Comparison of Receiver/Transmitter Algorithms for ODFMA with Dynamic Subcarrier Allocation

Carsten Bockelmann, Mark Petermann, Karl-Dirk Kammeyer
Department of Communications Engineering
University of Bremen, 28359 Bremen, Germany
Email:{bockelmann, petermann, kammeyer}@ant.uni-bremen.de

***Abstract*—We consider the downlink of a multiuser MIMO-OFDMA system with different transmitter and receiver concepts, where subcarriers are assigned to users according to some metric. This metric describes the channel quality and the user with the highest metric at a subcarrier is allowed to transmit. The examined transmission schemes are shown to behave differently based on the chosen metric. Furthermore, the influence of channel coding on this behavior is investigated.**

***Index Terms*—MIMO, OFDMA, Resource Allocation, Precoding**

I. Introduction

Orthogonal Frequency Division Multiple Access (OFDMA), especially for Multiple Input Multiple Output (MIMO) systems, has attracted much interest because of its abilitiy to exploit multiuser diversity. Several publications deal with the subcarrier assignment problem for the single antenna case, where only one channel coefficient per user and subcarrier determines the current channel state [1][2]. However, multiple antenna systems add the spatial dimension to be considered, which calls for different approaches. Information theory suggests the application of Dirty Paper Coding (DPC) [3] combined with joint power allocation over all users and subcarriers [4] to achieve the capacity of the system. In this paper, though, we will constrain to the case of subcarrier allocation with regard to some metric, which describes the channel quality, in contrast to the allocation of spatial transmission modes to (possibly different) users on a subcarrier [5]. The effect of dynamic subcarrier allocation metrics on specific spatial transmission modes is often neglected in favor of an information theoretic analysis [6]. We focus on a BER comparison of MIMO precoding schemes with perfect Channel State Information (CSI) at the transmitter in combination with OFDMA and dynamic subcarrier allocation. Furthermore, the influence of channel coding on subcarrier allocation will be investigated.

The remainder of the paper is organized as follows. Section II introduces the system model, which will be used throughout the paper. Section III discusses the possible channel metrics and their connection to specific transmission schemes, which are outlined in Section IV. In Section V we will show several simulation results and discuss the impact of the metrics on OFDMA systems. Finally, in Section VI the content and results of the paper will be summarized.

Notation

In the following, vectors and matrices are denoted by lower case and capital bold faced letters, respectively. We use $(\bullet)^T$ for the matrix transpose and $(\bullet)^H$ for conjugate transpose. The identity matrix of dimension n is denoted by $\mathbf{I}_n$. $\mathrm{Tr}\{\bullet\}$, $\mathrm{cond}\{\bullet\}$ and $\mathrm{diag}\{\bullet\}$ are used for the trace of a matrix, the l^2-norm condition number and a diagonal matrix with the elements of a vector in the argument on the main diagonal, respectively.

II. System Model

We consider the downlink of an OFDMA system with N_C subcarriers, N_U users, each equipped with N_R receive antennas, and a base station with N_T antennas assuming perfect CSI at both receiver and transmitter. Fig. 1 shows the general structure of the system. The received signal in frequency domain of user i at subcarrier k can be denoted as

$$\mathbf{y}_{k,i} = \mathbf{H}_{k,i}\mathbf{x}_k + \mathbf{n}_{k,i} \;, \tag{1}$$

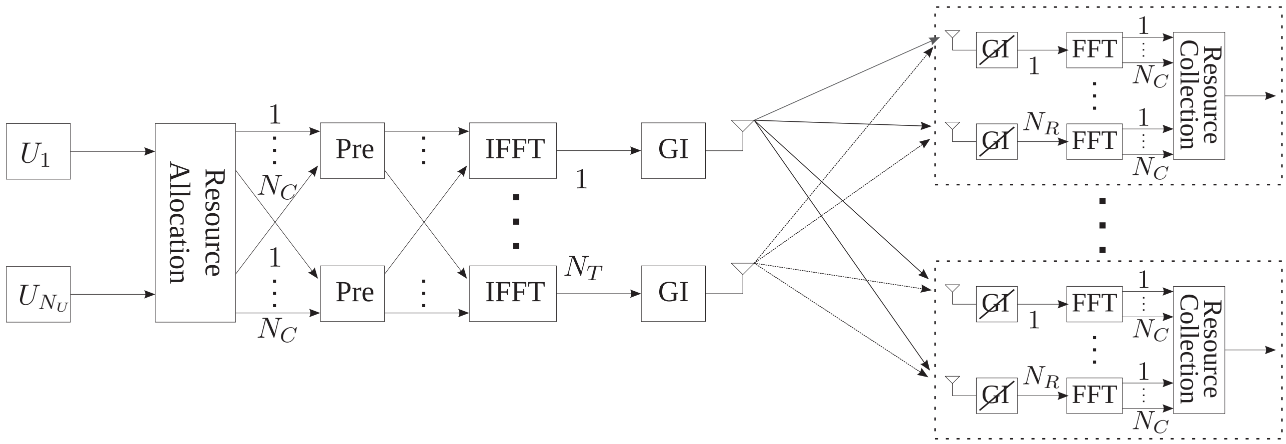

Fig. 1. System model with transmitter side precoding

where $\mathbf{y}_{k,i} \in \mathbb{C}^{N_R}$ and $\mathbf{n}_{k,i} \in \mathbb{C}^{N_R}$ are the receive vector and zero-mean circularly symmetric complex gaussian noise with covariance $\mathrm{E}\left\{\mathbf{n}_{k,i}\,\mathbf{n}_{k,i}^H\right\} = \mathbf{I}_{N_R}$, respectively. The transmit vector $\mathbf{x}_k \in \mathbb{C}^{N_T}$ is denoted without user index i as only one user per subcarrier will be served. The average subcarrier SNR is then simply defined by the average power per subcarrier P_k, where $\mathrm{E}\left\{\mathrm{Tr}\left\{\mathbf{x_k}\,\mathbf{x_k}^H\right\}\right\} = P_k$. The subcarrier channel matrices $\mathbf{H}_{k,i} \in \mathbb{C}^{N_R \times N_T}$ are obtained by the N_C-point Fourier transform of the coefficient matrices of the frequency selective channel $\mathbf{H}_i(\ell) \in \mathbb{C}^{N_R \times N_T}$, $0 \leq \ell \leq L_F - 1$, containing the delayed fading gains between the antennas. The elements of $\mathbf{H}_i(\ell)$ are i.i.d complex gaussian distributed with $\sigma_h^2 = 1$. Thus,

$$\mathbf{H}_{k,i} = \sum_{\ell=0}^{L_F-1} \mathbf{H}_i(\ell) e^{-j\Omega_k \ell}\,, \tag{2}$$

where Ω_k are the normalized equidistant sampling frequencies. The channel is assumed to be constant over one OFDM symbol, but changing independently between OFDM symbols. All other aspects of the downlink system are assumed to be perfect (e.g. long enough guard interval, perfect synchronization, etc.).

If channel coding is applied, a $[7,5]_{\text{oct}}$ convolutional code is used to encode the data of a user over one OFDM symbol only. Additionally, results for a half rate 3GPP Turbo Code [7] will be shown. Due to the constraint of coding over one OFDM symbol it may happen that a user will be assigned only a few subcarriers, which leads to short code words. This is especially disadvantageous for the turbo code as only a short interleaver can be used. Coding over more OFDM symbols will lessen this problem, but for the Rayleigh channel used, the channel allocation will inherently be fair on average.

III. Metrics

In order to decide which subcarrier is assigned to a specific user a decision rule is needed. In this paper a subcarrier will be assigned to the user i out of N_U users, whose channel metric $\zeta_{k,i} > \zeta_{k,j}\ \forall\ j \neq i$. The channel metric $\zeta_{k,i}$ characterizes the quality of the MIMO channel of user i at subcarrier k and can be chosen arbitrarily. An often considered metric for MIMO-OFDMA, especially if sum-rate results are considered, is given by the Frobenius norm of the channel matrix (3). Particularly eigenvalue based methods will be favoured by such a metric. Other possible choices are the condition number of the channel matrix (4) or the channel capacity (5).

$$\zeta_{k,i} = \sqrt{\mathrm{Tr}\left\{\mathbf{H}_{k,i}^H\,\mathbf{H}_{k,i}\right\}} \tag{3}$$

$$\zeta_{k,i} = -\mathrm{cond}\left\{\mathbf{H}_{k,i}\right\} \tag{4}$$

$$\zeta_{k,i} = \log_2\,\det\left(\mathbf{I} + P_k \mathbf{H}_{k,i}^H \mathbf{H}_{k,i}\right) \tag{5}$$

The condition number of the channel matrix promises good results for Zero Forcing (ZF) or Minimum Mean Square Error (MMSE) based transmission schemes, as this gives a measure of orthogonality of the channel matrix columns. A good condition numbers for example hints at low noise enhancement/low transmit power degradation for ZF receive/transmit filtering, respectively.

IV. Transmission Modes

The focus of this paper lies on preprocessing methods assuming perfect CSI at the receiver. As a comparison, also simple receiver centric schemes are considered, but mainly linear and non-linear precoding algorithms will be applied. Each of the schemes will be introduced shortly in the following. To ease notation the user and subcarrier indices are omitted in this section.

A. Linear Precoding

The general structure of linear precoding leads to a transmit vector $\mathbf{x} = \mathbf{G}_{\text{Tx}}\,\mathbf{d}$ with $\mathbf{d} \in \mathbb{C}^{N_T}$ being the symbol vector (with M-ary symbols). In order to control the transmit power, normalization has to be applied to the transmit filter.

$$\mathbf{x} = \sqrt{\frac{N_T}{\text{Tr}\left\{\mathbf{G}_{\text{Tx}}\,\mathbf{G}_{\text{Tx}}^H\right\}}}\mathbf{G}_{\text{Tx}}\,\mathbf{d}\;. \tag{6}$$

Hence, eq. (6) defines the transmit filter to be neutral with regard to the transmit power on average.

As the first linear precoding algorithm ZF pre-filtering will be introduced. Equivalent to receiver side ZF filtering the transmit filter matrix is chosen to be

$$\mathbf{G}_{\text{Tx,ZF}} = \mathbf{H}^H\left(\mathbf{H}\mathbf{H}^H\right)^{-1}\;. \tag{7}$$

This approach leads to a transmit power loss if the corresponding channel matrix is badly conditioned. The MMSE or regularized prefiltering [8] matrix, which can be expressed as

$$\mathbf{G}_{\text{Tx,MMSE}} = \mathbf{H}^H\left(\mathbf{H}\mathbf{H}^H + \frac{N_T}{P}\mathbf{I}_{N_T}\right)^{-1}\;, \tag{8}$$

takes this into account and leads to a compromise between power loss and interference suppression. These standard approaches are known to perform close to the according receiver side algorithms.

Another well known linear precoding technique is based on the Singular Value Decomposition (SVD) of the channel

$$\mathbf{H} = \mathbf{U}\mathbf{S}\mathbf{V}^H\;, \tag{9}$$

where $\mathbf{U}$ and $\mathbf{V}$ are unitary matrices and $\mathbf{S} = \text{diag}\left([\lambda_1, \ldots, \lambda_{N_T}]\right)$ is a diagonal matrix containing the singular values λ_j of the channel assuming $\lambda_j > 0\;\forall j \leq r$, r being the rank of the channel. At the transmitter side $\mathbf{G}_{\text{Tx}} = \mathbf{V}$ will be used. With the receiver side filter $\mathbf{G}_{\text{Rx}} = \mathbf{U}^H$ the MIMO system is then decomposed into r SISO systems which can be used to transmit data independently. Typically this decomposition is applied in combination with some rate and power allocation e.g. the Krongold algorithm [9].

B. Non-linear Precoding

Non-linear precoding structures apply the idea of Dirty Paper Coding (DPC) in practical ways. Tomlinson-Harashima-Precoding (THP) [10] has been widely applied to MIMO systems and will be the only non-linear scheme considered. The general structure as shown in Fig. 2 applies a modulo operation as the transmitter equivalent of a hard decision to the interference reduced signals. The Decision

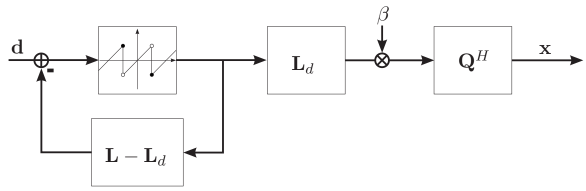

Fig. 2. General THP structure in DFE representation

Feedback structure of THP can be described via a QL-decomposition $\mathbf{H} = \mathbf{L}\mathbf{Q}$. Under the assumption of $N_R = N_T$ the elements of Fig. 2 result to

$$\beta = \sqrt{\frac{N_T}{\sum_{j=1}^{N_T} 1/L_{jj}}} \tag{10}$$

$$\mathbf{L_d} = \text{diag}\left([L_{11}, \ldots, L_{N_T N_T}]\right)\;. \tag{11}$$

Whether a ZF or MMSE approach is used depends on the QL-decomposition, which can be applied to an extended channel matrix $\underline{\mathbf{H}} = [\mathbf{H}^T\;\sqrt{\frac{N_T}{P}}\mathbf{I}_{N_R}]^T$ to achieve the MMSE solution. Furthermore, sorting can be used to enhance the performance of the scheme [11].

C. Receiver centric

In order to compare the precoding schemes to a receiver centric transmission scheme, we consider spatial multiplexing (SM) with ZF and MMSE receive filtering, where the receive filters are given by (12) and (13) respectively.

$$\mathbf{G}_{\text{Rx,ZF}} = \left(\mathbf{H}^H\mathbf{H}\right)^{-1}\mathbf{H}^H \tag{12}$$

$$\mathbf{G}_{\text{Rx,MMSE}} = \left(\mathbf{H}^H\mathbf{H} + \frac{N_T}{P}\mathbf{I}_{N_T}\right)^{-1}\mathbf{H}^H. \tag{13}$$

V. Simulation Results

The following mean BER results are achieved as the mean of the individual user's BER, which makes sense because the Rayleigh channel is fair on average, i.e. every user will perform equally on average. Throughout this section $L_F = 6$ channel taps will be used in the simulations. The equal distribution of subcarriers among all users is referred to as the ”static” case.

A. Uncoded Results

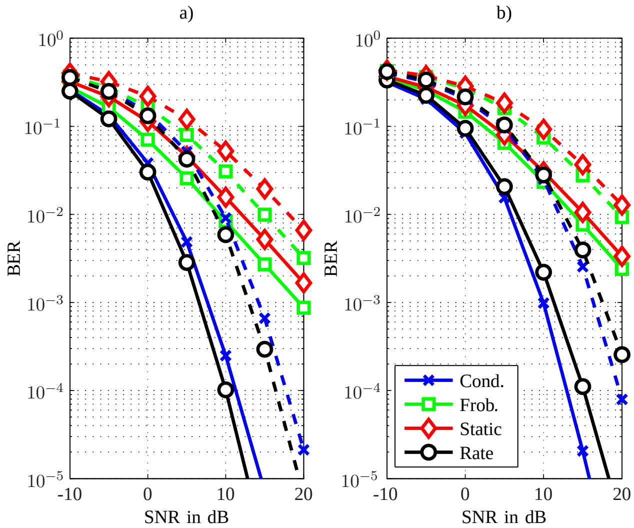

Fig. 3. a) Spatial multiplexing with receiver side ZF filtering and $N_R = N_T = 2$ antennas (— 4 bit/s/Hz, – – 8 bit/s/Hz), b) $N_R = N_T = 4$ antennas (— 8 bit/s/Hz, – – 16 bit/s/Hz); $N_C = 1024$ subcarriers, $N_U = 5$ users and $L_F = 6$.

Fig. 3 shows results for SM with receiver side ZF filtering for $N_T = N_R = 2$ and $N_T = N_R = 4$ using QPSK and 16-QAM. It can be seen, that for $N_T = N_R = 2$ the condition number metric performs well as expected, but the rate metric shows slightly better performance. For more antennas though it seems, that the condition number becomes a stronger metric with respect to the performance of ZF filtering, outperforming the rate metric. Since the performance of ZF based techniques is highly dependent on the structure (e.g. orthogonality of columns) of the channel matrix, a good condition number hints at low noise amplification which gets more important for a higher number of interfering signals. Similar results can be achieved for the SVD based linear precoding, where the Frobenius norm offers a slight advantage over the rate metric for higher numbers of antennas. Interestingly, sorting changes these results for THP as the original interference structure of the channel matrix is changed and the precoding processes the antennas in order of the channel gains (weakest first). This is very similar to the information theoretic approach of successive encoding, which may be the reason that the rate metric performs best.

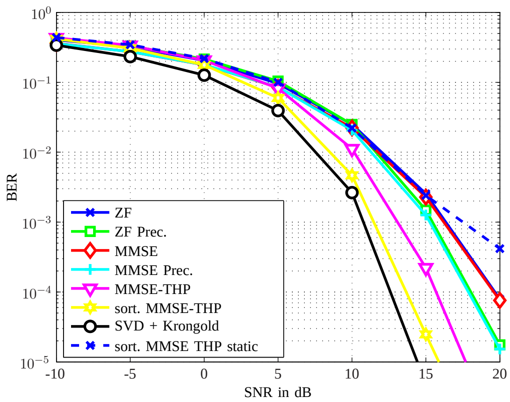

Fig. 4. a) Mean BER of different transmission schemes for $N_R = N_T = 4$ antennas at 16 bit/s/Hz; $N_C = 1024$ subcarriers, $N_U = 5$ users and $L_F = 6$.

Fig. 4 shows several transmission modes for $N_T = N_R = 4$ antennas. For every mode, the best metric was chosen (ZF/MMSE condition number, SVD Frobenius norm, sorted THP rate) to achieve the lowest possible BER. Sorted MMSE-THP offers the best BER performance without bit and power loading, whereas precoding via SVD with bit and power loading leads to the overall best performance. It can be observed that all schemes perform well with dynamic subcarrier allocation. Even ZF precoding with dynamic subcarrier allocation performs better then sorted MMSE THP in the static case.

B. Coded Results

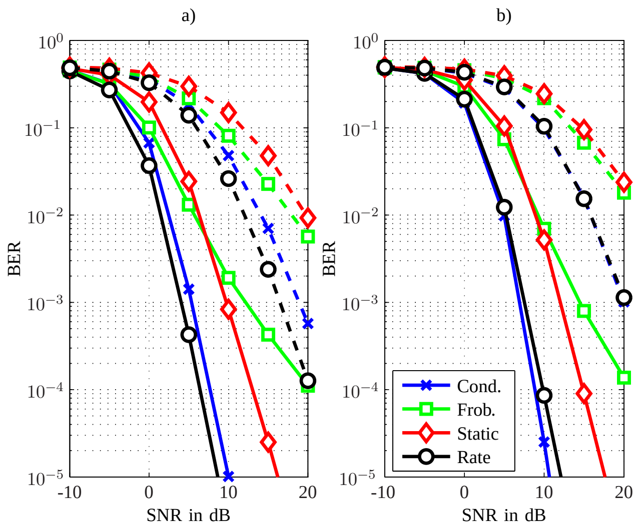

Fig. 5. a) Mean BER for a $[7, 5]$ coded spatial multiplexing system with receiver side ZF filtering at $N_R = N_T = 2$ antennas (— 4 bit/s/Hz, – – 8 bit/s/Hz), b) $N_R = N_T = 4$ antennas (— 8 bit/s/Hz, – – 16 bit/s/Hz); $N_C = 1024$ subcarriers, $N_U = 5$ users and $L_F = 6$.

The application of practical codes is mostly neglected if OFDMA resource allocation is optimized, therefore it is of much interest, to investigate the influence of codes of different strengths on several transmission schemes and channel quality metrics.

Fig. 5 shows coded results for equivalent parameters as denoted for Fig. 3. Obviously, for high

SNR the Frobenius norm leads to a performance degradation in contrast to the uncoded case where every dynamic allocation strategy performed better than the static scenario. Additionally, the observed behavior with regard to the condition number and the rate metric is preserved, but the difference in BER is smaller. Furthermore, it becomes obvious, that the used convolutional code leads to a performance gain due to the frequency diversity which can be exploited by the code. However, performance is actually degraded for $N_T = N_R = 2$ at 8 bit/s/Hz and $N_T = N_R = 4$ at 16 bit/s/Hz which leads to the conclusion, that stronger codes should be applied in such scenarios. Accordingly, Fig. 6a shows the

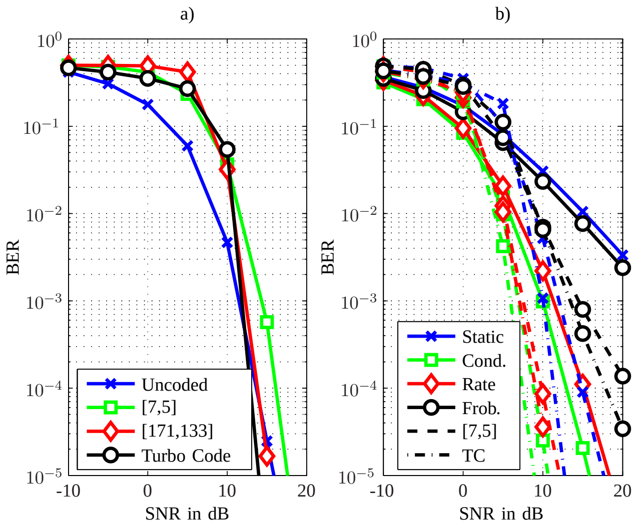

Fig. 6. Mean BER of a) sorted MMSE THP coded with different codes for $N_R = N_T = 4$ antennas and 16 bit/s/Hz and b) Spatial multiplexing for $N_R = N_T = 4$ with different codes at 8 bit/s/Hz; $N_C = 1024$ subcarriers, $N_U = 5$ users, $L_F = 6$ and rate metric.

performance of sorted MMSE THP if the rate metric is used to allocate subcarriers and different codes are applied. The general behavior does not change, but with a stronger channel code the performance can be actually enhanced in the BER region of interest. Usually one would anticipate the rate metric to outperfom other metrics if a stronger code is used, but as Fig. 6b shows this is not the case in that the condition number provides the best results. We conclude, that in terms of error rates, even for good error correcting codes, the subcarrier distribution has to be chosen well suited according to the transmission mode.

VI. Conclusions

We have shown the bit error rate performance of several MIMO transmission modes with respect to different channel quality metrics which were used to apply dynamic subcarrier allocation. In the uncoded case channel allocation with respect to the supportable channel rate, which inherently is an information theoretic measure under the assumption of ideal codes and infinite code word lengths, performs well. However, it is obvious that for higher number of antennas metrics which are well fit to the structure of the transmission scheme gain over the rate metric. Interestingly, sorted THP seems to be less constrained by the condition number of the channel matrix. With regard to the information theoretic analysis of dynamic OFDMA in the literature we can conclude that the often used Frobenius norm is a bad measure for schemes based on a ZF or MMSE design, a much more overall robust approach would be to use the rate for dynamic subcarrier allocation if no specific scheme is considered. Especially for BER optimization of coded transmission our results hint at a non rate based approach to subcarrier allocation problems as the difference to the other metrics seems not to decrease for stronger codes.

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