Multi-Carrier Two-Way Relaying with Non-Binary Coding

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***Abstract*—We apply non-binary channel coding to the symmetric multi-carrier two-way relay channel and evaluate two decoding approaches at the relay. *Complete-decode-and-forward* (CDF) decodes the packets of both users before forwarding the binary sum in the second phase, while *functional-decode-and-forward* (FDF) directly decodes the binary sum of both packets. For both approaches, we study suitable subchannel or rate allocation strategies and apply single-user and joint decoding of both packets.**

I. Introduction

The two-way relay channel, in which two users exchange information without the availability of a direct link, constitutes one of the canonical models for the application of network coding: while the relay receives the messages of both users, it is sufficient to retransmit a function of both messages such that each user can recover the other user's message using the knowlegde of its own message as visualized in Figure 1. The function which is typically applied to combine the two messages is the binary sum, which is also known as the XOR operation. It was soon recognized that for this reason there is no need at the relay to decode both messages and that it is sufficient to obtain the combined message [1], [2], which can be done in an elegant way by exploiting the linearity of the applied channel code. In this paper, we focus on two decoding approaches for the multi-carrier two-way relay channel, namely

- complete-decode-forward (CDF): the relay first decodes both messages and then forms the combined message
- functional-decode-forward (FDF): the relay decodes directly for the function, i.e. the XOR, of the two messages

For these two stragegies, some results in terms of achievable rates are available [3]–[6] while results for the practically important case of a discrete transmit alphabet and for channels with fading are still unknown. A very elegant approach for functional decoding and for the more general concept of *compute-forward* is given by lattice coding, which defines linear channel codes directly in the real number field and therefore can exploit the superposition of the received signals in a direct way [7]–[9]. While this approach has led to important insights in terms of achievable rates, it presents some difficulties for practical implementation, in particular for channels with fading.

Another line of research approaches the topic by considering the real or complex-valued transmit symbols and places less emphasis on channel coding [10]–[12]. While these results apply directly to coded systems with hard decoding, e.g., for algebraic codes like BCH or Reed-Solomon codes, they are not necessarily directly relevant for modern channel codes with soft decoding, like turbo and LDPC codes, which we consider in this paper. In the following, we focus on non-binary channel codes, which combine well with higher-order modulation schemes [13]–[16] and we restrict ourselves to relaying schemes which are downlink optimal [17], i.e. no noise from the uplink passes to the downlink. Since the broadcast (downlink) phase then simply transmits the combined message, the bottleneck of the two-way relay transmission lies in the uplink phase, on which we will focus in this paper.

Finally, we consider a broadband system where multicarrier modulation, e.g. OFDM or Filterbank Multi-Carrier (FBMC) is being used to combat the frequency selectivity of the channel. Under these circumstances, it is well known that it is generally preferable to use a single encoder and spread the codeword over the subcarriers, as opposed to encode each subcarrier separately [18], [19]. Beyond this basic result, the degrees of freedom offered by the subcarriers can be employed to implement adaptive resource allocation schemes that improve the data rate.

The remainder of this paper is organized as follows. In Section II we introduce the system model, discuss the principles of modulation with non-binary coding and introduce the necessary steps for estimating the relay codeword. The mutual information for CDF and FDF is investigated in Section III and link-level simulation results for a single-carrier TWRC are presented in Section IV. Additionally, we develop an adaptive resource allocation scheme for the multi-carrier TWRC in Section V and provide concluding remarks in Section VI.

II. System Model

In this paper, we consider a broadband two-way relay channel implementing a multi-carrier modulation such as OFDM or FBMC. More specifically, user-relay channels are modeled as

ISBN 978-3-8007-3659-1

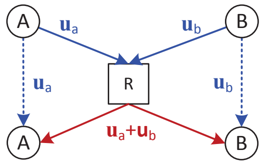

Figure 1. The two-way relay channel: Users A and B want to exchange their information packets $\mathbf{u}_\mathsf{a}$ and $\mathbf{u}_\mathsf{b}$ via the relay R. There is no direct link between the users.

frequency selective block-fading channels, meaning that the frequency response of the channel does not vary during the transmission of a codeword. Even though this implies reduced users' mobility, it is a reasonable assumption when targeting high rate broadband communications.

Almost static channels allow for a reasonable amount of channel state information at the transmitter side by means of a feedback channel. In the two-way relay case, channel reciprocity can also be exploited, since communications happen on both directions (users to relay and relay to users). Channel knowledge at the users' side yields two major benefits. First, channel equalizers can be implemented at the transmitters, thus releasing the relay from the difficult task of equalizing both channels at the same time. Second, and most important here, users can adapt their transmission parameters (power, rate, modulation) to the channel in order to increase their rate.

Before explaining all the details of the system model in Section II-E, we need to give a brief overview about non-binary coding and on how the resulting codewords are modulated.

A. Modulation with Non-Binary Coding

We represent the messages of both users as vectors of length K in the Galois field $\mathbb{F}_q = \mathrm{GF}(q)$, where the field order q is assumed to be a power of two. These messages $\mathbf{u}_\mathsf{a} \in \mathbb{F}_q^K$ and $\mathbf{u}_\mathsf{b} \in \mathbb{F}_q^K$ are encoded into the codewords $\mathbf{c}_\mathsf{a} = [c_{\mathsf{a},1}, \ldots, c_{\mathsf{a},N}] \in \mathbb{F}_q^N$ and $\mathbf{c}_\mathsf{b} = [c_{\mathsf{b},1}, \ldots, c_{\mathsf{b},N}] \in \mathbb{F}_q^N$ by

$$\mathbf{c}_\mathsf{a} = \mathbf{u}_\mathsf{a}\mathbf{G}, \; \mathbf{c}_\mathsf{b} = \mathbf{u}_\mathsf{b}\mathbf{G}, \tag{1}$$

where $\mathbf{G} \in \mathbb{F}_q^{K\times N}$ denotes the generator matrix. The codeword symbols $c_{\mathsf{a},n}$, $c_{\mathsf{b},n}$ are mapped to real-valued vectorial PAM symbols $\mathbf{x}_{\mathsf{a},n}$, $\mathbf{x}_{\mathsf{b},n}$, i.e.

$$\mathbf{x}_{\mathsf{a},n} = \boldsymbol{\mu}\left(c_{\mathsf{a},n}\right), \; \mathbf{x}_{\mathsf{b},n} = \boldsymbol{\mu}\left(c_{\mathsf{b},n}\right), \tag{2}$$

where $\boldsymbol{\mu} : \mathbb{F}_q \to \mathbb{R}^T$ denotes the mapping function. Each q-ary codeword symbol is mapped to T M_P-PAM symbols, i.e. it holds $q = M_\mathsf{P}^T$, being T a small integer. Note that for $q > 2$ this is different to the usual QAM mappings with binary codes, in which several coded bits are mapped to *one* QAM symbol. Here, instead, we map *one* coded symbol to several PAM symbols.

The advantage of including only one codeword symbol in the mapping is that in this way the equivalent channel between the encoder and the decoder remains memoryless, given that the physical channel does not introduce any memory. This property is implicitly assumed by a belief-propagation (BP) decoder which operates on a Tanner graph of the code. For binary codes, this assumption is only fulfilled exactly for BPSK while for higher-order modulations it does not hold.

For $q = 16$, we can map one codeword symbol to four BPSK symbols, two 4-PAM symbols or simply to one 16-PAM symbol, as listed in Table I. Although not used in the following, we note that for $T = 3$ it is possible to define a 3-dimensional constellation of 16 points by e.g. selecting 16 points from a suitable sphere packing [20], [21].

As a basis for the two-way relay channel, we briefly describe the soft demapping of a q-ary channel code in the case of a single user. The received signal of a single-user channel with fading is given by

$$\mathbf{y}_n = \mathbf{h}_n \circ \mathbf{x}_n + \mathbf{w}_n, \; \mathbf{w}_n \sim \mathcal{N}\left(0, \mathbf{I}_T\right), \mathbf{x}_n = \boldsymbol{\mu}\left(c_n\right), \tag{3}$$

where $\circ$ denotes the Hadamard product and $\mathbf{h}_n = [h_{n,1}, \ldots, h_{n,T}]$ stands for the channel coefficients.

As input for the decoder, we need to calculate the *a posteriori probabilities (APP)* for each codeword symbol:

$$p_n\left([\alpha]\right) \triangleq P\left[c_n = \alpha \mid \mathbf{y}_n\right], \; \alpha \in \mathbb{F}_q, \tag{4}$$

where we denote by $[\alpha] \in \mathbb{Z}_q = \{0, 1, \ldots, q-1\}$ the integer value which corresponds to the GF element α. With Bayes' theorem, we find

$$p_n\left([\alpha]\right) \propto p\left(\mathbf{y}_n \mid c_n = \alpha\right) \propto \exp\left(-\left\|\mathbf{y}_n - \mathbf{h}_n \circ \boldsymbol{\mu}\left(c_n\right)\right\|^2\right) \tag{5}$$

For binary codes, this can be reduced to a scalar value per coded bit, e.g. to the well-known L-values.

Table I
PARAMETERS FOR MAPPING FROM q-ARY CODEWORD SYMBOLS TO T-DIMENSIONAL M_P-PAM SYMBOLS FOR $q = 16$

T	1	2	3	4
M_P	16	4	–	2

The benefit of preserving a memoryless channel between encoder and decoder is also reflected in the modulation-constrained capacity: while for the binary case, the achievable rate is limited by the BICM capacity, for $q > 2$ with the described mappings the achievable rate is limited by the coded modulation (CM) capacity, which is higher (or equal for some few special cases) than the BICM capacity [22], [23]. Figure 2 shows the CM capacities for the modulations of Table I over the fast real-valued Rayleigh fading channel.

B. Uplink of the Two-Way Relay Channel

The uplink of the TWRC corresponds to the two-user multiple-access channel. We assume that the vectorial transmit symbols $\mathbf{x}_{\mathsf{a},n}, \mathbf{x}_{\mathsf{b},n} \in \mathbb{R}^T$, $T \in \mathbb{N}$, are multiplied by fading coefficients $\mathbf{h}_{\mathsf{a},n}, \mathbf{h}_{\mathsf{b},n} \in \mathbb{R}^T$ and are affected by unit-variance AWGN $\mathbf{w}_n \sim \mathcal{N}\left(0, \mathbf{I}_T\right)$, i.e.

$$\mathbf{y}_n = \mathbf{h}_{\mathsf{a},n} \circ \mathbf{x}_{\mathsf{a},n} + \mathbf{h}_{\mathsf{b},n} \circ \mathbf{x}_{\mathsf{b},n} + \mathbf{w}_n. \tag{6}$$

Further, we assume that the power gain of both channels is the same on average, such that the average SNR is given by

$$\mathrm{SNR} = \mathbb{E}\left[h_{\mathsf{a},n,t}^2\right] = \mathbb{E}\left[h_{\mathsf{b},n,t}^2\right]. \tag{7}$$

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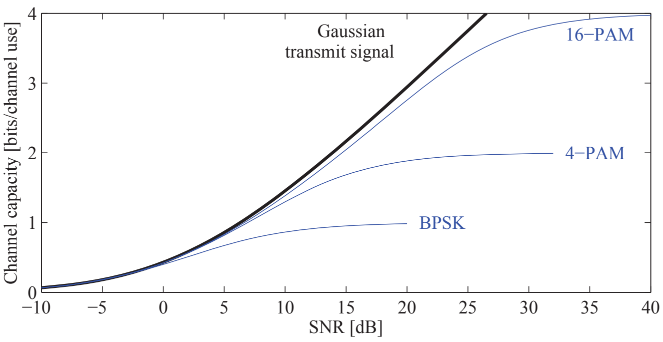

Figure 2. Channel capacities for fast Rayleigh fading

C. Demapping for Complete and for Functional Decoding

The conditional pdf of the TWRC according to (6) is given by

$$p\left(\mathbf{y}_n \mid c_{\mathsf{a},n}, c_{\mathsf{b},n}\right) \propto f_n\left(c_{\mathsf{a},n}, c_{\mathsf{b},n}\right)$$
$$\triangleq \exp\left(-\left\|\mathbf{y}_n - \mathbf{h}_{\mathsf{a},n} \circ \boldsymbol{\mu}\left(c_{\mathsf{a},n}\right) - \mathbf{h}_{\mathsf{b},n} \circ \boldsymbol{\mu}\left(c_{\mathsf{b},n}\right)\right\|^2\right). \quad (8)$$

For complete decoding, the relay tries to decode both messages $\mathbf{u}_\mathsf{a}$ and $\mathbf{u}_\mathsf{b}$. To this end, the APPs with respect to the coded symbols $c_{\mathsf{a},n}$ and $c_{\mathsf{b},n}$ have to be computed. In analogy to (4), this is given by

$$p_{\mathsf{a},n}\left([\alpha]\right) \triangleq P\left[c_{\mathsf{a},n} = \alpha \mid \mathbf{y}_n\right]$$
$$= \sum_{\beta\in\mathbb{F}_q} P\left[c_{\mathsf{a},n} = \alpha, c_{\mathsf{b},n} = \beta \mid \mathbf{y}_n\right] \propto \sum_{\beta\in\mathbb{F}_q} f_n\left(\alpha, \beta\right) \quad (9)$$

and

$$p_{\mathsf{b},n}\left([\beta]\right) \triangleq P\left[c_{\mathsf{b},n} = \beta \mid \mathbf{y}_n\right] \propto \sum_{\alpha\in\mathbb{F}_q} f_n\left(\alpha, \beta\right) \quad (10)$$

For functional decoding, the relay tries to decode directly for $\mathbf{u}_{\mathsf{ab}} \triangleq \mathbf{u}_\mathsf{a} + \mathbf{u}_\mathsf{b} \in \mathbb{F}_q^K$. Here, we can exploit the linearity of the code and directly decode for the sum of both codewords, since this is also a codeword:

$$\mathbf{c}_{\mathsf{ab}} \triangleq \mathbf{c}_\mathsf{a} + \mathbf{c}_\mathsf{b} = \mathbf{u}_\mathsf{a}\mathbf{G} + \mathbf{u}_\mathsf{b}\mathbf{G} = \left(\mathbf{u}_\mathsf{a} + \mathbf{u}_\mathsf{b}\right)\mathbf{G}$$

Note that this arithmetic is defined in $\mathbb{F}_q$. Since we assume that q is a power of two, i.e., the field $\mathbb{F}_q$ is an extension field of $\mathbb{F}_2$, the addition in $\mathbb{F}_q$ is the same as the vectorial addition in $\mathbb{F}_2$ and hence the sum in the field $\mathbb{F}_q$ corresponds to the XOR operator on the binary representation of the codeword symbols. The APP w.r.t. the combined codeword symbol $c_{\mathsf{ab},n} = c_{\mathsf{a},n} + c_{\mathsf{b},n}$ is therefore given by

$$p_{\mathsf{ab},n}\left([\alpha]\right) \triangleq P\left[c_{\mathsf{ab},n} = \alpha \mid \mathbf{y}_n\right] \propto \sum_{\beta\in\mathbb{F}_q} f_n\left(\alpha + \beta, \beta\right) \quad (11)$$

The simplest example for functional "decoding" is the AWGN TWRC, i.e. $h_\mathsf{a} = h_\mathsf{b}$, with uncoded BPSK. In this case, it is possible to recover the combined symbols c_{ab} but not the individual symbols c_a or c_b. However, for higher-order modulations, the situation is quite different. It has been shown that it is not possible to find a mapping $\mu : \mathbb{F}_q \rightarrow \mathbb{R}$ such that the sum of the PAM symbols, $x_\mathsf{a} + x_\mathsf{b}$ corresponds to a unique value of $c_\mathsf{a} + c_\mathsf{b}$[1] [12], [24]. Figure 3 shows the superposition of two Gray-labeled 4-PAM constellations and the resulting bit labels for c_{ab}. We see that the values $x_\mathsf{a} + x_\mathsf{b} \in \{-2, 2\}$ cannot be uniquely identified with a coded symbol c_{ab}. However, this is not necessarily a serious problem for a coded system with soft decoding: the APPs defined above automatically account for this peculiarity. Nevertheless, the search for multi-dimensional non-binary constellations which provide good performance for functional decoding remains an interesting research question.

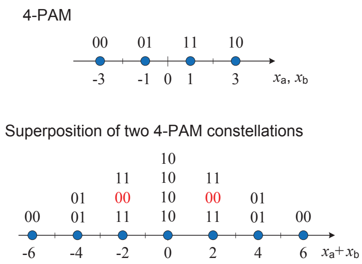

Figure 3. Superposition of two 4-PAM constellations. The bit labels for the combined symbols c_{ab} are not unique for some constellation points.

D. Joint Decoding of Non-Binary LDPC Codes

For the joint decoding of both codewords with a single decoder, we first define the joint codeword symbols by

$$d_n \triangleq q \cdot [c_{\mathsf{a},n}] + [c_{\mathsf{b},n}] \in \mathbb{Z}_{q^2} \quad (12)$$

and form the joint codeword $\mathbf{d} = [d_1, \ldots, d_N] \in \mathbb{Z}_{q^2}^N$. The mapping from GF symbols to integers is necessary to define the symbols in the larger range $\mathbb{Z}_{q^2}$. Alternatively, we could define the joint codeword symbols in $\mathbb{F}_q^2$, i.e. each codeword symbol as a two-dimensional vector of $\mathbb{F}_q$ elements [25]. It is, however, generally *not* possible to define the joint codeword symbols in the extension field $\mathbb{F}_{q^2}$. For the definition of a joint BP decoder on the code's Tanner graph, the representation of the codeword symbols as integers according to (12) is sufficient to apply the transform-based check node processing, which reduces the complexity of a decoder in $\mathbb{F}_q$ from $\mathcal{O}\left(q^2\right)$ to $\mathcal{O}\left(q \log q\right)$. Analogously, the complexity of a joint decoder of two q-ary codewords scales with $\mathcal{O}\left(q^2 \log q\right)$ [16].

For this joint decoder, the input is given by a vector of all APPs,

$$p_n\left(b\right) \triangleq P\left[d_n = b \mid \mathbf{y}_n\right] \propto f_n\left(\left\lfloor \frac{b}{q} \right\rfloor, b \bmod q\right) \quad (13)$$

for $b \in \mathbb{Z}_{q^2} = \left\{0, 1, \ldots, q^2 - 1\right\}$. From these APPs, we can

[1]Recall that arithmetic on $x_\mathsf{a}, x_\mathsf{b} \in \mathbb{R}$ is the usual one while for $c_\mathsf{a}, c_\mathsf{b}$ the Galois field arithmetic applies, i.e. the operator + appears in two meanings.

ISBN 978-3-8007-3659-1

obtain the APPs for the usual single-user decoder, (9)-(11), as

$$\begin{aligned} p_{\mathsf{a},n}\left(b_1\right) &= \sum_{b_2=0}^{q-1} p_n\left(q\cdot b_1 + b_2\right) \\ p_{\mathsf{b},n}\left(b_2\right) &= \sum_{b_1=0}^{q-1} p_n\left(q\cdot b_1 + b_2\right) \\ p_{\mathsf{ab},n}\left([\alpha]\right) &= \sum_{\beta\in\mathbb{F}_q} p_n\left(q\cdot[\alpha] + [\alpha+\beta]\right) \end{aligned} \quad (14)$$

Here, we again have to carefully distinguish between the GF elements $\alpha \in \mathbb{F}_q$ and their associated integer value $[\alpha] \in \mathbb{Z}_q$.

E. The Multi-Carrier Two-Way Relay Channel

For the multi-carrier TWRC, we have to map the vectorial transmit symbols to subcarriers. For adaptive modulation with non-binary coding as described above, one codeword symbol $c_{\mathsf{a},n}$ is mapped to one or more subcarriers, as depicted in Fig. 4. This model might correspond to an equalized filterbank with offset QAM or to the real and imaginary parts of OFDM subcarriers, as the subcarrier signals $s_{\mathsf{a},m}$ are real-valued PAM symbols. The modulation applied to each codeword symbol $c_{\mathsf{a},n}$ is identified by the parameter $T_n \in \{1,2,4\}$ according to Table I. Since in the following, we assume constant power allocation for all cases, the key difference between single-carrier and multi-carrier transmission is the assumption of channel state information at the transmitter for the latter case and henceforth the possibility for adaptive modulation. Note that in absence of power allocation, the number of subcarriers is not relevant for the adaptation of the modulation, also known as bit-loading in the context of multicarrier modulation.

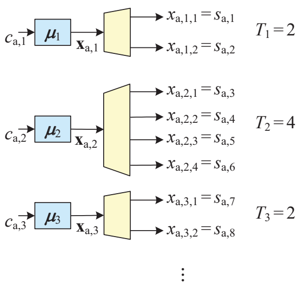

Figure 4. Symbol to subcarrier mapping in the uplink of the multi-carrier two-way relay channel

III. Mutual Information

For a coded system with soft decoding, which is state-of-the-art in virtually all wireless systems, the uncoded BER is *not* a meaningful performance indicator. Instead, the mutual information of the equivalent channel between the encoder output and the decoder input can be used as a precise performance metric [26]. From this mutual information, the packet error rate (PER) of the coded system can be predicted with very good accuracy [27]. For these reasons, we can use the mutual information to evaluate the performance of complete and functional decoding without considering the details of a particular coding scheme.

A. Complete Decoding

For complete decoding, the uplink of the TWRC corresponds to the multiple-access channel (MAC), for which the rate regions in the single and multi-carrier case are well known [28]. Since in this paper we focus on the symmetric case in which both users transmit at the same rate, the scalar channel capacity with equal rate for both users is the suitable performance bound. Taking into account the discrete transmit alphabet, we can write for the capacity of user A,

$$C_{\mathsf{a}} \triangleq I\left(c_{\mathsf{a}};\mathbf{y}\right) = \sum_{c_{\mathsf{a}}=0}^{q-1} \int p\left(c_{\mathsf{a}},\mathbf{y}\right) \log_2 \frac{p\left(c_{\mathsf{a}},\mathbf{y}\right)}{p\left(c_{\mathsf{a}}\right)p\left(\mathbf{y}\right)} \mathrm{d}\mathbf{y} \quad (15)$$

and analogously for user B. Since we are considering only symmetric channels, we have $C_{\mathsf{b}} = C_{\mathsf{a}}$. While there is no closed-form expression for C_{a}, for moderate values of q, it can be easily computed numerically. An alternative numerical method was given by Kliewer et al. in the context of EXIT charts [29]. This approach is based on the APPs according to (9) and leads to

$$C_{\mathsf{a}} = \log_2 q + \mathbb{E}\left[\sum_{b=0}^{q-1} p_{\mathsf{a},n}(b)\cdot\log_2 p_{\mathsf{a},n}(b)\right] \quad (16)$$

B. Functional Decoding

Since the relay is only interested in the sum of both packets, it seems straightforward to compute the mutual information of the sum of the two symbols, $c_{\mathsf{ab}} = c_{\mathsf{a}} + c_{\mathsf{b}} \in \mathbb{F}_q$. The mutual information $I\left(c_{\mathsf{ab}};\mathbf{y}\right)$ can be computed in a very similar way as C_{a} above, i.e.

$$C_{\mathsf{ab}} \triangleq I\left(c_{\mathsf{ab}};\mathbf{y}\right) = \sum_{c_{\mathsf{ab}}=0}^{q-1} \int p\left(c_{\mathsf{ab}},\mathbf{y}\right) \log_2 \frac{p\left(c_{\mathsf{ab}},\mathbf{y}\right)}{p\left(c_{\mathsf{ab}}\right)p\left(\mathbf{y}\right)} \mathrm{d}\mathbf{y} \quad (17)$$

and the numerical computation can be carried out without problems, but we note that we encounter difficulties when we try to define an equivalent channel from c_{ab} to $\mathbf{y}$. This difficulty is related to more fundamental problems in defining the "capacity" for the transmission of the combined packet. Actually, there is evidence that the mutual information $C_{\mathsf{ab}} = I\left(c_{\mathsf{ab}};\mathbf{y}\right)$ does *not* constitute an upper bound for the rate of the combined packet [15] and to the best of our knowledge, the capacity for functional decoding is not known. Nevertheless, we can use C_{ab} as an upper bound for single-user functional decoding based on the APPs $p_{\mathsf{ab},n}$ defined in (11), whereas with joint functional decoding based on (13) higher performance may be achieved.

The mutual informations $C_{\mathsf{a}} = C_{\mathsf{b}}$ and C_{ab} according to (15), (17) are plotted in Figure 5 for 2-PAM (BPSK), 4-PAM and 16-PAM over the real-valued fast Rayleigh TWRC. We note that for sufficient SNR, C_{ab} is higher than C_{a}. A noticeable difference to the single-user case is that modulations with higher rate do not always provide a higher mutual information.

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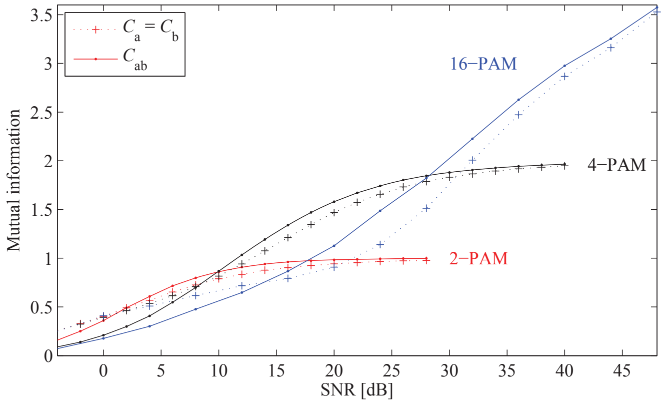

Figure 5. Mutual informations $C_\mathsf{a} = C_\mathsf{b}$ for complete and C_ab for functional decoding

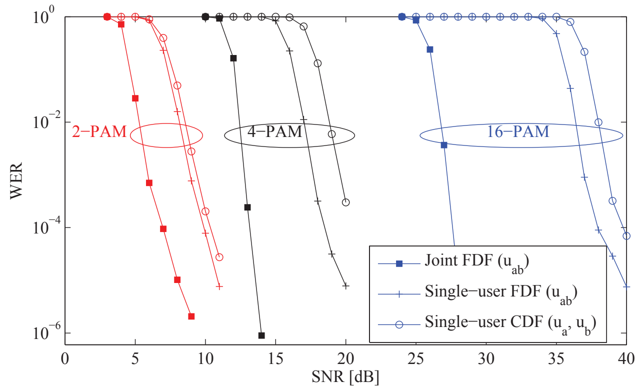

Figure 6. Word error rates for the single-carrier TWRC with fast Rayleigh fading for $\{2, 4, 16\}$-PAM

IV. Simulation Results for the TWRC with Fixed Modulations

In order to evaluate the different decoding approaches, we first performed simulations with fixed modulations for a fast fading TWRC. All simulations have been carried out with a rate $1/2$ non-binary LDPC code with field order $q = 16$ and message length $K = 180$, i.e. 720 bits. The channel coefficients $h_{\mathsf{a},n}, h_{\mathsf{b},n}$ undergo i.i.d. fast Rayleigh fading. Figure 6 shows the codeword (packet) error rates for the three considered modulations and for the following decoding methods

- Joint functional decoding: a joint decoder as described in Section II-D is applied on the APPs given by (13) and from its output, the estimate $\hat{\mathbf{u}}_\mathsf{ab}$ is derived.
- Singe-user functional decoding: a single-user decoder is applied on the APPs given by (11) to recover directly $\mathbf{u}_\mathsf{ab}$.
- Single-user complete decoding: two single-user decoders are applied on the APPs (9), (10) to recover $\mathbf{u}_\mathsf{a}$ and $\mathbf{u}_\mathsf{b}$. Due to symmetry, the error rates are identical.

As predicted by the mutual informations in Figure 5, we can observe that single-user FDF is slightly superior to CDF, which has the double complexity since it has to decode two packets. Joint decoding achieves a remarkable gain for all modulations, albeit at the prize of significantly more complexity ($\mathcal{O}\left(q^2 \log q\right)$ vs. $\mathcal{O}\left(q \log q\right)$.

V. Adaptation in the Multi-Carrier TWRC

This section shows how the users can adapt their transmission parameters to the channel in order to increase their rate, given that CSI is available at the transmitter side. Apart from power allocation, which we do not consider in this paper, this CSI can be exploited in two ways: by subchannel allocation, or by adaptive modulation (bit-loading), depending on the employed decoding approach.

For complete decoding, which corresponds directly to the multiple-access channel, we may apply known results for OFDMA and from information theory. From the latter, in the vector multiple-access channel, iterative water-filling is known as the optimum solution [28] in the sense of sum capacity. While iterative water-filling is a power-allocation scheme, it typically converges to an orthogonal subchannel allocation which is optimum when the number of subchannels tends to infinity [30]. For the TWRC, we have the additional constraint of equal rates for both users, for which reason we used a simple subchannel allocation method, that selects for each user half of the available subchannels. With this orthogonal allocation, we obtain two single-user problems for the rate selection, which are trivially solved by observing from Figure 2 that the highest modulation achieves the highest capacity.

For functional decoding, on the other hand, both users have to transmit with the same modulation per subchannel in order to allow the computation of the "functional" APPs $p_{\mathsf{ab},n}$ according to (11). This converts the multi-carrier problem into N single-carrier adaptive modulation selections, which lead to choosing the modulation with the highest mutual information C_ab. The average mutual informations per subchannel according to these simple adaptation strategies are plotted in Figure 7. Since the mutual informations are computed per subchannel, we have to consider additionally that with the orthogonal allocation for complete decoding, the mutual information per multicarrier symbol is divided by two for this case. Therefore, functional decoding is clearly superior for the entire SNR range.

This mutual information can be used to define the code rate of a suitable q-ary channel code in the same sense as for binary schemes, as e.g. in [19]. By adapting the code rate, the packet error rate can be limited to a pre-defined target error rate while the achieved rate is proportional to the mutual information in Fig. 7.

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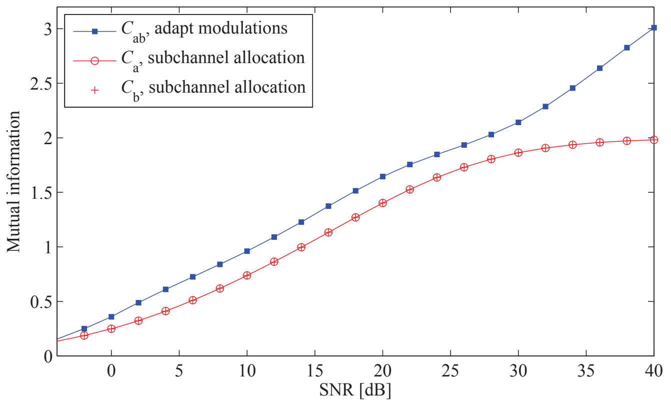

Figure 7. Achieved mutual informations with subchannel allocation for complete decoding and rate adaptation for functional decode

VI. Conclusion

In this paper[2], we have combined non-binary channel coding with higher-order modulations for the single-carrier and multi-carrier symmetric two-way relay channel. For this combination we investigated complete decoding as well as functional decoding for estimating the binary sum of both source packets at the relay. By investigating mutual information we demonstrated that functional decoding outperforms complete decoding for a sufficiently high SNR. The link-level simulation results indicate quite significant gains for joint decoding in all considered scenarios. Finally, a simple approach for adaptive modulation for the multi-carrier TWRC with functional decoding was presented yielding throughput gains compared to corresponding complete decoding schemes.

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[2]This work has been supported by the European Commission in the framework of the FP7 Network of Excellence in Wireless COMmunications NEWCOM\# (Grant agreement no. 318306), the FP7 EMPHATIC project (ICT-318362), the Catalan and Spanish Governments under SGR (2014 SGR 1567) and CICYT (TEC2011-29006-C03-01), respectively, and the German Research Foundation (DFG) under grant WU 499/10-1 within the priority programme "Communication in Interference Limited Networks (COIN)", SPP 1397.

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