Global Sum Rate Optimal Resource Allocation for Non-Regenerative 3-Way Relay Channels

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***Abstract*—Throughput in 3-way relay channels is studied in this paper. We focus on Gaussian channels, amplify-and-forward relaying, and treating interference as noise at the receivers. Unlike other contributions, we consider multiple unicast transmissions. First, we provide the achievable rate region. Then, we design two algorithms for joint power allocation at all terminals so as to maximize the system throughput. One algorithm has guaranteed convergence to the global optimal solution, while the other exhibits very low complexity. Numerical simulations evaluate the performance of both algorithms by means of a mmWave wireless board-to-board communication system. The results show that both algorithms converge to the same solution but with very different convergence speed. On average, obtaining the global optimal solution requires approximately 1,500x the iterations of the low complexity solution. Thus, we can either obtain a certificate of global optimality at the price of slow convergence or attain a stationary point that is most likely globally optimal within a few iterations.**

***Index Terms*—Multi-hop networks, multi-way relay channel, relay systems, resource allocation, monotonic optimization, mmWave communications, power control, global optimization, amplify-and-forward**

I. Introduction

Next generation High Performance Computing (HPC) systems are envisioned to move away from wired backplane communication and instead use wireless interfaces to communicate across boards [1], [2]. A promising candidate technology to obtain the required high data rates is millimeter wave (mmWave) communication [3]. It achieves much higher bandwidths than state-of-the-art wireless technologies and allows to place a high number of antennas on each computing node. This results in very narrow beams letting inter-node interference almost vanish. However, due to the high attenuation of mmWaves, communication is only feasible between adjacent boards. To overcome this obstacle nodes on intermediate boards can be used as relays. This is possible without additional hardware costs when layer 1 relaying is employed.

The multi-way relay channel (MWRC) models relay-aided communication across several nodes with no direct links between the users. One application of this model is the wireless board-to-board communication scenario outlined above, where multiple nodes exchange data across boards with the help of another node acting as relay. Communication over the relay also helps to relieve the on-board network, especially when routing over multiple hops is the alternative. Other applications are, for example, wireless sensor networks, Industry 4.0, heterogeneous dense small cell networks in modern and future 5G wireless networks, or communication of several ground stations over a satellite. It was first introduced in [4] as a model for clustered communication where multiple terminals use a single relay to multicast information to all other terminals in their cluster. Since the usual approach to share the relay across clusters is time sharing, most subsequent works only considered a single cluster. An extensive overview of results for the MWRC can be found in [5].

We focus on the Gaussian MWRC with 3 users, amplify-and-forward (AF) relaying, circular message exchanges, and treating interference as noise at the receivers. Achievable rate regions for this channel are derived in [6]. The energy efficiency (EE) of this channel with symmetric channels is analyzed in [7] for different relaying schemes. One important result in [7] is that, when taking hardware complexity into account, the EE of AF is quite good even though the throughput of other relaying schemes might be higher. As opposed to many other works on the MWRC, we are considering a circular message exchange where each node only wants to recover one message, and, thus, has to deal with interference. Some of this interference is self-induced and can be removed. While there are several ways to deal with the remaining interference at the receivers, in this paper we are limiting ourselves to treat the interference as additional noise so that conventional single user receivers can be employed.

In this paper, we determine the throughput optimal power allocation of the MWRC at hand. The resulting optimization problem is non-convex and, thus, can not be solved by standard convex optimization tools. While EE is considered to be the most important design criterion in future wireless networks, the throughput optimization problem considered here poses sufficient difficulties to be dealt with before tackling the EE problem. We present two algorithms to solve this optimization problem: 1) we leverage monotonic optimization theory to find the global optimal power allocation with exponential complexity; 2) we adapt a recently proposed unified successive pseudo-convex approximation framework [8] to find a stationary point

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of the problem exhibiting very fast convergence. Afterwards, we provide numeric evidence that both algorithm converge to the same solution.

Notation: $(a_k)_k$ denotes the row vector $(a_1, a_2, \dots)$. $\boldsymbol{e}_1, \boldsymbol{e}_2, \dots$ are the Euclidean unit vectors. For two vectors $\boldsymbol{x}, \boldsymbol{y}$, we say that $\boldsymbol{x} \geq \boldsymbol{y}$ if $x_i \geq y_i$, for all i, and $\boldsymbol{x} \leq \boldsymbol{y}$ if $x_i \leq y_i$. $\mathbb{R}_+^n$ denotes the set of n-dimensional positive real numbers.

II. System Model

We consider a 3-user single-input single-output (SISO) MWRC in which three users communicate with each other via an AF relay. Gaussian channels with quasi-static block flat fading and a circular (i.e. partial) message exchange, to be described in detail later, are considered. We assume full-duplex transmission and consider a scenario in which no direct user-to-user link is available. The users are denoted as node 1 to 3 and the relay as node 0. We define the set of all users as $\mathcal{K} = \{1, 2, 3\}$.

The signal received by the relay is given by

$$Y_0 = \sum_{k \in \mathcal{K}} h_k X_k + Z_0,$$

with X_k the channel input at node $k \in \mathcal{K}_R$ with power p_k, h_k the channel coefficient from user k to the relay, and Z_0 the independent and identically distributed (i.i.d.) zero-mean circularly symmetric complex Gaussian noise with power N_0. The relay scales the observed signal Y_0 by a positive constant and broadcasts it back to the users. The transmitted symbol at the relay is $X_0 = \alpha Y_0$, where α is a normalization factor chosen such that the transmit power at the relay is p_0, i.e., $\alpha = \sqrt{p_0 \,/\, \left(\sum_{k \in \mathcal{K}} |h_k|^2 p_k + N_0\right)}$.

Then, the signal received at user $k \in \mathcal{K}$ is given by

$$Y_k = g_k X_0 + Z_k,$$

with g_k the channel coefficient from the relay to user k, and Z_k the i.i.d. zero mean circularly symmetric complex Gaussian noise with power N. The channel inputs are subject to an average power constraint P on each X_k, $k \in \mathcal{K}$ and P_0 on X_0. The receiver first removes its self-interference from the received signal and then decodes for its desired message while treating the remaining interference as noise.

The message exchange is illustrated in Fig. 1 where the different line styles indicate different messages. We denote the receiver of node k's message as $q(k)$, $k \in \mathcal{K}$, and the user not interested in it as $l(k)$. Conversely, user k desires the message sent by user $l(k)$. From Fig. 1, we have $q(1) = l(3) = 2$, $q(2) = l(1) = 3$, and $q(3) = l(2) = 1$.

Lemma 1: A rate triple (R_1, R_2, R_3) is achievable for the Gaussian MWRC with AF relaying and treating interference as noise at the decoders if, for all $k \in \mathcal{K}$,

$$R_k(\boldsymbol{s}) < \log\left(1 + \frac{|h_k|^2 s_k}{\gamma_{l(k)}(\boldsymbol{s})}\right), \tag{1}$$

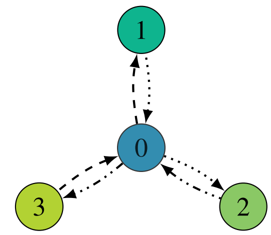


Fig. 1. Illustration of the system model where node 0 is the relay and nodes 1 to 3 are the users. Messages travel along the different line styles.

where $s_k = \frac{p_k}{N_0} \leq \frac{P}{N_0} = S_k$, $\boldsymbol{s} = (s_1, s_2, s_3)$, and

$$\gamma_{l(k)}(\boldsymbol{s}) = \underbrace{1}_{\substack{\text{sink}\\\text{noise}}} + \underbrace{\left|h_{l(k)}\right|^2 s_{l(k)}}_{\substack{\text{multiple access}\\\text{interference}}} + \underbrace{\widetilde{g}_{q(k)}^{-1}\left(1 + \sum_{i \in \mathcal{K}} |h_i|^2 s_i\right)}_{\text{relay noise amplification}},$$

with $\widetilde{g}_k = |g_k|^2 \frac{P_0}{N_k}$.

Proof sketch: Adapt [6, Corollary 1] to Gaussian channels using the standard procedure [9, Chapter 3]. Apply this to the channel defined above with $\mathbb{E}[X_k^2] = p_k$, $k \in \mathcal{K} \cup \{0\}$ to obtain (1) with $\widetilde{g}_k = |g_k|^2 \frac{p_0}{N_k}$. It is easily shown that the partial derivative of (1) with respect to p_0 is always non-negative. Thus, (1) is a monotonically increasing function of p_0 and $p_0 = P_0$ is optimal. ■

III. Sum rate optimal power allocation

Our goal is to determine the optimal power allocation that maximizes the system throughput, i.e., find the solution to the optimization problem

$$\begin{cases} \max\limits_{\boldsymbol{s}} & R_\Sigma(\boldsymbol{s}) = \sum\limits_{k \in \mathcal{K}} \log\left(1 + \dfrac{|h_k|^2 s_k}{\gamma_{l(k)}(\boldsymbol{s})}\right) \\ \text{s.t.} & \boldsymbol{s} \in [0; \boldsymbol{S}], \end{cases} \tag{P1}$$

where $\boldsymbol{S} = (S_1, S_2, S_3)$ and $[0; \boldsymbol{S}] = \times_k [0; S_k]$. Each rate R_k is monotonically increasing in s_k and decreasing in s_i, $i \neq k$. Thus, transmitting at maximum power at all nodes might be suboptimal. Moreover, each R_k is a concave function in s_k and convex in s_i, $i \neq k$. Hence, R_Σ is a non-concave function and (P1) can not be solved by standard convex optimization tools.

In general, no optimization tool to solve non-concave maximization problems with limited computational complexity is available. Thus, it appears difficult to solve (P1) with limited complexity. In the following, we present two approaches to solve (P1). The first is based on a novel successive pseudo-convex approximation framework [8]. It provides an efficient method to find a stationary point of (P1) with low computational complexity. Instead, the second approach utilizes monotonic optimization theory [10] to find a global optimal solution at the expense of exponential complexity. Interestingly, the numerical results presented in Section IV indicate that both algorithms converge to the same solution.

A. Successive convex approximation

The authors of [8] propose an iterative algorithm that solves the optimization problem $\min_{\boldsymbol{s}\in\mathcal{S}} f(\boldsymbol{s})$ as a sequence of successively refined approximate problems where, in each iteration t, a function $\widetilde{f}(\boldsymbol{s};\boldsymbol{s}^t)$ that approximates $f(\boldsymbol{s})$ is minimized.

First, note that maximizing $R_\Sigma(\boldsymbol{s})$ is equivalent to minimizing $-R_\Sigma(\boldsymbol{s})$. Further, observe that that the objective

$$\begin{aligned}R_\Sigma(\boldsymbol{s}) &= \sum_{k\in\mathcal{K}} \log\Big(\left|h_k\right|^2 s_k + \gamma_{l(k)}(\boldsymbol{s})\Big) - \sum_{k\in\mathcal{K}} \log\big(\gamma_{l(k)}(\boldsymbol{s})\big) \\ &= g(\boldsymbol{s}) - h(\boldsymbol{s}). \end{aligned} \tag{2}$$

is a DC function [11] since $g(\boldsymbol{s})$ and $h(\boldsymbol{s})$ are concave functions of $\boldsymbol{s}$.[1] A reasonable choice to approximate DC functions in general is

$$\widetilde{f}(\boldsymbol{s};\boldsymbol{s}^t) = -g(\boldsymbol{s}) + h(\boldsymbol{s}^t) + \nabla h(\boldsymbol{s}^t)^T\left(\boldsymbol{s}-\boldsymbol{s}^t\right). \tag{3}$$

With that approximation function, we obtain Algorithm 1.

Algorithm 1 Successive convex approximation algorithm

Initialize $t=0$, $\boldsymbol{s}^0\in[0;\boldsymbol{S}]$.
repeat
  $\boldsymbol{s}^{t+1} \leftarrow \underset{\boldsymbol{s}\in[0;\boldsymbol{S}]}{\arg\min}\ -g(\boldsymbol{s}) + h(\boldsymbol{s}^t) + \nabla h(\boldsymbol{s}^t)^T\left(\boldsymbol{s}-\boldsymbol{s}^t\right)$
  $t\leftarrow t+1$
until convergence.

Since $\widetilde{f}(\boldsymbol{s};\boldsymbol{s}^t)$ is a convex function the minimization problem in each iteration of Algorithm 1 can be solved by standard convex optimization techniques. Computation of $\widetilde{f}(\boldsymbol{s};\boldsymbol{s}^t)$ requires the gradient of $h(\boldsymbol{s})$ which is $\nabla h(\boldsymbol{s}) = \left(\mathbb{1}^T\boldsymbol{J}_h(\boldsymbol{s})\right)^T$ where $\boldsymbol{J}_h(\boldsymbol{s})$ is the Jacobian of $h(\boldsymbol{s})$. A straightforward computation yields

$$\boldsymbol{J}_h(\boldsymbol{s}) = \operatorname{diag}\left(\Gamma(\boldsymbol{s})\right)^{-1}\boldsymbol{J}_{\Gamma(\boldsymbol{s})}(s)$$

and

$$\boldsymbol{J}_{\Gamma(\boldsymbol{s})}(s) = (\widetilde{g}_{q(k)}^{-1})_k^T(|h_k|^2)_k + \Pi\operatorname{diag}((|h_k|^2)_k).$$

where $\Gamma(\boldsymbol{s}) = (\gamma_{l(k)}(\boldsymbol{s}))_{k\in\mathcal{K}}$ and $\Pi = \left[\begin{smallmatrix}\boldsymbol{e}_{q(1)} & \boldsymbol{e}_{q(2)} & \boldsymbol{e}_{q(3)}\end{smallmatrix}\right]$.

Corollary 1: Any limit point of $\{\boldsymbol{s}^t\}$ obtained by Algorithm 1 is a stationary point of (P1).

Proof: $R_\Sigma(\boldsymbol{s})$ is a proper and continuously differentiable function in $[0;\boldsymbol{S}]$, and $[0;\boldsymbol{S}]$ is a closed convex set. Thus, (P1) falls within the class of optimization problems considered in [8] and we can adapt [8, Algorithm 1] to solve (P1).

The approximate function $\widetilde{f}(\boldsymbol{s};\boldsymbol{s}^t)$ needs to fulfill the following technical conditions for guaranteed convergence to a stationary point:

1) $\widetilde{f}(\boldsymbol{x};\boldsymbol{y})$ is pseudo-convex in $\boldsymbol{x}$ for any $\boldsymbol{y}\in\mathcal{X}$.
2) $\widetilde{f}(\boldsymbol{x};\boldsymbol{y})$ is continuously differentiable in $\boldsymbol{x}$ for any $\boldsymbol{y}\in\mathcal{X}$ and continuous in $\boldsymbol{y}$ for any $\boldsymbol{x}\in\mathcal{X}$.
3) $\nabla_{\boldsymbol{x}}\widetilde{f}(y;y) = \nabla_{\boldsymbol{x}}f(y)$.
4) The solution set of $\min_{\boldsymbol{x}\in\mathcal{X}}\widetilde{f}(\boldsymbol{x};\boldsymbol{x}^t)$ is nonempty in every iteration t.
5) Given any convergent subsequence $\{x^t\}_{t\in\mathcal{T}}$ where $\mathcal{T}\subseteq\{1,2,\ldots\}$, the sequence $\{\arg\min_{\boldsymbol{x}\in\mathcal{X}}\ \widetilde{f}(\boldsymbol{x};\boldsymbol{x}^t)\}_{t\in\mathcal{T}}$ is bounded.

Since any convex function is also pseudo-convex, 1) is satisfied. It is obvious that condition 2) holds. With the remark below and $\nabla_{\boldsymbol{s}}(-R_\Sigma)(\boldsymbol{s}^t) = -\nabla g(\boldsymbol{s}^t) + \nabla h(\boldsymbol{s}^t)$, condition 3) holds as well. For conditions 4) and 5) to hold it is sufficient that $[0;\boldsymbol{S}]$ is bounded [8].

It is shown in [8, Section III-B] that $\widetilde{f}(\boldsymbol{s};\boldsymbol{s}^t)\geq f(\boldsymbol{s})$ and $\widetilde{f}(\boldsymbol{s}^t;\boldsymbol{s}^t) = f(\boldsymbol{s}^t)$ also holds for (3). In that case, the proposed algorithm can be simplified and we obtain Algorithm 1. ■

Remark 1: While many convex optimization tools are able to approximate the objective's gradient very well, explicit characterization is favorable for increased performance. The gradient of $\widetilde{f}(\boldsymbol{s};\boldsymbol{s}^t)$ is $\nabla\widetilde{f}(\boldsymbol{s};\boldsymbol{s}^t) = -\nabla g(\boldsymbol{s}) + \nabla h(\boldsymbol{s}^t)$ with $\nabla g(\boldsymbol{s}) = \left(\mathbb{1}^T\boldsymbol{J}_g(\boldsymbol{s})\right)^T$ and

$$\begin{aligned}\boldsymbol{J}_g(\boldsymbol{s}) = (\operatorname{diag}((\left|h_k\right|^2 s_k)_k) + \operatorname{diag}(\Gamma(\boldsymbol{s})))^{-1} \\ (\boldsymbol{J}_{\Gamma(\boldsymbol{s})}(\boldsymbol{s}) + \operatorname{diag}((|h_k|^2)_k)).\end{aligned}$$

B. Monotonic optimization

Solving general (non-convex) global optimization problems can involve examining every point of the feasible set. Monotonic optimization theory provides means to solve a broad class of such optimization problems in a much more efficient way but still with exponential complexity [10], [12]. More precisely, it allows to solve the general problem $\max_{\boldsymbol{x}\in\mathcal{G}\cap\mathcal{H}} f(\boldsymbol{x})$, where $f(\boldsymbol{x})$ is an increasing function, $\mathcal{G}$ is a normal set, and $\mathcal{H}$ is a conormal set. A function $f:\mathbb{R}^n\mapsto\mathbb{R}$ is called *increasing* on $\mathbb{R}_+^n$ if $f(\boldsymbol{x})\leq f(\boldsymbol{x'})$ whenever $0\leq\boldsymbol{x}\leq\boldsymbol{x'}$. A set $\mathcal{G}\subseteq\mathbb{R}_+^n$ is called *normal (conormal)* if for any two vectors $\boldsymbol{x},\boldsymbol{y}\in\mathbb{R}_+^n$ such that $\boldsymbol{y}\leq\boldsymbol{x}$ ($\boldsymbol{y}\geq\boldsymbol{x}$), if $\boldsymbol{x}\in\mathcal{G}$, then $\boldsymbol{y}\in\mathcal{G}$.

The key ideas behind monotonic optimization are that, due to f being increasing, the maximizer of f lies on the outer boundary of the feasible set, and that, since the feasible set is normal, if a point $\boldsymbol{x}$ is infeasible, then every point $\boldsymbol{x}'\geq\boldsymbol{x}$ is also infeasible. Thus, the feasible set can be approximated by a sequence of enclosing *polyblocks*. A set is called *polyblock* if it is a finite union of boxes in the nonnegative orthant. Since the maximizer of f over a polyblock is one of its vertices, the search space in each iteration is finite.

While the objective of (P1) is not increasing *per se*, the problem still has hidden monotonicity. Consider again the decomposition of $R_\Sigma(\boldsymbol{s})$ in (2). It is easily verified that $g(\boldsymbol{s})$ and $h(\boldsymbol{s})$ are increasing functions in $\boldsymbol{s}$. Thus, $R_\Sigma(\boldsymbol{s})$ belongs to the class of difference of increasing (d.i.) functions and we can use the following transformation from [10] to state (P1) as a monotonic optimization problem.

Due to $h(\boldsymbol{s})$ being increasing, for every $\boldsymbol{s}\in[0;\boldsymbol{S}]$ and some auxiliary variable $t\geq 0$, we have $h(\boldsymbol{s})+t = h(\boldsymbol{S})$. Replacing

[1] This property gives rise to another global optimal solution method, namely, DC programming [11]. However, since DC programming has the same computational complexity as monotonic optimization, we do not pursue this approach any further here.

TABLE I
LINK BUDGET FOR BOARD-TO-BOARD COMMUNICATIONS USING 16×16 ANTENNA ARRAYS [13]

	Unit	Value
Receiver noise figure	dB	15
Path loss exponent	–	2
Array gain	dB	24
Butler matrix inaccuracy	dB	20
Implementation loss	dB	5
Receiver temperature	K	323

$h(\boldsymbol{s})$ in the objective yields

$$\begin{cases} \max\limits_{\boldsymbol{s},t} & g(\boldsymbol{s}) + t - h(\boldsymbol{S}) \\ \text{s. t.} & t + h(\boldsymbol{s}) = h(\boldsymbol{S}) \\ & \boldsymbol{s} \in [0; \boldsymbol{S}]. \end{cases} \tag{P2}$$

Clearly, the objective is an increasing function now. Removing the inessential constant from the objective and relaxing the new constraint results in the equivalent monotonic optimization problem

$$\begin{cases} \max\limits_{\boldsymbol{s},t} & g(\boldsymbol{s}) + t \\ \text{s. t.} & 0 \leq t + h(\boldsymbol{s}) \leq h(\boldsymbol{S}) \\ & \boldsymbol{s} \in [0; \boldsymbol{S}]. \end{cases} \tag{P3}$$

After a shift of origin this can be solved in exponential time using the polyblock algorithm [12, Algorithm 3].

IV. NUMERICAL EVALUATION

We focus on wireless communication between computer boards for the numerical evaluation of the presented algorithms. This is a major issue addressed in the Collaborative Research Center 912 *Highly Adaptive Energy-Efficient Computing (HAEC)* initiated at TU Dresden in 2011 and funded by Deutsche Forschungsgemeinschaft (DFG) [1], [2]. Part of the *HAEC*-vision is a 1 liter HPC box housing 4 boards with 16 3D chip stacks each. Instead of wire based interconnects between boards, each chip stack is equipped with a wireless interface operating at 200 GHz carrier frequency with a bandwidth of 30 GHz. Recent hardware advancements towards this goal are reported in [13]. Therein, the link budget in Table I is given. The authors assume a 16×16 antenna array fed by a Butler matrix switching network on each node.

The system model considered in this paper and depicted in Fig. 1 is one possible communication scenario in this *HAEC Box*. Since node positions and beam directions are fixed, the antenna array and beamforming network can be regarded as a single antenna and due to the small beamwidth of massive MIMO mmWave communications, crosstalk between nodes can be neglected. Hence, the model assumptions are valid for the *HAEC* scenario.

Including the receiver noise figure in the channel gain, we obtain $G_c(d) = 2G_{\text{array}} - L_{\text{Impl. loss}} - L_{\text{Butler}} - L_{\text{Rx NF}} - PL_d[dB]$, where $PL_d[dB]$ is the free space path loss for the distance d in dB, i.e., $PL_d[dB] = 10n\log_{10}\left(\frac{4\pi d f_c}{c}\right)$, with n the path

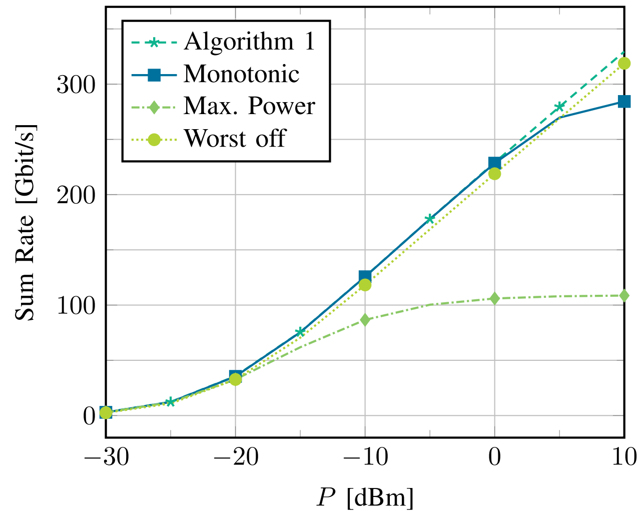
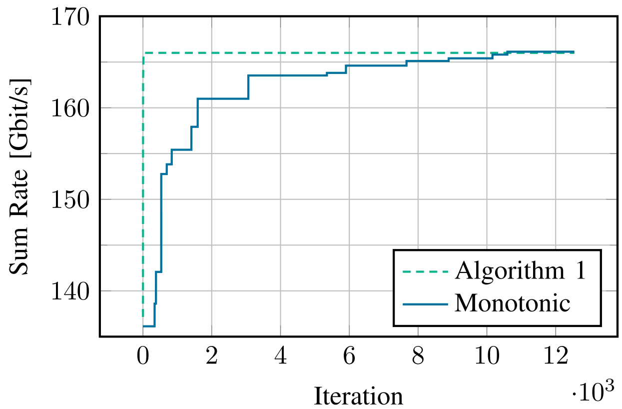


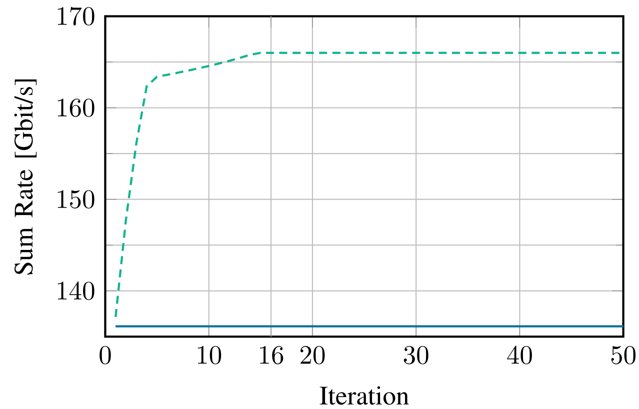
Fig. 2. Throughput in the 3-Way Relay Channel with amplify-and-forward relaying and multiple unicast transmissions for wireless board-to-board communication at 200 GHz averaged over 1,000 i.i.d. channel realizations. The optimal power allocation was computed with the polyblock algorithm ("monotonic") and is compared to the local optimal solution of Algorithm 1, and two heuristic power allocations where all users transmit at maximum power and where the user with the worst channel does not transmit, respectively.

loss exponent, f_c the carrier frequency, and c the speed of light in vacuum. For this setup, up- and downlink channel gains are equal, thus $|h_k|^2 = |g_k|^2 = G_c(d_k)$. The distances d_k, $k \in \mathcal{K}$, are chosen randomly and independently with uniform distribution between d_{min} and d_{max}. Simple geometrical computations yield minimum and maximum link lengths of $d_{min} = 2.5\,\text{cm}$ and $d_{max} = 11\,\text{cm}$, respectively. Moreover, all nodes have the same thermal noise and maximum transmit power constraint. The message exchange is as indicated by the different line styles in Fig. 1, i.e., node 1 transmits to node 2, 2 to 3, and 3 to 1.

Figure 2 shows the maximum average achievable sum rate for this scenario as a function of the transmit power obtained by monotonic optimization and the successive convex approximation in Algorithm 1 compared to two heuristic power allocations where all users transmit at the maximum allowed transmit power and where the user with the worst channel does not transmit at all, respectively. It can be seen that the optimal power allocation achieves higher throughput than both heuristic power allocations. Monotonic optimization yields a global optimal solution that is within an absolute tolerance of 3 Gbit/s of the optimal value, while Algorithm 1 converges only to a stationary point. However, the results in Fig. 2 show that apparently both algorithms converge to the same value for powers up to 0 dBm. After that, Algorithm 1 achieves better results than monotonic optimization because the polyblock algorithm was terminated after 50,000 iterations. At 0 dBm, about 10 % of the monotonic optimization runs were aborted due to that reason, and at 5 dBm and 10 dBm, 73 % and 100 %, respectively. Thus, in these cases the monotonic optimization algorithm does not yield the global optimal solution and



(a) All 12,544 iterations until regular termination of the polyblock algorithm are shown.



(b) Only the first 50 iterations are shown. Algorithm 1 terminates after 16 iterations while the first change in the polyblock algorithm happens at iteration 393 (not shown).

Fig. 3. Convergence of the polyblock algorithm ("monotonic") versus Algorithm 1 for $|h_1|^2 = -46.55\,\mathrm{dB}$, $|h_2|^2 = -39.46\,\mathrm{dB}$, $|h_3|^2 = -40.33\,\mathrm{dB}$ and $P = -10\,\mathrm{dBm}$.

Algorithm 1 obtains a better solution.[2]

Figure 3 shows the convergence of both algorithms for a fixed channel and transmit power. Figure 3a displays all iterations until convergence of monotonic optimization and Fig. 3b focuses on the convergence of Algorithm 1. It can be observed that monotonic optimization has very slow convergence, while Algorithm 1 converges after 16 iterations. Moreover, for monotonic optimization, with each iteration the size of the set of polyblock vertices increases leading to longer iteration times. Also, the number of iterations until convergence increases with P. Instead, the iterations needed for Algorithm 1 to terminate do not depend on P. While computation times for monotonic optimization were often of the order of seconds and minutes, for some parameter configurations (usually high transmit SNR) computation times can easily become hours or even days.

We can conclude that if performance is more important than a certificate of global optimality Algorithm 1 is a valid alternative to monotonic optimization with virtually no performance loss. In a static communication scenario like the *HAEC Box* precomputation of the optimal power allocation might be the way to go, making monotonic optimization a favorable choice. However, in (fast) changing environments, Algorithm 1 is most likely the better choice. Moreover, when increasing the number of users, monotonic optimization is not feasible at all due to its exponential complexity.

V. Conclusion

In this paper, we studied power allocation for the 3-user MWRC with AF relaying, multiple unicast transmission and single user receivers. We presented the achievable rate region for Gaussian channels and discussed throughput optimal power allocation. We obtained the global optimal solution of this problem leveraging monotonic optimization theory. To circumvent the high computational complexity of monotonic optimization, we derived an alternative successive convex approximation algorithm with very fast convergence to a stationary point. Numerical evaluations show that both algorithms converge to the same solution and that the successive convex approximation is indeed much faster than monotonic optimization.

[2] It is beyond question that monotonic optimization also yields the global optimal solution in these cases if the maximum iteration limit is increased accordingly.

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