Weighted Sum Rate Maximization for Non-Regenerative Multi-Way Relay Channels with Multi-User Decoding

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***Abstract*—This paper studies the maximization of the weighted sum rate in multi-way relay channels with simultaneous non-unique decoding at the receivers. We state the resource allocation problem as a global optimization problem of the transmit powers and achievable rates, and transform it into a monotonic optimization problem. The computational complexity of monotonic optimization problems is exponential in the number of variables. We observe that for fixed powers the problem is a linear program with much lower complexity and exploit this structural property by decomposing the optimization problem into an inner linear and an outer monotonic program. This reduces the computational complexity significantly and allows computing the global solution. We compare the achievable throughput with multi-user decoding and optimal power allocation numerically to state-of-the-art single-user decoding and to simply transmitting at maximum power. We observe that multi-user decoding performs much better than single-user decoding in terms of throughput and fairness for medium to high SNRs.**

***Index Terms*—Resource allocation, interference networks, 5G networks, monotonic optimization, power control, global optimization, multi-way relay channel, amplify-and-forward, simultaneous non-unique decoding, linear programming**

I. Introduction

Relays are fundamental building blocks of modern wireless networks. They help to increase coverage and reliability. Due to the high attenuation in mmWave communications, this is even more important in the era of 5G than it was in classical and current wireless systems. Thus, relaying is a key technology in emerging wireless technologies and its study is important to understand the fundamental limits of modern and future wireless communication systems. An integral part to advance the understanding of relaying in networks is the multi-way relay channel (MWRC). It models relay-aided communication across several nodes with no direct links between the users. Applications of this model are, e.g., heterogeneous dense small cell networks in modern and future wireless networks, wireless board-to-board communication in highly adaptive computing, wireless sensor networks, Industry 4.0, or communication of several ground stations over a satellite. An extensive overview of results for the MWRC is provided in [1].

We focus on the Gaussian MWRC with three users, amplify-and-forward (AF) relaying, and multiple unicast transmissions. Non-regenerative relaying introduces significantly less delay than state-of-the-art decode-and-forward (DF) relaying with the additional benefits of low energy consumption and reduced hardware cost. This comes at the price of noise amplification at the relay. To mitigate this effect, interference is not treated as additional noise at the receivers. Instead, simultaneous non-unique decoding (SND) [2, Chap. 6] is employed where all received messages are decoded simultaneously but without requiring correct decoding of interfering messages. Achievable rate regions for this channel are derived in [3]. Throughput and energy efficiency (EE) maximization is done in [4] for symmetric channels and in [5] for non-symmetric channels with treating interference as noise (IAN) at the receivers.

In this paper, we solve the weighted sum rate maximization problem for the MWRC at hand with global optimality. This optimization problem is non-convex and its global optimal solution requires computationally intensive algorithms. In general interference networks with single-user decoding the achievable rate region is often a hypercube and the weighted sum rate can be derived analytically [6]–[9]. Instead, the achievable rate region with SND is more involved making it impractical to determine the weighted sum rate analytically except for some special cases (e.g. [3]). Thus, we have to jointly optimize over the achievable rates and transmit powers instead of just the powers. This means that the number of optimization variables is roughly doubled compared to the case where the rate expressions can be eliminated. The computational complexity of global optimization algorithms usually grows exponentially with the number of variables making the resulting optimization problem significantly more complex than problems involving just the transmit powers.

However, it is not hard to notice that for fixed transmit powers the weighted sum rate maximization becomes a linear optimization problem while all the non-convexity is just due to the powers. Following the general rule in global optimization to exploit as much structure of the problem as possible, we show how to decompose the original problem into an easy to solve inner problem and a non-convex outer problem. Along the

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way, we transform the constraints such that they are increasing functions in the transmit powers making the outer problem a monotonic optimization problem [10].

In the next section, we develop the system model and derive the achievable rate region for AF relaying and SND at the receivers. In Section III, we state the weighted sum rate optimization problem and transform it as outlined above. Afterwards, in Section IV, we provide numeric evidence that the global optimal solution enables higher throughput and better fairness than state-of-the-art single-user decoding and is more energy-efficient than heuristic power allocation.

II. System Model

We consider a 3-user single-input single-output (SISO) MWRC where the users communicate in multiple unicast transmissions via an AF relay. Gaussian channels with quasi-static block flat fading, full-duplex transmission, SND at the receivers and no direct user-to-user links are assumed. Users are denoted as node 1 to 3 and the relay as node 0. We define the set of all users as $\mathcal{K} = \{1, 2, 3\}$.

The relay receives the signal

$$Y_0 = \sum_{k\in\mathcal{K}} h_k X_k + Z_0,$$

with X_k the channel input at node $k \in \mathcal{K}$ with power P_k, h_k the channel coefficient from user k to the relay, and Z_0 the independent and identically distributed (i.i.d.) zero-mean circularly symmetric complex Gaussian noise with power N_0 observed at the relay. The relay adjusts the power of the received symbol and broadcasts it back to the users, i.e., $X_0 = \alpha Y_0$ where $\alpha = \sqrt{P_0 \,/\, \left(\sum_{k\in\mathcal{K}} |h_k|^2 P_k + N_0\right)}$ is chosen such that the relay's transmit power is P_0.

User $k \in \mathcal{K}$ receives the signal

$$Y_k = g_k X_0 + Z_k,$$

with g_k the channel coefficient from the relay to user k, and Z_k the i.i.d. zero mean circularly symmetric complex Gaussian noise with power N. The channel inputs are subject to an average power constraint $\bar{P}_k$ on X_k, $k \in \mathcal{K} \cup \{0\}$. The receiver first removes its self-interference from the received signal and then decodes simultaneously for its desired and non-uniquely for the interfering message. Since the achievable rates do not depend on the absolute values of P_k and N_0 but on their ratio, we state all results in terms of the signal-to-noise ratio (SNR). We define the transmit SNR $S_k = \frac{P_k}{N_0}$, the maximum transmit SNR $\bar{S}_k = \frac{\bar{P}_k}{N_0}$, and the corresponding vectors $\boldsymbol{S} = (S_1, S_2, S_3)$ and $\bar{\boldsymbol{S}} = (\bar{S}_1, \bar{S}_2, \bar{S}_3)$.

The message exchange is illustrated in Fig. 1 where the different line styles indicate different messages. We denote the receiver of node k's message as $q(k)$, $k \in \mathcal{K}$, and the user not interested in it as $l(k)$. Conversely, user k desires the message sent by user $l(k)$. From Fig. 1, we have $q(1) = l(3) = 2$, $q(2) = l(1) = 3$, and $q(3) = l(2) = 1$.

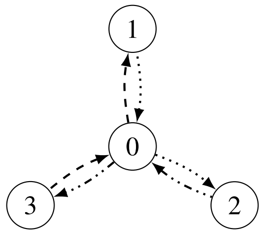

Fig. 1. Illustration of the system model where node 0 is the relay and nodes 1 to 3 are the users. Messages travel along the different line styles.

Lemma 1: A rate triple (R_1, R_2, R_3) is achievable for the Gaussian MWRC with AF relaying and SND if, for all $k \in \mathcal{K}$,

$$R_k < \log\left(1 + \frac{|h_k|^2 S_k}{\gamma_{l(k)}(\boldsymbol{S})}\right)$$

$$R_k + R_{l(k)} < \log\left(1 + \frac{|h_k|^2 S_k + |h_{l(k)}|^2 S_{l(k)}}{\gamma_{l(k)}(\boldsymbol{S})}\right)$$

where $S_k \leq \bar{S}_k$ and

$$\gamma_{l(k)}(\boldsymbol{S}) = \underbrace{1}_{\substack{\text{sink}\\\text{noise}}} + \underbrace{\widetilde{g}_{q(k)}^{-1}\left(1 + \sum_{i\in\mathcal{K}} |h_i|^2 S_i\right)}_{\text{relay noise amplification}},$$

with $\widetilde{g}_k = |g_k|^2 \frac{\bar{P}_0}{N_k}$.

Proof sketch: Adapt [3, Corollary 2] to Gaussian channels using the standard procedure in [2, Chap. 3]. Apply it to the considered channel with $\mathbb{E}[X_0^2] = P_0$ to obtain the rate expressions above with $\widetilde{g}_k = |g_k|^2 \frac{P_0}{N_k}$. The achievable rates are increasing in P_0 since the partial derivatives with respect to P_0 of the rate expression's right-hand sides (RHSs) are always non-negative. Thus, $P_0 = \bar{P}_0$ is optimal. ■

III. Weighted sum rate optimal power allocation

The optimal power allocation that maximizes the weighted system throughput is the solution to the optimization problem

$$\begin{cases} \max\limits_{\boldsymbol{R},\boldsymbol{S}} & \sum\limits_{k\in\mathcal{K}} w_k R_k \\ \text{s.t.} & \sum\limits_{k\in\mathcal{S}} R_k < h_{\mathcal{S}}(\boldsymbol{S}), \quad \text{for all } \mathcal{S} \in \mathfrak{S} \\ & \boldsymbol{R} \geq \underline{\boldsymbol{R}}, \quad \boldsymbol{S} \in [\boldsymbol{0}, \bar{\boldsymbol{S}}] \end{cases} \tag{P1}$$

with positive weights w_k, $k \in \mathcal{K}$, minimum rate and maximum power constraints $\underline{\boldsymbol{R}} \geq \boldsymbol{0}$ and $\bar{\boldsymbol{S}} > \boldsymbol{0}$, respectively, the family of sets $\mathfrak{S} = \bigcup_{k\in\mathcal{K}} \{\{k\}, \{k, l(k)\}\}$, and

$$h_{\mathcal{S}}(\boldsymbol{S}) = \log\left(1 + \frac{\sum_{k\in\mathcal{S}} |h_k|^2 S_k}{\gamma_{\mathcal{S}}(\boldsymbol{S})}\right)$$

where $\gamma_{\mathcal{S}} \coloneqq \gamma_{l(\kappa)}$ for κ such that $q(\kappa) \in \mathcal{K} \setminus \mathcal{S}$ if $|\mathcal{S}| = 2$ and $\kappa \in \mathcal{S}$ otherwise. This is a global optimization problem because $h_{\mathcal{S}}(\boldsymbol{S})$ is a non-concave function. Thus, standard tools from convex optimization are not applicable. Fortunately, (P1) has some structure that we can exploit to avoid examining every point in the feasible set.

First, observe that $h_{\mathcal{S}}(\boldsymbol{S})$ is a difference of increasing functions in $\boldsymbol{S}$, i.e.,

$$\begin{aligned} h_{\mathcal{S}}(\boldsymbol{S}) &= f_{\mathcal{S}}(\boldsymbol{S}) - g_{\mathcal{S}}(\boldsymbol{S}) \\ &= \log\left(\gamma_{\mathcal{S}}(\boldsymbol{S}) + \sum_{k\in\mathcal{S}} |h_k|^2 S_k\right) - \log\left(\gamma_{\mathcal{S}}(\boldsymbol{S})\right), \end{aligned}$$

where the monotonicity of $f_{\mathcal{S}}(\boldsymbol{S})$, $g_{\mathcal{S}}(\boldsymbol{S})$, and $\gamma_{\mathcal{S}}(\boldsymbol{S})$ is easily established by checking the non-negativity of their first derivatives. Since $\sum_i w_i R_i$ is also an increasing function (P1) belongs to the class of monotonic optimization problems [10].

Monotonic optimization theory provides a framework to solve the general optimization problem $\max_{\boldsymbol{x}\in\mathcal{G}\cap\mathcal{H}} f(\boldsymbol{x})$ where $f(\boldsymbol{x})$ is an increasing functions, $\mathcal{G}$ is a normal set, and $\mathcal{H}$ is a conormal set. Without going into detail, a set is normal if $\mathcal{G} = \{\boldsymbol{x}\in\mathbb{R}_+^n : g(\boldsymbol{x}) \le \alpha\}$ for some increasing function $g(\boldsymbol{x})$ and any $\alpha\in\mathbb{R}$, and conormal if $\mathcal{H} = \{\boldsymbol{x}\in\mathbb{R}_+^n : h(\boldsymbol{x}) \ge \alpha\}$ for some increasing function $h(\boldsymbol{x})$. For optimization problems of this type it is known that the optimal solution is a Pareto point of $\mathcal{G}$. Thus, we can restrict the search for the global optimizer to the upper boundary $\partial^+\mathcal{G}$ of $\mathcal{G}$. Several algorithms that exploit this property exist, whereas the polyblock algorithm is the most widely known [10], [11]. However, the computational complexity of these algorithms grows exponentially with the number of variables.

Note that problem (P1) is a linear optimization problem in $\boldsymbol{R}$ for fixed $\boldsymbol{S}$. We can exploit this structural property by transforming (P1) into an outer monotonic optimization problem

$$\begin{cases} \max\limits_{t,\boldsymbol{S}} & \rho(t,\boldsymbol{S}) \\ \text{s.t.} & t + g_\Sigma(\boldsymbol{S}) \le g_\Sigma(\bar{\boldsymbol{S}}) \\ & 0 \le t \le g_\Sigma(\bar{\boldsymbol{S}}) - g_\Sigma(\boldsymbol{0}) \\ & \boldsymbol{S}\in[\boldsymbol{0},\bar{\boldsymbol{S}}] \end{cases} \tag{P2}$$

where $g_\Sigma(\boldsymbol{S}) = \sum_{\mathcal{S}\in\mathfrak{S}} g_{\mathcal{S}}(\boldsymbol{S})$ and $\rho(t,\boldsymbol{S})$ is either the solution to the linear program

$$\begin{cases} \max\limits_{\boldsymbol{R}} & \sum_i w_i R_i \\ \text{s.t.} & \sum_{k\in\mathcal{S}} R_k < f_{\mathcal{S}}(\boldsymbol{S}) + \sum_{\mathcal{T}\in\mathfrak{S}\setminus\mathcal{S}} g_{\mathcal{T}}(\boldsymbol{S}) + t \\ & \qquad - g_\Sigma(\bar{\boldsymbol{S}}), \quad \text{for all } \mathcal{S}\in\mathfrak{S} \\ & \boldsymbol{R} \ge \underline{\boldsymbol{R}} \end{cases} \tag{P3}$$

or $-\infty$ if (P3) is infeasible. This reduces the number of variables in the outer monotonic optimization problem almost by a factor of two. Since the computational complexity of monotonic optimization is exponential in the number of variables, while the solution of a linear program only requires polynomial time, this transformation reduces the numerical complexity of solving (P1) significantly. We show that (P2) is a monotonic optimization problem in the next theorem. Afterwards, we establish that every optimal solution of (P2) also solves (P1).

Theorem 1: (P2) is a monotonic optimization problem and has at least one optimal solution.

Proof: Let $\mathcal{C}(\boldsymbol{S},t)$ be the constraint set of (P3) for fixed $(\boldsymbol{S},t)$. Since the RHS of the first constraint in (P3) is an increasing function, $\mathcal{C}(\boldsymbol{S},t) \subseteq \mathcal{C}(\boldsymbol{S}',t')$ whenever $0 \le (\boldsymbol{S},t) \le (\boldsymbol{S}',t')$. This implies $\max\{\sum_i w_i R_i : \boldsymbol{R}\in\mathcal{C}(\boldsymbol{S},t)\} \le \max\{\sum_i w_i R_i : \boldsymbol{R}\in\mathcal{C}(\boldsymbol{S}',t')\}$ since $w_i > 0$. Thus, $\rho(\boldsymbol{S},t)$ is an increasing function. Since $g_\Sigma(\boldsymbol{S})$ is increasing and continuous on $\boldsymbol{S}\ge 0$, and $g_\Sigma(\bar{\boldsymbol{S}})$ is constant, the constraint set of (P2) is closed and normal [10, Prop. 5]. This proves that (P2) is a monotonic optimization problem.

Moreover, the feasible set of (P2) is non-empty, $g_\Sigma(\boldsymbol{S})$ is a continuous function for $\boldsymbol{S}\ge 0$ and $\rho(t,\boldsymbol{S})$ is upper semi-continuous on the relevant interval. Thus, (P2) has at least one optimal solution [11, Prop. 11.11]. ■

Theorem 2: Let $(\boldsymbol{R}^*,\boldsymbol{S}^*,t^*)$ be a solution of (P2). Then, $(\boldsymbol{R}^*,\boldsymbol{S}^*)$ solves (P1).

Proof: First, consider the optimization problem

$$\begin{cases} \max\limits_{t,\boldsymbol{R},\boldsymbol{S}} & \sum_i w_i R_i & \text{(P4a)} \\ \text{s.t.} & \sum_{k\in\mathcal{S}} R_k < f_{\mathcal{S}}(\boldsymbol{S}) + \sum_{\mathcal{T}\in\mathfrak{S}\setminus\mathcal{S}} g_{\mathcal{T}}(\boldsymbol{S}) + t & \text{(P4b)} \\ & \qquad - g_\Sigma(\bar{\boldsymbol{S}}), \quad \text{for all } \mathcal{S}\in\mathfrak{S} & \\ & t + g_\Sigma(\boldsymbol{S}) \le g_\Sigma(\bar{\boldsymbol{S}}) & \text{(P4c)} \\ & 0 \le t \le g_\Sigma(\bar{\boldsymbol{S}}) - g_\Sigma(\boldsymbol{0}) & \text{(P4d)} \\ & \boldsymbol{R} \ge \underline{\boldsymbol{R}}, \quad \boldsymbol{S}\in[\boldsymbol{0},\bar{\boldsymbol{S}}]. & \text{(P4e)} \end{cases}$$

We know from Theorem 1 that (P2) has a solution. Hence, $\rho(t^*,\boldsymbol{S}^*)$ takes the value $\sum_i w_i R_i^*$ for every optimal solution $(\boldsymbol{R}^*,\boldsymbol{S}^*,t^*)$ satisfying the constraints in (P3). Hence, (P2) yields the maximum of $\sum_i w_i R_i^*$ under the constraints in (P3) and (P2). These are exactly the objective and constraints of (P4). Thus, $(\boldsymbol{R}^*,\boldsymbol{S}^*,t^*)$ solves (P4).

Next, observe that any optimal solution of (P4) satisfies (P4c) with equality because the RHS of (P4b) is increasing in t, and (P4d) is always inactive since g_Σ is an increasing function and $\boldsymbol{S}\ge\boldsymbol{0}$. Then, for (P4b), we have

$$\begin{aligned} \sum_{k\in\mathcal{S}} R_k^* &< f_{\mathcal{S}}(\boldsymbol{S}^*) + \sum_{\mathcal{T}\in\mathfrak{S}\setminus\mathcal{S}} g_{\mathcal{T}}(\boldsymbol{S}^*) + t^* - g_\Sigma(\bar{\boldsymbol{S}}^*) \\ &= f_{\mathcal{S}}(\boldsymbol{S}^*) + \sum_{\mathcal{T}\in\mathfrak{S}\setminus\mathcal{S}} g_{\mathcal{T}}(\boldsymbol{S}^*) + g_\Sigma(\bar{\boldsymbol{S}}^*) \\ &\quad - g_\Sigma(\boldsymbol{S}^*) - g_\Sigma(\bar{\boldsymbol{S}}^*) \\ &= f_{\mathcal{S}}(\boldsymbol{S}^*) - g_{\mathcal{S}}(\boldsymbol{S}^*). \end{aligned}$$

Thus, constraints (P4b)–(P4d) are equal to the first constraint of (P1) for any optimal solution of (P4). Because (P1) and (P4) also have the same objective and all other constraints are the same, any solution to (P4) is also an solution of (P1). ■

IV. Numerical evaluation

For the numerical evaluation, we assume equal maximum power constraints and noise power at the users and the relay, and no minimum rate constraint, i.e. $\underline{\boldsymbol{R}} = \boldsymbol{0}$. Channels are assumed reciprocal and chosen i.i.d. with circular symmetric complex Gaussian distribution, i.e., $h_k \sim \mathcal{CN}(0,1)$ and $g_k =$

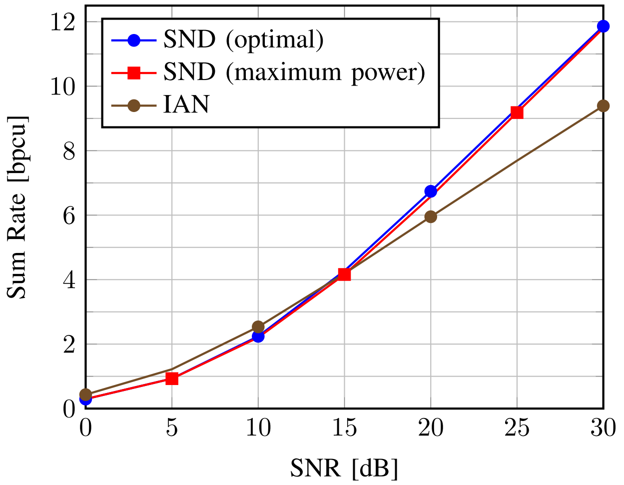

Fig. 2. Throughput in the 3-Way Relay Channel with amplify-and-forward relaying and 1) SND with optimal power allocation; 2) SND with maximum transmit power at all nodes; 3) IAN with optimal power allocation averaged over 1,000 i.i.d. channel realizations.

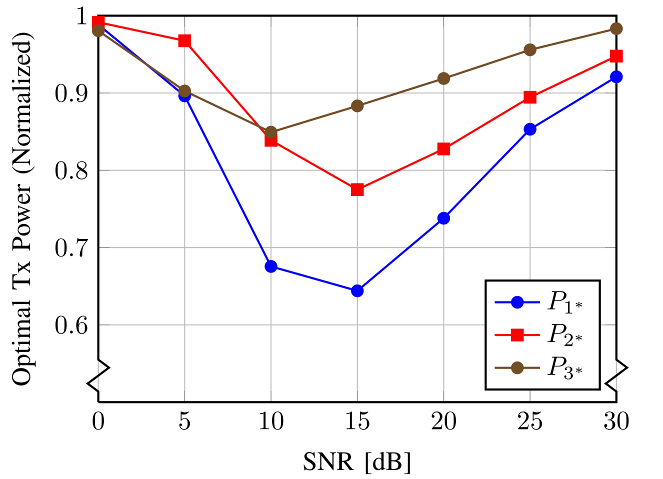

Fig. 3. Optimal power allocation per user for SND normalized to the maximum transmit power and ordered such that $|\widetilde{h}_{1^*}|^2 \geq |\widetilde{h}_{2^*}|^2 \geq |\widetilde{h}_{3^*}|^2$.

h_k^*. Results are averaged over 1,000 channel realizations. The outer problem (P2) is solved with the polyblock algorithm.

Figure 2 shows the maximum achievable sum rate in the MWRC with AF relaying and SND at the receivers, i.e., with all weights $w_k = 1$. Results are compared to simply using maximum transmit power at all nodes and to the achievable sum rate using single-user (IAN) receivers instead of the significantly more involved multi-user receivers. These were obtained by applying the algorithms from [5]. It can be observed that SND outperforms IAN in the interference-limited regime while IAN performs better when the SNR is low. Moreover, using maximum transmit power at all nodes is almost throughput optimal. This is quite surprising when looking at the optimal power allocation in Fig. 3.

Figure 3 shows the optimal power allocation for SND normalized to the maximum transmit power. For each drop, the users are re-ordered such that $|\widetilde{h}_{1^*}|^2 \geq |\widetilde{h}_{2^*}|^2 \geq |\widetilde{h}_{3^*}|^2$ where $\widetilde{h}_k = \alpha h_k g_{q(k)}$ is the effective channel of user k. It can be seen that the optimal transmit power for low and very high SNRs converges to using maximum transmit power. In between, all user transmit at reduced power where the user with the best channel uses the least power. This is in stark contrast to the optimal power allocation for IAN or water-filling based solutions where the worst user does not transmit at all for medium to high SNRs. Thus, SND has much better fairness than IAN while also achieving higher throughput. Figure 3 also reveals the advantage of the optimal power allocation over transmitting at maximum power. While there is no significant difference in the throughput, the optimal power allocation uses considerably less power and is, thus, more energy-efficient.

A. Implementation issues

The inner linear program (P3) is typically executed several times in each iteration of the outer algorithm. Hence, its implementation is performance critical. A simple benchmark comparing Matlab's `linprog` with the state-of-the-art Gurobi library [12] accessed through its Matlab interface already shows a 6x speed-up as can be seen from Table I. Further improvements are possible by noting that only the RHSs of the constraints in (P3) change between calls. Thus, we can setup the optimization model in Gurobi once and then just update it. With this approach, a total speed-up of factor 20 over `linprog` is possible achieving reasonable runtimes ("`mex`" entry in Table I). Please note that the runtimes reported in Table I are just for one operating point and do not reflect the average runtime to obtain Figs. 2 and 3.

TABLE I
RUNTIME OF THE POLYBLOCK ALGORITHM FOR DIFFERENT IMPLEMENTATIONS OF (P3).

Method	Runtime
`linprog`	6,004 s
`gurobi`	1,075 s
`mex`	300 s

V. CONCLUSION

We have determined the global weighted sum rate optimal power allocation for non-regenerative 3-user MWRCs. We proposed a transformation of the optimization problem that exploits the linearity in the rate variables and reduces the computational complexity significantly. Note that, although we consider a specialized problem, this approach can be applied easily to many similar resource allocation problems.

We compared the achievable throughput numerically to the state-of-the-art and a heuristic power allocation. It turns out that SND has much better fairness than IAN and that maximum transmit power at all nodes achieves almost the same throughput as the optimal resource allocation but is also less energy-efficient. Thus, the optimal power allocation not only maximizes the throughput but also helps to increase the energy efficiency, which are two key metrics in 5G networks.

References

[1] A. Chaaban and A. Sezgin, *Multi-way Communications: An Information Theoretic Perspective*, ser. Found. and Trends Commun. and Inf. Theory. Now Publishers, 2015, vol. 12, no. 3-4.

[2] A. El Gamal and Y.-H. Kim, *Network Information Theory*. Cambridge University Press, 2011.

[3] B. Matthiesen and E. A. Jorswieck, "Instantaneous relaying for the 3-way relay channel with circular message exchanges," in *49th Asilomar Conf. on Signals, Syst., and Comput.*, Pacific Grove, CA, Nov. 2015, pp. 475–479.

[4] B. Matthiesen, A. Zappone, and E. A. Jorswieck, "Resource allocation for energy-efficient 3-way relay channels," *IEEE Trans. Wireless Commun.*, vol. 14, no. 8, pp. 4454–4468, Aug. 2015.

[5] B. Matthiesen and E. A. Jorswieck, "Global sum rate optimal resource allocation for non-regenerative 3-way relay channels," in *Workshop Broadband Wireless Commun. Comput. Boards (Atto-Nets), IEEE Int. Conf, Ubiquitous Wireless Broadband (ICUWB)*, Salamanca, Spain, Sep. 2017.

[6] A. Zappone, L. Sanguinetti, G. Bacci, E. Jorswieck, and M. Debbah, "Energy-efficient power control: A look at 5G wireless technologies," *IEEE Trans. Signal Process.*, vol. 64, no. 7, pp. 1668–1683, Apr. 2016.

[7] M. Kobayashi and G. Caire, "An iterative water-filling algorithm for maximum weighted sum-rate of Gaussian MIMO-BC," *IEEE J. Sel. Areas Commun.*, vol. 24, no. 8, pp. 1640–1649, Jul. 2006.

[8] D. H. N. Nguyen and T. Le-Ngoc, "Sum-rate maximization in the multicell MIMO multiple-access channel with interference coordination," *IEEE Trans. Wireless Commun.*, vol. 13, no. 1, pp. 36–48, Jun. 2014.

[9] Y. Yang and M. Pesavento, "A unified successive pseudoconvex approximation framework," *IEEE Trans. Signal Process.*, vol. 65, no. 13, pp. 3313–3328, Jul. 2017.

[10] H. Tuy, "Monotonic optimization: Problems and solution approaches," *SIAM J. Optimization*, vol. 11, no. 2, pp. 464–494, Feb. 2000.

[11] ——, *Convex Analysis and Global Optimization*. Springer International Publishing, 2016.

[12] Gurobi Optimization. (2017) Gurobi optimizer 6.0.4. [Online]. Available: http://gurobi.com