R_{\mathrm{C}} = 0.5$, varying codeword length $N = 200, 400, 800$ and 2000, 16-QAM modulation and decoding by sum-product algorithm using 100 iterations. The relationship between the average input CBER P_e^{in} and the average output BER P_e^{out} of the decoder achieved by simulations is shown in Fig. 2 (marked curves).

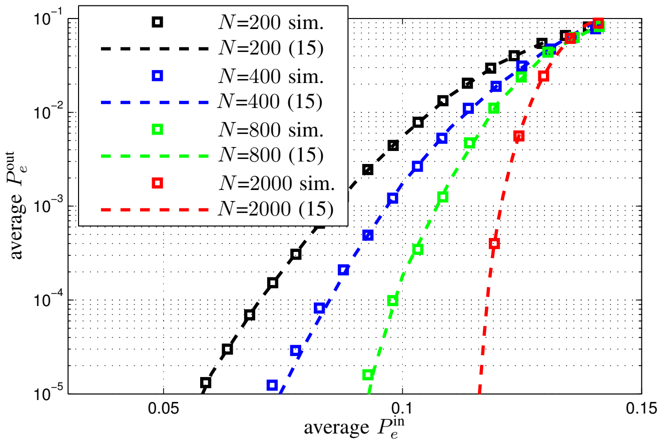

Fig. 2. Average input-output BER characteristic for an LDPC decoder of $R_{\mathrm{C}} = 0.5$ and codeword length $N = 200, 400, 800$ and 2000 in an AWGN channel with 16-QAM modulation.

In order to model the input-output BER behavior, the Lagrange interpolation method for polynomial approximation is used [9]

$$P_e^{\mathrm{out}} = f_{\mathcal{C}}(P_e^{\mathrm{in}}) = \sum_{i=1}^{K}\frac{\prod_{j\neq i}\left(P_e^{\mathrm{in}} - P_e^{\mathrm{in}}(j)\right)}{\prod_{j\neq i}\left(P_e^{\mathrm{in}}(i) - P_e^{\mathrm{in}}(j)\right)}P_e^{\mathrm{out}}(i) \tag{15}$$

with index $j = 1, 2, \ldots, K$. For a specific code the Lagrange polynomial is generated by inserting K simulation points $P_e^{\mathrm{out}}(i) = f_{\mathcal{C}}(P_e^{\mathrm{in}}(i))$, as marked for different curves in Fig. 2, into (15). Note that the degree of the polynomial achieved by Lagrange interpolation equals $K-1$. The approximated polynomials achieved by (15) are also shown in the figure in order to compare with the simulations. Irrespective of the codeword length, the approximations are well-suited to the simulations.

As $f_{\mathcal{C}}$ is a monotonic function, the model enables to determine for a given target BER P_e^{tar} at the output of the decoder, the required input CBER $P_e^{\mathrm{in}} = f_{\mathcal{C}}^{-1}(P_e^{\mathrm{tar}})$. Using (5), the corresponding target SNR calculates as

$$\gamma^{\mathrm{tar}} = \frac{2(M-1)}{3R_{\mathrm{C}}}\left[\mathrm{erfc}^{-1}\left(\frac{\mathrm{ld}(M)}{2(1-1/\sqrt{M})}f_{\mathcal{C}}^{-1}\left(P_e^{\mathrm{tar}}\right)\right)\right]^2 \,. \tag{16}$$

Thus, a mapping between the transmit power and the achieved performance after decoding has been achieved. Consequently, the optimization for coded AF with and without SD link can be solved as in the previous subsections using (16).

IV. Power Allocation for DF Relaying

A. SNR for DF Relaying

For DF relaying, the received message at R is first decoded or quantized to recover the transmitted information. The quality of this decision is directly related to the receive SNR at R given by γ_{SR} in (2). However, as decision errors are possible at the relay, it is not possible to determine generally the SNR at the receiver, neither with or without SD link. In order to achieve a SNR expression also for DF, one may assume error free decisions at R. Thus, the receive SNR at D for DF without SD link is directly given by γ_{RD} (2). Similarly, the total receive SNR equals the sum of the SNRs of the SD link and the RD link if the direct link is considered by MRC at D [10]

$$\gamma_{\text{D,MRC}}^{\text{DF}} = \gamma_{\text{RD}} + \gamma_{\text{SD}} = \frac{\mathcal{P}_{\text{R}} d_{\text{RD}}^{-\alpha}}{\sigma_n^2} + \frac{\mathcal{P}_{\text{S}} d_{\text{SD}}^{-\alpha}}{\sigma_n^2}. \quad (17)$$

Please notice, that more practical approximations of the receive SNR for error-prone decoding at R can be achieved and modified MRC schemes are possible [15]. Here, we restrict to these simple approximations for the receive SNR and common MRC.

B. Uncoded DF without SD Link

The BER P_e^{DF} at D for DF without SD link for BPSK and QPSK can be expressed and approximated as [11]

$$\begin{aligned} P_e^{\text{DF}} &= P_e^{\text{SR}}\left(1 - P_e^{\text{RD}}\right) + P_e^{\text{RD}}\left(1 - P_e^{\text{SR}}\right) && (18a)\\ &= P_e^{\text{SR}} + P_e^{\text{RD}} - 2P_e^{\text{SR}}P_e^{\text{RD}} && (18b)\\ &\approx P_e^{\text{SR}} + P_e^{\text{RD}}, && (18c)\end{aligned}$$

where P_e^{SR} and P_e^{RD} denote the BERs introduced by the SR link and the RD link, respectively. As shown by (18b) the BER at D corresponds to the sum of BERs of both component channels minus the impact of double error events. The term $2P_e^{\text{SR}}P_e^{\text{RD}}$ is quite small compared to $P_e^{\text{SR}}+P_e^{\text{RD}}$. By neglecting this term, the simplified upper bound (18c) can be achieved. Note that the BER expression (18a) does not hold for general M-QAM as outer symbols of the constellation have smaller error probabilities compared to inner symbols. An exact derivation of the BER expression at D for DF with M-QAM that distinguishes inner and outer symbols can be found in [12]. In this paper, we stick to the BER expression in (18) also for higher modulation schemes for simplicity.

Taking the BER approximation (18c), the power optimization problem can be formulated as

$$\begin{aligned} &\min \mathcal{P}_{\text{tot}} && (19a)\\ &\text{s.t. } P_e^{\text{SR}} + P_e^{\text{RD}} \leq P_e^{\text{tar}} && (19b)\\ &\mathcal{P}_{\text{S}}, \mathcal{P}_{\text{R}} > 0\ . && (19c)\end{aligned}$$

The error probabilities for the component channels are given by (4)

$$P_e^{\text{SR}} = P_e(\gamma_{\text{SR}}, M, 1) \text{ and } P_e^{\text{RD}} = P_e(\gamma_{\text{RD}}, M, 1) \quad (20)$$

and depend directly on the power components $\mathcal{P}_{\text{S}}$ and $\mathcal{P}_{\text{R}}$ by the corresponding SNRs γ_{SR} and γ_{RD}. In order to solve the optimization problem (19), we represent $\mathcal{P}_{\text{S}}$ and $\mathcal{P}_{\text{R}}$ by the error probabilities P_e^{SR} and P_e^{RD} by using (5)

$$\mathcal{P}_{\text{S}} = d_{\text{SR}}^{\alpha}\sigma_n^2 \frac{2(M-1)}{3}\left[\text{erfc}^{-1}\left(\frac{\text{ld}(M)}{2(1-1/\sqrt{M})}P_e^{\text{SR}}\right)\right]^2 \quad (21a)$$

$$\mathcal{P}_{\text{R}} = d_{\text{RD}}^{\alpha}\sigma_n^2 \frac{2(M-1)}{3}\left[\text{erfc}^{-1}\left(\frac{\text{ld}(M)}{2(1-1/\sqrt{M})}P_e^{\text{RD}}\right)\right]^2 . \quad (21b)$$

Thus, the required transmit powers to achieve given BERs P_e^{SR} and P_e^{RD} can be determined. By reaching the target BER in (19b), i.e., $P_e^{\text{DF}} = P_e^{\text{SR}} + P_e^{\text{RD}} = P_e^{\text{tar}}$, the optimization problem (19) is reformulated as

$$\begin{aligned}\min_{P_e^{\text{SR}}} \ & d_{\text{SR}}^{\alpha}\sigma_n^2 \frac{2(M-1)}{3}\left[\text{erfc}^{-1}\left(\frac{\text{ld}(M)}{2(1-1/\sqrt{M})}P_e^{\text{SR}}\right)\right]^2 + \\ & d_{\text{RD}}^{\alpha}\sigma_n^2 \frac{2(M-1)}{3}\left[\text{erfc}^{-1}\left(\frac{\text{ld}(M)}{2(1-1/\sqrt{M})}\left(P_e^{\text{tar}} - P_e^{\text{SR}}\right)\right)\right]^2 . \end{aligned} \quad (22)$$

Note that the minimization in (22) is executed with respect to P_e^{SR}. Thus, the optimization problem can be interpreted as finding the optimum value $P_{e,\text{opt}}^{\text{SR}}$ in the range $[0, P_e^{\text{tar}}]$ such that the total power is minimized.

In order to solve (22), its first derivative is calculated with respect to P_e^{SR} where the following derivation formula for the inverse complementary error function is used [13]

$$\frac{\mathrm{d}}{\mathrm{d}\,x}\text{erfc}^{-1}(x) = -\frac{1}{2}\sqrt{\pi}\mathrm{e}^{\left[\text{erfc}^{-1}(x)\right]^2}. \quad (24)$$

By setting this derivative to 0, equation (23a) on the top of the next page for P_e^{SR} is achieved. By solving this equation numerically, the optimum value $P_{e,\text{opt}}^{\text{SR}}$ can be calculated. Note that the left side and the right side of (23a) are symmetric and $P_{e,\text{opt}}^{\text{SR}}$ is the crosspoint of the two parts. The corresponding optimized values $\mathcal{P}_{\text{S,opt}}$ and $\mathcal{P}_{\text{R,opt}}$ can be calculated by using $P_{e,\text{opt}}^{\text{SR}}$ in (21). Additionally, when R has the same distance to S and D, i.e., $d_{\text{SR}} = d_{\text{RD}}$, it is easy to observe that the optimized value is achieved when $P_{e,\text{opt}}^{\text{SR}} = P_e^{\text{tar}}/2$, which results in equal power allocation $\mathcal{P}_{\text{S}} = \mathcal{P}_{\text{R}}$.

C. Uncoded DF with SD Link

When DF with SD link is considered, MRC in (3) suffers from wrong decisions at R as different symbols have been transmitted by S and R, i.e., $x_{\text{R}} \neq x_{\text{S}}$. Consequently, the errors made at R will influence the probability density function of the received signal at D after MRC. This becomes extremely severe when y_{RD} has a stronger impact on the output $\tilde{x}_{\text{D,MRC}}$ than y_{SD}, which is the case, if R is located close to D. In order to overcome this drawback of DF, modified versions of MRC have been proposed in [11], [14], [15]. However, in this paper common MRC as given in (3) is used for DF with SD link for simplicity. In another approach one could also analyze the BER at D including the impact of decision errors at R as derived in [12] for M-QAM. As this leads to rather complicated BER expressions, we will also not consider this approach here.

Taking these restrictions into account, the following investigation for DF with SD link is limited to relay networks where S, R and D are on a line (i.e., $d_{\text{SR}} + d_{\text{RD}} = d_{\text{SD}}$) leading to relatively simple BER approximations. When R is moving towards D, d_{SR} is increasing while d_{RD} is decreasing. By (3) the coefficients of the maximum ratio combiner depend on the path-loss effects $d_{\text{SD}}^{-\frac{\alpha}{2}}$ and $d_{\text{RD}}^{-\frac{\alpha}{2}}$ of the SD link and the RD link, respectively. Thus, the influence of the SD link becomes less important when R is close to D and correspondingly the performance of R dominates also the performance of D. However, if R is close to D its error probability is relatively

$$\frac{d_{\mathrm{SR}}^{\alpha}}{d_{\mathrm{RD}}^{\alpha}}\mathrm{erfc}^{-1}\!\left(\frac{\mathrm{ld}(M)}{2(1\text{-}1/\sqrt{M})}P_e^{\mathrm{SR}}\right)\mathrm{e}^{\left[\mathrm{erfc}^{-1}\left(\frac{\mathrm{ld}(M)}{2(1\text{-}1/\sqrt{M})}P_e^{\mathrm{SR}}\right)\right]^2}=\mathrm{erfc}^{-1}\!\left(\frac{\mathrm{ld}(M)}{2(1\text{-}1/\sqrt{M})}\left(P_e^{\mathrm{tar}}\text{-}P_e^{\mathrm{SR}}\right)\right)\mathrm{e}^{\left[\mathrm{erfc}^{-1}\left(\frac{\mathrm{ld}(M)}{2(1\text{-}1/\sqrt{M})}\left(P_e^{\mathrm{tar}}\text{-}P_e^{\mathrm{SR}}\right)\right)\right]^2} \tag{23a}$$

$$\frac{d_{\mathrm{SR}}^{\alpha}}{d_{\mathrm{RD}}^{\alpha}}\left(1\text{-}\left(\frac{d_{\mathrm{RD}}}{d_{\mathrm{SD}}}\right)^{\alpha}\right)\mathrm{erfc}^{-1}\!\left(\frac{\mathrm{ld}(M)}{2(1\text{-}1/\sqrt{M})}P_e^{\mathrm{SR}}\right)\mathrm{e}^{\left[\mathrm{erfc}^{-1}\left(\frac{\mathrm{ld}(M)}{2(1\text{-}1/\sqrt{M})}P_e^{\mathrm{SR}}\right)\right]^2}=\mathrm{erfc}^{-1}\!\left(\frac{\mathrm{ld}(M)}{2(1\text{-}1/\sqrt{M})}\left(P_e^{\mathrm{tar}}\text{-}P_e^{\mathrm{SR}}\right)\right)\mathrm{e}^{\left[\mathrm{erfc}^{-1}\left(\frac{\mathrm{ld}(M)}{2(1\text{-}1/\sqrt{M})}\left(P_e^{\mathrm{tar}}\text{-}P_e^{\mathrm{SR}}\right)\right)\right]^2} \tag{23b}$$

high. Consequently, when R is near to D, the SD link has only a minor impact on the decision at D. Based on this discussion we can still use the BER approximation in (18) for DF with SD link, i.e., $P_e^{\mathrm{DF}} \approx P_e^{\mathrm{SR}} + P_e^{\mathrm{RD}}$.

In Fig. 3 the BER approximation (18c) is compared with the corresponding simulations for both uncoded and coded DF with and without SD link. A triplet relaying system is considered where S, R and D are on one line and the distance proportion is $\frac{d_{\mathrm{SR}}}{d_{\mathrm{SD}}} = 0.3$ as an example. The path-loss exponent is set to $\alpha = 4$ and AWGN channels are assumed for all links. For coded DF, the same LDPC code of rate $R_{\mathrm{C}} = 0.5$ and codeword length $N = 2000$ is used at S and R with 16-QAM modulated symbols. Note that E_b is defined as the total transmit energy per information bit. Here, equal power allocation with $\mathcal{P}_{\mathrm{S}} = \mathcal{P}_{\mathrm{R}}$ was assumed. It can be observed, that the used approximation for the BER is very precise for this scenarios. Note, that the SNR gain for systems with SD link denotes the impact of the additional received signal at D.

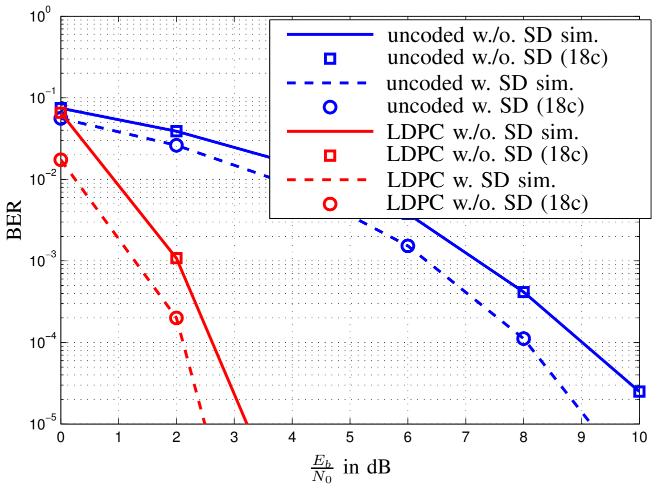

Fig. 3. BER obtained by simulation and by approximation (18c) for uncoded and coded DF with and without SD link. R joins the line between S and D with $\frac{d_{\mathrm{SR}}}{d_{\mathrm{SD}}} = 0.3$.

When the SD link is considered in the system, the receive SNRs at R and D are γ_{SR} and $\gamma_{\mathrm{D,MRC}}^{\mathrm{DF}}$, as defined in (2) and (17), respectively. To exploit the BER constraint in the optimization problem (19), we use γ_{SR} and $\gamma_{\mathrm{D,MRC}}^{\mathrm{DF}}$ in (20), to represent the power components $\mathcal{P}_{\mathrm{S}}$ and $\mathcal{P}_{\mathrm{R}}$ with respect to the BERs P_e^{SR} and P_e^{RD} as

$$\mathcal{P}_{\mathrm{S}} = d_{\mathrm{SR}}^{\alpha}\sigma_n^2\frac{2(M{-}1)}{3R_{\mathrm{C}}}\left[\mathrm{erfc}^{-1}\!\left(\frac{\mathrm{ld}(M)}{2(1-1/\sqrt{M})}P_e^{\mathrm{SR}}\right)\right]^2 \tag{25a}$$

$$\begin{aligned}\mathcal{P}_{\mathrm{R}} &= d_{\mathrm{RD}}^{\alpha}\sigma_n^2\frac{2(M{-}1)}{3R_{\mathrm{C}}}\left[\mathrm{erfc}^{-1}\!\left(\frac{\mathrm{ld}(M)}{2(1-1/\sqrt{M})}P_e^{\mathrm{RD}}\right)\right]^2 - \mathcal{P}_{\mathrm{S}}\left(\frac{d_{\mathrm{RD}}}{d_{\mathrm{SD}}}\right)^{\alpha}\\ &= d_{\mathrm{RD}}^{\alpha}\sigma_n^2\frac{2(M{-}1)}{3R_{\mathrm{C}}}\left[\mathrm{erfc}^{-1}\!\left(\frac{\mathrm{ld}(M)}{2(1-1/\sqrt{M})}P_e^{\mathrm{RD}}\right)\right]^2 - \\ &\quad\left(\frac{d_{\mathrm{SR}}d_{\mathrm{RD}}}{d_{\mathrm{SD}}}\right)^{\alpha}\sigma_n^2\frac{2(M{-}1)}{3R_{\mathrm{C}}}\left[\mathrm{erfc}^{-1}\!\left(\frac{\mathrm{ld}(M)}{2(1-1/\sqrt{M})}P_e^{\mathrm{SR}}\right)\right]^2\end{aligned} \tag{25b}$$

Note that the term $-\mathcal{P}_{\mathrm{S}}\left(\frac{d_{\mathrm{RD}}}{d_{\mathrm{SD}}}\right)^{\alpha}$ in the expression of $\mathcal{P}_{\mathrm{R}}$ is due to the presence of the SD link. This term points out the possible reduction of transmit power $\mathcal{P}_{\mathrm{R}}$ due to the consideration of the SD link.

Finally, the optimization problem (19) can be reformulated to minimize $\mathcal{P}_{\mathrm{tot}}$ using the expressions of $\mathcal{P}_{\mathrm{S}}$ and $\mathcal{P}_{\mathrm{R}}$ in (25). As for uncoded DF without SD link, the first derivative of $\mathcal{P}_{\mathrm{tot}}$ is calculated with respect to P_e^{SR} and set to 0 to find $P_{e,\mathrm{opt}}^{\mathrm{SR}}$, as shown in (23b) on the top of this page. By comparing (23a) and (23b) for DF with and without SD link, it can be observed that the influence of SD link to the power allocation scheme is an extra distance-dependent component $\left(1-\left(\frac{d_{\mathrm{RD}}}{d_{\mathrm{SD}}}\right)^{\alpha}\right)$. This will not complicate the optimization process and thus can be solved numerically as before.

D. Coded DF with and without SD Link

For coded DF without SD link, we again adopt the modeling method of the applied channel code using polynomials to represent the input-output BER property of the decoder, as is used for coded AF. Note that since the message is decoded at both R and D for DF, the modeling polynomial function f_C is imposed on the SR link and the RD link separately, i.e.,

$$P_e^{\mathrm{SR}} = f_C(P_e(\gamma_{\mathrm{SR}}, M, R_{\mathrm{C}})) \tag{26a}$$

$$P_e^{\mathrm{RD}} = f_C(P_e(\gamma_{\mathrm{RD}}, M, R_{\mathrm{C}}))\,. \tag{26b}$$

When the SD link is considered, the involved SNR γ_{RD} in (26b) is substituted with $\gamma_{\mathrm{D,MRC}}^{\mathrm{DF}}$. Note that the optimization problem for coded DF remains unchanged as for uncoded DF except for the inclusion of the polynomial function f_C as shown in (26). Since f_C is monotonically increasing, the optimization can be again solved as described before.

V. Performance Evaluation

A. System Description

For the performance evaluation, AWGN channels are assumed for all links with noise power density $N_0 = -174\,\text{dBm/Hz}$ and bandwidth $W = 1\text{MHz}$, leading to $\sigma_n^2 = N_0 W$. The path-loss exponent α is ranged from 2 to 5. In a Cartesian coordinate system with a metric unit, S is situated at $(0,0)$ and D at $(0,1000)$. The position of R is determined by (x,y), $-1000 \le x \le 1000$, $0 \le y \le 1000$. In order to obtain a fair comparison, 16-QAM is adapted for relaying systems when 4-QAM is adapted for the direct transmission without relay due to the half-duplex constraint. LDPC codes with $R_\text{C} = 0.5$ and codeword length $N = 2000$ with a maximum of 100 iterations are used for coded systems.

B. Total Power for AF and DF without SD Link

Let $\mathcal{P}_\text{tot}^\text{AF}$, $\mathcal{P}_\text{tot}^\text{DF}$ and $\mathcal{P}_\text{tot}^\text{DT}$ denote the total transmit power after optimization for AF, DF and DT, respectively. For both uncoded and coded systems without SD link the optimized power $\mathcal{P}_\text{tot}^\text{AF}$ and $\mathcal{P}_\text{tot}^\text{DF}$ are drawn in Fig. 4 with $\mathcal{P}_\text{tot}^\text{DT}$ as reference. As expected, both AF and DF require the minimum total transmit power when R is located in the middle between S and D, and DF is superior to AF. The superiority of DF degrades as R moves from the middle to S or D. Additionally, the coded systems save power tremendously, e.g., 9 dB for DF when R is at $(0,500)$, compared to the uncoded system on the sacrifice of halved spectral efficiency due to $R_\text{C} = 0.5$.

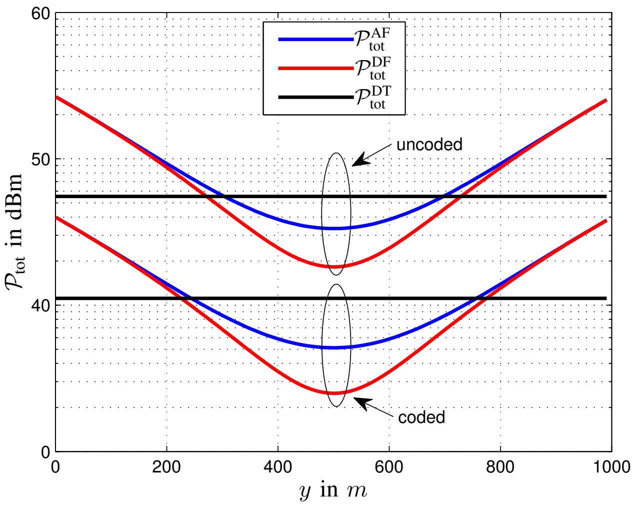

Fig. 4. $\mathcal{P}_\text{tot}$ of AF and DF without SD link for $P_e^\text{tar} = 10^{-4}$ with $\alpha = 4$ in a one-dimensional region ($x = 0$).

C. FRR for AF and DF without SD Link

The relaying strategies AF and DF are compared with the direct transmission without relay in this subsection. By requiring the same target BER P_e^tar at D for AF, DF and DT, we define the Feasible Relay Region (FRR) [4] as the geographical area in which a relaying system consumes less transmit power than the direct transmission under the same target BER. With the help of the following definitions

$$\Delta\mathcal{P}^\text{AF} = \mathcal{P}_\text{tot}^\text{AF} - \mathcal{P}_\text{tot}^\text{DT}, \tag{27a}$$

$$\Delta\mathcal{P}^\text{DF} = \mathcal{P}_\text{tot}^\text{DF} - \mathcal{P}_\text{tot}^\text{DT}, \tag{27b}$$

FRRs can be ascertained in the sense that a relaying system is superior to the direct transmission when $\Delta\mathcal{P}^\text{AF} < 0$ for AF and $\Delta\mathcal{P}^\text{DF} < 0$ for DF. Note that both $\Delta\mathcal{P}^\text{AF}$ and $\Delta\mathcal{P}^\text{AF}$ are functions of d_SR, d_RD and d_SD.

As already visualized in Fig. 4 for the one-dimensional region in which $\mathcal{P}_\text{tot}^\text{AF}$ or $\mathcal{P}_\text{tot}^\text{DF}$ is smaller than $\mathcal{P}_\text{tot}^\text{DT}$, the FRRs of coded AF and DF without SD link for different values of α are presented in a contour plot in Fig. 5. The FRRs are the geographical areas embraced by the contour curves, which are ellipses centered at $(0,500)$ and of different sizes depending on relaying strategies and α. Comparing Fig. 5(a) and (b), we can draw the conclusion that DF achieves larger FRR than AF without SD link in such a scenario. It is also shown that the FRR expands with growing α, which implies that in a strong path-loss environment, positioning the relay is less crucial.

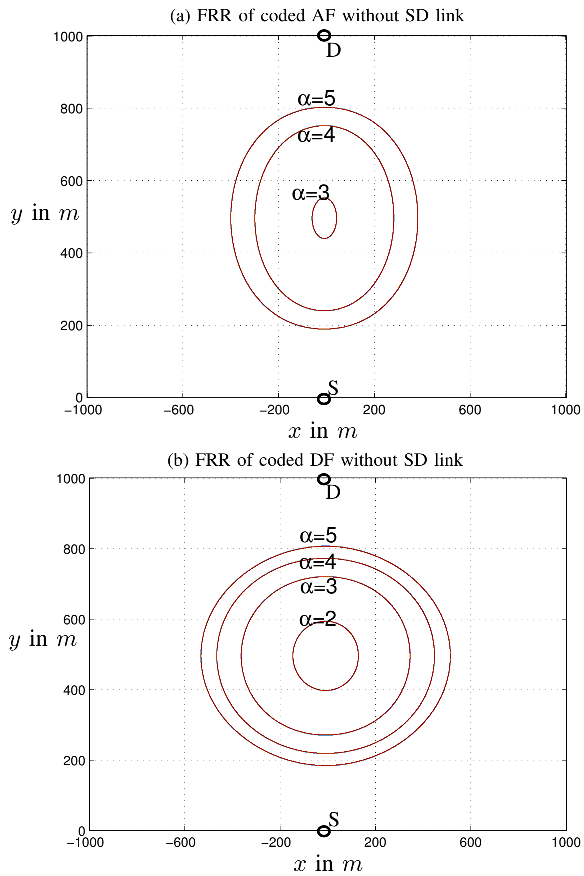

Fig. 5. FRR for different values of α and $P_e^\text{tar} = 10^{-4}$.

D. Total Power for AF and DF with SD Link

The total power consumption after optimization is plotted for coded AF and DF with and without SD link in Fig. 6. We can observe that AF with SD link saves total transmit power compared to AF without SD link over the whole one-dimensional range $0 \leq y \leq 1000$. The amount of saved power achieves a minimum when R is in the middle. However, DF with SD link is only capable of saving transmit power when R is nearer to S compared to DF without SD link. As is shown in the figure, the SD link contributes no improvement for $400 \leq y \leq 1000$ because the RD link that may carry errors takes dominance over the SD link for MRC when R moves towards D. Note that the performance for DF with SD link can be improved by using modified MRCs other than that in (3).

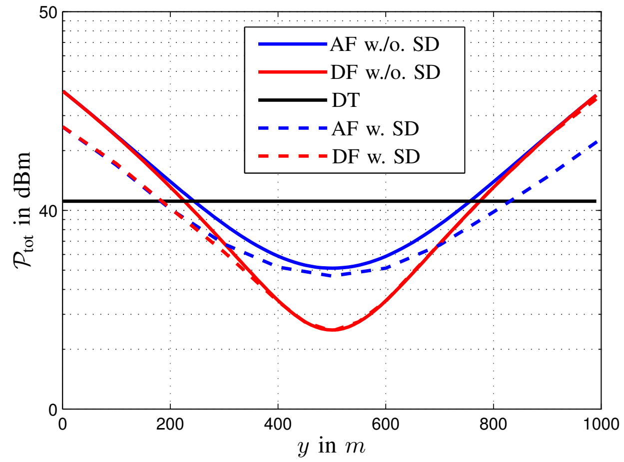

Fig. 6. $\mathcal{P}_{\text{tot}}$ of coded AF and DF with SD link and without SD link for $P_e^{\text{tar}} = 10^{-4}$ with $\alpha = 4$ in a one-dimensional region ($x = 0$). An LDPC code of rate $R_{\text{C}} = 0.5$ and codeword length $N = 2000$ with 16-QAM is used.

E. Performance of LDPC Codes with different N

Using (15), the input-output characteristic of LDPC codes of different codeword lengths can be modeled and utilized in the power optimization for coded systems. The optimized total power consumption $\mathcal{P}_{\text{tot}}$ of coded DF without SD link is plotted in Fig. 7 for LDPC codes of rate $R_{\text{C}} = 0.5$ and varying codeword length $N = 200, 400, 800$ and 2000. It is shown that the total transmit power can be saved with the increase of the codeword length wherever R is positioned in the one-dimensional region, which corresponds to the property of LDPC codes.

F. Power proportion ξ

The proportion of power allocated to S and R to achieve a given P_e^{tar} is investigated in this subsection. Considering uncoded AF and DF relaying systems with and without SD link, the power proportion $\xi = \mathcal{P}_{\text{S,opt}}/\mathcal{P}_{\text{R,opt}}$ is visualized in Fig. 8 for different relay positions in a one-dimensional region. When R moves towards D, a growing proportion of power is allocated to S, and vice versa. Additionally, ξ changes faster for DF than AF when R moves from the middle to S or D, which indicates that DF is more sensitive to power allocation. Also note that $\mathcal{P}_{\text{S,opt}}$ is equal to $\mathcal{P}_{\text{R,opt}}$ when R is in the middle for both AF and DF without SD link. However, $\mathcal{P}_{\text{S,opt}}$ is

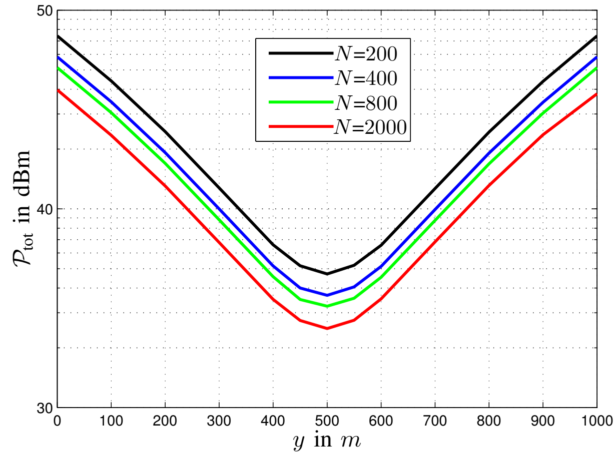

Fig. 7. $\mathcal{P}_{\text{tot}}$ of coded DF without SD link for $P_e^{\text{tar}} = 10^{-4}$ with $\alpha = 4$ in a one-dimensional region ($x = 0$). LDPC codes of rate $R_{\text{C}} = 0.5$ and codeword length $N = 200, 400, 800$ and 2000 with 16-QAM are used.

slightly greater than $\mathcal{P}_{\text{R,opt}}$ when the SD link is considered, as shown in the scaled central region of Fig. 8. As discussed for (25), the consideration of the SD link results in a reduced transmit power for R.

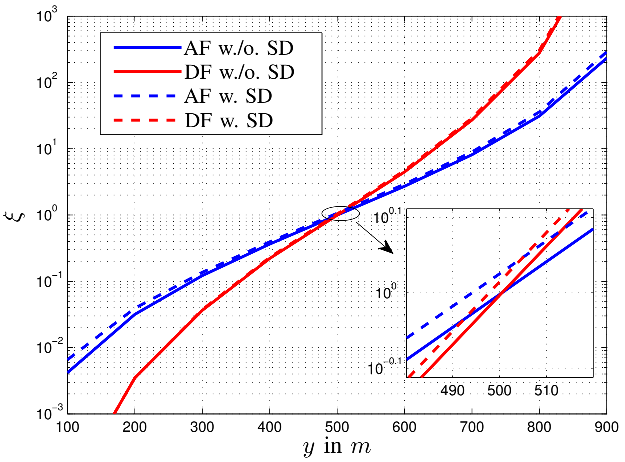

Fig. 8. Power proportion ξ of uncoded AF and DF for $P_e^{\text{tar}} = 10^{-4}$ with $\alpha = 4$ in a one-dimensional region ($x = 0$).

G. Link-level Evaluation

Finally, the optimized power components $\mathcal{P}_{\text{S,opt}}$ and $\mathcal{P}_{\text{R,opt}}$ are used in the link-level simulations. The simulated BERs normalized by the corresponding targets BERs for uncoded and coded AF and DF without SD link are plotted in Fig. 9 for different $P_e^{\text{tar}} = 10^{-2}, 10^{-3}, 10^{-4}$ and 10^{-5}. The simulated BERs reach the corresponding targets with great precision for both uncoded AF and DF, meaning that the optimization works fine for uncoded systems. However, the BERs achieved by the simulations are not in correspondence with the targets but lower for coded systems. This leads to the conclusion that the optimization for coded systems still works to achieve a given QoS but is loose and not able to reach the QoS precisely. The imprecision becomes greater as P_e^{tar} grows larger. This is caused by the imprecision of the polynomial-based approximation in small output BER regions, as shown in Fig. 2. Additionally, the imprecision for DF is greater than that of AF because the modeling function f_C is used twice, i.e., at R and D, for DF, which influences the performance of the optimization.

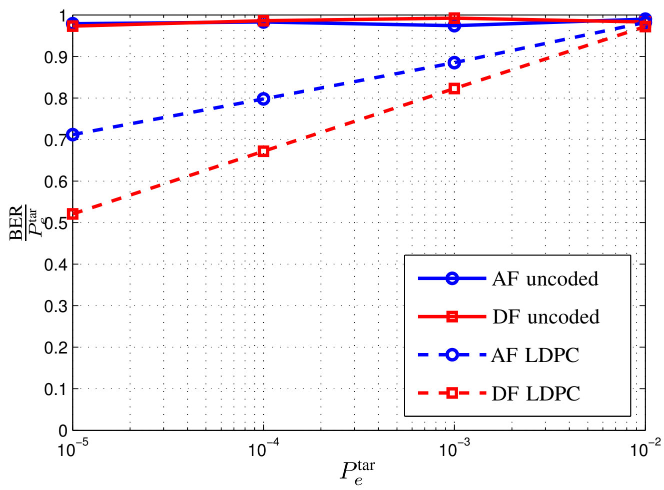

Fig. 9. Comparison of BERs achieved by simulations and the corresponding $P_e^{\text{tar}} = 10^{-2}, 10^{-3}, 10^{-4}$ and 10^{-5} with $\alpha = 4$, $x = 0$ and $y = 500$.

VI. Conclusion

In this paper a closed-form solution is proposed for a BER-based power allocation scheme in a triplet relaying network with AWGN channels. The total transmit power for the source and the relay is minimized to achieve a given target BER at the destination. We have adapted the power allocation scheme for both uncoded and coded AF and DF relaying strategies. For coded systems, the input-output BER characteristic for the decoder of the applied channel code is modeled by a polynomial, which is a monotonically increasing function and thus the optimization problem can still be solved without much difficulty. To compare the total consumed power of AF and DF with the direct transmission without relay, the Feasible Relay Region (FFR) is investigated, which visualizes the geographical area where a relaying system is superior to the direct transmission.

It has been shown that DF seems to be a better choice with optimized power allocation than AF by saving total transmit power and possessing larger FFR without SD link. Additionally, the results of the optimization are used in the link-level simulations, which indicates that the required QoS can be achieved by using the optimized power components, and thus verifies the validity of the proposed power allocation scheme.

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