Robust Distributed MMSE Precoding in Satellite Constellations for Downlink Transmission

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Abstract—Satellite communication systems are expected to play an important role in the fifth generation mobile network (5G) for global broadband coverage and service continuity. Constellations with many cooperating small satellites offer great advantages in terms of scalability and flexibility over traditional satellite communication systems with few large satellites. Small satellites means less power and low complexity per satellite while delivering 5G services as enhanced Mobile Broadband (eMBB) by exploiting the network of cooperating satellites. Thus, we derive a Distributed Precoding (DiP) algorithm by solving an user averaged Minimum Mean Square Error (MMSE) optimization problem subject to individual satellite power constraints. To come up with a robust precoder, the proposed design is based on a contamination model taking channel estimation accuracy into account. Simulation results show that the cooperation and processing of the satellite swarm leads to an increase of the Downlink (DL) sum rate.

I. INTRODUCTION

Satellite networks offer a promising and cost efficient solution to complement terrestrial networks due to reduced vulnerability to natural disasters and their capabilities of wide service coverage [1], [2]. In the fifth generation mobile network (5G), satellite networks are expected to play an important role to enable enhanced Mobile Broadband (eMBB) service in unserved areas, e.g., on board aircrafts or vessels, and to improve the performance of terrestrial networks in underserved areas. Furthermore, the possibility to provide global coverage with satellite networks seems promising to increase the reliability of massive Machine Type Communications (mMTC) by providing service continuity [3].

State-of-the-Art (SotA) precoding techniques for satellite communication are mostly focusing on large satellites with many antennas [4], [5]. However, due to the success of micro-electronics and microsystems technology in recent years, there has been a growing interest in small satellites and distributed architectures for spacecraft systems, i.e., satellite swarms [6] or fractionated spacecraft [7]. Such systems offer increased flexibility and robustness against the failure of single satellites. Additionally, these systems are easy to evolve and scale by deploying additional satellites and replacing old ones without disrupting the system [6]. Furthermore, the decreasing costs for developing and launching small satellites into the Low Earth Orbit (LEO) gives private companies the opportunity to provide satellite services. Thus, small satellites have become a core component of the so-called "NewSpace" [8] and several

activities have been started to combine 5G and "NewSpace" to "5G-NewSpace" [9]. The challenge in designing radio technologies implemented in small satellites are the strong power and complexity limitations of small satellites [6]. Hence, we propose a Distributed Precoding (DiP) algorithm where several small satellites are transmitting the user data cooperatively to compensate the limited transmit power of single satellites. Moreover, the calculations to design the precoder are split over the satellites resulting in low computational complexity per satellite.

DiP under individual power constraints has already been investigated for terrestrial networks. E.g., in [10], [11] distributed precoder designs are proposed for maximizing the sum rate. Distributed algorithms for joint optimization of precoding matrix and receive filter, i.e., transceiver optimization, based on the Minimum Mean Square Error (MMSE) criterion under individual transmit power constraints are shown in [12]–[16].

The main contribution of this paper is to provide a DiP algorithm for satellite constellations consisting of small satellites. The proposed DiP alogrotihm can be seen as an extension of [16] with increased robustness against channel estimation errors. Following a similar approach as in [13]–[15], the precoder design problem is formulated as a constrained MMSE optimization problem with per-satellite power constraints (PSPC) and then a DiP algorithm for satellite constellations is derived utilizing the Karush-Kuhn-Tucker (KKT) conditions. Different to the algorithm in [15], the local precoders in this paper are not updated in a sequential way but in parallel in order to reduce the latency due to calculation. To keep the computational complexity low, a large matrix inversion is avoided at the cost of a higher communication overhead between the satellites compared to [12], [13].

The rest of this paper is organized as follows. In the following section, the system model is introduced and the optimization problem is formulated. The DiP algorithm for satellite constellations is presented in Section III and numerically evaluated for a satellite constellation in the LEO in Section IV. Finally, the paper is concluded in Section V.

II. PRELIMINARIES

A. System Model

We consider joint DL transmission of N_S small satellites all serving the same N_U single antenna Non-Terrestrial Network (NTN) terminals on earth assigned to the same time and frequency slots, as depicted in Fig. 1. The NTN terminals

are multiplexed based on their spatial distribution, i.e., space-division multiple access (SDMA) is performed by linearly precoding the data symbols. Furthermore, each satellite is assumed to be equipped with N_t antennas. By employing cooperation between the satellites, a total antenna array of $N_{\text{Tx}} = N_t N_{\text{S}} > N_{\text{U}}$ antennas is created. The communication between the satellites can be implemented via optical inter-satellite links (ISLs), allowing reliable and high data rate communication [17] and are assumed to be perfect.

The type of NTN terminals is not further specified here and can be either some mobile devices, a Very Small Aperture Terminal (VSAT) or a relay node forwarding the data towards the User Equipments (UEs) [3], i.e., the satellites may serve the UEs directly or indirectly by transmitting the data to a relay node. The Doppler shift occurring due to the high relative speed of the satellites compared to the NTN terminals is assumed to be already compensated at the satellites.

The data symbols, intended for the NTN terminals, are stacked into the multi-user vector $\mathbf{s} = [s_1, ..., s_{N_{\text{U}}}]^T \in \mathbb{A}^{N_{\text{U}}}$ with $\text{E}\left\{\mathbf{s}\mathbf{s}^H\right\} = \mathbf{I}$, where $\mathbf{I}$ is the identity matrix of appropriate dimension and $\mathbb{A} \subseteq \mathbb{C}$ denotes the symbol alphabet. The data vector $\mathbf{s}$ is linearly precoded with precoding matrix $\mathbf{G}_j \in \mathbb{C}^{N_t \times N_{\text{U}}}$ at each satellite $j \in \{1, ..., N_{\text{S}}\}$ to form the transmitted signal

$$\mathbf{x}_j = \mathbf{G}_j \mathbf{s} \in \mathbb{C}^{N_t}. \tag{1}$$

The precoding matrices $\{\mathbf{G}_j\}_{j=1}^{N_{\text{S}}}$ can be stacked to form the global precoding matrix

$$\mathbf{G} = \left[\mathbf{G}_1^T, ..., \mathbf{G}_{N_{\text{S}}}^T\right]^T = [\mathbf{g}_1, ..., \mathbf{g}_{N_{\text{U}}}] \in \mathbb{C}^{N_{\text{Tx}} \times N_{\text{U}}} \tag{2}$$

with $\mathbf{g}_u = \left[\mathbf{g}_{1,u}^T, ..., \mathbf{g}_{N_{\text{S}},u}^T\right]^T \in \mathbb{C}^{N_{\text{Tx}}}$ being the precoding vector of all satellites intended for terminal $u \in \{1, ..., N_{\text{U}}\}$. Then, we can write the received signal y_u at the NTN terminal u as

$$y_u = \mathbf{h}_u^H \mathbf{g}_u s_u + \sum_{v \neq u} \mathbf{h}_u^H \mathbf{g}_v s_v + n_u = \mathbf{h}_u^H \mathbf{G} \mathbf{s} + n_u \tag{3}$$

where $\mathbf{h}_u = \left[\mathbf{h}_{1,u}^T, ..., \mathbf{h}_{N_{\text{S}},u}^T\right]^T \in \mathbb{C}^{N_{\text{Tx}}}$ is the instantaneous frequency-flat channel vector from all transmit antennas to terminal u and $n_u \sim \mathcal{CN}(0, \sigma_n^2)$ denotes independent and identically distributed (i.i.d.) complex white Gaussian distributed noise. Each NTN terminal u re-scales the received signal by $\beta_u > 1$ only, due to the path loss affecting the signal during the transmission. Thus, we can express the estimated signal before hard decision $\hat{s}_u \in \mathbb{C}$ at the NTN terminal u to be

$$\hat{s}_u = \beta_u \left(\mathbf{h}_u^H \mathbf{G} + n_u\right). \tag{4}$$

Furthermore, we can write the channel $\mathbf{H}_j$ from satellite j to all NTN terminals as

$$\mathbf{H}_j = [\mathbf{h}_{j,1}, ..., \mathbf{h}_{j,N_{\text{U}}}]^H. \tag{5}$$

Due to the long delay and the fast relative speed between the satellites and the NTN terminals, accurate CSI at the satellites is difficult to obtain. Therefore, we assume a contamination
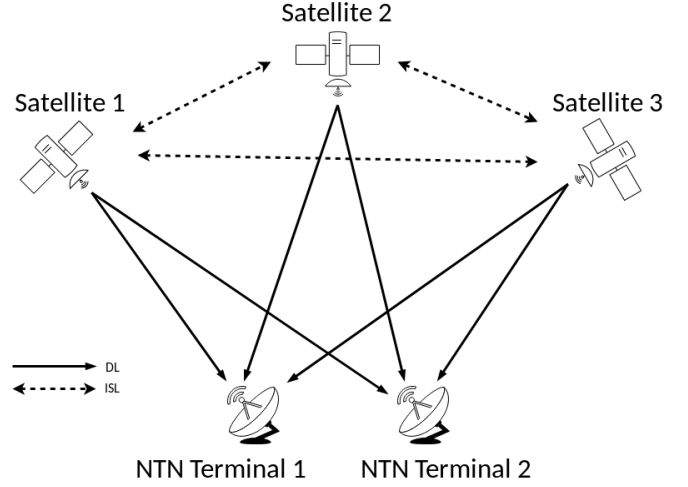

Fig. 1. Satellite Network

model, where at each satellite j only an erroneous estimate $\hat{\mathbf{H}}_j$ of the channel matrix $\mathbf{H}_j$ is available. The estimated channel $\hat{\mathbf{H}}_j$ and the true channel $\mathbf{H}_j$ are related by

$$\hat{\mathbf{H}}_j = \mathbf{H}_j + \tilde{\mathbf{H}}_j \,\forall j \in \{1, ..., N_{\text{S}}\} \tag{6}$$

where $\tilde{\mathbf{H}}_j \sim \mathcal{CN}(0, N_{\text{U}}\sigma_{\tilde{h}}\mathbf{I})$ is the i.i.d. additive channel estimation error.

B. Problem Formulation

In this paper, we propose a DiP algorithm to minimize the sum MSE over all terminals between the estimated symbols $\{\hat{s}_u\}_{u=1}^{N_{\text{U}}}$ and the intended symbols $\{s_u\}_{u=1}^{N_{\text{U}}}$ under individual power constraints. Thus, we can write the precoder design problem as a constraint optimization problem

$$\min_{\{\mathbf{G}_j\}_{j=1}^{N_{\text{S}}}, \{\beta_u\}_{u=1}^{N_{\text{U}}}} \sum_{u=1}^{N_{\text{U}}} \text{E}\left\{\|s_u - \hat{s}_u\|_2^2\right\} \tag{7}$$
$$\text{s.t. } \text{tr}\left\{\mathbf{G}_j \mathbf{G}_j^H\right\} \leq P_j \,\forall j \in \{1, ..., N_{\text{S}}\}$$

where $\text{tr}\{\cdot\}$ denotes the trace operator and P_j is the maximum allowed transmit power of satellite j. Note that the optimal precoding matrices $\{\mathbf{G}_j\}_{j=1}^{N_{\text{S}}}$ are depending on the re-scaling factors $\{\beta_u\}_{u=1}^{N_{\text{U}}}$ and vice versa. Therefore, we propose to update the precoding matrices $\{\mathbf{G}_j\}_{j=1}^{N_{\text{S}}}$ and re-scaling factors $\{\beta_u\}_{u=1}^{N_{\text{U}}}$ alternatingly as in [11]–[16].

Let $\mathbf{B} = \text{diag}\left(\beta_1, ..., \beta_{N_{\text{U}}}\right)$ be the global re-scaling matrix and

$$\mathbf{H} = [\mathbf{H}_1, ..., \mathbf{H}_{N_{\text{S}}}] = [\mathbf{h}_1, ..., \mathbf{h}_{N_{\text{U}}}]^H \in \mathbb{C}^{N_{\text{U}} \times N_{\text{Tx}}} \tag{8}$$

be the global channel matrix. The global estimated channel matrix $\hat{\mathbf{H}}$ and the erroneous channel vector $\hat{\mathbf{h}}_u$ for terminal u are defined in the same way as (8). Using (4) and (8), we can express the stacked estimated multi-user data vector $\hat{\mathbf{s}} = [\hat{s}_1, ..., \hat{s}_{N_{\text{U}}}] \in \mathbb{C}^{N_{\text{U}}}$ to be

$$\hat{\mathbf{s}} = \mathbf{B}\left(\mathbf{H}\mathbf{G}\mathbf{s} + \mathbf{n}\right) \tag{9}$$

with $\mathbf{n} = [n_1, ..., n_{N_U}]^T$. Then, we can rewrite the optimization problem in a more compact way as

$$\min_{\mathbf{G},\mathbf{B}} \mathrm{E}\left\{\|\mathbf{s} - \hat{\mathbf{s}}\|_2^2\right\}$$
$$\text{s.t. } \mathrm{tr}\left\{\mathbf{G}_j\mathbf{G}_j^H\right\} \leq P_j \,\forall j \in \{1, ..., N_S\}. \tag{10}$$

In order to solve the global optimization problem from a per-satellite perspective, we rewrite the product of the channel matrix $\mathbf{H}$ and the precoding matrix $\mathbf{G}$ as

$$\mathbf{HG} = \sum_{j=1}^{N_S} \mathbf{H}_j\mathbf{G}_j \tag{11}$$

which directly follows from (2) and (8).

III. DISTRIBUTED PRECODING

A. Algorithm

The DiP algorithm proposed in this section is based on the KKT conditions. Let $\lambda_j \geq 0$ be the Lagrange multiplier associated with the power constraint of the jth satellite, $\boldsymbol{\Lambda} = \mathrm{diag}\,(\lambda_1, ..., \lambda_{N_S}) \otimes \mathbf{I}$ and $\mathbf{P} = 1/N_t\,\mathrm{diag}\,(P_1, ..., P_{N_S}) \otimes \mathbf{I}$, where $\otimes$ denotes the Kronecker product. Then, the Lagrangian of (10) can be stated as

$$\mathcal{L}(\mathbf{G}, \mathbf{B}, \boldsymbol{\Lambda}) = \mathrm{E}\left\{\|\mathbf{s} - \hat{\mathbf{s}}\|_2^2\right\} + \mathrm{tr}\left\{\boldsymbol{\Lambda}\left(\mathbf{GG}^H - \mathbf{P}\right)\right\} \tag{12a}$$

$$= \sum_{j=1}^{N_S}\left(\mathrm{tr}\left\{\mathrm{E}\left\{\sum_{i=1}^{N_S}\mathbf{G}_i^H\mathbf{H}_i^H\mathbf{B}^H\mathbf{BH}_j\mathbf{G}_j\right\}\right\}\right.$$
$$- \mathrm{tr}\left\{\mathrm{E}\left\{\mathbf{G}_j^H\mathbf{H}_j^H\mathbf{B}^H + \mathbf{BH}_j\mathbf{G}_j\right\}\right\}$$
$$+ \lambda_j\left(\mathrm{tr}\left\{\mathbf{G}_j\mathbf{G}_j^H\right\} - P_j\right)\right) \tag{12b}$$
$$+ \mathrm{E}\left\{\mathbf{n}^H\mathbf{B}^H\mathbf{Bn}\right\} + N_U.$$

By noting that $\mathbf{B} = \mathbf{B}^H$, we can define $\Upsilon = \sum_{u=1}^{N_U}\beta_u^2$ and rewrite the expectation of the first term in (12b) to be

$$\mathrm{tr}\left\{\mathrm{E}\left\{\sum_{i=1}^{N_S}\mathbf{G}_i^H\mathbf{H}_i^H\mathbf{B}^H\mathbf{BH}_j\mathbf{G}_j\right\}\right\} =$$
$$\mathrm{tr}\left\{\sum_{i=1}^{N_S}\mathbf{G}_i^H\hat{\mathbf{H}}_i^H\mathbf{BB}\hat{\mathbf{H}}_j\mathbf{G}_j\right\} + \Upsilon\sigma_{\tilde{h}}^2\,\mathrm{tr}\left\{\mathbf{G}_j\mathbf{G}_j^H\right\}. \tag{13}$$

According to the KKT conditions [18], the first derivatives of the Lagrangian have to vanish for the optimal precoding matrices $\{\mathbf{G}_j^*\}_{j=1}^{N_S}$, re-scaling factors $\{\beta_u^*\}_{u=1}^{N_U}$ and Lagrange multipliers $\{\lambda_j^*\}_{j=1}^{N_S}$:

$$\nabla_{\mathbf{G}_j}\mathcal{L}\left(\mathbf{G}^*, \mathbf{B}^*, \boldsymbol{\lambda}^*\right) = \mathbf{0}\,\forall j \in \{1, ..., N_S\} \tag{14a}$$
$$\nabla_{\beta_u}\mathcal{L}\left(\mathbf{G}^*, \mathbf{B}^*, \boldsymbol{\lambda}^*\right) = 0\,\forall u \in \{1, ..., N_U\} \tag{14b}$$

Thus, necessary conditions for $\{\mathbf{G}_j^*\}_{j=1}^{N_S}$ and $\{\beta_u^*\}_{u=1}^{N_U}$ to be optimal solutions of (10) are given by

$$\mathbf{G}_j^* = \mathbf{T}_j^*\hat{\mathbf{H}}_j^H\mathbf{B}^*\left(\mathbf{I} - \sum_{i\neq j}\mathbf{B}^*\hat{\mathbf{H}}_i\mathbf{G}_i^*\right)\,\forall j \tag{15a}$$

$$\beta_u^* = \frac{\mathrm{Re}\left\{\hat{\mathbf{h}}_u^H\mathbf{g}_u^*\right\}}{\left\|\hat{\mathbf{h}}_u^H\mathbf{G}^*\right\|_2^2 + \sigma_{\tilde{h}}^2\left\|\mathbf{G}^*\right\|_F^2 + \sigma_n^2}\,\forall u \tag{15b}$$

with $\mathbf{T}_j^* = \left(\hat{\mathbf{H}}_j^H\mathbf{B}^*\mathbf{B}^*\hat{\mathbf{H}}_j + (\Upsilon^*\sigma_{\tilde{h}}^2 + \lambda_j^*)\mathbf{I}\right)^{-1}$. Further KKT conditions are for all $j \in \{1, ..., N_S\}$ [18]:

$$\left\|\mathbf{G}_j^*\right\|_F^2 - P_j \leq 0\,\forall j \tag{16a}$$
$$\lambda_j^* \geq 0\,\forall j \tag{16b}$$
$$\lambda_j^*\left(\left\|\mathbf{G}_j^*\right\|_F^2 - P_j\right) = 0\,\forall j \tag{16c}$$

Note that (15) and (16) do not represent an explicit solution. According to (15a), the optimal precoding matrix $\mathbf{G}_j^*$ of satellite j depends on the Lagrange multiplier λ_j^*, the matrices $\{\hat{\mathbf{H}}_i\mathbf{G}_i^*\}_{i\neq j}$ from the other satellites $i \neq j$ and the re-scaling matrix $\mathbf{B}$, i.e.,

$$\mathbf{G}_j^* = f\left(\{\hat{\mathbf{H}}_i\mathbf{G}_i^*\}_{i\neq j}, \lambda_j^*, \mathbf{B}^*\right). \tag{17}$$

Therefore, we seek for an iterative procedure where at each satellite j its precoding matrix $\mathbf{G}_j$, the re-scaling factors $\{\beta_u\}_{u=1}^{N_U}$ and its Lagrange multiplier λ_j are updated in an alternating fashion. Instead of directly updating the precoding matrix $\mathbf{G}_j$ according to (15a), we introduce the relaxation parameter $0 < \omega \leq 1$ to ensure the convergence of the DiP algorithm. The update of the precoding matrix at satellite j and iteration $k = 0, ..., K-1$ is thus given by

$$\mathbf{G}_j^{(k+1)} = (1-\omega)\mathbf{G}_j^{(k)} + \omega f\left(\{\hat{\mathbf{H}}_i\mathbf{G}_i^{(k)}\}_{i\neq j}, \lambda_j^{(k)}, \mathbf{B}^{(k)}\right)$$
$$= \mathbf{G}_j^{(k)} - \omega\mathbf{T}_j^{(k)}\left(\hat{\mathbf{H}}_j^H\mathbf{B}^{(k)}\mathbf{B}^{(k)}\sum_{i=1}^{N_S}\hat{\mathbf{H}}_i\mathbf{G}_i^{(k)}\right.$$
$$\left.+ \left(\Upsilon^{(k)}\sigma_{\tilde{h}}^2 + \lambda_j^{(k)}\right)\mathbf{G}_j^{(k)} - \hat{\mathbf{H}}_j^H\mathbf{B}^{(k)}\right). \tag{18}$$

The re-scaling factor $\beta_u^{(k)}$ for each NTN terminal u at iteration k is determined by plugging the global precoding matrix $\mathbf{G}^{(k)}$ into (15b), i.e.,

$$\beta_u^{(k)} = \frac{\mathrm{Re}\left\{\hat{\mathbf{h}}_u^H\mathbf{g}_u^{(k)}\right\}}{\left\|\hat{\mathbf{h}}_u^H\mathbf{G}^{(k)}\right\|_2^2 + \sigma_{\tilde{h}}^2\left\|\mathbf{G}^{(k)}\right\|_F^2 + \sigma_n^2}. \tag{19}$$

Then, the Lagrange multiplier $\lambda_j^{(k)}$ has to be chosen in order to satisfy the KKT conditions (16). To the best of our knowledge, there is no analytic solution for $\lambda_j^{(k)}$ but it can be found numerically. As we can see in (16), either $\lambda_j^{(k)}$ has to be zero or the power constraint (16a) has to be fulfilled with equality. Therefore, a root-finding algorithm like Newton's method or the bisection method can be used in each iteration k to find the $\lambda_j^{(k)}$ which satisfies

$$\left\|\mathbf{G}_j^{(k+1)}\left(\lambda_j^{(k)}\right)\right\|_F^2 = P_j. \tag{20}$$

If no positive $\lambda_j^{(k)}$ is found, that multiplier is set to zero.

The precoding matrix $\mathbf{G}_j$ of satellite j is initialized by the local MMSE precoder without taking the precoding matrices $\{\mathbf{G}_i\}_{i \neq j}$ of the other satellites $i \neq j$ into account and assuming the same re-scaling factor for all NTN terminals $\beta = \beta_u^{(0)} = \beta_v^{(0)}, \forall u, v \in \{1, ..., N_U\}$, i.e.,

$$\mathbf{G}_j' = \left(\hat{\mathbf{H}}_j^H \hat{\mathbf{H}}_j + N_U \left(\sigma_{\hat{h}}^2 + \frac{\sigma_n^2}{P_j} \right) \mathbf{I} \right)^{-1} \hat{\mathbf{H}}_j^H \quad (21a)$$

$$\beta^{-1} = \sqrt{\frac{P_j}{\operatorname{tr}\left\{ \mathbf{G}_j' \mathbf{G}_j'^H \right\}}} \quad (21b)$$

$$\mathbf{G}_j^{(0)} = \beta^{-1} \mathbf{G}_j'. \quad (21c)$$

In Algorithm 1, the proposed robust DiP algorithm under individual PSPC is summarized.

B. Computation and Inter-Satellite-Communication

In each iteration k, each satellite j needs the complete matrix $\hat{\mathbf{H}}\mathbf{G}^{(k)}$ in order to update its precoding matrix $\mathbf{G}_j^{(k+1)}$ as well as the Lagrange multiplier $\lambda_j^{(k)}$ and the re-scaling matrix $\mathbf{B}^{(k)}$. Therefore, we propose that in each iteration k each satellite j transmits its matrix $\hat{\mathbf{H}}_j \mathbf{G}_j^{(k)} \in \mathbb{C}^{N_U \times N_U}$ via ISLs to all other satellites $i \neq j$ and receives the matrices $\{\hat{\mathbf{H}}_i \mathbf{G}_i^{(k)}\}_{i \neq j}$. Alternatively, only $\mathbf{G}_j^{(k)} \in \mathbb{C}^{N_t \times N_U}$ can be transmitted in order to reduce the communication overhead. However, if only the precoding matrices are exchanged, each satellite has to perform a larger matrix multiplication in each iteration k to calculate $\hat{\mathbf{H}}\mathbf{G}^{(k)}$, which significantly increases the computational complexity for a large number of satellites N_S.

Note that $\hat{\mathbf{h}}_u^H \mathbf{G}^{(k)}$ in (19) is a submatrix of $\hat{\mathbf{H}}\mathbf{G}^{(k)}$ and the total radiated power of the satellites can be well approximated by

$$\left\| \mathbf{G}^{(k)} \right\|_F^2 \approx \sum_{j=1}^{N_S} P_j. \quad (22)$$

Thus, the re-scaling matrix $\mathbf{B}^{(k)}$ can be calculated locally at each satellite without any further information exchange.

After the final precoding matrix $\mathbf{G}^{(K)}$ is calculated, each NTN terminal u has to estimate its re-scaling factor $\hat{\beta}_u$. This can be done by an automatic gain control (AGC) or by estimating the effective channel $\mathbf{h}_u^H \mathbf{G}^{(K)}$ with pilot signals. At the terminals, it is less challenging to obtain accurate CSI than at the satellites. Therefore we assume, that each NTN terminal u has perfect knowledge about the effective channel $\mathbf{h}_u^H \mathbf{G}^{(K)}$ and calculates its re-scaling factor $\hat{\beta}_u$ by

$$\hat{\beta}_u = \frac{\operatorname{Re}\left\{ \mathbf{h}_u^H \mathbf{g}_u^{(K)} \right\}}{\left\| \mathbf{h}_u^H \mathbf{G}^{(K)} \right\|_F^2 + \sigma_n^2}. \quad (23)$$

IV. PERFORMANCE EVALUATION

In this section, we evaluate the performance of the proposed DiP algorithm through numerical simulations. The simulation scenario consists of $N_S = 6$ satellites, each equipped with $N_t = 2$ antennas, randomly deployed in a circular area of $A_{\text{space}} = 2000 \, \text{km}^2$ with an altitude of $d_0 = 1000 \, \text{km}$. The

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Algorithm 1 Robust DiP under PSPC

For each SC $j = 1, ..., N_S$ in parallel
1: Initialize local precoding matrix $\mathbf{G}_j^{(0)}$ according to (21);
2: **for** $k = 0, ..., K - 1$ **do**
3:    **Transmit** the data matrix $\hat{\mathbf{H}}_j \mathbf{G}_j^{(k)}$ to other SCs;
4:    **Update** re-scaling matrix $\mathbf{B}^{(k)}$ according to (19);
5:    **Update** $\lambda_j^{(k)}$ by numerically solving (20);
6:    **Update** the precoding matrix $\mathbf{G}_j^{(k+1)}$ according (18);
7: **end for**
8: Transmit $\mathbf{x}_j = \mathbf{G}_j^{(K)} \mathbf{s}$ to all user terminals

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$N_U = 8$ NTN terminals are randomly deployed in a circular area of $A_{\text{earth}} = 50 \, \text{km}^2$. The center of the area where the NTN terminals are deployed is at the same position as the projection of the satellite's area center on the earth surface. It is further assumed that all satellites have a dominant line-of-sight (LOS) path to all NTN terminals. The path loss (PL) is therefore modeled as the free-space loss [19] times a log-normal random variable. Thus, the PL from satellite j to NTN terminal u is modeled in decibels as

$$PL_{j,u} = 20 \log_{10} \left(\frac{4\pi d_{j,u} f_c}{c} \right) - (G_{Tx} + G_{Rx}) + \xi_{j,u} \quad (24)$$

where $G_{Tx} = 20 \, \text{dBi}$ and $G_{Rx} = 40 \, \text{dBi}$ is the antenna gain of the satellites and users, respectively, and $d_{j,u}$ is the distance between satellite j and terminal u. The carrier frequency is assumed to be $f_c = 18 \, \text{GHz}$ and $c = 3 \times 10^8 \, \text{m/s}$ is the propagation speed of the electromagnetic wave. $\xi_{j,u}$ is a Gaussian random variable with variance $\sigma_{\xi_{j,u}}^2 = 9 \, \text{dB}$ representing large scale fading effects due to the atmosphere. Let $a_{j,u} = 10^{-PL_{j,u}/20}$, then the channel from satellite j to NTN terminal u is modeled as

$$\mathbf{h}_{j,u} = a_{j,u} e^{-j2\pi \frac{f_c d_{j,u}}{c}} + \boldsymbol{\chi}_{j,u} \quad (25)$$

where the random vector $\boldsymbol{\chi}_{j,u} \sim \mathcal{CN}(0, 1/a_{j,u}^2 \mathbf{I})$ represents small scale fading effects and phase distortions due to atmospheric effects. The maximum transmit power of each satellite is limited to $P_1 = P_2 = ... = P_{N_S} = 1/N_S$. Let further $\sigma_H^2 = \operatorname{tr}\{\mathbf{H}^H \mathbf{H}\}/(N_{Tx} N_U)$ be the average gain of the transmitted signal power due to the channel, $\sigma_{\hat{h}}^2$ is chosen such that $10 \log_{10}\left(\sigma_H^2/\sigma_{\hat{h}}^2\right) = 10 \, \text{dB}$. The signal-to-noise ratio (SNR) is defined as the ratio of the average signal power received by the NTN terminals and the noise power σ_n^2.

The relaxation parameter ω for updating the precoding matrices $\{\mathbf{G}_j^{(k+1)}\}_{j=1}^{N_S}$ is empirically chosen to $\omega = 0.6$ to ensure convergence of the DiP algorithm. The Lagrange multiplier $\lambda_j^{(k)}$ are determined using the bisection method at the first iteration $k = 0$ and with three steps of Newton's method for $k \geq 1$ taking the previous $\lambda_j^{(k-1)}$ as initial guess.

All simulations are averaged over $L = 100$ randomly chosen satellite and terminal positions, each with $M = 60$ different realizations of the noise, the fading variables and channel estimation errors. Let $\mathbf{s}_{l,m}$ be the multi-user data vector of the lth constellation and mth realization and $\hat{\mathbf{s}}_{l,m}$ be its stacked
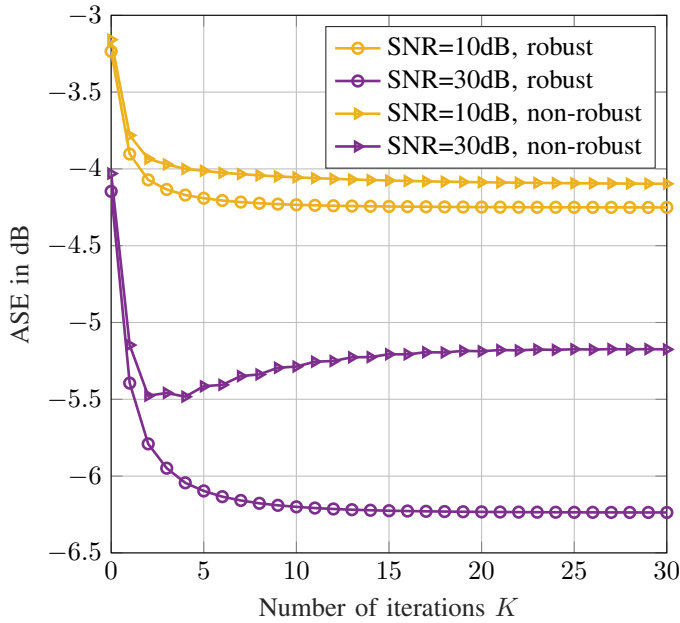
Fig. 2. ASE w.r.t. number of iterations for the proposed DiP algorithm with and without considering the channel estimation error

Fig. 3. Sum rate w.r.t. SNR for the proposed robust DiP algorithm for $N_U = 8$ users

estimates at the NTN terminals. The average squared error (ASE), i.e., an estimate of the MSE, is then given by

$$\text{ASE} = \frac{1}{N_U} \frac{1}{LM} \sum_{l=1}^{L} \sum_{m=1}^{M} \|\mathbf{s}_{l,m} - \hat{\mathbf{s}}_{l,m}\|_2^2 \quad (26)$$

where the factor $1/N_U$ is included to normalize the ASE to the number of NTN terminals. In Fig. 2, the ASE is shown against the executed number of iterations K of the proposed DiP algorithm. We compare the performance of the robust DiP algorithm with its non-robust counterpart, where the erroneous channel estimate is assumed to be the true channel without considering the statistics of the estimation error, i.e, assuming $\sigma_{\tilde{h}}^2 = 0$. Furthermore, the algorithm is evaluated for SNR $= 10\,\text{dB}$ and SNR $= 30\,\text{dB}$. It is shown that the DiP algorithm converges after roughly $K \approx 20$ iterations and the robust design approach outperforms the non-robust approach. This becomes more clear for high SNR, i.e., SNR $= 30\,\text{dB}$, where the channel estimation error is large compared to the noise power. Then, the ASE of the non-robust approach even increases after a few iteration, because the interference is tried to be reduced based on erroneous CSI.

From a system design perspective, the sum rate R may be of larger interest than the ASE. The signal-to-interference-plus-noise ratio (SINR) at the NTN terminal u is given by

$$\text{SINR}_u = \frac{|\mathbf{h}_u^H \mathbf{g}_u|^2}{\sum_{v \neq u} |\mathbf{h}_u^H \mathbf{g}_v|^2 + \sigma_n^2} \quad (27)$$

and the sum rate R is then given by
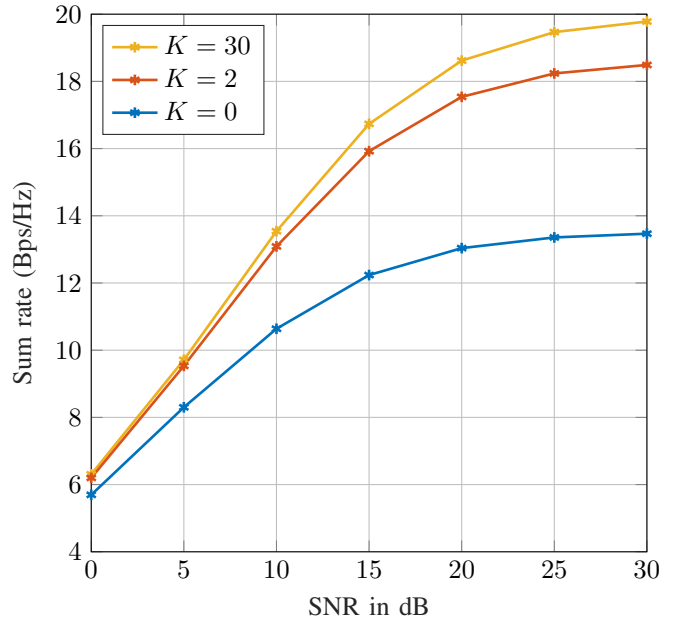
$$R = \sum_{u=1}^{N_U} \log_2 \left(1 + \text{SINR}_u\right). \quad (28)$$

In Fig. 3 and 4, the sum rate achieved with the proposed robust DiP algorithm is shown for $N_U = 8$ and $N_U = 4$ NTN terminals, respectively. For the setup with $N_U = 4$, the relaxation parameter is reduced to $\omega_{N_U=4} = 0.5$ to avoid divergence of the DiP algorithm. Note that the precoding matrices $\mathbf{G}_j^{(0)}$ are initialized by solving the local MMSE precoder design problem and therefore, $K = 0$ represents the performance of the DL transmission if no information between the satellites regarding the precoder design are exchanged. With only $K = 2$ iterations, the sum rate can be significantly increased for both setups. If the processing time to calculate the precoding matrices is limited due to some latency constraints, still a good performance can be achieved by performing only a few iterations. Because the Lagrange multiplier λ_j are optimized in each iteration, the power constraints are fulfilled after each iteration as well.

V. CONCLUSION

In this paper, a robust DiP algorithm is proposed for joint DL transmission from a LEO satellite constellation towards NTN terminals on the ground. Satellite communication is expected to become an essential part for 5G in order to provide global coverage and satellite swarms offer great advantages over traditional satellite systems in terms of scalability and robustness against the failure of single satellites. Simulation results show a noticeable improvement of the sum rate at low computational costs by allowing communication between the satellites for the precoder design. Moreover, its fast convergence makes the algorithm a promising precoding approach for satellite constellation in 5G consisting of many small satellites.
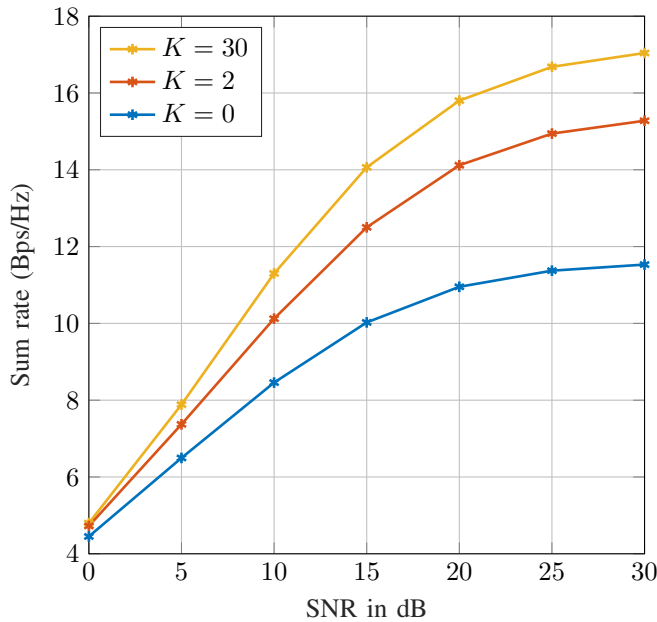
Fig. 4. Sum rate w.r.t. SNR for the proposed robust DiP algorithm for $N_{\mathrm{U}} = 4$ users

However, there are open questions to be clarified for practical realization such as synchronization of the satellites and quantization of the exchanged information.

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