OFDM-IDM Space-Time Coding in Two-Hop Relay-Systems with Error-Prone Relays

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***Abstract*—In this paper, the combination of distributed Interleave-Division-Multiplexing-Space-Time Codes (IDM-STCs) with Orthogonal Frequency Division Multiplexing (OFDM) in two-hop relay systems applying Decode-and-Forward (DF) is investigated. In order to cope with erroneous decoding at the relays, a model to capture the decoding reliabilities of the relays is formulated and a method to incorporate these reliabilities in the iterative detection process at the destination is presented. In addition, different approaches to keep the required signaling overhead low are discussed and compared with each other. It is shown, that the proposed scheme achieves significant gains over the common DF detection scheme assuming perfect decoding at the relays. Even with only one bit signaling per frame and relay an improvement of 3.6 dB can be obtained in certain scenarios.**

I. Introduction

In the last years, relay systems have gained much attention, as they introduce spatial diversity and, therefore, allow to cope with pathloss and shadowing in mobile communication scenarios. In relay systems, distributed relays can form so-called virtual antenna arrays (VAA) allowing the application of techniques known from MIMO systems, like Space-Time Coding (STC), which has been identified as a very promising and flexible technique to exploit transmit diversity.

In this work, Space-Time Codes based on the multiple access scheme Interleave-Division Multiple-Access (IDMA) [1], so-called IDM-Space-Time Codes (IDM-STCs) [2], [3], are investigated. For IDM-STCs, an antenna-specific interleaving is applied to achieve temporal correlation among the transmitted signals. In [4], IDM-STCs have been applied to relay systems in a distributed fashion using uncoded transmission and the Decode-and-Forward (DF) protocol. In [5], the principles of distributed IDM-STC have been extended to coded systems and further relay protocols like Amplify-and-Forward (AF) and Decode-Estimate-and-Forward (DEF). Although the detection complexity of IDM-STCs is very modest and grows only linearly with the number of channel taps, IDM-STCs have shown severe performance degradations for strong frequency-selective channels in comparison to other schemes like, e.g., cyclic delay diversity, as was shown in [6]. In order to overcome these performance degradations and to greatly simplify the overall detection process at the destination, here, IDM-STCs are combined with OFDM resulting in OFDM-IDM-STCs. Specifically, IDM-STCs are applied in frequency-domain to the subcarriers of an OFDM transmission. The resulting OFDM-IDM-STCs allow for a detection independent of the number of channel taps. Furthermore, their performance does not suffer from strong frequency-selectivity. While IDM-STCs, as well as the combination of IDMA and OFDM (OFDM-IDMA), have been addressed repeatedly in literature, e.g., in [7], OFDM-IDM-STCs have been given less attention. To the authors' knowledge, the only work combining distributed IDM-STCs with OFDM is [8], calling the result IDM-Space-Frequency Codes (IDM-SFCs). However, since the coding is both in time and frequency, we stick with the name OFDM-IDM-STCs.

While the investigations in [8] were restricted to Amplify-and-Forward systems, in this paper, OFDM-IDM-STCs are applied to Decode-and-Forward relay networks, allowing for more sophisticated methods at the relays, like, e.g., error detection. Since, depending on the number of falsely decoded bits per frame, erroneous relays can still contribute to the overall transmission, all relays are allowed to always forward to the destination, regardless of the decoding success. Instead, the decoding error probability of the relays is determined and signaled to the destination, where it is used as an indicator for the relays' reliabilities. By formulating a joint model for the source-relay transmission, the decoding, and the re-encoding at the relay, based on binary symmetric channels, these reliabilities can be incorporated in the detection at the destination, allowing for a weighting of the relays' messages, depending on their reliabilities. The required signaling overhead to the destination can be kept very low. As will be shown, even a signaling overhead of one bit per relay and frame can significantly improve the overall performance of the system.

The remainder of this paper is structured as follows. In Section II the system model is presented and its components are discussed in detail. Here, the focus is on the joint model and the modified soft-value calculation at the destination, which are the essential parts of this work. Section III is dedicated to the relay-destination signaling. Different approaches to keep the introduced signaling overhead low are discussed. In Section IV some numerical results are given, where the proposed scheme is compared to the conventional Decode-and-Forward scheme. In addition, the different signaling approaches are compared with each other. In Section V, finally, some conclusions are drawn.

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II. System Model

A. Overview

A two-hop relay system as depicted in Fig. 1 is considered. One source S communicates with one destination D via N parallel relays R_ν, $1 \leq \nu \leq N$.

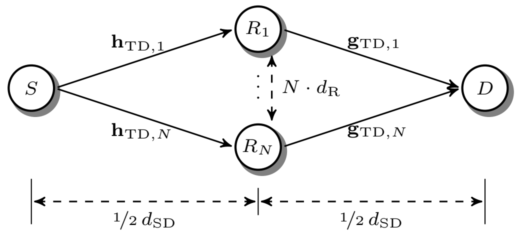

Fig. 1. Topology of the considered two-hop relay system.

No direct link from source to destination is assumed and the channel impulse responses from S to R_ν and from R_ν to D in time-domain are given by $\mathbf{h}_{\mathrm{TD},\nu}$ and $\mathbf{g}_{\mathrm{TD},\nu}$, respectively. Frequency-selective block Rayleigh fading channels with L_h and L_g channel taps, respectively, are assumed and the pathloss on each hop is given by d^ϵ such that

$$\mathbf{h}_{\mathrm{TD},\nu} = d^{-\epsilon/2}\tilde{\mathbf{h}}_{\mathrm{TD},\nu} \tag{1a}$$

$$\mathbf{g}_{\mathrm{TD},\nu} = d^{-\epsilon/2}\tilde{\mathbf{g}}_{\mathrm{TD},\nu}\,, \tag{1b}$$

where d denotes the distance between the corresponding nodes and ϵ ist the pathloss exponent. The components of $\tilde{\mathbf{h}}_{\mathrm{TD},\nu}$ and $\tilde{\mathbf{g}}_{\mathrm{TD},\nu}$ are i.i.d. and $\mathrm{E}\{||\tilde{\mathbf{h}}_{\mathrm{TD},\nu}||^2\} = \mathrm{E}\{||\tilde{\mathbf{g}}_{\mathrm{TD},\nu}||^2\} = 1$ holds, such that the total received power only depends on the pathloss but not on L_h and L_g, respectively. Also, each receiver, i.e., R_ν and D experiences additive white gaussian noise (AWGN) with power σ_n^2. Due to the half-duplex constraint, communication happens in the following two phases.

- In the first phase, the source broadcasts its information to the relays using OFDM to overcome frequency-selectivity on the first hop and to simplify the calculation of Log-Likelihood-Ratios (LLRs) of the codebits at the relays. Using the locally calculated LLRs, each relay performs decoding and re-encoding of the hard quantized output of the channel decoder, applying the same channel code as the source.
- In the second phase, the relays transmit the re-encoded messages simultaneously to the destination. In order to exploit the offered spatial diversity, a distributed IDM-STC across the N relays is applied. To keep the detection at the destination simple and its complexity independent of the number of channel taps, the distributed IDM-STC is combined with OFDM. At the destination soft-RAKE-detection in frequency-domain is applied.

Since OFDM is applied on both hops, the following explanations will be carried out in frequency-domain under the assumption of a sufficiently long cyclic prefix (CP), i.e., $L_{\mathrm{CP}} \geq \max\{L_\mathrm{h}, L_\mathrm{g}\}$. Therefore, the channels can be described in frequency-domain by

$$\mathbf{h}_{\mathrm{FD},\nu} = \mathrm{IDFT}_{N_\mathrm{c}}\{\mathbf{h}_{\mathrm{TD},\nu}\} \tag{2a}$$

$$\mathbf{g}_{\mathrm{FD},\nu} = \mathrm{IDFT}_{N_\mathrm{c}}\{\mathbf{g}_{\mathrm{TD},\nu}\}\,. \tag{2b}$$

For the sake of completeness, the OFDM-specific blocks, i.e., discrete fourier transform (DFT), inverse discrete fourier transform (IDFT) and addition or cancelation of the cyclic prefix (CP, CP^{-1}) will be given in the figures, as they are used in the system. They will, however, not be discussed any further.

B. Source

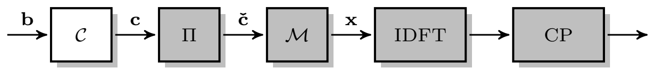

Fig. 2. Structure of the source.

In Fig. 2, the structure of the source is shown. The vector containing the infobits $\mathbf{b} \in \mathbb{F}_2^{L_\mathrm{b}}$ of length L_b is encoded with a forward error correcting code (FEC) $\mathcal{C}$ resulting in the coded sequence $\mathbf{c} \in \mathbb{F}_2^{L_\mathrm{c}}$ of length L_c. As common for IDMA-based systems [1], a serial concatenation of a convolutional code of rate R_{conv} and a repetition code of rate R_{rep} is chosen. The coded sequence $\mathbf{c}$ is interleaved using Π leading to the sequence $\check{\mathbf{c}} \in \mathbb{F}_2^{L_\mathrm{c}}$. The interleaved sequence $\check{\mathbf{c}}$ is then mapped onto the symbol vector $\mathbf{x} \in \mathcal{A}^{L_\mathrm{x}}$, whose length L_x is chosen as an even multiple of the number of subcarriers N_c. For the sake of notational simplicity, the following explanations throughout Section II assume a normalized BPSK alphabet $\mathcal{A}$ with $\sigma_\mathrm{x}^2 = 1$. The extension to higher order modulation schemes, however, can easily be achieved [1]. Finally, the symbol vector in frequency-domain $\mathbf{x}$ is broadcasted to the relays.

C. Relay

By stacking the channel's frequency response

$$\mathbf{h}_\nu = \begin{bmatrix} \mathbf{h}_{\mathrm{FD},\nu} \\ \mathbf{h}_{\mathrm{FD},\nu} \\ \vdots \\ \mathbf{h}_{\mathrm{FD},\nu} \end{bmatrix} \in \mathbb{C}^{L_\mathrm{x}}\,, \tag{3}$$

the received signal at R_ν is given by

$$\mathbf{y}_\nu = \mathbf{h}_\nu \odot \mathbf{x} + \mathbf{n}_\nu\,, \tag{4}$$

where "$\odot$" denotes the Hadamard product, i.e., the elementwise multiplication, and $\mathbf{n}_\nu \in \mathbb{C}^{L_\mathrm{x}}$ is a vector of additive white gaussian noise of power σ_n^2. The structure of the relay R_ν is depicted in Fig. 3.

First, the received signal in frequency-domain $\mathbf{y}_\nu$ is matched-filtered with $\mathbf{h}_\nu^\mathrm{H}$, leading to the matched-filtered receive signal $\bar{\mathbf{y}}_\nu$. Based on $\bar{\mathbf{y}}_\nu$, LLRs $\mathbf{\Lambda}_{\check{\mathrm{c}}_\nu}$ for the interleaved

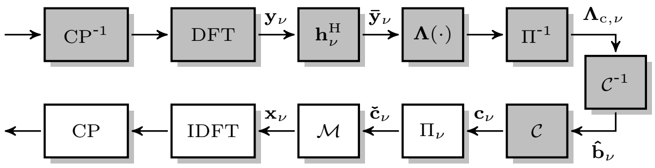

Fig. 3. Structure of the relay R_ν.

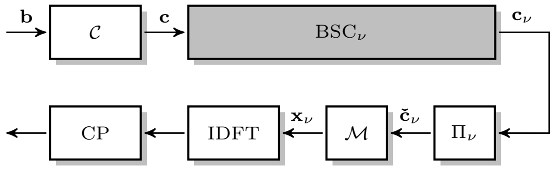

Fig. 4. Equivalent joint model for S and R_ν based on a binary symmetric channel describing the relation between the source codeword $\mathbf{c}$ and the relay codeword $\mathbf{c}_\nu$.

codebits are calculated. For the ith codebit $\check{c}_i$, $1 \leq i \leq L_{\mathrm{c}}$, the LLR is given by

$$\Lambda_{\check{c}_\nu,i} = 2\,\frac{|h_{\nu,i}|^2}{\sigma_{\mathrm{n}}^2}\,\bar{y}_{\nu,i}\,, \tag{5}$$

where, $h_{\nu,i}$ denotes the ith element of $\mathbf{h}_\nu$. After deinterleaving, channel decoding, denoted as $\mathcal{C}^{-1}$, is performed. The decoding process consists of the decoding of the repetition code, which is a summation of the corresponding LLRs, followed by the decoding of the convolutional code, using the well-known BCJR-algorithm [9]. After decoding, the resulting hard quantized info bits $\hat{\mathbf{b}}_\nu$ are re-encoded using *the same* channel code $\mathcal{C}$ as the source, leading to the code sequence $\mathbf{c}_\nu$. Relay-specific interleaving Π_ν and successive mapping $\mathcal{M}$ then lead to the νth relay's transmit signal $\mathbf{x}_\nu$. At the relays, again, the same alphabet $\mathcal{A}$ is used.

To investigate the detection in Decode-and-Forward (DF) systems, often error-free decoding at the relays is assumed. Due to varying channel conditions, i.e., fading and noise, however, this is usually a too optimistic assumption. As a consequence, the reliabilities of the relays' forwarded messages are overestimated, which can cause severe performance degradations when applying soft-detection at the destination. To avoid this overestimation, we propose a method to incorporate the relays' decoding reliabilities in the detection process at the destination. As proposed in [10] for information-theoretic investigations, the relation between $\mathbf{c}$ and $\mathbf{c}_\nu$, i.e., the source codeword and the codeword at relay R_ν, can be modeled using a binary symmetric channel (BSC) with a certain error or crossover probability q_ν as

$$\mathbf{c}_\nu = \mathrm{BSC}_\nu\{\mathbf{c}, q_\nu\}\,. \tag{6}$$

This crossover probability is zero for perfect decoding at the relay and tends to $q_\nu = 0.5$ as the relay's decoding reliability decreases. Using this way of describing the relays' reliabilities allows for the formulation of an equivalent joint model composing the $S - R_\nu$ transmission, the decoding and the re-encoding at R_ν as depicted in Fig. 4. This equivalent joint model is the basis for a modified LLR-calculation at the destination, introduced in II-D.

In the following, the error probability q_ν is assumed to be perfectly known at R_ν, that is

$$q_\nu = \frac{d_{\mathrm{H}}(\mathbf{c}, \mathbf{c}_\nu)}{L_{\mathrm{c}}}\,, \tag{7}$$

where $d_{\mathrm{H}}(\cdot)$ denotes the Hamming distance and L_{c} is the lenght of the code sequence. Obviously, this would require perfect knowledge of $\mathbf{c}$ at R_ν, which is assumed here. In practice, however, q_ν can be estimated using the output LLRs of the code bits generated by the channel decoder at the relay [11].

D. Destination

The received signal at the destination in frequency-domain $\mathbf{y}$ consists of the superposition of the relays' transmit signals $\mathbf{x}_\nu \in \mathbb{C}^{L_{\mathrm{x}}}$ and additive white gaussian noise $\mathbf{n} \in \mathbb{C}^{L_{\mathrm{x}}}$ of power σ_{n}^2. By stacking the channels' frequency responses $\mathbf{g}_{\mathrm{FD},\nu}$ according to eq. (3), it can be written as

$$\mathbf{y} = \sum_{\nu=1}^{N} \mathbf{g}_\nu \odot \mathbf{x}_\nu + \mathbf{n}\,. \tag{8}$$

In order to seperate the signals $\mathbf{x}_\nu$, an iterative turbo detection [1], [2] is applied, as shown in Fig. 5. After soft-interference-cancelation (IC) and relay-specific de-interleaving by Π_ν^{-1}, the LLRs for the same codebits transmitted via different relays are summed up

$$\boldsymbol{\Lambda}_{\mathrm{c}}^{\mathrm{IC}} = \sum_{\nu=1}^{N} \Pi_\nu^{-1}\big(\boldsymbol{\Lambda}_{\check{\mathrm{c}}_\nu}^{\mathrm{IC}}\big) \tag{9}$$

and channel decoding $\mathcal{C}^{-1}$ is performed. The LLRs of the extrinsic information $\boldsymbol{\Lambda}_{\mathrm{ext,c}_\nu}^{\mathrm{DEC}}$ from the channel decoder are interleaved again and fed back as a-priori-information $\boldsymbol{\Lambda}_{\mathrm{a},\check{\mathrm{c}}_\nu}^{\mathrm{IC}}$ to the interference canceler for the next iteration.

Usually, for Decode-and-Forward (DF), perfect decoding at the relays is assumed and the interference canceler would deliver LLRs based on the conditional pdfs $p(y_i|\check{c}_{\nu,i} = 0)$ and $p(y_i|\check{c}_{\nu,i} = 1)$ as

$$\Lambda_{\check{c}_\nu,i}^{\mathrm{IC,DF}} = \ln\left(\frac{p(y_i|\check{c}_{\nu,i} = 0)}{p(y_i|\check{c}_{\nu,i} = 1)}\right)\,. \tag{10}$$

However, since decoding errors at the relays cannot be ruled out completely, (10) leads to an overestimation of the true LLRs. By taking the correlation between $\mathbf{c}$ and $\mathbf{c}_\nu$ into account, this overestimation can be avoided, as the probabilities for decoding errors at the relays can easily be included in the LLR calculation. Based on the equivalent BSC model, the pdfs used for LLR calculation can be formulated conditioned on the source codebits $\mathbf{c}$. Considering only relay R_ν and using the law of total probabilities [12]

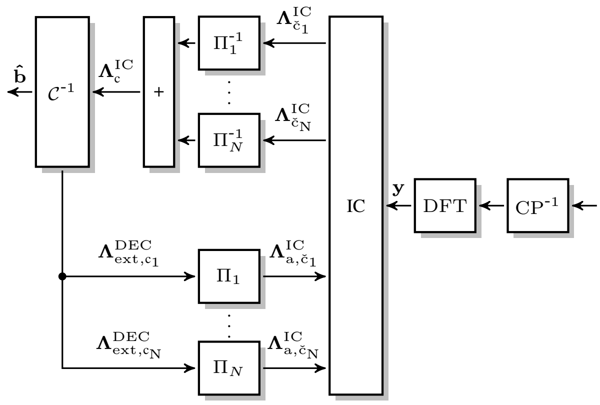

Fig. 5. Structure of the iterative OFDM-IDM-STC detector at the destination.

$$p_\nu(y_i|\check{c}_i = 0) = p(y_i|\check{c}_{\nu,i} = 0) \cdot \Pr\{\check{c}_{\nu,i} = 0|\check{c}_i = 0\} + p(y_i|\check{c}_{\nu,i} = 1) \cdot \Pr\{\check{c}_{\nu,i} = 1|\check{c}_i = 0\} \quad (11)$$

$$p_\nu(y_i|\check{c}_i = 1) = p(y_i|\check{c}_{\nu,i} = 1) \cdot \Pr\{\check{c}_{\nu,i} = 1|\check{c}_i = 1\} + p(y_i|\check{c}_{\nu,i} = 0) \cdot \Pr\{\check{c}_{\nu,i} = 0|\check{c}_i = 1\}\,. \quad (12)$$

The probabilities in (11) and (12) are directly given from the crossover probability q_ν of the BSC as

$$\Pr\{\check{c}_{\nu,i} = 0|\check{c}_i = 0\} = \Pr\{\check{c}_{\nu,i} = 1|\check{c}_i = 1\} = 1 - q_\nu \quad (13a)$$
$$\Pr\{\check{c}_{\nu,i} = 1|\check{c}_i = 0\} = \Pr\{\check{c}_{\nu,i} = 0|\check{c}_i = 1\} = q_\nu\,. \quad (13b)$$

Inserting (13a)-(13b) into (11) and (12) leads to

$$p_\nu(y_i|\check{c}_i = 0) = p(y_i|\check{c}_{\nu,i} = 0) \cdot (1 - q_\nu) + p(y_i|\check{c}_{\nu,i} = 1) \cdot q_\nu \quad (14)$$

$$p_\nu(y_i|\check{c}_i = 1) = p(y_i|\check{c}_{\nu,i} = 1) \cdot (1 - q_\nu) + p(y_i|\check{c}_{\nu,i} = 0) \cdot q_\nu\,. \quad (15)$$

Therefore, taking the BSC model into account, the LLRs delivered by the interference canceler can be estimated based on the source codebits $\mathbf{c}$ as

$$\Lambda_{\check{c}_{\nu,i}}^{\text{IC,DFq}} = \ln\left(\frac{p_\nu(y_i|\check{c}_i = 0)}{p_\nu(y_i|\check{c}_i = 1)}\right) = \ln\left(\frac{p(y_i|\check{c}_{\nu,i} = 0) \cdot (1 - q_\nu) + p(y_i|\check{c}_{\nu,i} = 1) \cdot q_\nu}{p(y_i|\check{c}_{\nu,i} = 1) \cdot (1 - q_\nu) + p(y_i|\check{c}_{\nu,i} = 0) \cdot q_\nu}\right). \quad (16)$$

In order to differentiate this scheme from the conventional Decode-and-Forward (DF), it is denoted as DFq, indicating the consideration of the relays' error probabilities q_ν. The pdfs $p(y_i|\check{c}_{\nu,i} = 0)$ and $p(y_i|\check{c}_{\nu,i} = 1)$ can be obtained as usual, e.g., using soft-RAKE-detection [1], [2] with respect to $\mathbf{g}_\nu$ and $\mathbf{x}_\nu$. By including the error probabilities q_ν into the detection process following (16), an overestimation of the LLRs, as for DF, can be avoided. As will be shown in Section IV by numerical examples, this can result in significant gains in terms of frame-error-rates (FER) at the destination.

For the destination to perform the calculation (16), signaling of q_ν from all R_ν to D is necessary. In the next section, different approaches are discussed, in order to reduce the introduced overhead for this signaling.

III. Relay-Destination Signaling

In total and without further simplifications, each of the N relays has to transmit its error probability q_ν to the destination for every transmitted frame. For small frame lengths or a high precision of q_ν this can easily result in a considerable overhead. It is, therefore, resonable, to reduce the amount of signaling without degrading the performance of the overall scheme too much.

Mainly, two different effects influence the decoding reliability of the relays, namely the fading state of the channel and the pathloss between source and relay. Hence, the decoding reliability q_ν can be seen as a measure of the current channel state. Depending on how fast the channel state changes, it may not be necessary to signal the error probabilities for every frame to the destination.

In the following, several approaches are presented and discussed, which aim at decreasing the overall signaling overhead by applying different simplifications.

A. Instantaneous error statistics (INST)

If no simplifications are made, the unquantized error probabilities q_ν are calculated and signaled for every frame. Since these error probabilities describe the current channel state best, this scheme is denoted as instantaneous error statistics (INST) and is used as a benchmark for the other schemes.

B. Long-term statistics (LT)

If the channel state is only changing slowly, one might replace the instantaneous error probabilities q_ν with their long-term averages q_ν^{LT}, leading to an average reliability of the relays. Especially for static scenarios, i.e., fixed positions of source and relays, this is a reasonable approach, since the pathlosses between source and relays don't change over time. As a consequence, averaging q_ν should converge to a steady-state q_ν^{LT} after some time, which then would have to be signaled only once to the destination. Afterwards, no further signaling from R_ν to D would be necessary at all.

C. Moving Average Model (MA)

If the pathlosses between source and relays change, e.g., due to movement of the former or the latter, long-term statistics of q_ν may be too conservative. Calculating the moving average q_ν^{MA} of q_ν for a number of recently transmitted frames seems to be a good compromise between using the instantaneous error probabilities q_ν and using their long-term statistics q_ν^{LT}, as it allows to exploit the knowledge of the current pathlosses

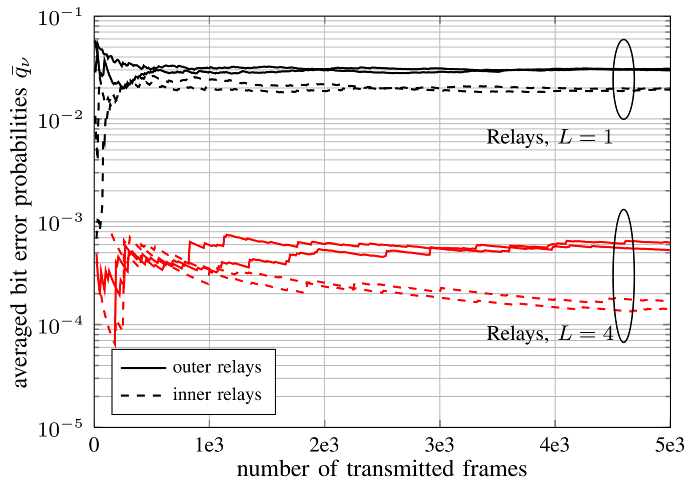

Fig. 6. Averaged bit error probabilities $\bar{q}_\nu$ at the relays versus the number of transmitted frames for $L = 1$ and $L = 4$ channel taps at $1/\sigma_n^2 = 0\,\mathrm{dB}$. Solid: outer relays, dashed: inner relays.

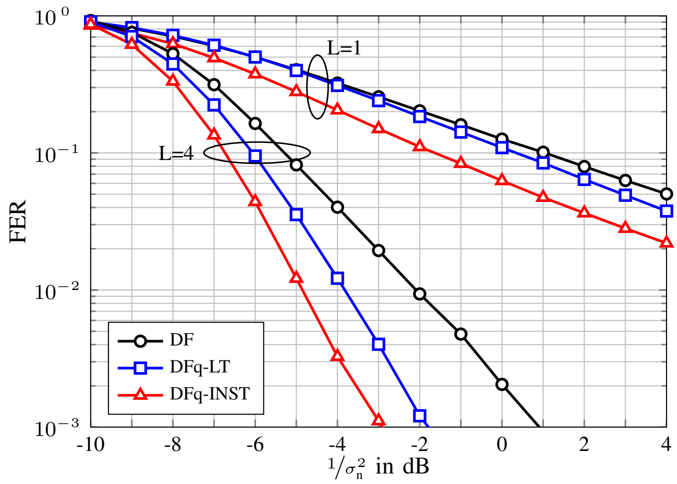

Fig. 7. FER comparison at the destination for common Decode-and-Forward (DF) and the proposed scheme (DFq) using the long-term (LT) or the instantaneous error statistic (INST).

without requiring a framewise signaling to the destination. Depending on the size of the applied window, the calculated average is very likely to change only slightly from one frame to the next. Hence, it is not necessary to update this value at the destination for every frame. By adjusting the signaling interval of q_ν^{MA}, the introduced overhead of this scheme can be adapted to the particular transmission szenario.

D. *Quantization (QT)*

Another possibility to reduce the introduced overhead is the quantization of the error probabilies q_ν at the relays. Naturally, the error probabilities have to be quantized anyways in order to be processible by the system. For the MA model, however, a quantization with a sufficiently high resolution was implicitly assumed, so that no quantization effects occur. In contrast, for this method, a quantization with only a few bits shall be considered.

As will be seen in Section IV, the distribution of q_ν is highly non-uniform. Therefore, a non-uniform quantization seems more appropriate than a uniform quantization. A non-uniform quantizer that minimizes the power of the quantization error is the well-known Lloyd-Max quantizer [13], [14]. In order to calculate the optimum partitioning with respect to the distortion power, the distribution of q_ν, which is dependent on the SNR, has to be known. For the numercial analysis in Section IV, however, only a single distribution at a specific target FER is used, as it allows to use a single codebook for all SNRs at a negligible performance loss.

IV. Numerical Results

A two-hop relay system with one source S, one destination D and $N = 4$ parallel relays R_ν, distributed equidistantly along an imaginary line with an inter-relay distance of $d_{\mathrm{R}} = 0.2$, as depicted in Fig. 1, is considered. Frequency-selective block Rayleigh fading with $L = L_{\mathrm{h}} = L_{\mathrm{g}}$ i.i.d. channel taps is assumed on both hops and the pathloss exponent is set to $\epsilon = 3$. For channel coding, a combination of the half-rate $(5,7)_8$ convolutional code and the half-rate repetition code is applied and the codeword length is set to $L_{\mathrm{c}} = 1024$ codebits. The QPSK alphabet $\mathcal{A}$ with $\sigma_x^2 = 1$ is chosen. The resulting symbols are mapped onto $N_{\mathrm{c}} = 64$ subcarriers and a cyclic prefix of length $L_{\mathrm{CP}} = 10$ is applied. For detection at the destination, $N_{\mathrm{it}} = 10$ iterations are performed.

A. *Static source*

First, in order to to present the general advantage of the proposed scheme, a static system with fixed positions for all nodes and, hence, fixed pathlosses among the nodes, is considered. The distance between S and D is normalized to $d_{\mathrm{SD}} = 1$. Due to the given topology, the two outer relays R_1 and R_4 have a higher distance to the source than the two inner relays R_2 and R_3 and, therefore, experience a higher pathloss. For preliminary analysis, the averaged bit error probabilities $\bar{q}_\nu$ at the relays for different degrees of frequency-selectivity $L = 1, 4$ dependent on the number of transmitted frames are given in Fig. 6. Here, the averaging is always from the first to the last transmitted frame. The dashed lines represent the two inner relays R_2 and R_3, while the solid lines describe the two outer relays R_1 and R_4. Since the positions of all nodes are fixed, the reliabilities of the relays only change due to the fading of the channel and the noise. It can be seen, that, for this specific simulation, the influences of the fading and noise are averaged out very well after approximately 2000 frames and the error rates already show clear tendencies regarding the relays' reliabilities. The two outer relays R_1 and R_4 (solid), i.e., the relays with higher distance to the source, apparently lead to a higher error probability and, hence, to a lower reliability, compared to the inner relays R_2 and R_3 (dashed). After approximately 4000 frames the error rates almost reach a steady-state which corresponds to their long-term average q_ν^{LT}.

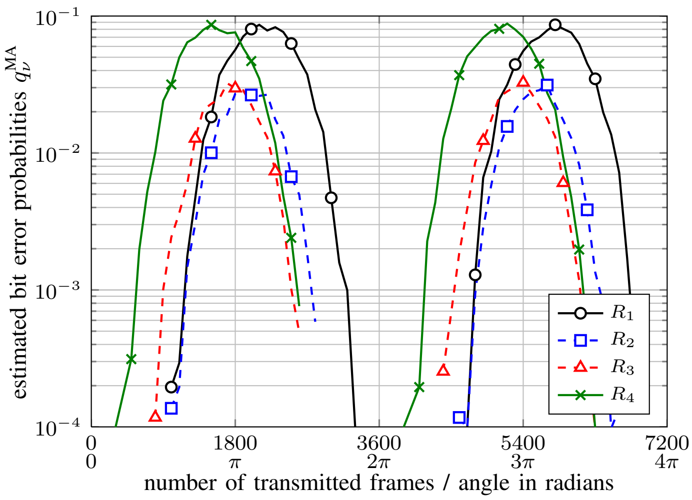

Fig. 8. Moving average q_ν^{MA} (MA) of the error probabilities q_ν at the relays for a circular moving source versus the number of transmitted frames for $L = 4$ channel taps at $1/\sigma_n^2 = -6$ dB and an averaging over 100 frames.

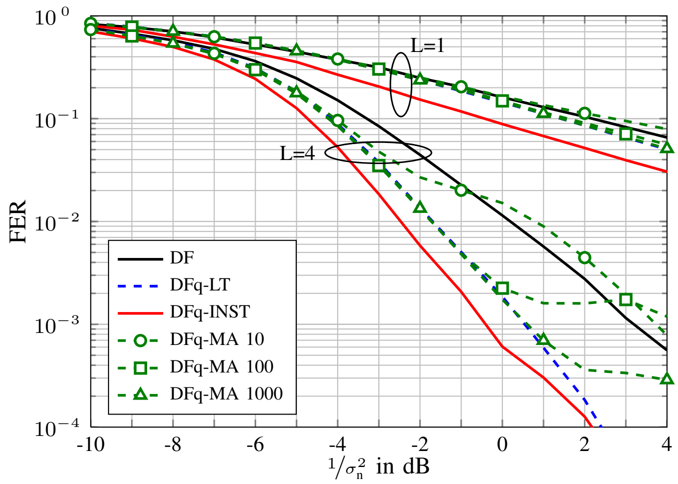

Fig. 9. FER comparison at the destination for the moving average scheme (DFq-MA) with different averaging window sizes.

Clearly, the error rates at the relays are significantly lower for $L = 4$ channel taps, compared to $L = 1$ channel tap. This is due to the higher frequency diversity offered by the former.

Fig. 7 shows the resulting frame error rates (FERs) at the destination for conventional DF, as well as for the proposed DFq using the instantaneous error statistic q_ν (DFq-INST) or the long-term statistic q_ν^{LT} (DFq-LT) determined in Fig. 6. The proposed DFq clearly outperforms DF in both cases, due to the higher LLR reliability in the detection process at the destination. Naturally, using the instantaneous error statistic leads to the best performance, as it not only describes the ergodic channel, but the current channel realization, i.e., including fading and noise. Nevertheless, using the long-term statistic of the relays' reliabilities also clearly leads to a gain compared to the conventional DF scheme. As the long-term statistic doesn't change much after an initial setup phase, it only has to be signaled once during the connection establishment, while the instantaneous error statistic requires a continuous signaling to the destination. Again, due to the higher frequency diversity, the performance of all schemes is significantly higher for $L = 4$ channel taps in comparison to $L = 1$ channel tap.

B. Moving source - moving average

Now, the system setup is modified in order to obtain a more dynamic szenario. Specifically, the source is no longer fixed, but moved counterclockwise around its former position with a radius of $r = 0.2$. Due to the movement, the distances between the source and the relays change constantly, i.e., from frame to frame. The movement is normalized such, that a full cycle takes 3600 transmitted frames, starting with the closest position to the relays.

Obviously, calculating the long-term error statistic of the relays, as before for the static scenario, should also lead to some performance improvement, as the chosen movement is periodic and the long-term statistic describes the relations at the average position, namely, the center of the circle. Yet, the question arises, if there is a better way to capture the relays' reliabilities in such a case.

In order to consider the changing pathlosses during the movement, a moving average (MA) technique is applied, averaging the error probabilities at the relays over a number of recently transmitted frames. While it is favorable to keep the size of the averaging window small, in order to capture the current pathlosses as good as possible, the window should be large enough to average out the channel fading sufficiently.

Fig. 8 depicts the averaged error probabilities q_ν^{MA} at the relays for an averaging over 100 frames. As expected, the two outer relays R_1 and R_4 perform worse than the inner ones most of the time. For a certain time period, however, they alternatingly achieve the same or even better performance as the inner relays.

In Fig. 9, the FERs at the destination for this scenario are shown. As a reference, common DF, as well as the instantaneous (DFq-INST) and the long-term scheme (DFq-LT) are given again. Furthermore, the DFq-MA scheme for different window sizes is shown. The averaged error probability is signaled every tenth frame to the destination for all three DFq-MA curves. Apparently, the DFq-MA scheme never performs better than the DFq-LT scheme, independent of the size of the window. Obviously, the averaging of the fading has a bigger impact on the overall performance than the knowledge of the current pathlosses. Above a certain SNR the DFq-MA scheme leads to a step in der FER curve, resulting in a performance even worse than common DF. This is due to the inherent delay this scheme possesses. In the higher SNR region, errors occur only sporadicly. Since the signaling to the destination is not instantaneous, it is very likely, that no signaling takes place exactly the moment, a decoding error at a relay occurs. The frame sent from this relay is, hence, not considered erroneous at the destination and the corresponding relay's

TABLE I
CODEBOOKS FOR $L = 4$ CHANNEL TAPS.

resolution in bits	codebooks			
1	0.0038	0.1642		
2	0.0013	0.0542	0.1416	0.2691
3	0.0006	0.0270	0.0600	0.1012
	0.1496	0.2061	0.2707	0.3511
4	0.0000	0.0145	0.0368	0.0615
	0.0879	0.1155	0.1437	0.1730
	0.2023	0.2316	0.2616	0.2901
	0.3194	0.3494	0.3821	0.4245

reliability is overestimated, as for DF. On the other hand, once the signaling took place, the previously detected error influences the detection of the next frames, even if no further errors occur. The reliability of the corresponding relay is then underestimated as long as the error is taken into account for the averaging. Both events occuring simultaneously, i.e., one or more relays' reliabilities being overestimated while other ones' being underestimated, can obviously result in a performance worse than not considering the relays' reliabilities at all (DF). Note, that increasing the rate at which the calculated average is signaled to the destination has almost no impact on the performance, unless the signaling is done with every frame. In that case, the mentioned effects could be reduced, though not avoided completely. The introduced signaling overhead, however, then would be the same compared to the DFq-INST scheme, which clearly outperforms DFq-MA.

C. Moving source - quantization

Since averaging doesn't seem to be an appropriate method to decrease the signaling overhead, the impact of quantizing the instantaneous error probabilities at the relays and only signaling the quantized values to the destination is investigated. Fig. 10 shows the joint histogram of the unquantized error q_ν of all relays for $L = 4$ at $1/\sigma_n^2 = -2$ dB which corresponds to a FER of approximately 10^{-2} at the relays. As the distribution of q_ν is highly non-uniform, a non-uniform quantization using the Lloyd-Max algorithm is applied. Exemplarily, the resulting partition borders (solid) and the corresponding codebook entries (dashed) for a 2-bit quantization are given in the figure. The codebooks for all quantization resolutions for $L = 4$ are given in Tab. I.

In Fig. 11, the resulting FERs for the quantization scheme (DFq-QT) with different resolutions, i.e., 1 up to 4 bits, are given and compared to DF, DFq-INST and DFq-LT. The DFq-QT scheme clearly leads to better performances compared to the DFq-LT scheme for all quantization resolutions. Even with 1-bit quantization a gap of only 0.3dB to the unquantized case (DFq-INST) at FER $= 10^{-2}$ is achieved. For 3-bit quantization, almost no gap to the unquantized case is left. Interestingly, in the higher SNR region the frame error curves for the unquantized case, as well as for the 4-bit quantization flatten, such that a 1- up to 3-bit quantization leads to an even better performance than in the unquantized case. This

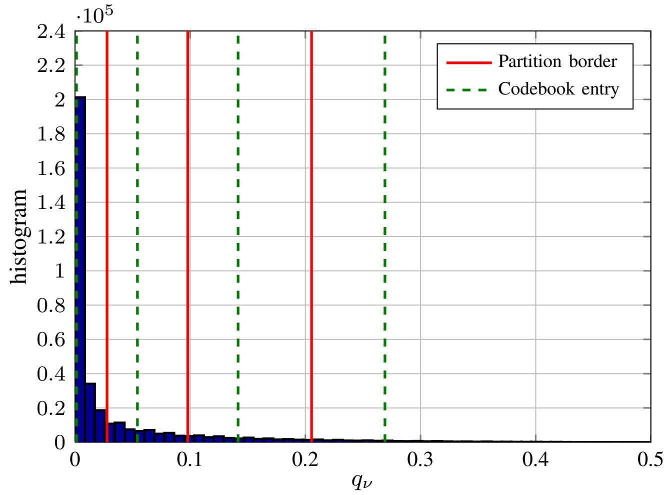

Fig. 10. Joint histogram for the error probabilities q_ν, $1 \leq \nu \leq 4$ at the relays for $L = 4$ at $1/\sigma_n^2 = -2$ dB. Partition borders and codebook entries for non-uniform 2-bit quantization.

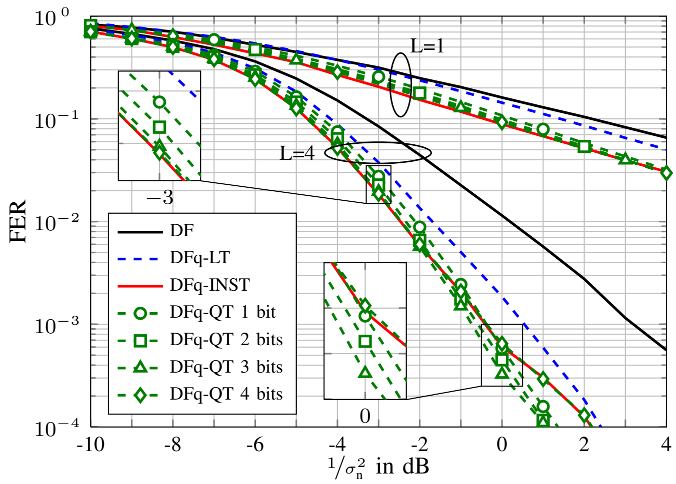

Fig. 11. FER comparison at the destination for the quantization scheme (DFq-QT) with different quantization resolutions.

seems contradictory at first, but inspecting the coresponding codebooks for the different quantizations, given in Tab. I, shows, that the codebooks for 1 up to 3 bits don't contain a zero entry. That means, that even if no errors at the relays are detected, the detection at the destination would be carried out slightly more conservative than it would be for the unquantized case. Generally, iterative detection schemes have been known to gain from smaller convergence steps, even if convergence requires more steps in total. In fact, first investigations, in which a small delta was added to the determined error probabilities, avoiding probabilities of zero, showed an improved performance for the unquantized (DFq-INST), as well as for the 4-bit quantization scheme. To gain more insight in the actual behaviour of the iterative detection process at this point, an EXIT analysis seems resonable, which will be part of our future work.

V. Conclusion

In this paper, the combination of distributed IDM-Space-Time Codes with OFDM in two-hop Decode-and-Forward relay systems is presented. The resulting OFDM-IDM Space-Time Codes allow for a detection in frequency-domain with a complexity independent of the number of channel taps. By describing the correlation of the source codeword and the re-encoded sequence at the relays by a joint model using a binary symmetric channel, a method to incorporate the relays' decoding reliabilities in the iterative detection process at the destination was obtained. In addition, different methods aiming at the reduction of the signaling overhead for the proposed scheme were introduced. It was shown, that the proposed method allows for considerable performance gains compared to the common Decode-and-Forward scheme assuming perfect decoding at the relays. Even with only one bit signaling per frame and relay an improvement of $3.6\,\mathrm{dB}$ at $\mathrm{FER} = 10^{-3}$ was obtained.

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