COMPARISON OF RECEIVER ARCHITECTURES FOR HIGH-RATE SPACE-TIME CODES

Ronald Böhnke, Carsten Bockelmann, and Karl-Dirk Kammeyer

Department of Communications Engineering
University of Bremen
Otto-Hahn-Allee, 28359 Bremen, Germany
Email: {boehnke, bockelmann, kammeyer}@ant.uni-bremen.de

ABSTRACT

Several high-rate space-time codes combining spatial multiplexing with transmit diversity have been proposed in recent years. They are usually optimized for uncoded transmission with maximum likelihood detection at the receiver. In this paper, also suboptimal receiver structures and the combination with channel coding are investigated.

1. INTRODUCTION

The performance of wireless communication systems can be significantly improved by using multiple transmit and receive antennas. Layered architectures like V-BLAST [1] are a practical way to reach unprecedented data rates in rich scattering environments that are typical for indoor scenarios. On the other hand, spatial diversity can be exploited using space-time block codes (STBC) in order to mitigate the adverse effects of fading [2]. A heuristic combination of these two concepts termed multistratum space-time (MSST) codes was proposed in [3] and extended to quasi-orthogonal space-time codes in [4]. The linear dispersion codes in [5] provide a more general framework, but require a numerical optimization of the codeword matrices for any given data rate. Recently, code designs based on number theory have attracted a great deal of attention, although the main concepts already appeared in [6]. Diagonal algebraic space-time (DAST) codes [7] achieve full diversity by multiplying a vector of QAM information symbols with an appropriate rotation matrix. The extension to multiple layers is known as threaded algebraic space-time (TAST) coding [8]. Here, each layer is multiplied by a unique complex number in order to maintain full diversity. The (layered) complex field coding from [9] and [10] is equivalent to DAST and TAST codes, respectively.

Most publications on high-rate space-time codes deal with uncoded transmission with maximum likelihood (ML) detection at the receiver. However, even with efficient search algorithms the computational complexity may be very high due to the large number of symbols that have to be jointly detected. This is especially true if additionally channel coding is employed, as soft estimates for the code bits are required. Hence, in this paper we study the performance degradation when using different kinds of suboptimal interference cancellation receivers. It will be shown that a layer-wise encoding of the data, which can not exploit transmit diversity in combination with V-BLAST, turns out to be beneficial for other space-time coding schemes.

After a brief description of the system model in the following section, the considered space-time coding schemes and detection algorithms will be explained in Sections 2 and 4, respectively. Simulation results are provided in Section 5, and concluding remarks can be found in Section 6.

This work was supported in part by the German Research Foundation (DFG) under grant KA 841/15-1.

2. SYSTEM MODEL

Consider the system model sketched in Fig. 1. The first block includes the optional channel encoding of the information bits $\mathbf{b}$ followed by bit-wise interleaving. The resulting code bits $\mathbf{c}$ are mapped onto QAM symbols $\mathbf{d}$ which are subsequently fed to the space-time encoder. We assume uncorrelated and quasi-static flat Rayleigh fading, so the $N_R \times N_T$ equivalent baseband channel matrix $\mathbf{H}$ contains independent complex Gaussian entries and remains constant during the transmission of one frame. The receive vector at time k is given by

$$\mathbf{y}[k] = \mathbf{H}\,\mathbf{x}[k] + \mathbf{n}[k] \; , \tag{1}$$

where $\mathbf{x}[k]$ contains the corresponding transmit signals and $\mathbf{n}[k]$ represents white Gaussian noise. The detector finally yields estimates $\hat{\mathbf{b}}$ of the information bits based on all observations $\mathbf{y}$. In the following sections, the individual components of the system model will be described in more detail.

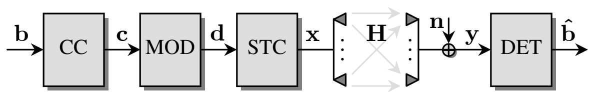


Fig. 1. Illustration of the general system model.

3. HIGH-RATE SPACE-TIME CODES

In general, a space-time encoder maps K_{ST} information symbols onto N_{ST} consecutive transmit vectors. We are mainly interested in linear codes that are able to achieve the maximum possible rate of $K_{ST}/N_{ST} = \min\{N_T, N_R\}$ symbols per channel use. In this section, some codes known from the literature are reviewed.

3.1. BLAST

The Bell Labs layered space-time architectures (BLAST) represent a very simple way to realize high data rates using multiple antenna systems [1,11]. The QAM symbols $\mathbf{d}$ are just demultiplexed into $N_L = N_T$ parallel layers and directly transmitted without any further processing, hence N_T symbols are transmitted per channel use. For uncoded transmission, BLAST can not take advantage of transmit diversity. The same holds if the layers are encoded individually as depicted in Fig. 2, which will turn out to be beneficial for successive detection schemes (cf. Section 4). In both cases, the weakest layer will dominate the overall error rate. This can be avoided by applying an outer channel code instead that spreads the information bits across all transmit antennas. It was demonstrated in [12] that such an approach has the potential to perform close to the capacity limits for ergodic channels.

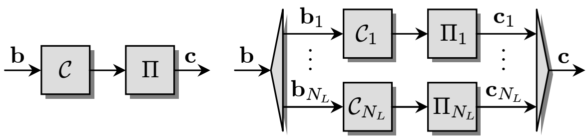


Fig. 2. Outer code (left) and layer-wise encoding (right).

3.2. Multistratum Space-Time Codes

While receive diversity can easily be obtained by maximum ratio combining, appropriate space-time coding is required in order to achieve also transmit diversity. Orthogonal space-time block codes [13] are especially attractive as the information symbols do not interfere, which results in a very low decoding complexity. A simple example is the well-known Alamouti scheme [2] for $N_T = 2$ transmit antennas

$$\mathbf{X} = \begin{pmatrix}\mathbf{x}[1] & \mathbf{x}[2]\end{pmatrix} = \begin{pmatrix} d_1 & -d_2^* \\ d_2 & d_1^* \end{pmatrix}, \qquad (2)$$

where $K_{ST}/N_{ST} = 1$ symbol is transmitted per channel use. However, for more than two transmit antennas there exists no linear orthogonal space-time block code with rate one. The transmission of one symbol per channel use for $N_T > 2$ is only possible if either the linearity or the orthogonality constraint is dropped. In [14], the following quasi-orthogonal space-time code with full diversity

$$\begin{aligned}\mathbf{X} &= \begin{pmatrix}\mathbf{x}[1] & \mathbf{x}[2] & \mathbf{x}[3] & \mathbf{x}[4]\end{pmatrix} \\ &= \begin{pmatrix} d_1 & -d_2^* & -\tilde{d}_3^* & \tilde{d}_4 \\ d_2 & d_1^* & -\tilde{d}_4^* & -\tilde{d}_3 \\ \tilde{d}_3 & -\tilde{d}_4^* & d_1^* & -d_2 \\ \tilde{d}_4 & \tilde{d}_3^* & d_2^* & d_1 \end{pmatrix} \end{aligned} \qquad (3)$$

was proposed, where $\tilde{d}_i = e^{j\pi/4} d_i$ are rotated versions of the actual information symbols. Comparing (3) with (2) we see that the quasi-orthogonal code follows from a recursive application of the Alamouti scheme. Due to this structure, d_1 only interferes with $\tilde{d}_4$ and d_2 with $\tilde{d}_3$, respectively. Therefore, a maximum likelihood detector can estimate these two symbol pairs independently, so the complexity is smaller than for general non-orthogonal schemes.

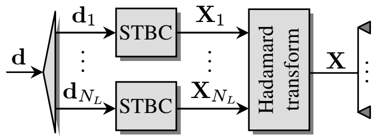


Fig. 3. Multistratum space-time codes.

The multistratum space-time codes proposed in [3] combine (quasi-)orthogonal space-time codes with the multi-layer concept of BLAST, aiming at high rate and transmit diversity at the same time. As shown in Fig. 3, all layers are first encoded with the same space-time block code and subsequently superimposed by the Hadamard transform. This results in the transmit vectors

$$\mathbf{x}[k] = \sum_{l=1}^{N_L} \mathcal{H}_{k,l}\, \mathbf{x}_l[k]\ , \qquad (4)$$

where $\mathcal{H}_{k,l}$ denotes the element at row k and column l of an $N_{ST} \times N_{ST}$ Hadamard matrix. With the Alamouti scheme from (2) and $N_L = 2$ layers, we get

$$\begin{aligned}\mathbf{X} &= \begin{pmatrix}+\mathbf{x}_1[1] + \mathbf{x}_2[1] & +\mathbf{x}_1[2] - \mathbf{x}_2[2]\end{pmatrix} \\ &= \begin{pmatrix} d_1 + d_3 & -d_2^* + d_4^* \\ d_2 + d_4 & d_1^* - d_3^* \end{pmatrix}. \end{aligned} \qquad (5)$$

A multistratum code for $N_T = 4$ transmit antennas with up to four layers can be constructed in a similar fashion from the quasi-orthogonal code in (3).

Although each layer enjoys full diversity, this is not necessarily true for the whole code. E.g., the determinant of (5) vanishes whenever $d_1 = \pm d_3$ and $d_2 = \pm d_4$. This may be avoided by proper constellation rotations as for the quasi-orthogonal design or more sophisticated superposition of the layers. However, we did not try to optimize the existing codes.

In contrast to BLAST, multistratum space-time codes can also be applied in MIMO systems with less receive than transmit antennas, as layers can be switched off without transmission breaks. This enables a flexible trade-off between data rate and decoding complexity. The special case $N_L = 1$ corresponds to ordinary space-time block coding.

3.3. Threaded Algebraic Space-Time Codes

The basic concept of threaded algebraic space-time codes is quite similar to that of multistratum codes. Instead of using a space-time block code, the N_T symbols $\mathbf{d}_l$ of the l-th layer are multiplied with a layer-specific unitary precoding matrix

$$\mathbf{x}_l = \mathbf{\Theta}_l \, \mathbf{d}_l = \phi^{l-1} \, \mathbf{\Theta} \, \mathbf{d}_l \quad , \; 1 \leq l \leq N_L \; . \tag{6}$$

Afterwards, the elements of the vectors $\mathbf{x}_l$ are mapped onto the transmit antennas at $N_{ST} = N_T$ successive time slots in a cyclic manner according to

$$x_l[k] = x_{[l-k]_{N_T},k} \tag{7}$$

with the shorthand notation $[l-k]_{N_T} = (l-k) \bmod N_T + 1$. For $N_T = 4$, this results in the transmit matrix

$$\mathbf{X} = \begin{pmatrix} x_{1,1} & x_{4,2} & x_{3,3} & x_{2,4} \\ x_{2,1} & x_{1,2} & x_{4,3} & x_{3,4} \\ x_{3,1} & x_{2,2} & x_{1,3} & x_{4,4} \\ x_{4,1} & x_{3,2} & x_{2,3} & x_{1,4} \end{pmatrix} . \tag{8}$$

If the matrix $\mathbf{\Theta}$ is chosen such that the superposition of the M-QAM symbols in $\mathbf{d}_l$ leads to M^{N_T} different constellation points, it is possible to estimate the whole vector $\mathbf{d}_l$ from a single transmit symbol $x_{l,k}$. Hence, as for orthogonal space-time block codes, it suffices if at least one of the transmit antennas is not in a deep fade. This property was termed signal space diversity in [15], where the real-valued rotation matrix for $N_T = 2$

$$\mathbf{\Theta} = \frac{1}{\sqrt{1+n_g^2}} \begin{pmatrix} 1 & -n_g \\ n_g & 1 \end{pmatrix} \tag{9}$$

with the Golden number $n_g = (1+\sqrt{5})/2$ was found by maximizing the coding gain. An alternative complex rotation

$$\mathbf{\Theta} = \begin{pmatrix} 1 & \theta \\ 1 & -\theta \end{pmatrix} , \quad \theta = e^{j\pi/4} \tag{10}$$

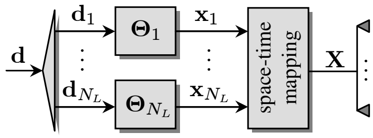


Fig. 4. Threaded algebraic space-time codes.

was derived in [16] using tools from number theory. The resulting transmit constellations for 4-QAM and $N_T = 2$ are shown in Fig. 5. It can be observed that a symbol pair $\mathbf{d}_l$ is mapped onto a vector $\mathbf{x}_l$, where each entry is taken from a constellation with 16 points.

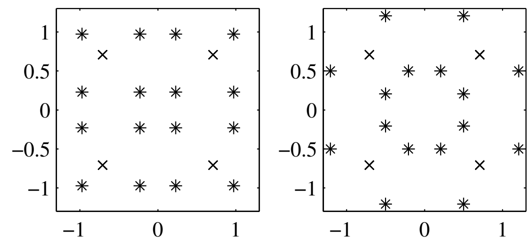


Fig. 5. Original 4-QAM constellations and transmit symbols for the optimal real (left) and complex (right) rotation.

For four transmit antennas, we restrict to the complex rotation matrix from [16]

$$\mathbf{\Theta} = \begin{pmatrix} 1 & \theta & \theta^2 & \theta^3 \\ 1 & -j\theta & -\theta^2 & j\theta^3 \\ 1 & -\theta & \theta^2 & -\theta^3 \\ 1 & j\theta & -\theta^2 & -j\theta^3 \end{pmatrix} , \quad \theta = e^{j\pi/8} \; . \tag{11}$$

Note that (10) and (11) can be generalized to

$$\mathbf{\Theta} = \boldsymbol{\mathcal{F}}_{N_T} \mathrm{diag}\{1, \theta, \ldots, \theta^{N_T-1}\} \; , \quad \theta = e^{j\pi/(2N_T)} \tag{12}$$

with the N_T-point discrete Fourier transform matrix $\boldsymbol{\mathcal{F}}_{N_T}$ if the number of transmit antennas is a power of two [9].

Different layers are separated by the complex factor ϕ in (6). It must be chosen such that full diversity is maintained for the total code. We will use $\phi = e^{j\pi/4}$ for the real matrix (9), which results in a code that is equivalent to the Golden code [17], and $\phi = e^{j\pi/(6N_T)}$ for the complex rotations [8].

4. DETECTION ALGORITHMS

After the description of the transmit schemes, we now turn to the receiver. Let us indicate complex quantities by an underline for the moment. Some of the discussed space-time codes require a complex conjugation, which is not a linear operation. However, by splitting the QAM symbols $\underline{d} = d^{\Re} + jd^{\Im}$ up into their real and imaginary part we can write

$$\underline{d} = \begin{pmatrix} 1 & j \end{pmatrix} \begin{pmatrix} d^{\Re} \\ d^{\Im} \end{pmatrix} , \quad \underline{d}^* = \begin{pmatrix} 1 & -j \end{pmatrix} \begin{pmatrix} d^{\Re} \\ d^{\Im} \end{pmatrix} \; . \tag{13}$$

Thus, with the definition of the vector

$$\mathbf{d} = \begin{pmatrix} d_1^{\Re} & d_1^{\Im} & \cdots & d_{K_{ST}}^{\Re} & d_{K_{ST}}^{\Im} \end{pmatrix}^T \tag{14}$$

that contains $K'_{ST} = 2K_{ST}$ real-valued ASK symbols for a certain space-time codeword and stacking the columns of

$\underline{\mathbf{X}}$ on top of each other, every linear space-time code can be described by a generator matrix $\underline{\mathbf{G}}$ of size $N_{ST}N_T \times K'_{ST}$

$$\underline{\mathbf{x}} = \left(\underline{\mathbf{x}}^T[1] \quad \cdots \quad \underline{\mathbf{x}}^T[N_{ST}]\right)^T = \underline{\mathbf{G}}\,\mathbf{d}\,. \tag{15}$$

Let us consider the Alamouti scheme as an example. Comparing (2) with (15) and (14), we can easily determine the generator matrix

$$\underline{\mathbf{G}} = \begin{pmatrix} \underline{\mathbf{G}}[1] \\ \underline{\mathbf{G}}[2] \end{pmatrix} = \left(\begin{array}{cccc} 1 & j & 0 & 0 \\ 0 & 0 & 1 & j \\ \hline 0 & 0 & -1 & j \\ 1 & -j & 0 & 0 \end{array}\right)\,, \tag{16}$$

where the implicitly defined submatrices $\underline{\mathbf{G}}[k]$ characterize the k-th transmit vector of one codeword. With the Kronecker product $\otimes$, the corresponding vector of observations at the receiver is given by

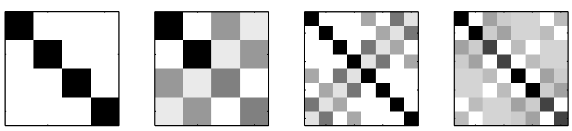
$$\begin{aligned} \underline{\mathbf{y}} &= \left(\underline{\mathbf{y}}^T[1] \quad \cdots \quad \underline{\mathbf{y}}^T[N_{ST}]\right)^T \\ &= (\mathbf{I}_{N_{ST}} \otimes \underline{\mathbf{H}})\,\underline{\mathbf{x}} + \underline{\mathbf{n}} = (\mathbf{I}_{N_{ST}} \otimes \underline{\mathbf{H}})\,\underline{\mathbf{G}}\,\mathbf{d} + \underline{\mathbf{n}} \\ &= \underline{\mathbf{A}}\,\mathbf{d} + \underline{\mathbf{n}}\,. \end{aligned} \tag{17}$$

In (17), the $N_{ST}N_R \times K'_{ST}$ system matrix $\mathbf{A}$ summarizing the joint effects of space-time coding and channel was introduced. As the symbols in $\mathbf{d}$ are real, we can also partition all complex matrices and vectors in (17) into their real and imaginary parts similar to (14) and finally arrive at the completely real-valued linear system model

$$\begin{aligned} \mathbf{y} &= \left(y_1^{\Re}[1] \quad y_1^{\Im}[1] \quad \cdots \quad y_{N_R}^{\Re}[N_{ST}] \quad y_{N_R}^{\Im}[N_{ST}]\right)^T \\ &= \mathbf{A}\,\mathbf{d} + \mathbf{n}\,. \end{aligned} \tag{18}$$

which will be used subsequently. Without loss of generality, we may let $\sigma_d^2 = \sigma_n^2 = 1/2$, so that the signal to noise ratio per receive antenna becomes SNR $= K'_{ST}\sigma_a^2$.

The structure of the system matrix $\mathbf{A}$ is illustrated in Fig. 6 for two transmit and receive antennas. After matched filtering with $\mathbf{A}^T$, the ASK symbols in $\mathbf{d}$ transmitted by the Alamouti scheme during $N_{ST} = 2$ time slots are received with the same power and do not interfere, which is just opposite for BLAST. By construction, the multistratum code (5), where each codeword contains 8 real-valued information symbols, is also partly orthogonal, while the TAST code based on the real rotation (9) is again rather unstructured.



Alamouti BLAST MSST TAST

Fig. 6. Illustration of $\mathbf{A}^T\mathbf{A}$ for different space-time codes.

4.1. Sphere Detection

The optimum detector looks for the vector $\hat{\mathbf{d}}$ that has been transmitted with highest probability for given observations $\mathbf{y}$. With equiprobable information symbols, this corresponds to the maximum likelihood criterion

$$\hat{\mathbf{d}} = \underset{\mathbf{d}'}{\operatorname{argmax}}\, p(\mathbf{y}|\mathbf{d}) = \underset{\mathbf{d}'}{\operatorname{argmin}}\, \|\mathbf{y} - \mathbf{A}\,\mathbf{d}'\|^2\,. \tag{19}$$

For the brute force approach, $M^{K_{ST}}$ Euclidean distances have to be evaluated. This complexity can be significantly reduced by using a sphere detector [4]. Introducing the QL decomposition of the system matrix $\mathbf{A} = \mathbf{Q}\,\mathbf{L}$, where $\mathbf{Q}$ has orthogonal columns of unit norm and $\mathbf{L}$ is lower triangular, it can be shown that minimizing

$$\|\mathbf{z} - \mathbf{L}\,\mathbf{d}'\|^2 = \sum_{k=1}^{K'_{ST}} \left| z_k - \sum_{m=1}^{k} l_{km}\, d'_m \right|^2 \tag{20}$$

with $\mathbf{z} = \mathbf{Q}^T\,\mathbf{y}$ is equivalent to (19). In contrast to the original criterion, the sum in (20) can be calculated recursively

$$\Delta_k = \Delta_{k-1} + \left| z_k - \sum_{m=1}^{k} l_{km}\, d'_m \right|^2 \quad \text{with} \quad \Delta_0 = 0\,. \tag{21}$$

Now, for fixed $d'_1, \ldots, d'_{k-1}$, the second term in (21) is minimized over d'_k. Whenever the partial sum Δ_k exceeds the squared radius ϱ^2, a new hypothesis will be tested for d'_{k-1}. If for some vector $\mathbf{d}'$ there is $\Delta_{K'_{ST}} < \varrho^2$, we have found a new improved estimate and the search radius can be decreased.

For coded transmission, it is advisable to make use of reliability information. The a-posteriori probability (APP) of the l-th code bit can be determined according to

$$L(c_l|\mathbf{y}) = \ln \frac{\sum_{\mathbf{d}' \in \mathbb{D}_l^0} e^{-\|\mathbf{y} - \mathbf{A}\,\mathbf{d}'\|^2/2\sigma_n^2 - \mathbf{L}_A^T \mathbf{c}'}}{\sum_{\mathbf{d}' \in \mathbb{D}_l^1} e^{-\|\mathbf{y} - \mathbf{A}\,\mathbf{d}'\|^2/2\sigma_n^2 - \mathbf{L}_A^T \mathbf{c}'}}\,, \tag{22}$$

where the set $\mathbb{D}_l^\xi$ contains all vectors $\mathbf{d}$ for which $c_l = \xi$, and $\mathbf{L}_A$ is a-priori information about the code bits $\mathbf{c}'$ corresponding to $\mathbf{d}'$ that may stem from the channel decoder in an iterative receiver. Here, only the extrinsic information $L_E(c_l) = L(c_l|\mathbf{y}) - L_A(c_l)$ is exchanged.

In order to reduce the computational effort, the max-APP approximation considers only the largest term in each sum of (22), thus avoiding the calculation of exponentials and logarithms. Furthermore, the sphere detector can be used to limit the sets $\mathbb{D}_l^\xi$ to those vectors with small Euclidean distance $\|\mathbf{y} - \mathbf{A}\,\mathbf{d}\|^2 < \varrho^2$. The choice of the radius ϱ is a critical task, as it should be large enough to ensure that for each bit c_l both hypotheses 0 and 1 are taken into account. In contrast to the uncoded case where a hard decision suffices, it can not be reduced after a new point is found inside the sphere. Therefore, even approximate APP detection becomes impractical for large K_{ST}.

4.2. Successive Interference Cancellation

For each layer, interference caused by already detected layers is subtracted from the receive signal, while the remaining interference is suppressed by a linear filter. With $\mathbf{z} = \mathbf{Q}^T\mathbf{y}$ from the previous section and assuming correct hard decisions $\hat{d}_m = d_m$, this leads to

$$\tilde{d}_k = z_k - \sum_{m=1}^{k-1} l_{km}\,\hat{d}_m = l_{kk}\,d_k + \tilde{n}_k\,. \tag{23}$$

The performance can be significantly improved if an MMSE filter is used for interference suppression. Furthermore, the order of detection can be optimized by exchanging elements of $\mathbf{d}$ and the corresponding columns of the system matrix $\mathbf{A}$ such that the signal to interference and noise ratio (SINR) is maximized in each detection step. To this end, $\mathbf{Q}$ and $\mathbf{L}$ just have to be replaced by $\tilde{\mathbf{Q}}$ and $\tilde{\mathbf{L}}$ obtained from the QL decomposition of the extended system matrix [18]

$$\begin{pmatrix} \mathbf{A}\mathbf{\Pi} \\ \frac{\sigma_n}{\sigma_d}\mathbf{I} \end{pmatrix} = \begin{pmatrix} \tilde{\mathbf{Q}} \\ \frac{\sigma_n}{\sigma_d}\tilde{\mathbf{L}}^{-1} \end{pmatrix} \tilde{\mathbf{L}} \tag{24}$$

with the permutation matrix $\mathbf{\Pi}$. In order to reduce the risk of error propagation, the channel decoding should be integrated into the interference cancellation. However, this is only possible if the layers are encoded individually.

Fig.7 shows the outage capacities per layer for BLAST and the multistratum code based on (3). Note that in the latter case there are $N'_L = 2N_L = 8$ sub-layers consisting of four real-valued symbols each due to the quasi-orthogonality of the constituent space-time code. It can be observed that sorting is crucial for BLAST, although then the data rates must be chosen according to the weakest layer. This is different for the multistratum code, where an adaptive rate allocation is much more important.

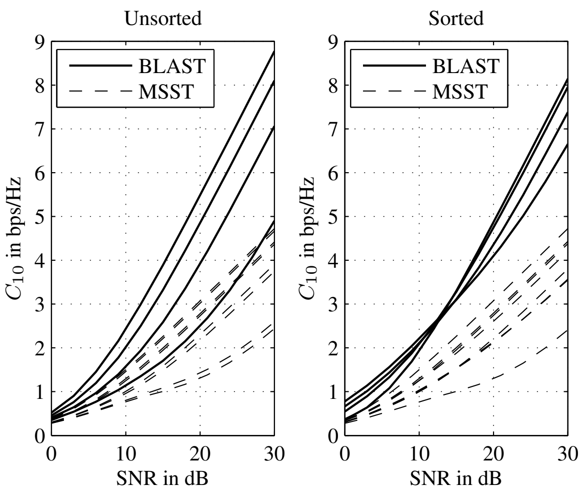


Fig. 7. Layer-wise 10% outage capacities for $N_T = N_R = 4$.

4.3. Iterative Interference Cancellation

Assume that in an iterative receiver the channel decoder feeds back a-priori information $\mathbf{L}_{A,k}$ about the code bits of the ASK symbol d_k. Using $\Pr(d_k = d'_k\,|\,\mathbf{L}_{A,k}) \propto e^{-\mathbf{L}_{A,k}^T \mathbf{c}'_k}$, the conditional mean

$$\bar{d}_k = \mathrm{E}\left\{d_k\,|\,\mathbf{L}_{A,k}\right\} = \sum_{d'_k \in \mathbb{D}} d'_k \cdot \Pr(d_k = d'_k\,|\,\mathbf{L}_{A,k}) \tag{25}$$

as well as the error variance

$$\begin{aligned}\sigma^2_{e,k} =& \mathrm{E}\left\{|d_k - \bar{d}_k|^2\,|\,\mathbf{L}_{A,k}\right\} \\ =& \sum_{d'_k \in \mathbb{D}} |d'_k|^2 \cdot \Pr(d_k = d'_k\,|\,\mathbf{L}_{A,k}) - |\bar{d}_k|^2\end{aligned} \tag{26}$$

can be obtained. With $\bar{\mathbf{n}} = \mathbf{y} - \mathbf{A}\bar{\mathbf{d}}$, the receive vector after soft interference cancellation for the k-th symbol is given by

$$\bar{\mathbf{y}}_k = \mathbf{y} - \sum_{m \neq k} \mathbf{a}_m \bar{d}_m = \mathbf{a}_k \bar{d}_k + \bar{\mathbf{n}}\,. \tag{27}$$

The corresponding MMSE filter can be determined from the covariance matrix of $\bar{\mathbf{n}}$

$$\mathbf{w}_k = (\mathbf{A}\mathbf{\Phi}_e\mathbf{A}^T + \sigma_n^2\mathbf{I})^{-1}\mathbf{a}_k\,, \tag{28}$$

where the diagonal matrix $\mathbf{\Phi}_e$ contains the residual error variances from (26) for all K'_{ST} symbols. Applying this to (27) results in the unbiased estimate

$$\tilde{d}_k = \frac{\mathbf{w}_k^T \tilde{\mathbf{y}}_k}{\mathbf{w}_k^T \mathbf{a}_k} = d_k + \tilde{n}_k \tag{29}$$

of d_k. It can easily be verified that the effective noise $\tilde{n}_k$ in (29) has variance $\sigma^2_{\tilde{n},k} = (\mathbf{w}_k^T\mathbf{a}_k)^{-1} - \sigma^2_{e,k}$. The a-posteriori information for the code bits is then calculated similar to (22), but symbol-wise, i.e. only d'_k is considered in the sums and not the whole vector $\mathbf{d}'$. Hence, the complexity is not exponential in K'_{ST} anymore.

Without a-priori information, (28) corresponds to a conventional MMSE filter, but with increasing reliability of the code bit estimates, linear interference suppression is more and more replaced by interference cancellation. In [19], it was suggested to use the complete a-posteriori information from the decoder in (25) and (26), while the soft demapping should still restrict to the extrinsic one. There, the described iterative detector was applied to multistratum codes in combination with an outer channel code, so (29) has to be calculated for all layers in parallel before channel decoding can be performed. However, in contrast to BLAST, a layer-wise encoding does not incur a performance loss for these space-time coding schemes. The big advantage of this approach is that a-priori information can be exploited immediately once a single layer has been decoded. Thus, it is preferable to detect the layers successively. Note that in this case the inverse in (28) can be updated efficiently using the matrix inversion lemma, so the complexity per iteration is comparable to parallel interference cancellation.

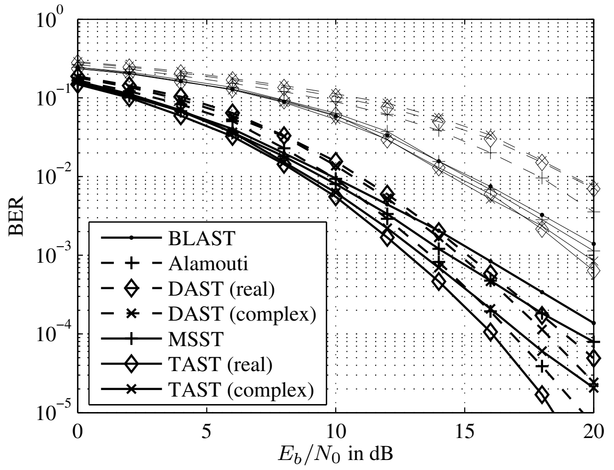


Fig. 8. BER for uncoded transmission with ML detection, rate $R = 4$ (thick) and $R = 8$ (thin), $N_T = N_R = 2$.

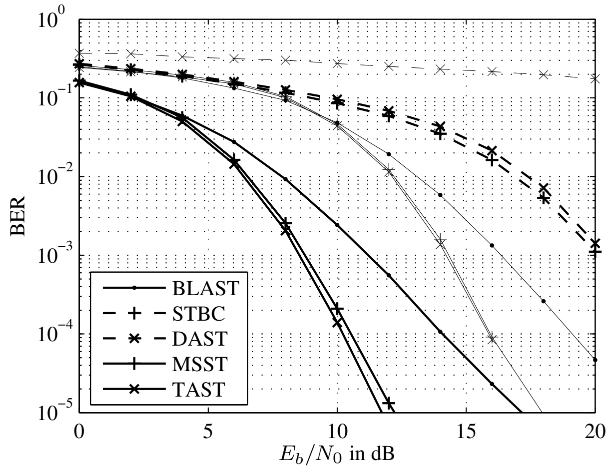


Fig. 9. BER for uncoded transmission with ML detection, rate $R = 8$ (thick) and $R = 16$ (thin), $N_T = N_R = 4$.

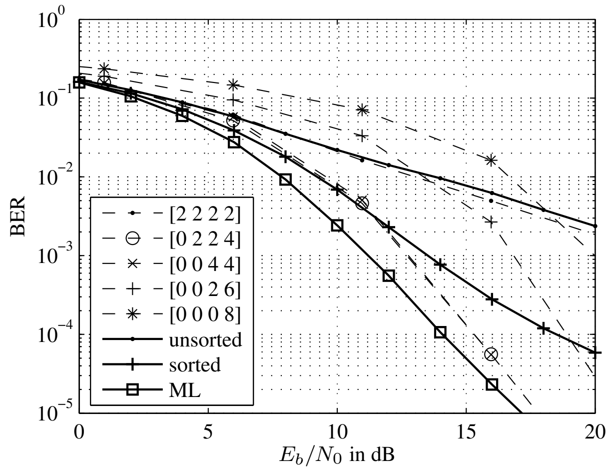


Fig. 10. BER for uncoded transmission with SIC detection, BLAST (thick) and MSST code with various rate allocations (thin), $R = 8$, $N_T = N_R = 4$.

5. SIMULATION RESULTS

In this section, the different space-time codes and receiver architectures are compared by means of Monte-Carlo simulations. The bit or frame error rates (BER / FER) are plotted versus

$$\frac{E_b}{N_0} = \frac{N_{ST}\, N_R\, \mathrm{SNR}}{K_{ST}\, \log_2(M)\, R_c}\,, \tag{30}$$

where E_b is the total received energy per bit, and N_0 the one-sided spectral power density of the noise.

Let us start with uncoded transmission. Fig. 8 shows the performance of the considered space-time mappings for two transmit antennas and optimal maximum likelihood detection. The dashed lines correspond to transmitting only one layer. The orthogonal Alamouti scheme is superior to the two diagonal algebraic space-time codes (DAST) based on either the real or complex rotation matrices from (9) and (10). For $R = 4$ bps/Hz and high SNR, it is only outperformed by the TAST code using the real-valued precoder. As already mentioned in Section 3.2, the multistratum code does not achieve full diversity and only shows a coding gain when compared to BLAST. At higher data rate, the advantage of multi-layer transmission becomes obvious.

With four transmit and receive antennas, the MSST and TAST codes behave quite similar in Fig. 9 and much better than BLAST. Note that the partly orthogonal structure of the MSST code (cf. Fig 6) can be exploited to reduce the decoding complexity, which is not possible for the TAST code. Single-layer transmission is not appropriate here, because for $R = 8$ bps/Hz this already requires a 256-QAM.

In Fig. 10, the successive interference cancellation with MMSE filtering based on the QL decomposition of the extended system matrix from Section 4.2 is investigated. The importance of optimizing the detection order for BLAST is clearly visible. From Fig. 7 we already concluded that this does not hold for MSST codes. Thus, the rate allocation is optimized instead. With 4-QAM on $N_L = 4$ layers, the performance is only slightly better than unsorted BLAST detection. However, switching off two of the layers and using 16-QAM on the remaining ones is better than BLAST with ordered SIC and comes close to the maximum likelihood curve at sufficiently high SNR. With layer-wise channel coding, the data rates can be adjusted in much smaller steps, e.g. by puncturing a rate $R_c = 1/2$ code. Furthermore, powerful turbo or LDPC codes may be employed, which is usually not a good choice for iterative receivers. Note that statistical rate allocation works best if the layer capacities vary only little with the current channel realization. To this end, other sources of diversity like that provided by a frequency-selective channel may be additionally exploited.

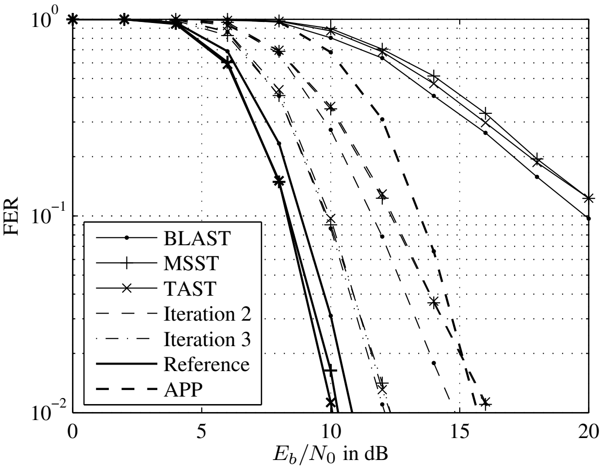


Fig. 11. FER for convolutionally coded transmission with iterative PIC detection, $R = 8$, $N_T = N_R = 4$.

We now turn to coded transmission with iterative interference cancellation. A frame consists of $N_b = 800$ information bits, and a simple convolutional code with rate $R_c = 1/2$ and constraint length $L_c = 3$ is used as an outer code. Fig. 11 depicts the frame error rates achieved by the parallel interference cancellation from [19]. It can be observed that BLAST converges faster than the other two space-time codes in the first iterations, but the reference curve for perfectly cancelled interference is slightly worse since the channel code is not able to fully exploit the transmit diversity. After the second iteration this simple linear receiver is already better than non-iterative APP detection, which was only applied for BLAST; for the other schemes containing $K'_{ST} = 32$ different 4-ASK symbols per codeword, even the approximate list sphere detector has an impractical computational complexity.

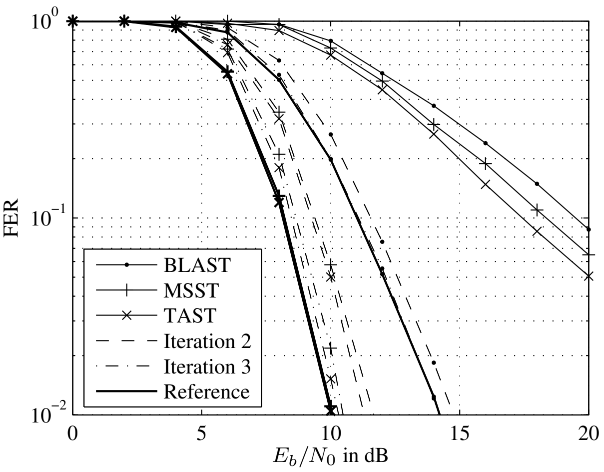


Fig. 12. FER for convolutionally coded transmission with iterative SIC detection, $R = 8$, $N_T = N_R = 4$.

The results for our proposed successive interference cancellation is illustrated in Fig. 12. Channel coding is applied per sub-layer consisting of non-interfering ASK symbols. Consequently, there are $N'_L = 4$, 8, or 16 individually encoded data streams for BLAST, MMST, and TAST codes, respectively. BLAST can not make use of transmit diversity anymore, so the performance is degraded significantly. However, the other more sophisticated space-time codes highly benefit from the layer-wise encoding. After two iterations the successive interference cancellation is better than the parallel one after three, where the interference-free case is now nearly reached. Due to the partial orthogonality, the MSST code is again preferable.

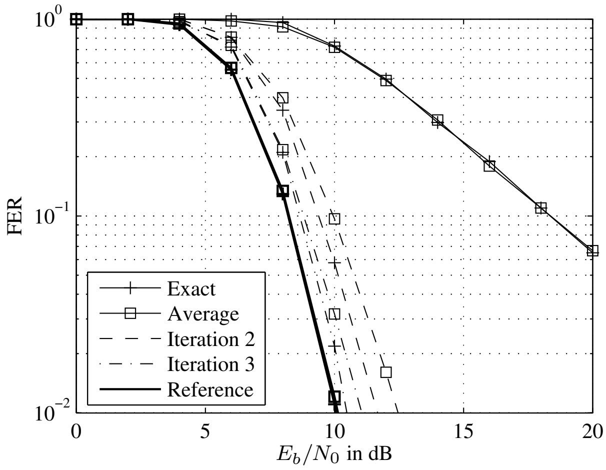


Fig. 13. Comparison of exact MMSE filter and an average one for MSST code, $R = 8$, $N_T = N_R = 4$.

Up to now, the MMSE filters were determined according to (28) for each of the $N_b/(R\,N_{ST}) = 25$ different space-time codewords per frame. Fig. 13 demonstrates that the degradation when using only one filter per layer and iteration based on the average error variances $\bar{\sigma}^2_{e,k}$ is rather small and can easily be made up for by some additional iterations. However, for fast fading or, equivalently, frequency-selective channels in a MIMO-OFDM system this simplification is not possible anymore.

6. CONCLUSION

In this paper, different space-time coding schemes for high-rate transmission have been reviewed. It was shown that multistratum codes do not necessarily achieve the full degree of transmit diversity. Nevertheless, for uncoded transmission with four transmit antennas they are much better than BLAST and come close to threaded space-time codes while exhibiting a partially orthogonal structure. Furthermore, layers can be easily switched off, which does not only allow for the application in systems with less receive than transmit antennas, but also improves the performance of simple successive interference cancellation with linear interference suppression, where in contrast to V-BLAST a proper rate allocation is more important for MSST codes than sorting. For iterative detection algorithms exploiting a-priori information from the channel decoder, it was demonstrated that much better convergence properties can be obtained if coding is applied per layer instead of using an outer code; the interference was almost completely removed after only three iterations. However, in this case BLAST can not take advantage of transmit diversity.

An interesting field for future research is the gain of high-rate space-time codes in MIMO-OFDM systems with multiple users, where some kind of channel knowledge at the base station can be used for scheduling or even precoding.

7. REFERENCES

[1] P. W. Wolniansky, G. J. Foschini, G. D. Golden, and R. A. Valenzuela, "V-BLAST: An Architecture for Realizing Very High Data Rates Over the Rich-Scattering Wireless Channel," in *Proc. ISSE*, Pisa, Italy, September 1998.

[2] S.M. Alamouti, "A Simple Transmit Diversity Technique for Wireless Communications," *IEEE Journal on Selected Areas in Commununications*, vol. 16, no. 8, pp. 1451–1458, October 1998.

[3] U. Wachsmann, J. Thielecke, and H. Schotten, "Exploiting the Data-Rate Potential of MIMO Channels: Multi-Stratum Space-Time Coding," in *Proc. IEEE VTC Spring 2001*, Rhodes, Greece, May 6-9 2001.

[4] R. Böhnke, V. Kühn, and K.D. Kammeyer, "Efficient Near Maximum-Likelihood Decoding of Multistratum Space-Time Codes," in *Proc. IEEE VTC Fall 2004*, Los Angeles, USA, September 2004.

[5] B. Hassibi and B.M. Hochwald, "High-Rate Codes that are Linear in Space and Time," *IEEE Transactions on Information Theory*, vol. 48, no. 7, pp. 1804–1824, July 2002.

[6] J. Boutros, E. Viterbo, C. Rastello, and J.-C. Belfiore, "Good lattice constellations for both Rayleigh fading and Gaussian channels," *IEEE Transactions on Information Theory*, vol. 42, no. 2, pp. 502–518, 1996.

[7] M.O. Damen, K.A.-M., and J.-C. Belfiore, "Diagonal Algebraic Space-Time Block Codes," *IEEE Transactions on Information Theory*, vol. 48, no. 3, pp. 628–636, March 2002.

[8] H. El Gamal and M.O. Damen, "Universal Space-Time Coding," *IEEE Transactions on Information Theory*, vol. 49, no. 5, pp. 1097–1119, May 2003.

[9] Y. Xin, Z. Wang, and G.B. Giannakis, "Space-Time Diversity Systems Based on Linear Constellation Precoding," *IEEE Transaction on Wireless Communications*, vol. 2, no. 2, pp. 294–309, March 2003.

[10] X. Ma and G.B. Giannakis, "Full-Diversity Full-Rate Complex-Field Space-Time Coding," *IEEE Transactions on Signal Processing*, vol. 51, no. 11, pp. 2917–2930, November 2003.

[11] G.J. Foschini, "Layered Space-Time Architecture for Wireless Communication in a Fading Environment when Using Multiple Antennas," *Bell Labs Technical Journal*, vol. 1, no. 2, pp. 41–59, Autumn 1996.

[12] S. ten Brink and B.M. Hochwald, "Achieving near-capacity on a multiple-antenna channel," *IEEE Transactions on Communications*, vol. 51, no. 3, pp. 389–399, March 2003.

[13] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, "Space-Time Block Codes from Orthogonal Designs," *IEEE Transactions on Information Theory*, vol. 45, no. 5, pp. 1456–1467, July 1999.

[14] W. Su and X.-G. Xia, "Signal Constellations for Quasi-Orthogonal Space-Time Block Codes with Full Diversity," *IEEE Transactions on Information Theory*, vol. 50, no. 10, pp. 2331–2347, October 2004.

[15] J. Boutros and E. Viterbo, "Signal Space Diversity: A Power- and Bandwidth-Efficient Diversity Technique for the Rayleigh Fading Channel," *IEEE Transactions on Information Theory*, vol. 44, no. 4, pp. 1453–1467, 1998.

[16] X. Giraud, E. Boutillon, and J.-C. Belfiore, "Algebraic Tools to Build Modulation Schemes for Fading Channels," *IEEE Transactions on Information Theory*, vol. 43, no. 3, pp. 938–952, 1997.

[17] J.-C. Belfiore, G. Rekaya, and E. Viterbo, "The Golden Code: A 2 x 2 Full-Rate Space-Time Code with Non-Vanishing Determinants," *IEEE Transactions on Information Theory*, vol. 51, no. 4, pp. 1432–1436, April 2005.

[18] R. Böhnke, D. Wübben, V. Kühn, and K.D. Kammeyer, "Reduced Complexity MMSE Detection for BLAST Architectures," in *Proc. IEEE Global Communications Conference (Globecom'03)*, San Francisco, California, USA, December 2003.

[19] M. Witzke, *Iterative Detektion von MIMO-Signalen mit linearen und erweitert-linearen Filtern*, Ph.D. thesis, Lehrstuhl für Nachrichtentechnik - Technische Universität München, 2005.