Spectral and Energy Efficiency in 3-Way Relay Channels with Circular Message Exchanges

Bho Matthiesen, Alessio Zappone, and Eduard A. Jorswieck
Communications Theory, Communications Laboratory
Department of Electrical Engineering and Information Technology
Technische Universität Dresden, Germany
Email: {bho.matthiesen, alessio.zappone, eduard.jorswieck}@tu-dresden.de

***Abstract*—Spectral and energy efficiency in 3-way relay channels are studied in this paper. First, achievable sum rate expressions for 3-way relay channels are derived for different relaying protocols. Moreover, an outer bound for the capacity of the 3-way relay channel is presented. Next, leveraging the derived achievable sum rate expressions, two algorithms for joint power allocation at the users and at the relay are designed so as to maximize the system energy efficiency. Numerical results are provided to corroborate and provide insight on the theoretical findings.**

***Index Terms*—Multi-way networks, relay systems, energy efficiency, resource allocation, fractional programming, 5G networks.**

I. Introduction

Relays are fundamental building blocks of wireless networks. One recently proposed channel model for relay networks is the multi-way relay channel (MWRC). Such a model applies to many communication architectures like the communication of several ground stations over a satellite, or wireless board-to-board communication in highly adaptive computing [1] where multiple chips exchange data with the help of another chip acting as relay. The MWRC was first introduced in [2], where all users in the cluster send a message and are interested in decoding the messages of all other users in the cluster. In [3] the common-rate capacity of the additive white Gaussian noise (AWGN) MWRC with full message exchange is given and it is shown that for three and more users this capacity is achieved by decode-and-forward (DF) for signal-to-noise ratios (SNRs) below 0 dB and compute-and-forward otherwise. In [4] a constant gap approximation of the capacity region of the Gaussian 3-user MWRC with full message exchange is given.

Besides spectral efficiency, another key performance metric in modern and future 5G wireless networks is energy efficiency (EE). From a mathematical standpoint, one well-established definition of the EE of a communication system is the ratio between the system capacity or achievable rate and the total consumed power [5], [6]. With this definition, the EE is measured in bit/Joule. Previous results on EE in relay systems mainly focus on regular amplify-and-forward (AF) or DF schemes and do not consider the MWRC. In [7] the optimal placement of relays in cellular networks is investigated and is seen to provide power-saving gains. [8] considers the bit/Joule definition of EE and devises energy-efficient power control algorithms in interference networks. A cooperative approach is considered in [9], where a multiple-input multiple-output (MIMO) AF relay-assisted system is considered.

In this paper a 3-way relay channel is considered and both spectral and energy efficiency are analyzed and optimized. In contrast to most other works on MWRCs, we focus on a partial message exchange where each message is only destined for one receiver and, also, not every user sends a message to each other user. This makes it necessary to deal with interference at the receivers which complicates the analysis. However, it might also result in higher achievable rates due to less decoding constraints. The contributions of the paper can be summarized as follows: 1) achievable sum rate expressions are derived for the AF, DF, and noisy network coding (NNC) relaying protocols; 2) an outer bound for the capacity of the 3-way relay channel is derived and used for benchmarking purposes; 3) building on the derived achievable sum rate expressions, two algorithms for energy efficiency optimization are provided to jointly allocate the users' and the relays transmit powers.

We define the function $C(x) = \log_2(1+x)$ for $x \geq 0$.

II. System Model

We consider the symmetric 3-user single-input single-output (SISO) MWRC with circular (i.e., partial) message exchange, AWGN, no direct user-to-user links, and full-duplex transmission. The users are denoted as node 1 to 3 and the relay is node 0. We define the set of all users as $\mathcal{K} = \{1,2,3\}$ and the set of all nodes as $\mathcal{K}_0 = \mathcal{K} \cup \{0\}$.

The 3-user MWRC consists of an uplink channel $Y_0 = \sum_{k\in\mathcal{K}} X_i + Z_0$, and downlink channels $Y_k = X_0 + Z_k, \quad k \in \mathcal{K}$, where X_k and Y_k are the complex valued channel input and output at node $k \in \mathcal{K}_0$, respectively, and Z_k is zero mean circularly symmetric complex Gaussian noise with power N_0 at the relay and N at all other nodes.

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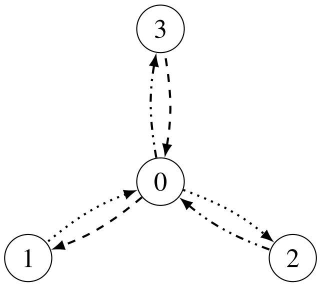

Fig. 1. Illustration of the system model where node 0 is the relay and nodes 1 to 3 are the users. Messages travel along the different line styles.

All noise variables are mutually independent and the channel inputs are independent and identically distributed over time. All channel inputs have zero mean and an average power constraint $\mathbb{E}\,|X_0|^2 \leq P_0$ and $\mathbb{E}\,|X_k|^2 \leq P$, for $k \in \mathcal{K}$.

We consider a circular message exchange as illustrated in Fig. 1 where user $q(k)$ wants message m_k with $q = [2, 3, 1]$. We also define $l(k)$ as the index of the interfering (i.e., unwanted) message at user $q(k)$ as $l = [3, 1, 2]$.

A $(2^{nR_1}, 2^{nR_2}, 2^{nR_3}, n)$ code for the 3-user MWRC consists of three message sets $\mathcal{M}_k = [1 : 2^{nR_k}]$, one for each user $k \in \mathcal{K}$, three encoders, where encoder $k \in \mathcal{K}$ assigns a symbol $x_{ki}(m_k, y_k^{i-1})$ to each message $m_k \in \mathcal{M}_k$ and received sequence y_k^{i-1} for $i \in [1 : n]$, a relay encoder that assigns a symbol $x_{0i}(y_0^{i-1})$ to every past received sequence y_0^{i-1} for $i \in [1 : n]$, and three decoders, where decoder $k \in \mathcal{K}$ assigns an estimate $\hat{m}_{q(k)} \in \mathcal{M}_{q(k)}$ or an error message e to each pair (m_k, y_k^n).

We assume that the message triple (M_1, M_2, M_3) is uniformly distributed over $\mathcal{M}_1 \times \mathcal{M}_2 \times \mathcal{M}_3$. The average probability of error is defined as $P_e^{(n)} = \Pr\left\{\hat{M}_k \neq M_k \text{ for some } k \in \mathcal{K}\right\}$.

A rate triple (R_1, R_2, R_3) is said to be achievable if there exists a sequence of $(2^{nR_1}, 2^{nR_2}, 2^{nR_3}, n)$ codes such that $\lim_{n\to\infty} P_e^{(n)} = 0$. The capacity region of the 3-user MWRC is the closure of the set of achievable rates. The sum rate is defined as $R_\Sigma = \max\{R_1 + R_2 + R_3 : (R_1, R_2, R_3) \in \mathcal{R}\}$, where $\mathcal{R}$ is an achievable rate region. Whenever $\mathcal{R}$ is the capacity region, we call R_Σ the sum capacity C_Σ.

III. Bounds on the Sum Capacity

We start our treatment of the symmetric 3-user MWRC by deriving an upper bound on the sum capacity and then continue with several inner bounds.

A. Outer Bound

This outer bound consists of the cut set bound in the uplink and a downlink bound [10] that takes the side information at the receivers into account.

Lemma 1: The sum capacity of the symmetric 3-user MWRC is upper bounded as

$$C_\Sigma \leq \min\left\{\frac{3}{2}\,\mathrm{C}\left(\frac{P_0}{N}\right), 3\,\mathrm{C}\left(\frac{P}{N_0}\right)\right\}. \quad (1)$$

Proof: The proof is omitted due to space constraints. ■

B. Amplify-and-Forward

We first consider AF relaying where the relay scales the observed signal by a positive constant and broadcasts it back to the users. The transmitted symbol at the relay is $X_0 = \alpha Y_0$, where α is a normalization factor chosen such that the transmit power constraint at the relay is met, i.e., $\alpha = \sqrt{P_0' \,/\, \left(\sum_{k\in\mathcal{K}} P_k' + N_0\right)}$, where P_k', $k \in \mathcal{K}_0$, is the actual transmit power of node k satisfying the average power constraints. The receiver first removes its self-interference from the received signal and then decodes for its desired message while treating the remaining interference as noise.

We split the transmission into three equal length blocks and switch off user i in time slot i. This reduces interference and allows for higher transmission powers in the other two time slots while still meeting the average power constraint.

Lemma 2: In the 3-user MWRC, the sum rate

$$R_\Sigma^{AF} = \mathrm{C}\left(\frac{3PP_0}{N_0P_0 + 3PN + NN_0}\right) \quad (2)$$

is achievable with AF relaying and treating interference as noise at the receivers.

Proof: Omitted due to space limitations. ■

C. Decode-and-Forward

In DF relaying, the relay completely decodes the messages of each user and then broadcasts them back to all users. The achievable rate region is the intersection of the capacity regions of the 3-user multiple-access channel and the broadcast channel with receiver side information and partial decoding at the receivers.

Lemma 3: In the 3-user MWRC, the sum rate

$$R_\Sigma^{DF} = \min\left\{\frac{3}{2}\,\mathrm{C}\left(\frac{P_0}{N}\right),\ \mathrm{C}\left(\frac{3P}{N_0}\right)\right\} \quad (3)$$

is achievable with DF relaying.

Proof sketch: The achievable rate region is given in [2, Proposition 2]. Using the simplex algorithm and the fact that $\mathrm{C}\left(\frac{2P}{N_0}\right) < \mathrm{C}\left(\frac{P_0}{N}\right)$ implies $\mathrm{C}\left(\frac{3P}{N_0}\right) < \frac{3}{2}\,\mathrm{C}\left(\frac{P_0}{N}\right)$ we can prove (3). ■

Remark 1: The result from [2] implements a full message exchange. However, from the outer bound in [10] it can be seen that in the symmetric case the relaxed decoding requirements due to the partial message exchange considered here can not result in higher rates for DF.

Remark 2: For a completely symmetric scenario, DF is sum rate optimal in the low SNR regime. To see this, let $P = P_0$ and $N = N_0$ and define $S = \frac{P}{N}$. Then the bound in Lemma 1 is $C_\Sigma \leq \frac{3}{2}\,\mathrm{C}(S)$, and $R_\Sigma^{DF} = \min\left\{\frac{3}{2}\,\mathrm{C}(S),\ \mathrm{C}(3S)\right\}$. It is easily shown that for $S \leq 3 + 2\sqrt{3}$ the first term in the minimum is dominant, i.e., $R_\Sigma^{DF} = \frac{3}{2}\,\mathrm{C}(S)$. Since this is equal to the outer bound, we have $C_\Sigma = \frac{3}{2}\,\mathrm{C}(S)$ for SNRs up to $3 + 2\sqrt{3} \approx 8.1\,\mathrm{dB}$.

D. Noisy Network Coding

NNC [11] generalizes compress-and-forward to discrete memoryless networks. For general multi-message networks there are two different decoding methods to choose from: simultaneous non-unique decoding (SND) and treating interference as noise (IAN). Since, in general, none of the methods is superior to the other, we evaluate both bounds. However, it turns out that treating interference as noise (IAN) is strictly worse than simultaneous non-unique decoding (SND) and even than AF.

Lemma 4: In the 3-user MWRC, the sum rate

$$R_{\Sigma}^{NNC-SND} = \frac{3}{2}\,\mathrm{C}\left(\frac{2PP_0}{N_0P_0 + 2PN + NN_0}\right) \qquad (4)$$

is achievable with NNC and simultaneous non-unique decoding.

Proof sketch: Use [11, Theorem 2] and identify $\mathcal{D}_0 = \emptyset$ and $\mathcal{D}_k = \{q(k)\}$ for $k \in \mathcal{K}$. Assume $\hat{Y}_i = Y_i + \hat{Z}_i$ with $\hat{Z}_i \sim \mathcal{CN}(0, Q_i)$ for $i \in \mathcal{K}_0$, and $Q = \emptyset$, i.e., no time-sharing is used. Then, the achievable rate region is

$$R_k < \mathrm{C}\left(\frac{P}{N_0 + Q_0}\right),$$
$$\sum_{i \in \mathcal{K}\setminus\{k\}} R_i < \min\left\{\mathrm{C}\left(\frac{2P}{N_0 + Q_0}\right),\ \mathrm{C}\left(\frac{P_0}{N}\right) - \mathrm{C}\left(\frac{N_0}{Q_0}\right)\right\},$$

for each $k \in \mathcal{K}$.

With the simplex algorithm [12] and after maximization over Q_0 we get (4). ■

Remark 3: It can be shown that $R_{\Sigma}^{NNC-SND} \geq R_{\Sigma}^{AF}$.

Lemma 5: In the 3-user MWRC, the sum rate

$$R_{\Sigma}^{NNC-IAN} = 3\,\mathrm{C}\left(\frac{PP_0}{2PP_0 + N_0P_0 + 3PN + NN_0}\right)$$

is achievable with NNC and treating interference as noise.

Proof: Similar to Lemma 4. ■

Remark 4: It can be shown that $R_{\Sigma}^{NNC-IAN} \leq R_{\Sigma}^{AF}$. Thus, with Remark 3, $R_{\Sigma}^{NNC-IAN} \leq R_{\Sigma}^{NNC-SND}$.

IV. Energy Efficiency

The EE of the system is defined as the ratio between the achievable sum rate and the consumed power. The power consumed in the system is given by the sum of the transmit power of each user and of the relay plus the circuit power that is dissipated in each terminal to operate the devices. Moreover, the transmit power of each terminal should be scaled by a factor larger than 1 to model the nonidealities of the power amplifier [6], [13]. Namely, we can express the total power P_t consumed in the network as $P_t = \phi P + \psi P_0 + P_c$, with P_c denoting the total circuit power consumed in all nodes, $\psi \geq 1$ being the inefficiency of the relay amplifier, and $\phi \geq 3$, accounting for the inefficiency of the amplifier of the three users. Accordingly, the EE can be defined as the ratio between the achievable sum rate and P_t. Then, given the achievable sum rate expressions from Section III, the EE can be expressed in two different functional forms. For the upper-bound and for the DF case we have

$$\mathrm{EE}_1 = \frac{\min\left\{a_1\,\mathrm{C}\left(\alpha_1 \frac{P_0}{N}\right),\ a_2\,\mathrm{C}\left(\alpha_2 \frac{P}{N_0}\right)\right\}}{\phi P + \psi P_0 + P_c},$$

with a_1, a_2, α_1, and α_2 non-negative parameters. For the AF and NNC cases we have

$$\mathrm{EE}_2 = \frac{\alpha\,\mathrm{C}\left(\frac{PP_0}{aP + bP_0 + c}\right)}{\phi P + \psi P_0 + P_c},$$

with α, a, b, and c non-negative parameters. In the following, EE maximization will be carried out by means of fractional programming tools. In particular, we recall the following result from [14], [15]. Consider the generic fractional problem $\max_{\boldsymbol{x} \in \mathcal{S}} \frac{f(\boldsymbol{x})}{g(\boldsymbol{x})}$ where $\mathcal{S} \in \mathbb{R}^n$, $f,g : \mathcal{S} \to \mathbb{R}$, with $f(\boldsymbol{x}) \geq 0$ and $g(\boldsymbol{x}) > 0$. Define the function $F(\lambda) = \max_{\boldsymbol{x} \in \mathcal{S}} (f(\boldsymbol{x}) - \lambda g(\boldsymbol{x}))$. Then, maximizing $f(\boldsymbol{x})/g(\boldsymbol{x})$ is equivalent to finding the unique zero of $F(\lambda)$. This can be accomplished by means of Dinkelbach's algorithm [15], which only requires the solution of a sequence of convex problems, provided $f(\boldsymbol{x})$ and $g(\boldsymbol{x})$ are concave and convex, respectively, and that $\mathcal{S}$ is a convex set. Moreover, it can be shown that the convergence rate of Dinkelbach's algorithm is superlinear [15].

A. Maximization of EE_1

The maximization of EE_1 is a non-concave and non-smooth problem. However, it can be reformulated as a smooth problem introducing the auxiliary variable t as follows.

$$\begin{cases} \max\limits_{P,P_0} \quad \dfrac{t}{\phi P + \psi P_0 + P_c} \\ \text{s.t.} \quad P \in [0; P^{max}],\ P_0 \in [0; P_0^{max}] \\ a_1\,\mathrm{C}\left(\alpha_1 \frac{P_0}{N}\right) - t \geq 0\,, \quad a_2\,\mathrm{C}\left(\alpha_2 \frac{P}{N_0}\right) - t \geq 0 \end{cases} \qquad (5)$$

The numerator and denominator of the objective of (5) are both linear and the constraints are convex. As a consequence, (5) can be solved by means of Dinkelbach's algorithm with an affordable complexity.

B. Maximization of EE_2

In this case, the optimization problem is formulated as

$$\begin{cases} \max\limits_{P,P_0} \dfrac{\alpha\,\mathrm{C}\left(\frac{PP_0}{aP + bP_0 + c}\right)}{\phi P + \psi P_0 + P_c} \\ \text{s.t.} \quad P \in [0; P^{max}]\,, \quad P_0 \in [0; P_0^{max}] \end{cases} \qquad (6)$$

Problem (6) is more challenging than Problem (5) because the numerator of the objective function is not jointly concave in the optimization variables. However, we observe that the numerator of the objective is separately concave in P for fixed P_0 and vice versa. This suggests that a convenient way to tackle Problem (6) is by means of the alternating maximization algorithm [16], according to which we can alternatively optimize with respect to P fixing the value of P_0, and with respect to P_0 for a fixed value of P.

Algorithm 1 Alternating maximization for Problem (6)

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Initialize P_0^(0) ∈ [0,P_0^max]. Set a tolerance ϵ.
Set n = 0;
while |EE_2^(n) − EE_2^(n−1)| ≤ ϵ do
    Given P_0^(0), solve Problem (6) with respect
to P to obtain the optimal P^(n+1);
    Given P^(n+1), solve Problem (6) with
respect to P_0 to obtain the optimal P_0^(n+1);
    n = n + 1;
end while
Output (P,P_0).
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The formal algorithm is reported next and labeled Algorithm 1.

Each subproblem in Algorithm 1 can be globally solved by means of Dinkelbach's algorithm. Moreover, the following proposition holds.

Proposition 1: Algorithm 1 converges to a stationary point of Problem (6).

Proofsketch: After each iteration of Algorithm 1 the objective is not decreased. Hence, convergence follows since the objective is upper-bounded. Convergence to a stationary point holds by virtue of [16, Proposition 2.7.1], which states that alternating maximization converges to a stationary point if: 1) the feasible set is the Cartesian product of closed and convex sets; 2) the objective is continuously differentiable on the feasible set; 3) the solution to each subproblem is unique. In our case, 1) and 2) are apparent. As for 3) it also holds because the objective function of each subproblem can be shown to be strictly pseudo-concave [8]. ■

V. Discussion & Numerical Results

For a discussion and numerical evaluation of the presented transmission schemes, we consider a completely symmetric scenario with $N = N_0$. We assume $P = P_0$ for the spectral efficiency, and for the EE evaluation, we assume $P^{max} = P_0^{max}$, unit noise variance and no power loss at the transmitter, i.e., $\psi = 1$ and $\phi = 3$. The shown performance has been obtained using the algorithms proposed in Section IV.

Fig. 2 shows the achievable sum rates from Section III as a function of the SNR. As noted in Remarks 3 and 4, it can be observed that NNC with SND achieves a higher sum rate than AF and AF achieves a higher sum rate than NNC with IAN. In the low SNR regime, the sum capacity is achieved by DF (see Remark 2). Starting at approximately 8 dB, DF stops being sum rate optimal but is still better than all other considered transmission schemes. Starting from approximately 14.27 dB NNC SND is the best in terms of spectral efficiency. Its gap to the outer bound is at most $1.5 \log_2(1.5)$ bit ≈ 0.877 bit. In contrast, for all other considered schemes this gap grows unbounded as $\text{SNR} \to \infty$. Furthermore, the gap between DF and AF approaches 2 bit as $\text{SNR} \to \infty$. NNC IAN is clearly worse than every other employed scheme and should not be considered in terms of spectral efficiency.

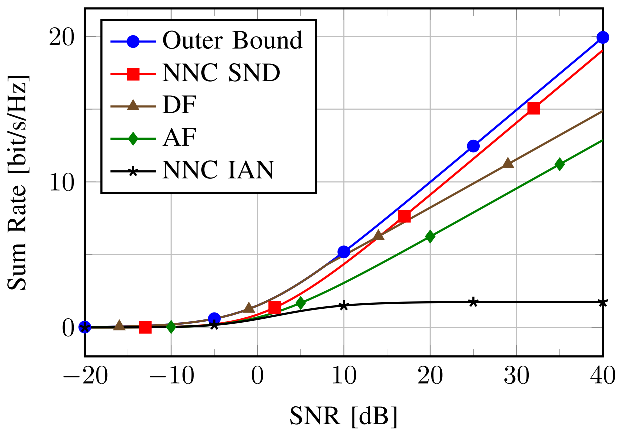

Fig. 2. Spectral efficiency in the 3-user MWRC; 1) Outer bound from Lemma 1, 2) noisy network coding (NNC) with simultaneous non-unique decoding (SND) and 3) treating interference as noise (IAN), 4) amplify-and-forward (AF) and 5) decode-and-forward (DF) plotted as a function of the SNR.

Fig. 3 shows the EE as a function of the SNR for a fixed circuit power $P_c = 1$ W. First of all, it can be seen that the EE saturates when P_{max} exceeds a given value, which is lower than 0 dB for all considered schemes. This is explained recalling that, unlike the achievable rate, the EE is not increasing with the transmit powers, but instead admits an optimum transmit power level. If P_{max} is larger than such power level, then it is not optimal to transmit at full power. This also explains why DF performs significantly better than all other schemes, including NNC SND. Indeed, due to the saturation of the EE, the SNR range for which NNC SND yields a larger achievable sum rate than DF is not reached when EE is optimized. Finally, as expected, NNC SND is better than AF, which is better than NNC IAN.

However, as opposed to DF, AF does not require power-hungry analog to digital conversion and digital signal processing at the relay which results in significantly less power consumption. Furthermore, the decoders at the users are also expected to consume less power due to the use of a (considerably simpler) single user receiver. Thus, the higher achievable rates and the resulting better EE of DF over AF are obtained at the cost of a more complex hardware and, hence, of a larger consumed circuit power. This suggests that the comparison in Fig. 3 might be unfair and that the large gap between DF and AF might in fact be smaller when the comparison is done on equal grounds. Some insight on this issue is given in Fig. 4, which shows the EE of DF as a function of its circuit power P_c^{DF} and the EE of AF for a fixed circuit power $P_c^{AF} = 1$ W. It can be seen that, as expected, the gap to AF gets smaller with increasing P_c^{DF} and that AF might even outperform DF given a significantly large P_c^{DF}.

VI. Conclusion

In this paper, we studied both the spectral and the energy efficiency of the 3-user MWRC with a partial message exchange. We provided analytic sum rate expressions for the most common relaying schemes and discussed the solution of

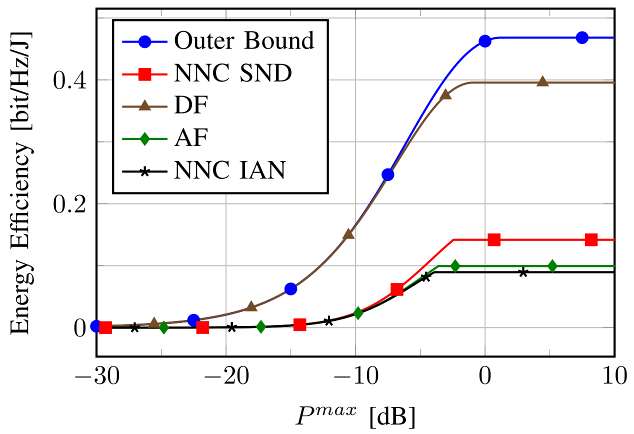

Fig. 3. Energy efficiency in the 3-user MWRC of 1) noisy network coding (NNC) with simultaneous non-unique decoding (SND) and 2) treating interference as noise (IAN), 3) amplify-and-forward (AF), 4) decode-and-forward (DF), and 5) the outer bound from Lemma 1 as a function of the SNR for fixed circuit power $P_c = 1\,\text{W}$.

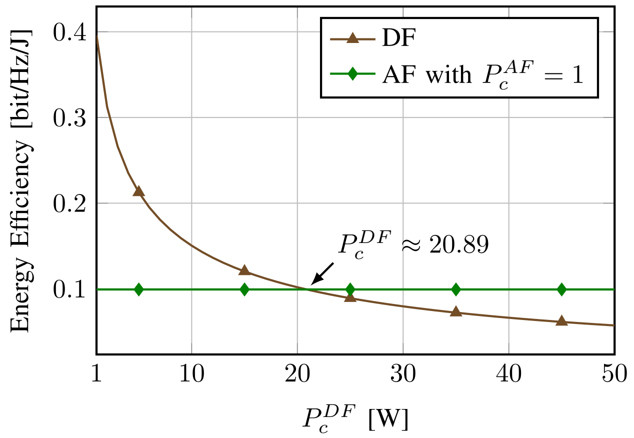

Fig. 4. Energy efficiency in the 3-user MWRC of decode-and-forward (DF) as a function of the circuit power P_c^{DF} compared to amplify-and-forward (AF) with a fixed circuit power $P_c^{AF} = 1\,\text{W}$ for an operating point of $P^{max} = 10\,\text{W}$. The intersection is at $P_c^{DF} \approx 20.89\,W$.

the optimization problems arising in the calculation of the EE.

We have seen that if we assume the same power consumption for all schemes, DF performs best in terms of EE. Moreover, the energy-efficient performance of NNC is not satisfactory due to the fact that NNC achieves a higher spectral efficiency only in the high SNR regime, which is not the operating regime when EE is optimized. Furthermore, we have shown that AF might have better EE than the more complex DF if different circuit powers are assumed. This assumption is reasonable since different hardware complexities imply different circuit powers. Thus, to compare the EE of the presented relaying schemes in a fairer way, circuit power consumption models are necessary. This issue will be addressed further in future work.

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