Beamspace MIMO for Satellite Swarms

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***Abstract*—Systems of small distributed satellites in low Earth orbit (LEO) transmitting cooperatively to a multiple antenna ground station (GS) are investigated. These satellite swarms have the benefit of much higher spatial separation in the transmit antennas than traditional big satellites with antenna arrays, promising a massive increase in spectral efficiency. However, this would require instantaneous perfect channel state information (CSI) and strong cooperation between satellites. In practice, orbital velocities around 7.5 km/s lead to very short channel coherence times on the order of fractions of the inter-satellite propagation delay, invalidating these assumptions. In this paper, we propose a distributed linear precoding scheme and a GS equalizer relying on local position information. In particular, each satellite only requires information about its own position and that of the GS, while the GS has complete positional information. Due to the deterministic nature of satellite movement this information is easily obtained and no inter-satellite information exchange is required during transmission. Based on the underlying geometrical channel approximation, the optimal inter-satellite distance is obtained analytically. Numerical evaluations show that the proposed scheme is, on average, within 99.8 % of the maximum achievable rate for instantaneous CSI and perfect cooperation.**

***Index Terms*—low Earth orbit (LEO), small-satellite swarms, MIMO satellite communications, distributed precoding, angle division multiple access**

I. Introduction

Integrating non-terrestrial networks (NTNs) into terrestrial communication systems is an important step towards truly ubiquitous connectivity [1], [2]. An essential building block are small satellites in low Earth orbit (LEO) that are currently deployed in private sector mega constellations [3]–[5]. Their main benefits are much lower propagation delays and deployment costs due to the LEO when compared to more traditional high-throughput satellites [6]–[8] in medium Earth orbit (MEO) and geostationary orbit (GEO). While current systems focus on connecting ground stations (GSs) to a single satellite, combining several low cost satellites in swarms leads to increased flexibility and scalability [9].

Especially the joint transmission of multiple satellites forming large virtual antenna arrays promises tremendous spectral efficiency gains solely due to the increased spatial separation of antennas [10]–[12]. However, the straightforward implementation of conventional multiple-input-multiple-output (MIMO) transmission schemes requires complete instantaneous channel state information (CSI) and inter-satellite coordination of joint beamforming. This is infeasible due to very short channel coherence times resulting from high orbital velocities in combination with comparably large propagation delays, both in ground-to-satellite and in inter-satellite links. In this paper, we show that this is not an obstacle if positional information is exploited. In contrast to complete CSI, this information is often readily available or easily determined from the deterministic movement of satellites. This leads to an approximate channel model, which is employed to derive a beamspace MIMO [13], [14] based distributed linear precoder and equalizer. The precoder has low complexity, requires, at each satellite, only knowledge of the own rotation and the position of itself and the GS, and achieves close to optimal spectral efficiency. Similarly, the equalizer only needs angle of arrival (AoA) information for the satellites and, given proper design of the satellite swarm, shows nearly optimal performance. We obtain an analytical solution on the optimal swarm layout and numerically evaluate the system performance.

The related literature can be summarized as follows: In [10], the downlink (DL) from a satellite swarm with more than 50 nano-satellites towards a single antenna ground station (GS) is studied. It is shown that, if the signals of all satellites add up in phase at the GS a high array gain is achieved. Communication between multiple satellites and a GS with multiple antennas is studied in [15], where an iterative interference cancellation algorithm is considered to deal with the large spatial correlation between two close GEO satellites. Furthermore, in [16] and [17], the capacity of multi-satellite systems are studied. In [18], a distributed precoding algorithm, based on the minimum mean-squared error (MMSE) criterion and exploiting information exchange between the satellites, is proposed for a multi-user DL scenario. In [11] a zero-forcing (ZF) equalizer at the ground terminal is proposed while receiving from two satellites. In [19]–[21], beamspace MIMO is adapted to ground to satellite communications, focusing on scenarios involving a single satellite.

II. System Model and Performance Bounds

Consider a swarm of N_{S} satellites flying in a trail formation with constant inter-satellite distance D_{S}. They have a common circular orbit at an altitude of d_0 that is assumed to be ideal Keplerian and aligned within the xy-plane. Then, the polar coordinates of satellite ℓ in the Earth-centered coordinate system are denoted by $\mathbf{r}_\ell = [r_0, \vartheta_\ell]^T$, where r_0 is the orbital radius, i.e., the distance from the center of the Earth to the satellite, and ϑ_ℓ is the polar angle. Given the Earth's radius $r_{\mathrm{E}} = 6371\,\mathrm{km}$, the orbital radius is $r_0 = r_{\mathrm{E}} + d_0$. The GS

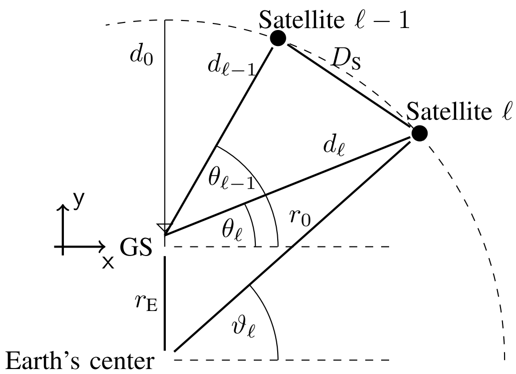

Fig. 1. Geometric relation between Satellites and GS

is located within the orbital plane at position $\mathbf{r}_{\mathrm{Rx}} = [r_{\mathrm{E}}, \pi/2]$. This setup is illustrated in Fig. 1.

It is convenient to describe the satellites position in a GS centered coordinate system. The relative polar coordinates of satellite ℓ to the GS are denoted as $\mathbf{d}_\ell = [d_\ell, \theta_\ell]$, where $\theta_\ell \in [0, \pi]$ is the elevation angle, i.e., the polar angle in the GS centered coordinate frame. This angle is equivalent to the AoA of the signal from satellite ℓ at the GS. Considering the triangle between satellite ℓ, the GS and the Earth's center, we obtain from the law of sines $\vartheta_\ell = \theta_\ell + \arcsin(r_{\mathrm{E}} \cos(\theta_\ell)/r_0)$. Correspondingly, the distance d_ℓ between satellite ℓ and the GS is $d_\ell = \sqrt{d_0^2 + 2 r_{\mathrm{E}} r_0 \left(1 - \sin\left(\vartheta_\ell\right)\right)}$. The angle of departure (AoD) Θ_ℓ from satellite ℓ is readily obtained from the elevation angle θ_ℓ and the satellite's rotation η_ℓ as $\Theta_\ell = \theta_\ell - \eta_\ell - \frac{\pi}{2}$, where η_ℓ is defined such that the antenna arrays at the satellite and the GS are parallel to each other at $\eta_\ell = 0$. Hence, if the satellite points perfectly towards the GS, the AoD is $\Theta_\ell = 0$. Assume that the satellite can transmit only in the directions $\Theta_\ell \in [-\pi/2, \pi/2]$. Then, its rotation must be within the interval $\eta_\ell \in [\min(-\pi/2, \theta_\ell - \pi), \min(\pi/2, \theta_\ell)]$.

Observe that r_0 and r_{E} are constant over time, while d_ℓ, ϑ_ℓ, θ_ℓ, η_ℓ, and Θ_ℓ are time variant. All of these values are either known a priori at the satellites and the GS or can be obtained easily because the satellites are moving on predefined orbits.

A. Communication Model

Satellite ℓ is equipped with an uniform linear array (ULA) consisting of N_{t} antennas spaced $D_{\mathrm{A}} = \frac{c_0}{2 f_{\mathrm{c}}}$ apart, where f_{c} is the carrier frequency and c_0 is the speed of light. The satellites jointly transmit M independent messages that are known a priori at all satellites and encoded in $\mathbf{s} \in \mathbb{C}^M$ using uncorrelated unit variance Gaussian codebooks. Satellite ℓ employs linear precoding to transmit the signal $\mathbf{x}_\ell = \mathbf{G}_\ell \mathbf{s}$ with $\mathbf{G}_\ell \in \mathbb{C}^{N_{\mathrm{t}} \times M}$. The transmission is subject to an average power constraint ρ_ℓ, i.e.,

$$\operatorname{tr}\left\{\mathbf{G}_\ell \mathbf{G}_\ell^H\right\} \leq \rho_\ell \,. \tag{1}$$

The GS is equipped with an ULA consisting of $N_{\mathrm{r}} \geq N_{\mathrm{S}}$ antenna elements. Its received signal is $\mathbf{y} = \sum_{\ell=1}^{N_{\mathrm{S}}} \mathbf{H}_\ell \mathbf{x}_\ell + \mathbf{n}$, where $\mathbf{n}$ is independent and identically distributed (i.i.d.) complex circularly symmetric white Gaussian noise with power σ_{n}^2 and $\mathbf{H}_\ell \in \mathbb{C}^{N_{\mathrm{r}} \times N_{\mathrm{t}}}$ is the channel from satellite ℓ to the GS. Due to the collaborative transmission, this is effectively a point-to-point channel. In particular, let $\mathbf{H} = [\mathbf{H}_1, ..., \mathbf{H}_{N_{\mathrm{S}}}]$ and $\mathbf{x} = \left[\mathbf{x}_1^T, ..., \mathbf{x}_{N_{\mathrm{S}}}^T\right]^T$ to obtain the equivalent channel

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}.$$

The capacity of this channel is

$$R_{\mathrm{opt}} = \max_{\operatorname{tr}\{\mathbf{G}\mathbf{G}^H\} \leq \sum_{\ell=1}^{N_{\mathrm{S}}} \rho_\ell} \log_2 \left| \mathbf{I}_{N_{\mathrm{r}}} + \sigma_{\mathsf{n}}^{-2} \mathbf{H}\mathbf{G}\mathbf{G}^H \mathbf{H}^H \right|, \tag{2}$$

where $\mathbf{G} = [\mathbf{G}_1^T, ..., \mathbf{G}_{N_{\mathrm{S}}}^T]^T \in \mathbb{C}^{N_{\mathrm{Tx}} \times M}$ with $N_{\mathrm{Tx}} = N_{\mathrm{S}} N_{\mathrm{t}}$ [22]. The maximum in (2) is achieved for

$$\mathbf{G}_{\mathrm{opt}} = \mathbf{V}\mathbf{P}^{\frac{1}{2}}, \tag{3}$$

where $\mathbf{U}\mathbf{\Sigma}\mathbf{V}^H$ is the singular value decomposition (SVD) of $\mathbf{H}$, and $\mathbf{P} = \operatorname{diag}\left(p_1, ..., p_{N_{\mathrm{Tx}}}\right)$ with p_μ the transmit power of the μth beam. Combining (2) and (3), we obtain

$$R_{\mathrm{opt}} = \sum_{\mu=1}^{M} \log_2 \left(1 + \lambda_\mu \frac{p_\mu}{\sigma_{\mathsf{n}}^2}\right), \tag{4}$$

where λ_μ the μth Eigenvalue of $\mathbf{H}\mathbf{H}^H$. The optimal power allocation $\mathbf{P}$ is obtained from the waterfilling algorithm [22].

III. Geometry Based DL Transmission

Observe that several implicit assumptions are made in the derivation of (4). In particular, perfect instantaneous CSI is required at all involved communication nodes and the beamforming matrices are computed centralized. Moreover, the per-satellite power constraints (1) are not necessarily met as a relaxed sum power constraint over all satellites is considered in (2). Obtaining the necessary CSI requires accurate channel estimation at the satellites, which then has to be shared with all other satellites in the swarm via inter-satellite links. Consequently, especially in the LEO, where the coherence time of the channel is very short, SVD based optimal precoding is not feasible.

However, the channels from the antenna elements of a single satellite to the GS are highly correlated [12] and thus, although the matrix $\mathbf{V} \in \mathbb{C}^{N_{\mathrm{Tx}} \times N_{\mathrm{Tx}}}$ has N_{Tx} columns, there are only $M \leq N_{\mathrm{S}} \leq N_{\mathrm{Tx}}$ singular values significantly larger than zero. Accordingly, only the right singular vectors corresponding to the M largest singular values are of interest for designing the precoding matrix $\mathbf{G}$. Furthermore, the communication between satellites and GSs is usually done under line-of-sight (LOS). Thus, the channel matrix $\mathbf{H}$ is fully determined by the distances $d_{m,n}^\ell$ between transmit and receive antennas as well as atmospheric effects [23], [24].

In this section, we exploit readily available position information to estimate the dominant large-scale components of $\mathbf{H}$. Based on this geometrical channel model, we design a distributed linear precoder that does not require any inter-satellite coordination and a linear equalization scheme at the GS that does not rely on traditional CSI acquisition.

A. Geometrical Channel Approximation

Due to the large distance between satellite ℓ and the GS in relation to the antenna spacing, the AoA and AoD between antennas in the transmit and receive arrays are approximately equal. Thus, the distance $d_{m,n}^{\ell}$ from the ℓth satellite's nth antenna to the mth GS antenna is approximately

$$d_{m,n}^{\ell} \approx d_{\ell} - D_{\mathrm{A}}(m-1)\cos(\theta_{\ell}) - D_{\mathrm{A}}(n-1)\sin(\Theta_{\ell}) \quad (5)$$

where d_{ℓ} is the distance from the first transmit antenna at satellite ℓ to the first receive antenna. Moreover, the mn channels from satellite ℓ to the ground station are subject to the same atmospheric effects [24]. Thus, it is reasonable to assume that the entries in $\mathbf{H}_{\ell}$ have equal magnitude and differ only in their phase. In particular, let $\nu = 2\pi f_{\mathrm{c}}/c_0$ be the wavenumber of the radiated carrier signal. Then, the phase difference between channels from adjacent transmit antennas to the same receive antenna is $\nu D_{\mathrm{A}} \sin(\Theta_{\ell}) = \pi \sin(\Theta_{\ell})$. Likewise, the phase difference between channels from a single transmit antenna to adjacent receive antennas is $\pi\cos(\theta_{\ell})$ [13], [19]. Thus, the (m,n)th entry of $\mathbf{H}_{\ell}$ is approximately $\alpha_{\ell} e^{j\pi((m-1)\cos(\theta_{\ell})+(n-1)\sin(\Theta_{\ell}))}$, where α_{ℓ} is the i. i. d. complex channel gain from satellite ℓ to the GS with $\mathrm{E}\{\alpha_{\ell}\} = 0$ and $\mathrm{Var}\{\alpha_{\ell}\} = \sigma_{\alpha}^2$. As the satellites are following the same trajectory, the statistics of $\{\alpha_{\ell}\}_{\ell=1}^{N_{\mathrm{S}}}$ are assumed to be the same for all satellites.

Define the steering vectors

$$\mathbf{a}_{\ell}^T = \left[e^{j\pi m\cos(\theta_{\ell})}\right]_{m=0}^{N_{\mathrm{r}}-1}, \quad \mathbf{b}_{\ell}^T = \left[e^{-j\pi n\sin(\Theta_{\ell})}\right]_{n=0}^{N_{\mathrm{t}}-1}. \quad (6)$$

Then, the approximated channel matrix is

$$\tilde{\mathbf{H}}_{\ell} = \alpha_{\ell}\mathbf{a}_{\ell}\mathbf{b}_{\ell}^H \approx \mathbf{H}_{\ell} \quad (7)$$

and has rank one. Due to $N_{\mathrm{r}} \geq N_{\mathrm{S}}$ and the satellites having distinct positions in the orbital plane, i.e., $\theta_i \neq \theta_{\ell}$, for all $i \neq \ell$, the overall channel matrix $\tilde{\mathbf{H}} = [\alpha_1\mathbf{a}_1\mathbf{b}_1^H, ..., \alpha_{N_{\mathrm{S}}}\mathbf{a}_{N_{\mathrm{S}}}\mathbf{b}_{N_{\mathrm{S}}}^H]$ has rank N_{S}. This allows for the parallel transmission of $M = N_{\mathrm{S}}$ independent streams.

In the following, the precoder and equalizer are designed based on $\tilde{\mathbf{H}}$. This only requires knowledge of the differential phases between the antennas that is straightforward to obtain from local position information, as shown above and in Section II.

B. Precoding

Based on the observation, that we can transmit $M = N_{\mathrm{S}}$ independent data streams in parallel, we propose the following geometry based precoder

$$\mathbf{G}_{\mathrm{geo}} = \sqrt{\frac{1}{N_{\mathrm{t}}}}\,\mathrm{blkdiag}\left(\sqrt{\rho_1}\mathbf{b}_1, \ldots, \sqrt{\rho_{N_{\mathrm{S}}}}\mathbf{b}_{N_{\mathrm{S}}}\right). \quad (8)$$

Thus, satellite ℓ transmits into the direction of the eigenvector of $\mathrm{E}\left\{\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}\right\}$. Let $\mathbf{g}_{\ell,\mathrm{geo}} = \sqrt{\rho_{\ell}/N_{\mathrm{t}}}\mathbf{b}_{\ell}$, then, due to the block diagonal precoding matrix, satellite ℓ transmits $\mathbf{x}_{\ell} = \mathbf{g}_{\ell,\mathrm{geo}} s_{\ell}$, i.e., it needs not know $\mathbf{s}$ but only their part s_{ℓ} of the stream. Note that according to (8), the per satellite average power constraint (1) is always satisfied. Furthermore, satellite ℓ only has to know its AoD Θ_{ℓ} and no cooperation between the satellites is needed to determine the precoding matrix $\mathbf{G}_{\mathrm{geo}}$. In addition, the proposed precoding is based on manipulating only the phase at each antenna and thus, an efficient implementation with a single RF chain per satellite is possible [13].

C. Linear Equalization

In a satellite swarm, all satellites are usually of the same type [9] and thus, it is assumed that all satellites transmit with the same power, in the following, i.e., $\rho_{\ell} = \rho$, for all ℓ. Assuming the previously proposed precoder (8) and employing a linear equalizer $\mathbf{W} = [\mathbf{w}_1, ..., \mathbf{w}_{N_{\mathrm{S}}}]^H \in \mathbb{C}^{N_{\mathrm{S}}\times N_{\mathrm{r}}}$ at the GS, the estimated signal is $\tilde{\mathbf{s}} = \mathbf{W}\mathbf{H}\mathbf{G}_{\mathrm{geo}}\mathbf{s} + \mathbf{W}\mathbf{n}$. Consequently, the signal transmitted by satellite i interferes with the signal transmitted by satellite ℓ. Then, the signal-to-interference-and-noise ratio (SINR) of the ℓth stream is

$$\Gamma_{\ell} = \frac{\left|\mathbf{w}_{\ell}^H\mathbf{H}_{\ell}\mathbf{g}_{\ell,\mathrm{geo}}\right|^2}{\sum_{i\neq\ell}\left|\mathbf{w}_{\ell}^H\mathbf{H}_i\mathbf{g}_{i,\mathrm{geo}}\right|^2 + \sigma_{\mathsf{n}}^2\mathbf{w}_{\ell}^H\mathbf{w}_{\ell}} \quad (9)$$

$$= \frac{\mathbf{w}_{\ell}^H\mathbf{H}_{\ell}\mathbf{g}_{\ell,\mathrm{geo}}\mathbf{g}_{\ell,\mathrm{geo}}^H\mathbf{H}_{\ell}^H\mathbf{w}_{\ell}}{\mathbf{w}_{\ell}^H\left(\sum_{i\neq\ell}\mathbf{H}_i\mathbf{g}_{i,\mathrm{geo}}\mathbf{g}_{i,\mathrm{geo}}^H\mathbf{H}_i^H + \sigma_{\mathsf{n}}^2\mathbf{I}_{N_{\mathrm{r}}}\right)\mathbf{w}_{\ell}^H} \quad (10)$$

and the achievable rate R_{lin} is given by the sum of the individual rates

$$R_{\mathrm{lin}} = \sum_{\ell=1}^{N_{\mathrm{S}}}\log_2\left(1+\Gamma_{\ell}\right) \quad (11)$$

Observe that Γ_{ℓ} is independent of $\mathbf{w}_i$ for all $i \neq \ell$. Thus, (11) is maximized by optimizing each Γ_{ℓ} separately. Since Γ_{ℓ} is a generalized Rayleigh quotient [25], it's maximizer is [26]

$$\mathbf{w}_{\ell,\mathrm{opt}}^H = \mathbf{g}_{\ell,\mathrm{geo}}^H\mathbf{H}_{\ell}^H\left(\sum_{i=1}^{N_{\mathrm{S}}}\mathbf{H}_i\mathbf{g}_{i,\mathrm{geo}}\mathbf{g}_{i,\mathrm{geo}}^H\mathbf{H}_i^H + \sigma_{\mathsf{n}}^2\mathbf{I}_{N_{\mathrm{r}}}\right)^{-1}. \quad (12)$$

However, acquiring perfect instantaneous CSI $\mathbf{H}$ is costly. Instead, we obtain the equalizer based on the approximated channel in Section III-A. Since $\tilde{\mathbf{H}}_{\ell}\mathbf{g}_{\ell,\mathrm{geo}} = \alpha_{\ell}\sqrt{N_{\mathrm{t}}\rho}\mathbf{a}_{\ell}$, the proposed equalizer is

$$\mathbf{w}_{\ell,\mathrm{geo}}^H = \mathbf{a}_{\ell}^H\left(\sum_{i=1}^{N_{\mathrm{S}}}\mathbf{a}_i\mathbf{a}_i^H + \bar{\sigma}_{\mathsf{n}}^2\mathbf{I}_{N_{\mathrm{r}}}\right)^{-1} \quad (13a)$$

$$= \mathbf{a}_{\ell}^H\left(\mathbf{A}\mathbf{A}^H + \bar{\sigma}_{\mathsf{n}}^2\mathbf{I}_{N_{\mathrm{r}}}\right)^{-1} \quad (13b)$$

where $\bar{\sigma}_{\mathsf{n}}^2 = \sigma_{\mathsf{n}}^2/(\sigma_{\alpha}^2 N_{\mathrm{t}}\rho)$ and $\mathbf{A} = [\mathbf{a}_1, ..., \mathbf{a}_{N_{\mathrm{S}}}]$. Note that the proposed equalizer only requires the knowledge of the AoAs $\{\theta_{\ell}\}_{\ell=1}^{N_{\mathrm{S}}}$ from all satellites as well the signal-to-noise ratio (SNR) $1/\bar{\sigma}_{\mathsf{n}}^2$ at the GS.

IV. Optimal Inter-Satellite Distance

Based on the channel approximation (7), the interconnection between the inter-satellite distance D_{S} and the achievable rate is now analyzed. Assuming perfect CSI at the GS and the fixed precoder $\mathbf{G}_{\text{geo}}$, the ergodic rate $\tilde{R}$ for $\tilde{\mathbf{H}}$ is upper bounded by

$$\tilde{R}_{\text{opt}} \leq \log_2 \left| \mathbf{I}_{N_\text{r}} + \frac{1}{\sigma_\text{n}^2} \mathrm{E}\left\{ \tilde{\mathbf{H}} \mathbf{G}_{\text{geo}} \mathbf{G}_{\text{geo}}^H \tilde{\mathbf{H}}^H \right\} \right| \tag{14a}$$

$$= \log_2 \left| \mathbf{I}_{N_\text{r}} + \frac{1}{\bar{\sigma}_\text{n}^2} \mathbf{A}\mathbf{A}^H \right| \tag{14b}$$

Thus, the achievable rate for $\tilde{\mathbf{H}}$ is determined by the matrix $\mathbf{A}$, which is composed of the steering vectors $\{\mathbf{a}_\ell\}_{\ell=1}^{N_\text{S}}$.

Due to the trail formation, the swarm is fully described by two parameters: The inter-satellite distance D_S and the number of satellites N_S. Choosing a proper inter-satellite distance D_S is crucial, as it directly impacts the angular spread of the AoAs between the satellites, which can be used to tune the matrix $\mathbf{A}$ such that the achievable rate is maximized, as stated in the following proposition.

Proposition 1. *The optimal inter-satellite distance w.r.t. the upper bound of the rate* (14) *is achieved, if the following relation for the AoA between every two satellites ℓ and i holds*

$$\forall \ell \neq i : |\cos(\theta_\ell) - \cos(\theta_i)| = \frac{2k}{N_r} \tag{15}$$

where k can be any positive integer number which is not a multiple of N_r, i.e., k must fulfill $\mod(k, N_r) \neq 0$.

Proof. Observe that (14) is equivalent to

$$\tilde{R} \leq \log_2 \left| \mathbf{I}_{N_\text{S}} + \frac{1}{\bar{\sigma}_\text{n}^2} \mathbf{A}^H \mathbf{A} \right| = \log_2 \left(\prod_{\ell=1}^{N_\text{S}} \left(1 + \frac{\tilde{\lambda}_\ell}{\bar{\sigma}_\text{n}^2} \right) \right) \tag{16}$$

where $\tilde{\lambda}_\ell$ are the positive eigenvalues of $\mathbf{A}^H\mathbf{A}$. Keeping the trace of $\mathbf{A}^H\mathbf{A}$ constant, this is maximized if all eigenvalues have the same value [27, Thm. 2.21]. In other words, any $N_S \times N_S$ matrix $\mathbf{B} = \mathbf{A}^H\mathbf{A}$ maximizing (16) has a single eigenvalue λ with multiplicity N_S.

Further, observe that $\mathbf{B}$ is a normal matrix. By [25, Thm. 2.5.4], $\mathbf{B}$ is similar matrix to a diagonal matrix, i.e., there exists a nonsingular matrix $\mathbf{S}$ such that $\mathbf{S}^{-1}\mathbf{\Lambda}\mathbf{S} = \mathbf{B}$ with $\mathbf{\Lambda}$ diagonal. Since similar matrices have the same eigenvalues [25, Cor. 1.3.4], $\mathbf{\Lambda}$ must be $\lambda\mathbf{I}$. Then, for every nonsingular $\mathbf{S}$, we have $\mathbf{B} = \mathbf{S}^{-1}\lambda\mathbf{I}\mathbf{S} = \lambda\mathbf{S}^{-1}\mathbf{S} = \lambda\mathbf{I}$. It follows that $\mathbf{B} = \lambda\mathbf{I}$ is the unique maximizer of (16).

If the AoAs θ_i and θ_ℓ of satellites ℓ and i satisfy (15), the steering vectors $\mathbf{a}_i$ and $\mathbf{a}_\ell$ can be represented as different columns of the $N_\text{r} \times N_\text{r}$ discrete Fourier transform (DFT) matrix and are thus orthogonal as the DFT is an orthogonal matrix, i.e.,

$$\mathbf{a}_i^H \mathbf{a}_\ell = \sum_{m=0}^{N_\text{r}-1} e^{j\pi m (\cos(\theta_\ell) - \cos(\theta_i))} \tag{17a}$$

$$= \sum_{m=0}^{N_\text{r}-1} e^{j 2\pi \frac{km}{N_\text{r}}} = 0\,. \tag{17b}$$

Consequently, the matrix $\mathbf{A}^H\mathbf{A}$ becomes a scaled identity matrix

$$\mathbf{A}^H\mathbf{A} = N_\text{r}\mathbf{I}_{N_\text{S}} \tag{18}$$

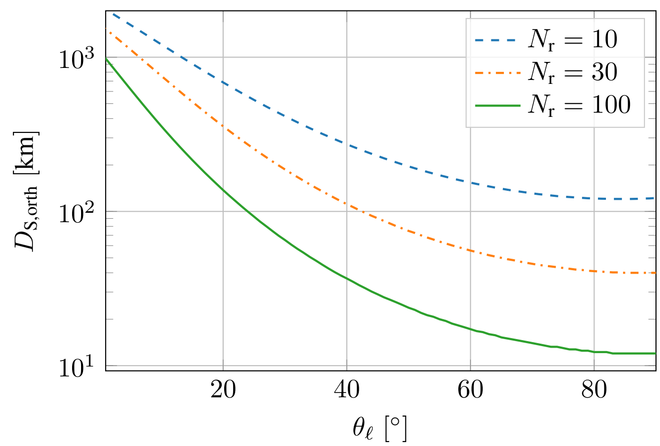

Fig. 2. Required inter-satellite distances $D_{\text{S,orth}}$ for different elevation angles θ_ℓ and number of receive antennas N_r at altitude $d_0 = 600\,\text{km}$ to achieve orthogonal steering vectors

which maximizes the achievable rate (14), as all eigenvalues are identical. □

Consider now two neighbouring satellites ℓ and $\ell - 1$. The difference of the cosine terms $\Delta\phi$ is given by

$$\Delta\phi = \cos(\theta_\ell) - \cos(\theta_{\ell-1}) \tag{19a}$$

$$= \cos(\theta_\ell) - \frac{r_0 \cos(\vartheta_\ell + \Delta\vartheta)}{\sqrt{d_0^2 + 2r_\text{E}r_0\left(1 - \sin(\vartheta_\ell + \Delta\vartheta)\right)}}\,. \tag{19b}$$

where $\Delta\vartheta = \vartheta_{\ell-1} - \vartheta_\ell = \arccos\left(1 - D_\text{S}^2/(2r_0^2)\right)$ is the angular distance between both satellites $\ell - 1$ and ℓ, measured from the Earth's center, which is constant over time and identical for all neighbouring satellites. In Fig. 2, the dependency between the required inter-satellite distance $D_{\text{S,orth}}$ and the AoA θ_ℓ in degree to fulfill (15) is shown for an altitude $d_0 = 600\,\text{km}$, $k = 1$ and different numbers of receive antennas N_r.

Obviously, it is not possible to ensure orthogonal channels between all satellites during the whole flight with a constant inter-satellite distance D_S, as the AoA changes over time. Adjusting the inter-satellite distance during the flight requires additional fuel and increased complexity for flight control and should thus be avoided.

However, as evaluated numerically in the next section, the capacity is not decreasing dramatically for $\Delta\Phi > 2/N_\text{r}$. Therefore, as a close to optimal heuristic, the condition (15) can be relaxed, such that the average capacity over the whole flight is maximized if

$$\min_\ell \Delta\phi = \min_\ell \cos(\theta_\ell) - \cos(\theta_{\ell-1}) \geq \frac{2}{N_\text{r}} \tag{20}$$

holds for each time instance.

V. Numerical Evaluations

A. Channel Model

In this paper, a pure LOS based channel model between the GS and each satellite is assumed and the carrier frequency is $f_\text{c} = 20\,\text{GHz}$.

The (m,n)th element of the channel matrix $\mathbf{H}_\ell$ is modeled by

$$[\mathbf{H}_\ell]_{m,n} = h^\ell_{m,n} = \frac{1}{\sqrt{L^\ell_{m,n}}} e^{-j\left(\nu d^\ell_{m,n} + \phi_{\text{atm},\ell}\right)} \tag{21}$$

where $\phi_{\text{atm},\ell} \in [0, 2\pi]$ is a uniformly distributed phase shift caused by the atmosphere, and L is the path loss, which is given in decibel as [23]

$$\begin{aligned} L^\ell_{m,n|\text{dB}} =& 20\log_{10}(2\nu d_{m,n}) - (\zeta_{\text{Tx,dB}} + \zeta_{\text{Rx,dB}}) \\ &+ L_{\text{sf},\ell} + L_{\text{cl},\ell} + L_{\text{gas},\ell} + L_{\text{ts},\ell} \end{aligned} \tag{22}$$

where $L_{\text{sf},\ell} \sim \mathcal{N}(0, \sigma^2_{\text{sf},\ell})$ and $L_{\text{cl},\ell}$ are the shadow fading and clutter loss, repsectively. In LOS $L_{\text{cl},\ell} = 0\,\text{dB}$ and $\sigma^2_{\text{sf},\ell}$ depends on the AoA. The specific values can be found in [23], while in this paper the rural scenario has been considered. $L_{\text{gas},\ell}$ includes atmospheric gas absorption described in [28] using the reference standard atmosphere [29]. Eventually, $L_{\text{ts},\ell}$ includes the losses due tropospheric scintillation summarized in [30], [31] and $\zeta_{\text{Tx,dB}}$ and $\zeta_{\text{Rx,dB}}$ are the transmit and receive antenna gains, respectively.

The proposed precoding approach for satellite swarms is evaluated numerically in terms of the achievable rate.

The sum transmit power P_{Tx} of the satellite swarm and the total number of transmit antennas $N_{\text{Tx}} = 60$ is independent of the number of satellites N_{S} inside the swarm, i.e., the power and transmit antennas of each satellite is $\rho = P_{\text{Tx}}/N_{\text{S}}$ and $N_{\text{t}} = 60/N_{\text{S}}$, respectively. The GS is assumed to be equipped with $N_{\text{r}} = 100$ receive antennas and the minimum elevation angle is $\theta_{\text{min}} = 30°$. For better comparison, the transmission is assumed to start if the mean AoA of all satellites $\theta_{\text{mean}} = 1/N_{\text{S}} \sum_{\ell=1}^{N_{\text{S}}} \theta_\ell$ equals the minimum elevation angle θ_{min}, i,e, the evaluation is done for $30° \leq \theta_{\text{mean}} \leq 150°$. Furthermore, the noise power is assumed as $P_{\text{N,dB}} = -120\,\text{dBW}$ and the the transmit and receive antenna gains are $G_{\text{Tx,dB}} = 17.8\,\text{dBi}$ and $G_{\text{Rx,dB}} = 20\,\text{dBi}$, respectively. The altitude of the orbital plane is $d_0 = 600\,\text{km}$ during all simulations.

B. Inter-satellite distance

In section IV, the optimal inter-satellite distance $D_{\text{S,orth}}$ based on the channel approximation (7) has been derived. In Fig. 3, the achievable rate (2) is shown in dependence of the inter-satellite distance D_{S} for a total transmit power $P_{\text{Tx}} = 10\,\text{dBW}$. It can be observed that the rate increases with an increasing inter-satellite distance up to $D_{\text{S}} = 12\,\text{km}$. Afterwards, it periodically increases and decreases, slightly, independent on the number of satellites N_{S}. The local maxima are every $12\,\text{km}$, which corresponds to section IV. In Fig. 4, the achievable rate is averaged over the considered time. Then, there are no more periodic variations of the achievable rate, but the achievable rate does not further increase for $D_{\text{S}} \geq 65\,\text{km}$, which justifies the heuristic (20).

C. Geometry Based DL

Throughout this paper, two different precoder and two different equalizer approaches have been discussed. In Fig. 5 three possible combinations are compared.

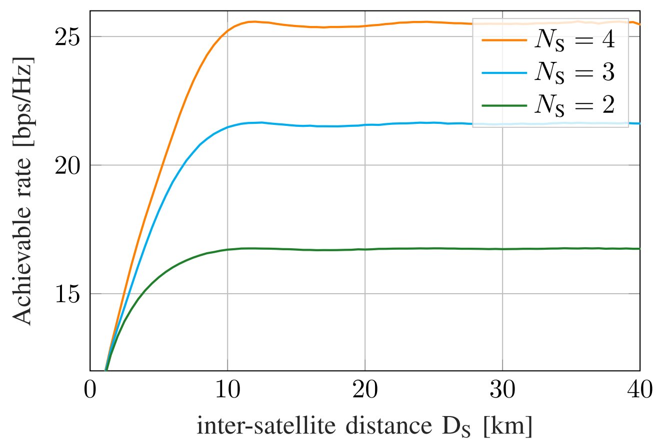

Fig. 3. Achievable rate performance for different inter-satellite distances D_{S} and number of satellites N_{S} for fixed AoA $\theta_{\text{mean}} = 90°$

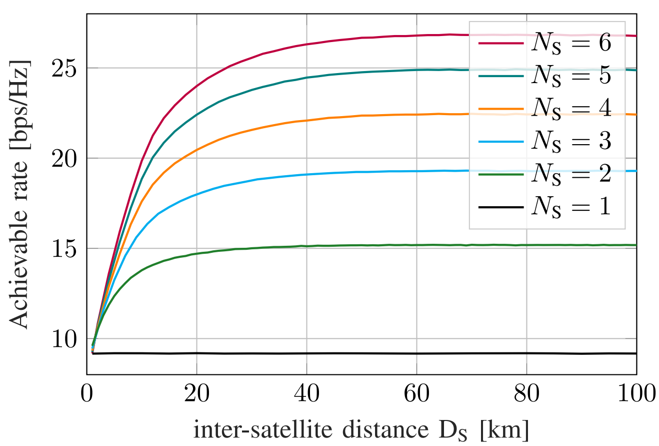

Fig. 4. Achievable rate performance for different inter-satellite distances D_{S} and number of satellites N_{S} averaged over time

First, the optimal precoder is given by the SVD (3). Further, assume an ideal, and in general non-linear, equalizer, the achievable rate R_{opt} is given by (4). Second, consider the geometry based precoder (8) and again, an ideal equalizer. Then, the achievable rate is given by

$$R_{\text{per}} = \log_2 \left| \mathbf{I}_{N_{\text{r}}} + \frac{1}{\sigma^2_{\text{n}}} \mathbf{H}\mathbf{G}_{\text{geo}}\mathbf{G}^H_{\text{geo}}\mathbf{H}^H \right| . \tag{23}$$

Finally, consider again the geometric precoder (8) and the linear equalizer (13). The corresponding achievable rate is given by R_{lin} in (11). In Fig. 5 these three approaches are compared. If the inter-satellite distance is $D_{\text{S}} = 70\,\text{km}$, i.e., $D_{\text{S}} > D_{\text{S,orth}} = 65\,\text{km}$ for $\theta_{\text{mean}} = 30°$, all three approaches achieve almost the same performance. If on the other hand $D_{\text{S}} = 10\,\text{km}$, the achievable rate decreases with a linear equalizer. However, with the proposed precoder (8), still, the optimum achivable rate can be reached.

VI. Discussion

In this paper, we developed a low complexity distributed precoder for satellite swarms and a linear equalizer, both

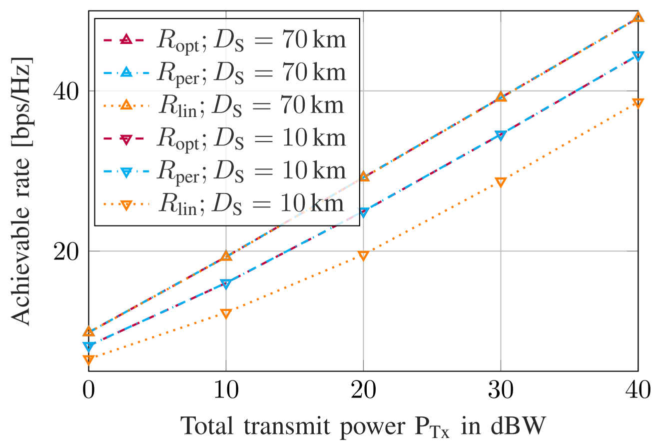

Fig. 5. Achievable rate performance for different transmit powers P_{Tx} and inter-satellite distances D_{S}. Number of satellites is $N_{\mathrm{S}} = 3$

utilizing only the geometric relation between the satellites and GS positions. Given that the inter-satellite distances are chosen adequately, we have shown that the proposed precoder-equalizer combination achieves a performance very close to the capacity upper bound obtained by assuming perfect CSI and instantaneous coordination between satellites. This only requires positional knowledge at all terminals, a small amount of CSI at the GS, i.e., tracking of one scalar channel coefficient per satellite, no CSI at the transmitter and no active coordination between satellites. Of course, in a real world system even these assumptions might not hold. In particular, the satellite positions are subject to small perturbations and channel coefficients are difficult to track perfectly in this high mobility scenario. However, these aspects can be incorporated in the system design to make it robust against such imperfections, as we will show in the journal extension of this paper.

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References

[1] 3GPP, "Study on using satellite access in 5G," *TR 22.822 V16.0.0*, 2018.
[2] O. Kodheli *et al.*, "Satellite communications in the new space era: A survey and future challenges," *IEEE Communications Surveys Tutorials*, vol. 23, no. 1, pp. 70–109, 2021.
[3] I. del Portillo, B. G. Cameron, and E. F. Crawley, "A technical comparison of three low earth orbit satellite constellation systems to provide global broadband," *Acta Astronautica*, vol. 159, pp. 123 – 135, 2019.
[4] B. Di, L. Song, Y. Li, and H. V. Poor, "Ultra-dense LEO: Integration of satellite access networks into 5G and beyond," *IEEE Wireless Communications*, vol. 26, no. 2, pp. 62–69, 2019.
[5] I. Leyva-Mayorga, B. Soret, M. Röper, D. Wübben, B. Matthiesen, A. Dekorsy, and P. Popovski, "LEO small-satellite constellations for 5G and beyond-5G communications," *IEEE Access*, vol. 8, pp. 184 955–184 964, 2020.
[6] G. Zheng, S. Chatzinotas, and B. Ottersten, "Generic optimization of linear precoding in multibeam satellite systems," *IEEE Trans. Wireless Commun.*, vol. 11, no. 6, pp. 4695–4707, Jun. 2012.
[7] V. Joroughi, M. A. Vazquez, and A. I. Perez-Neira, "Precoding in multigateway multibeam satellite systems," *IEEE Transactions on Wireless Communications*, vol. 15, no. 7, pp. 4944–4956, 2016.
[8] A. I. Perez-Neira, M. A. Vazquez, M. B. Shankar, S. Maleki, and S. Chatzinotas, "Signal processing for high-throughput satellites: Challenges in new interference-limited scenarios," *IEEE Signal Process. Mag.*, vol. 36, no. 4, pp. 112–131, 2019.
[9] C. Verhoeven, M. Bentum, G. Monna, J. Rotteveel, and J. Guo, "On the origin of satellite swarms," *Acta Astronautica*, vol. 68, no. 7, pp. 1392 – 1395, 2011.
[10] A. Budianu, A. Meijerink, and M. Bentum, "Swarm-to-earth communication in OLFAR," *Acta Astronautica*, vol. 107, pp. 14 – 19, 2015.
[11] R. Richter, I. Bergel, Y. Noam, and E. Zehavi, "Downlink cooperative mimo in leo satellites," *IEEE Access*, vol. 8, pp. 213 866–213 881, 2020.
[12] R. T. Schwarz, T. Delamotte, K. Storek, and A. Knopp, "MIMO applications for multibeam satellites," *IEEE Trans. Broadcast.*, 2019.
[13] H. Lin, F. Gao, S. Jin, and G. Y. Li, "A new view of multi-user hybrid massive MIMO: Non-orthogonal angle division multiple access," *IEEE J. Sel. Areas Commun.*, vol. 35, no. 10, pp. 2268–2280, 2017.
[14] I. Ahmed *et al.*, "A survey on hybrid beamforming techniques in 5G: Architecture and system model perspectives," *IEEE Commun. Surveys Tuts.*, vol. 20, no. 4, pp. 3060–3097, 2018.
[15] F. Yamashita, K. Kobayashi, M. Ueba, and M. Umehira, "Broadband multiple satellite MIMO system," in *Proc. IEEE Veh. Tech. Conf.*, vol. 4, 2005, pp. 2632–2636.
[16] D. Goto, H. Shibayama, F. Yamashita, and T. Yamazato, "LEO-MIMO satellite systems for high capacity transmission," in *2018 IEEE Global Communications Conference (GLOBECOM)*, 2018, pp. 1–6.
[17] K. Liolis, A. Panagopoulos, and P. Cottis, "Multi-satellite MIMO communications at Ku-band and above: Investigations on spatial multiplexing for capacity improvement and selection diversity for interference mitigation," *EURASIP J. Wireless Commun. Netw.*, vol. 2007, 12 2007.
[18] M. Röper and A. Dekorsy, "Robust distributed mmse precoding in satellite constellations for downlink transmission," in *Proc. IEEE 2nd 5G World Forum (5GWF)*, 2019, pp. 642–647.
[19] L. You, K. X. Li, J. Wang, X. Gao, X. G. Xia, and B. Ottersten, "Massive MIMO transmission for LEO satellite communications," *IEEE J. Sel. Areas Commun.*, vol. 38, no. 8, pp. 1851–1865, 2020.
[20] W. Guo, A.-A. Lu, X. Gao, and X.-G. Xia, "Broad coverage precoder design for synchronization in satellite massive mimo systems," *IEEE Trans. Commun.*, pp. 1–1, 2021.
[21] Z. Lin, M. Lin, B. Champagne, W. Zhu, and N. Al-Dhahir, "Robust hybrid beamforming for satellite-terrestrial integrated networks," in *ICASSP 2020 - 2020 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, 2020, pp. 8792–8796.
[22] E. Telatar, "Capacity of multi-antenna gaussian channels," *Eur. Trans. Telecommun.*, vol. 10, no. 6, pp. 585–595, 1999.
[23] 3GPP, "Study on new radio (NR) to support non-terrestrial networks," *TR 38.811 V15.2.0*, Sep. 2019.
[24] K. Storek, C. A. Hofmann, and A. Knopp, "Measurements of phase fluctuations for reliable MIMO space communications," in *Proc. IEEE Asia-Pac. Conf. Wireless Mob.*, Aug. 2015, pp. 157–162.
[25] R. A. Horn and C. R. Johnson, *Matrix Analysis*. Cambridge, U.K.: Cambridge Univ. Press, 1990.
[26] M. Sadek, A. Tarighat, and A. H. Sayed, "Active antenna selection in multiuser MIMO communications," *IEEE Transactions on Signal Processing*, vol. 55, no. 4, pp. 1498–1510, 2007.
[27] E. A. Jorswieck and H. Boche, *Majorization and Matrix Monotone Functions in Wireless Communications*, ser. FnT Commun. Inf. Theory. Boston, MA, USA: Now, 2007, vol. 3, no. 6.
[28] ITU-R, "Attenuation by atmospheric gases and related effects," *ITU-R P.676-12*, Aug. 2019.
[29] ——, "Reference standard atmospheres," *ITU-R P.835-6*, Dec. 2017.
[30] ——, "Propagation data and prediction method required for the design of Earth-space telecommunication systems," *ITU-R P.618-13*, Dec. 2017.
[31] ——, "Inospheric propagation data and prediction methods required for the design of satellite networks and systems," *ITU-R P.531-14*, Aug. 2019.