Improved HARQ based on Network Coding and its Application in LTE

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Abstract—In this paper, a novel HARQ transmission scheme based on network coding is proposed for wireless unicast scenarios. Instead of retransmitting erroneous packets individually, a network coded packet constructed by the XOR of two erroneous packets is transmitted similar to network coding. In order to fully exploit the network coded packet in combination with the previously received erroneous packets, soft combining methods with respect to Chase Combing (CC) and Incremental Redundancy (IR) are developed. The expected throughput gain of 33% for one retransmission for the proposed solution compared to common HARQ transmission is confirmed by LTE link-level simulations.

Index Terms—Network coding, HARQ, XOR, Chase Combining, Incremental Redundancy.

I. INTRODUCTION

Recently, network coding (NC) that has been proposed in [1] to increase the system throughput for error-free networks, has been found to be useful also for unreliable wireless communications [2]. Meanwhile, hybrid automatic repeat request (HARQ) techniques are applied in many modern communication systems to improve the reliability of received data in unreliable and lossy wireless channels. There are two favorable types of HARQ soft combining approaches, i.e., Chase Combining (CC) and Incremental Redundancy (IR) [3]. Chase Combining explores retransmissions of identical packets while Incremental Redundancy combines packets determined by varying puncturing patters, i.e., different redundancy versions [4], [5].

More recently, the combination of network coding and HARQ techniques has been studied for multicast multiuser scenarios [6]. The network coding principle is applied across the packets of different users. Similar ideas have been applied for multicast and broadcast schemes in [7], [8]. Herein, the authors have demonstrated that the throughput efficiency can be improved by network coded HARQ (NC-HARQ).

In this paper we propose the application of network coded HARQ for point-to-point transmission without intermediate nodes. Furthermore, we develop iterative detection methods for CC and IR by combining soft information for previously received packages utilizing the box-plus operation. Fig. 1 illustrates the proposed application of NC-HARQ for LTE uplink unicast transmissions. The user equipment (UE) desires to transmit packets c_k to an evolved Node B (eNB) in time slots t_k controlled by an individual HARQ processes. However, in case of decoding errors the retransmission is done

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Fig. 1. Network coded HARQ in LTE uplink unicast scenario, where in the third time slot a network coded packet $c_1 \oplus c_2$ is transmitted to eNB.

for two erroneous packages jointly and not separately. If, for example, the packets c_1 and c_2 could not be decoded by the eNB correctly, a network coded packet given by the XOR $c_1 \oplus c_2$ is transmitted in time slot t_3 in case of CC. In case of IR, the network coded packet is determined by the XOR of the applied redundancy versions for retransmission. The main idea is to apply the principle of NC-HARQ in case of two unsuccessfully decoded packets and use common HARQ for single erroneous packets. Since the overall transmission requires only 3 time slots compared to 4 time slots in common HARQ for two erroneous packets and one retransmission, a throughput enhancement of up to 33% due to NC-HARQ can be expected by using adequate decoding algorithms.

The main contribution of this paper is the iterative soft detection schemes developed for CC- and IR-aided network coding HARQ. More specifically, for IR appropriate schemes to consider different redundancy versions for the retransmissions have been designed. The concept is presented for application in the LTE uplink, however, can generally been used for any point-to-point communications.

The remainder of the paper is organized as follows. In Section II the system model of the network coded HARQ for unicast uplink transmission is introduced. In Section III we present two soft combining methods for CC and IR. Finally, the performance is investigated by means of simulation results and conclusions are given in Section IV and V, respectively.

II. SYSTEM MODEL OF NC-HARQ

A simple application of NC-HARQ in LTE uplink transmission is illustrated by Fig. 1. Note that in order to present the idea of network coded HARQ for unicast transmission clearly, we concentrate on main aspects so that several functional components such as cyclic redundancy check (CRC)



Fig. 2. LTE turbo encoder which consists of two convolutional encoders and one turbo code internal interleaver.

attachment, coding/rate matching, scrambing/interleaving etc. are not discussed in this paper.

A. Mathematical Description

At the UE the information vector \mathbf{a}_k in time slot t_k is encoded by the standard LTE turbo encoder shown in Fig. 2 yielding the codeword \mathbf{b}_k . This systematic encoder consists of the parallel concatenation of two recursive convolutional codes with constraint length 4 and mother code rate $R_c = 1/3$ and an internal interleaver [9]. By applying LTE rate-matching (i.e., sub-interleaving, bit selection, puncturing, etc.) determined by the current transport format and the redundancy version the vector \mathbf{c}_k is achieved. The code bits $c_{k,i}$ are mapped to symbols $x_{k,j} \in \mathbb{X}$ of the modulation alphabet \mathbb{X} (e.g., QPSK, 16-QAM or 64-QAM.) and transmitted to the eNB. In case of block fading the received signal vector in time slot t_k is given by

$$\mathbf{y}_k = h_k \mathbf{x}_k + \mathbf{n}_k \;, \tag{1}$$

where \mathbf{n}_k is the noise vector whose elements are identically distributed (i.i.d) zero-mean Gaussian random variables with variance σ_n^2 . The coefficient h_k , $\forall k$ denotes complex zeromean circular symmetric Gaussian distributed variable with variance 1. Please note, that more general channel models are assumed for the performance evaluations.

Based on the received signal $y_{k,j}$ the Log-Likelihood-Ratios (LLRs) for the participating coded bit $c_{k,i}$ can be calculated by the soft demapper [10]

$$L_{\text{Dem}}(c_{k,i}) = \ln \frac{\Pr\{c_{k,i} = 0 | y_{k,j}, h_k\}}{\Pr\{c_{k,i} = 1 | y_{k,j}, h_k\}}$$
(2)
$$= \ln \frac{\sum_{x \in \mathbb{X}, c_{k,i} = 0} \exp\left(-\frac{|y_{k,j} - h_k x|^2}{\sigma_n^2}\right)}{\sum_{x \in \mathbb{X}, c_{k,i} = 1} \exp\left(-\frac{|y_{k,j} - h_k x|^2}{\sigma_n^2}\right)}.$$

To calculate the probability of $c_{k,i} = 0$ (or $c_{k,i} = 1$), all possible symbols $x \in \mathbb{X}$ related to $c_{k,i} = 0$ (or $c_{k,i} = 1$) need to be considered.

In the sequel, we introduce the encoding procedure of network coded HARQ applying Chase Combining (CC) or Incremental Redundancy (IR). To simply the description, it is assumed that the eNB is not able to decode the two successive packets c_1 and c_2 correctly. If, on the other hand, only one of these packets is not correctly decoded, the NC-HARQ scheme simplifies to a common HARQ, i.e., only the erroneous packet will be retransmitted. The only difference compared to common HARQ is that in NC-HARQ two erroneous packets from two HARQ processes are combined by means of network coding and transmitted in one time slot. By this way, the throughput efficiency can be improved.

B. Network Coded HARQ with Chase Combining (NC-HARQ-CC)

In NC-HARQ-CC, the transmission procedure is listed as follows

- 1) c_1 is transmitted in the first time slot t_1 controlled by the first HARQ process and y_1 is received and decoded. Due to decoding failure the eNB sends NACK₁ to the UE.
- 2) c_2 is transmitted in the second time slot t_2 controlled by the second HARQ process and y_2 is received and decoded. Due to decoding failure the eNB sends NACK₂ to the UE.
- A network coded retransmission based on the XOR combination of the two packets given by c₃ = c₁ ⊕ c₂ is transmitted by the UE and y₃ is received at the eNB.
- 4) From the noisy estimates of y_1 , y_2 at the first transmissions and y_3 at the retransmission, the eNB aims to reconstruct the codewords b_1 and b_2 by means of adequate detection algorithms, which will be accessed in the next section.
- 5) Based on the decoding success ACK/NACK messages are transmitted for both packets to the UE.
- 6) If any packet is still incorrectly decoded, there are several possibilities for additional retransmissions. For example, the network coded packet can be transmitted again or the retransmission can be done according to a retransmission pattern, e.g., c₁, c₂, c₃ cyclically.

C. Network Coded HARQ with Incremental Redundancy (NC-HARQ-IR)

In case of NC-HARQ-IR, the coded bits \mathbf{b}_1 and \mathbf{b}_2 are rate-matched to \mathbf{c}_1^r and \mathbf{c}_2^r for retransmission according to the different redundancy versions defined for LTE uplink [9]. A network coded packet based on the XOR operation of both redundancy versions is transmitted, i.e., $\mathbf{c}_3^r = \mathbf{c}_1^r \oplus \mathbf{c}_2^r$, where r indexes the different redundancy version according to a predefined redundancy version sequence. Therefore, contrary to NC-HARQ-CC, the network coded packet \mathbf{c}_3^r contains different redundancy version as the initial transmission.

The challenge of NC-HARQ-IR is that we are not able to obtain the information bits from the network coded packet since only subset of systematic bits may be transmitted during the retransmission phase. To this end, the all systematic bits from the previous transmitted packets need to be reconstructed. This aspect will be addressed by the discussion of the decoding approach for NC-HARQ-IR.

Compared to common HARQ, herein, we need to redesign decoding methods regarding to CC and IR to exploit the information of each packet contained in the combined transmission. Hence, the task of this paper is to develop adequate soft detection algorithms with respect to NC-HARQ-CC and NC-HARQ-IR, which will assessed in detail in the next section.

III. DECODING ALGORITHMS FOR NC-HARQ

In this section, we describe the decoding procedure for NC-HARQ-CC and NC-HARQ-IR, respectively. To fully preserve the soft information contained in the network coded packet c_3 , the well-known box-plus operation also known as soft XOR instead of common bit-level XOR is used to extract the soft information of each packet. For instance, soft XOR of two LLR values L_1 and L_2 can be calculated by [10]

$$L_1 \boxplus L_2 = 2\operatorname{atanh}(\operatorname{tanh}(L_1/2) \cdot \operatorname{tanh}(L_2/2)) \tag{3a}$$

$$\approx \operatorname{sign}(L_1) \cdot \operatorname{sign}(L_2) \cdot \min(|L_1|, |L_2|), \quad (3b)$$

where \boxplus denotes bit-wise soft XOR operation and tanh, atanh and sign indicates hyperbolic tangent function, inverse hyperbolic tangent function and sign function, respectively. Note that with consideration of the implementation complexity, we use the approximated form in the LTE uplink simulations.

A. Decoding Approach for NC-HARQ-CC

Fig. 3 depicts the decoding procedure of network coded HARQ based on CC. From the received signals \mathbf{y}_1 and \mathbf{y}_2 , we can calculate with (2) the corresponding LLR vectors, i.e., $L_{Dem}(\mathbf{c}_1)$ and $L_{Dem}(\mathbf{c}_2)$. Based on these LLRs, the LLRs of the corresponding mother codeword $L_{Dem}(\mathbf{b}_1)$ and $L_{Dem}(\mathbf{b}_2)$ is obtained by the means of rate de-matching according to [9]. Then, the turbo decoder estimates the LLRs for the codebits, namely $L_{Dec}(\mathbf{b}_1)$ and $L_{Dec}(\mathbf{b}_2)$.

For the retransmission, the LLRs of the network coded signal y_3 can be calculated by (2), i.e., $L_{Dem}(c_3)$. To obtain nearly the same complexity of CC and IR, we introduce an optional turbo decoder for CC as shown in Fig. 3. We will also use the LLR output of this turbo decoder denoted by $L_{Dec}(b_3)$ for further processing. $L_{Dec}(b_3)$ contains additional information about b_1 and b_2 . By using the soft XOR operation in (3b), the additional soft information of each packet can be achieved if the transmissions of two initial packets are independent,

$$L_{\rm NC}(\mathbf{b}_1) = L_{\rm Dec}(\mathbf{b}_2) \boxplus L_{\rm Dec}(\mathbf{b}_3) , \qquad (4a)$$

$$L_{\rm NC}(\mathbf{b}_2) = L_{\rm Dec}(\mathbf{b}_1) \boxplus L_{\rm Dec}(\mathbf{b}_3) , \qquad (4b)$$

Now, similar to [11], [12], the LLR values from the network coded packet and the first transmission can be combined by means of bit-level LLR combination, which is simply addition of two LLR values, i.e., $L_{Dec}(\mathbf{b}_1) + L_{NC}(\mathbf{b}_1)$ and $L_{Dec}(\mathbf{b}_2) + L_{NC}(\mathbf{b}_2)$ as the inputs of the turbo decoders shown in Fig. 3. After decoding, the payload of each packet can be estimated

and the LLR values of both packets are updated by the ouput LLRs of the decoders $L'_{Dec}(\mathbf{b}_1)$ and $L'_{Dec}(\mathbf{b}_2)$.

If any packet can not be successfully decoded, for simplicity, the same network coded packet can be retransmitted again. The decoding process is the same as in the first retransmission. The only difference is that the updated LLR values $L'_{Dec}(\mathbf{b}_1)$ and $L'_{Dec}(\mathbf{b}_2)$ are used for bit-level combination. The NC-HARQ continues until both packets are correctly decoded or a maximum number of transmission is reached. In comparison with common CC based HARQ, NC-HARQ-CC has the similar complexity.

B. Decoding Approach for NC-HARQ-IR

In case of NC-HARQ-IR, since the network coded packet is constructed by XOR-adding two codewords of different redundancy versions, the network coded packet alone can not be decoded in general. In this paper a decoding strategy for NC-HARQ-IR is shown in Fig. 4. As illustrated, the systematic bits are reconstructed manually by soft XOR of the LLRs of both original packets

$$L_{\rm NC}(\mathbf{b}_3) = L_{\rm Dec}(\mathbf{b}_1) \boxplus L_{\rm Dec}(\mathbf{b}_2). \tag{5}$$

Then the bit-level maximum ratio combining of $L_{\rm NC}({\bf b}_3)$ and $L_{\rm Dem}({\bf b}_3)$ is applied. Herein, this combination is equivalent to common HARQ-IR that two different redundancy version w.r.t. the XOR construction of the two packets. By this way, the reliability of the network coded packet is approved. The rest of the algorithm is similar to NC-HARQ-CC.

In comparison to common HARQ-IR, the complexity of NC-HARQ-IR is slightly increased due to the additional decoder.

C. Decoding Approach for NC-HARQ-IR with Soft Reencoding

As mentioned, it is not guaranteed that the systematic bits can be decoded in retransmission phase for IR, since only part of the code bits are retransmitted. The idea of soft re-encoding is that based on the soft information (LLR) of the systematic bits in the first transmission, the LLRs of the whole codeword can be generated by means of the BCJR algorithm [13], [14]. Note that BCJR algorithm is originally used for decoding the convolutional codes, herein, it is used to generate the soft code bits of a convolutional code based on the soft information of the data bits, i.e., for encoding. The BCJR algorithm for softreencoding can be found in [13].

The soft re-encoding is applied for convolutional code, however, as shown in Fig. 5, we extend it to the turbo code case. Considering the fact that the rate matching is doing nevertheless permutation, interleaving and puncturing, once the soft information about the whole codeword **b** is obtained, the soft information of any rate-matched codeword is also achieved.

The decoding process for NC-HARQ-IR with soft reencoding is illustrated in Fig. 6. By using the soft re-encoder, the LLRs of the codeword with any redundancy version can be generated, e.g. from the LLRs of the systematic bits



Fig. 3. Block diagram of NC-HARQ-CC. By using soft XOR, additional information of each packet can be exploited. Here, \oplus indicates the bit-level LLR combination and \boxplus denotes soft XOR.



Fig. 4. Block diagram of NC-HARQ-IR. The systematic bits of network coded packet need to be reconstructed in order to exploit the information contained in the network coded packet. Here, \boxplus denotes soft XOR.



Fig. 6. NC-HARQ-IR by soft re-encoding.



Fig. 5. Soft re-encoding based on modified BCJR algorithms to generate the LLRs of the whole codeword from the LLRs of the information bits.

 $L_{Dec}(\mathbf{a}_1)$, the LLRs of the second redundancy version can be obtained, denoted by $L_{En}(\mathbf{b}_1)$. Therefore, the LLRs of codeword \mathbf{c}_2 can be calculated by the box-plus operation from the network-coded packet $L_{Dec}(\mathbf{b}_3)$ and $L_{En}(\mathbf{b}_1)$, i.e., $L_{NC}(\mathbf{b}_2) = L_{Dec}(\mathbf{b}_3) \boxplus L_{En}(\mathbf{b}_1)$. Similarly, $L_{NC}(\mathbf{b}_2)$ can be achieved by the same way. Consequently, the soft information extracted from the network-coded packet is combined with the soft information obtained in the first transmission, i.e., bit-level maximum ratio combining.

IV. PERFORMANCE EVALUATION

In this section, we compare the proposed NC-HARQ schemes by simulation results given by a standard link-level simulator. The detailed network parameters setup is summarized in Tab.I.

Parameters	Setup
Bandwidth	20 MHz
Carrier frequency	2.3 GHz
Modulation	16-QAM
Turbo code rate	1/3
Codeword length	2640
Effective code rate	0.4
Number of antennas (UE)	1
Number of antennas (eNB)	2
Max. number of retransmission	4

TABLE I Network parameters for LTE uplink simulations.

Moreover, the normalized throughput is considered as key performance indicator

$$T_{\text{normalized}} = \frac{N_{\text{correct}}}{N_{\text{total}}} , \qquad (6)$$

where N_{correct} denotes the number of packets that are correctly decoded and N_{total} indicates the total number of transmitted packets.

Fig. 7 shows the remaining block error rate (BLER) after one retransmission for common HARQ and NC-HARQ transmission over AWGN channel. Note that the BLER curves of common HARQ and NC-HARQ are with different throughput efficiency, since for common HARQ 4 time slots are used for 2 packets while only 3 time slots are used for NC-HARQ. The main purpose of this figure is to compare the performance of the proposed soft combining algorithm. Fig. 7 shows that common HARQ achieves better performance than NC-HARQ after the first retransmission, e.g. about 0.6 dB gain with IR and 0.4 dB gain with CC at BLER of 10^{-3} . However, as shown in Fig. 8 it can be observed that for SNR= 3dB that NC-HARQ reaches the 2/3 normalized throughput while common HARQ achieves 1/2 normalized throughput. NC-HARQ outperforms common HARQ with respect to the effective throughput, e.g., the theoretical gain of 33% is achieved. The reason is that NC-HARQ can recover both packets by only one network coded retransmission at this SNR level while common HARO needs two retransmissions.



Fig. 7. The remaining BLER after one retransmission of common HARQ and network coded HARQ for AWGN channel.

Fig. 9 depicts the throughput for a typical urban channel with UE speed 30km/h [15]. It can be observed that about 33% throughput gain is achieved for NC-HARQ-IR while about 25% for NC-HARQ-CC at the SNR of 2 dB. In comparison to AWGN channel considered in Fig. 8, the randomness of the typical urban channel results in relative flat behavior of the throughput w.r.t SNR. Fig. 10 illustrates the throughput for a 1-path Rayleigh fading channel with UE speed of 50km/h. Herein, throughput enhancements up to 33% can be observed for NC-HARQ at the moderate SNR. Therefore, the proposed NC-HARQ schemes promise throughput enhancements with slightly increased complexity.

Fig.11 shows the throughput of the simple NC-HARQ-IR and the NC-HARQ-IR with soft re-encoding for AWGN channel. The NC-HARQ-IR with soft re-encoding performs



Fig. 8. Throughput performance of common HARQ and network coded HARQ for AWGN channel.



Fig. 9. Throughput performance of common HARQ and network coded HARQ for a typical urban channel with UE speed 30kmh.

worse than the simple NC-HARQ-IR, especially at low SNR. The reason is that the re-encoding with a LTE turbo encoder consisting of two recursive convolutional code can lead to error propagation, which leads to dramatically amplitude reduction of the LLR values, i.e., if any bit is not reliable, the whole codeword can be influenced. In comparison to the simple NC-HARQ-IR, the NC-HARQ-IR with soft re-encoding is not favorable.

Fig.12 shows the throughput of the NC-HARQ-CC with/without the optional turbo decoder shown in Fig. 3 for AWGN channel. By introducing the optional turbo decoder, the throughput is slightly enhanced especially for low SNR.

V. CONCLUSION

In this paper, a novel network coded HARQ approach is proposed for wireless unicast transmission. Although the design is related to LTE uplink, the proposed scheme is not limited to LTE uplink but can be applied to any HARQ transmission. In NC-HARQ, two erroneous packets from different HARQ processes are combined by means of XOR.



Fig. 10. Throughput performance of common HARQ and network coded HARQ for 1-path Rayleigh fading channel with UE speed 50kmh.



Fig. 11. Throughput comparison of simple NC-HARQ-IR and NC-HARQ-IR with soft re-encoding for AWGN channel.

We introduced the decoding procedure for Chase Combining and Incremental Redundancy and designed a soft-reencoding with BCJR algorithm. As shown by simulation results, the theoretical throughput gain up of to 33% in comparison to common HARQ can be achieved by NC-HARQ.

VI. ACKNOWLEDGEMENT

The authors would like to thank Aydar Zimaliev for assistance with the LTE simulation chain.

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Fig. 12. Throughput comparison of the NC-HARQ-CC with/without the optional turbo decoder for AWGN channel.

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