Resource Allocation for Distributed MIMO Multi-hop Wireless Networks

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Abstract: Distributed MIMO technology has gained significant attention in industry and academia recently, due to its ability to increase capacity drastically and its inherent attribute of scalability for wireless mesh networks. In this paper we briefly overview the concept of distributed MIMO and investigate the end-to-end ergodic channel capacity of a distributed MIMO multi-hop network. By formulating the resource allocation problem as a concave optimization problem, we are able to obtain the solution of optimal power and bandwidth allocation in a very efficient way.

1 Introduction

In this paper an end-to-end scenario in a wireless multi-hop network is considered, where a source communicates with the destination via a number of relays. In order to avoid interference between the relaying hops, orthogonal access schemes like frequency-division multiple access (FDMA) or time-division multiple access (TDMA) are usually used. However, it can be shown that both access schemes achieve the same capacities [2], so that only FDMA will be considered for simplicity. At each relaying node the decode-and-forward relaying protocol is applied, where the data will be first detected and decoded completely, then re-encoded and transmitted to the next relaying nodes [3]. Recently, it was shown that the channel capacity of a wireless mesh network can drastically be increased by applying MIMO techniques with respect to spatially separated relaying nodes [1]. To this end, several relays are used to form a virtual antenna array (VAA). The end-to-end connection is therefore accomplished through a number of topologically imposed VAAs.

Since the data will be transmitted to the destination through a number of hops, an optimal resource allocation strategy should assign fractional power and bandwidth to each hop such that the end-to-end capacity is maximized. In this paper the end-to-end ergodic capacity for a distributed MIMO multi-hop network will be studied. With respect to an approximated expression of the ergodic capacity, we will derive the optimal resource (power and bandwidth) allocation strategy for a given distributed MIMO multi-hop network. This strategy is shown to be of low complexity and to achieve near-maximum end-to-end ergodic capacity.

The remainder of the paper is organized as follows. In Section 2 the concept of distributed MIMO scheme is briefly overviewed. A concave optimization problem for maximizing the end-to-end capacity is formulated in Section 3. Some results are presented in Section 4. Finally, conclusions are given in Section 5.

2 Distributed MIMO Multi-hop Networks

A system model of a distributed MIMO multi-hop network is depicted in Figure 1, where a source node communicates with a destination node via a number of relaying nodes. Some spatially separated relaying nodes are formed into virtual antenna arrays (VAAs), which allows to increase capacity by applying space-time processing techniques, e.g space-time block codes [1]. For the further investigation a fixed network topology is assumed, i.e. the task of combining nodes to a VAA is not considered. As the data is transmitted from the source node through a number of VAAs to the destination node, such a network is referred to as a distributed MIMO multi-hop network. Note that there is no receive cooperation but only transmit cooperation between the relaying nodes of one VAA. In other words, each node in kth VAA receives signals transmitted by the nodes in the $(k-1)$th VAA, where the signals are space-time encoded cooperatively. Thus, the transmission can be modeled as a multiple-input single output (MISO) scheme. Note that the kth VAA serves as receive antenna array at the kth hop while as transmit antenna array at the $(k+1)$th hop.

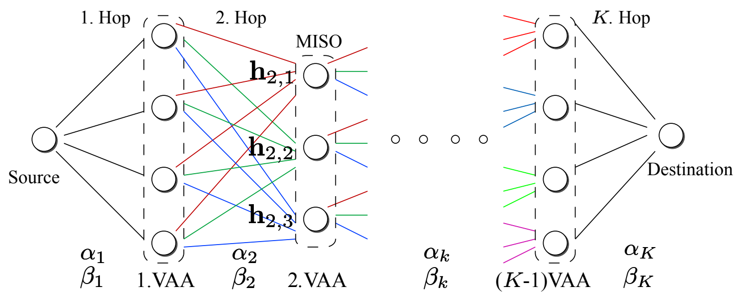

Figure 1: System model of distributed MIMO multi-hop networks

We summarize the encoding, relaying and decoding process for a given distributed MIMO network topology shortly as follows,

- **Source node**: Broadcasts the data to the nodes of the first VAA with bandwidth fraction α_1 and power fraction β_1.
- **Relaying nodes at the kth hop**: The data is decoded at each node at the kth VAA and re-encoded according to a given space-time code of length T with bandwidth

fraction α_k (FDMA) and power fraction β_k. All transmit nodes of one VAA use same bandwidth and transmission power.

- **Destination node**: Finally, the data is space-time decoded.

To produce a mathematical representation of the distributed MIMO multi-hop system, let k index the hop, t_k, r_k denote the number of the transmit nodes and the receive nodes within the kth hop, respectively. Let $\mathbf{X}_k \in \mathbb{C}^{t_k \times T}$ denote the space-time encoded signal matrix from the t_k nodes in the kth hop, then the received signal at the jth node $\mathbf{y}_{k,j} \in \mathbb{C}^{1\times T}$ can be represented by the equation

$$\mathbf{y}_{k,j} = \sqrt{\frac{\gamma_k \beta_k P}{t_k}} \mathbf{h}_{k,j} \mathbf{X}_k + \mathbf{n}_{k,j}, \tag{1}$$

where $\mathbf{n}_{k,j} \sim \mathcal{N}_C(0,\, N_0) \in \mathbb{C}^{1\times T}$ is the Gaussian noise vector, P is the total power available for the network and N_0 is the power spectral density of the noise. The complex channel realization from the transmit nodes to the jth receive node within the kth hop is denoted as $\mathbf{h}_{k,j} \in \mathbb{C}^{1\times t_k}$. The elements of $\mathbf{h}_{k,j}$ obey the same uncorrelated Rayleigh fading statistics, i.e. complex zero-mean circular symmetric Gaussian distribution with variance 1. The pathloss at the kth hop is given by $\gamma_k = (\frac{1}{d_k})^{\epsilon}$, where d_k is the distance between the transmit nodes and the receive nodes at the kth hop and ϵ denotes the pathloss exponent within range of 2 to 5 for most wireless channels.

According to the relaying process discussed above, the optimization problem to maximize the end-to-end ergodic capacity C_{e2e} results in finding the optimal bandwidth fraction $\boldsymbol{\alpha}^* = [\alpha_1^*, \ldots, \alpha_K^*]^T$ and power fraction $\boldsymbol{\beta}^* = [\beta_1^*, \ldots, \beta_K^*]^T$ where $\alpha_k^*,\, \beta_k^* \in [0,\, 1], k = 1, \ldots, K$ that satisfy

$$(\boldsymbol{\alpha}^*, \boldsymbol{\beta}^*) = \arg\max_{\boldsymbol{\alpha},\, \boldsymbol{\beta}} C_{e2e}(\boldsymbol{\alpha},\, \boldsymbol{\beta}). \tag{2}$$

Note that the Shannon capacity forms an upper bound and is therefore a useful measurement of the performance of the distributed MIMO multi-hop system.

3 Maximization of End-to-end Channel Capacity

The ergodic capacity of a MIMO channel was elegantly derived by Telatar [4]. The Shannon capacity of a MISO system according to (1) can be expressed as

$$C_{k,j} = \alpha_k W \, \mathrm{E}_{\mathbf{h}_{k,j}} \left\{ \log_2 \left(1 + \mathbf{h}_{k,j} \mathbf{h}_{k,j}^H \frac{\beta_k P \gamma_k}{\alpha_k t_k W N_0} \right) \right\}, \tag{3}$$

where W denotes the total bandwidth of the system. The ergodic capacity of the kth hop is dictated by the worst MISO channel $C_k = \min_j (C_{k,1}, \ldots, C_{k,j}, \ldots, C_{k,r_k})$. It is assumed that the relaying nodes belonging to the same VAA are spatially sufficiently close as to justify a common pathloss γ_k. Hence, each MISO system within the same hop has the same ergodic capacity, so that $C_k = C_{k,j},\, \forall\, j$.

Using $\log_2(1+x) \approx \sqrt{x}$ [5], the MISO channel capacity (3) can be approximated by

$$C_{k,j} \approx \sqrt{\frac{\beta_k P \alpha_k W \gamma_k}{t_k N_0}} \mathrm{E}_{\mathbf{h}_{k,j}} \left\{ \sqrt{\mathbf{h}_{k,j} \mathbf{h}_{k,j}^H} \right\} = \sqrt{\frac{\beta_k P \alpha_k W \gamma_k}{t_k N_0}} \frac{\Gamma(t_k + 1/2)}{\Gamma(t_k)} \tag{4}$$

where $\mathbf{h}_{k,j} \mathbf{h}_{k,j}^H$ is a gamma distributed random variable with $2t_k$ degrees of freedom. It is well-known that $\mathrm{E}_{\mathbf{h}_{k,j}} \left\{ \sqrt{\mathbf{h}_{k,j} \mathbf{h}_{k,j}^H} \right\} = \frac{\Gamma(t_k+1/2)}{\Gamma(t_k)}$ holds [5], where $\Gamma(\cdot)$ denotes the complete Gamma function. We now check the concavity of (4) in the joint arguments, the power fraction β_k and bandwidth fraction α_k. For simplicity we describe (4) as

$$C_k = \sqrt{\frac{\beta_k P \alpha_k W \gamma_k}{t_k N_0}} \frac{\Gamma(t_k + 1/2)}{\Gamma(t_k)} = \sqrt{\alpha_k \beta_k} \cdot A \tag{5}$$

where $A = \sqrt{\frac{P W \gamma_k}{t_k N_0}} \frac{\Gamma(t_k+1/2)}{\Gamma(t_k)}$. So that, the first-order partial derivatives, second-order partial derivatives and second-order mixed derivatives of C_k with respect to α_k, β_k are given as follows

$$\begin{aligned} \frac{\partial C_k}{\partial \alpha_k} &= \frac{A}{2}\sqrt{\frac{\beta_k}{\alpha_k}} \\ \frac{\partial C_k}{\partial \beta_k} &= \frac{A}{2}\sqrt{\frac{\alpha_k}{\beta_k}} \\ \frac{\partial^2 C_k}{\partial \alpha_k^2} &= -\frac{A}{4}\frac{\sqrt{\beta_k}}{{\alpha_k}^{3/2}} \\ \frac{\partial^2 C_k}{\partial \beta_k^2} &= -\frac{A}{4}\frac{\sqrt{\alpha_k}}{{\beta_k}^{3/2}} \\ \frac{\partial^2 C_k}{\partial \alpha_k \partial \beta_k} &= \frac{\partial^2 C_k}{\partial \beta_k \partial \alpha_k} = \frac{A}{4\sqrt{\alpha_k \beta_k}} \end{aligned} \tag{6}$$

To show the concavity of the C_k, we note that (for $\alpha_k > 0,\ \beta_k > 0$) the Hessian matrix is

$$\begin{aligned} \nabla^2 C_k(\alpha_k,\ \beta_k) &= \begin{bmatrix} -\frac{A}{4}\frac{\sqrt{\beta_k}}{{\alpha_k}^{3/2}} & \frac{A}{4\sqrt{\alpha_k\beta_k}} \\ \frac{A}{4\sqrt{\alpha_k\beta_k}} & -\frac{A}{4}\frac{\sqrt{\alpha_k}}{{\beta_k}^{3/2}} \end{bmatrix} \\ &= -\frac{A}{4\alpha_k^{3/2}\beta_k^{3/2}} \begin{bmatrix} \beta_k^2 & -\alpha_k\beta_k \\ -\alpha_k\beta_k & \alpha_k^2 \end{bmatrix} \\ &= -\frac{A}{4\alpha_k^{3/2}\beta_k^{3/2}} \begin{bmatrix} \beta_k \\ -\alpha_k \end{bmatrix} \begin{bmatrix} \beta_k \\ -\alpha_k \end{bmatrix}^T \preceq 0 \end{aligned} \tag{7}$$

hence, C_k is proven to be jointly concave in the power faction β_k and band fraction α_k.

Due to decode-and-forward relaying protocol, the destination node can decode the signals correctly if and only if the signals are correctly decoded at each hop. Thus, the end-to-end ergodic capacity C_{e2e} is determined by the smallest capacity C_k [1]

$$C_{e2e} = \min_k (C_1, \ldots, C_k, \ldots, C_K). \tag{8}$$

Furthermore, the $\min$ function is concave and nondecreasing. According to the theory of the concavity of a composition function [6], a composition function $f(x) = h(g(x))$ is concave if h is concave and nondecreasing, and g is concave. Here, f is C_{e2e}, h is the min function, g is C_k. Clearly, C_{e2e} is jointly concave in $(\boldsymbol{\alpha}, \boldsymbol{\beta})$. Then, a concave optimization problem for maximizing the end-to-end channel capacity can be formulated as follows

$$\begin{aligned} &\text{maximize} \quad C_{e2e} = \min_k (C_1, \ldots, C_k, \ldots, C_K) \\ &\text{subject to} \quad \sum_{k=1}^{K} \beta_k = 1 \quad \text{and} \quad \sum_{k=1}^{K} \alpha_k = 1. \end{aligned} \tag{9}$$

With the total power and total bandwidth constraints, increasing any one capacity C_k inevitably reduces the others. The minimum is therefore maximized if all capacities $C_k, \forall\, k$ are equated, i.e. $C_1 = C_2 = \cdots = C_K$. By using the constraints in (9) and the approximation (4) a simple expression of the optimal bandwidth and power fraction follows

$$\alpha_k = \beta_k = \frac{\sqrt{d_k^\epsilon} G_k}{\sum_{m=1}^{K} \sqrt{d_m^\epsilon} G_m}, \tag{10}$$

where $G_m = \frac{\Gamma(t_m)\sqrt{t_m}}{\Gamma(t_m+1/2)}$ is introduced for convenience. It can be shown that $G_k \approx 1$ holds [7] and consequently a suboptimal but simpler solution of the power and bandwidth fraction can be obtained

$$\alpha_k = \beta_k \approx \frac{\sqrt{d_k^\epsilon}}{\sum_{m=1}^{K} \sqrt{d_m^\epsilon}}, \tag{11}$$

which only depends on the distances d_k.

4 Results

In order to analyze the proposed optimization strategy, a distributed MIMO multi-hop system consisting of 5 hops with $[1, 2, 3, 4, 5, 1]$ denoting the number of nodes per VAA is investigated. The distances between the hops are $[1, 1, 2, 2, 1]$ km. Figure 2 shows the ergodic capacity for different resource allocation strategies. We can see that the optimized power and bandwidth allocation according to (10) for the distributed MIMO system clearly outperforms the equal power and bandwidth allocation ($\alpha_k = \beta_k = \frac{1}{K}, \forall\, k$), the traditional SISO multi-hop transmission ($t_k = r_k = 1, \forall\, k$) and the direct transmission (the source node communicates with destination node directly without any relaying nodes). Note that even the suboptimal solution based on (11) achieves near-optimum performance.

Table 1 shows the optimal power and bandwidth fraction according to the closed form solution (10). The same results can also be achieved by applying common optimization tools for (9). We can see, that hops with large distance require more power and bandwidth than others.

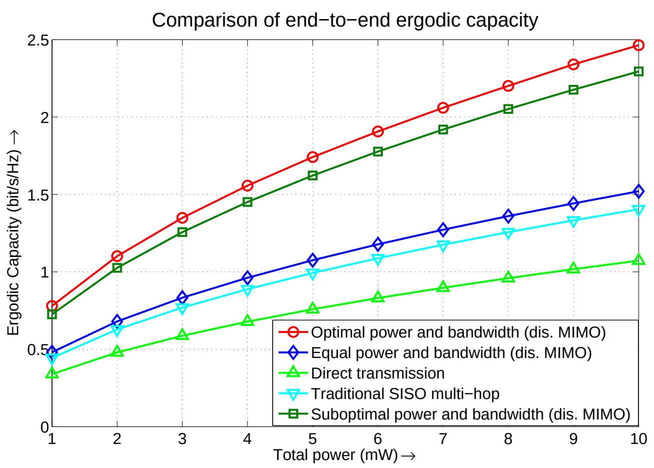

Figure 2: Ergodic channel capacity of a distributed MIMO multi-hop network for different resource allocation strategies. Network topology: 5 hops with nodes assignment $[1,\ 2,\ 3,\ 4,\ 5,\ 1]$ per VAA and distance $\mathbf{d} = [1,\ 1,\ 2,\ 2,\ 1]$ km, pathloss exponent $\epsilon = 3$.

Hop	1. Hop	2. Hop	3. Hop	4. Hop	5. Hop
Distance	1 km	1 km	2 km	2 km	1 km
Fractions $\alpha_k = \beta_k$	0.1263	0.1189	0.3250	0.3175	0.1124

Table 1: Power and band fraction according to (10).

5 Conclusion

In this paper we have briefly introduced the concept of distributed MIMO schemes, which allows the application of MIMO capacity enhancement techniques over spatially adjacent nodes. A concave optimization problem has been formulated for optimal resource allocation to maximize the end-to-end capacity of distributed MIMO multi-hop networks. Finally, we demonstrate that the optimal resource allocation strategy leads to a strong increase in ergodic capacities.

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