On Allocation Strategies for Dynamic MIMO-OFDMA with Multi-User Beamforming

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Abstract— **This paper considers the impact of different user allocation metrics and strategies on Orthogonal Frequency Division Multiple Access schemes with multi-user beamforming and receive diversity. Here, only a selected subset of all existing users is scheduled on each subcarrier. Thus, if the number of transmit antennas is less than the number of users, user selection is required. Employing strong forward error correcting codes significantly improves the performance of the systems. However, intending to maximize the sum-rate of the system does not automatically yield the best performance in terms of error rates. We analyse the difference of user allocation strategies for uncoded and coded cases with linear beamforming methods and illustrate the performances in terms of perfect and imperfect channel state information.**

Index Terms— **OFDMA, MIMO, Maximum Ratio Transmission, beamforming, multi-user communications, user selection**

I. Introduction

The utilization of Orthogonal Frequency Division Multiple Access (OFDMA) in wideband communication systems has attracted attention over the past years and is now intended for the downlink of the 3GPP Long Term Evolution (LTE) [1]. By separation of the frequency-selective channel into several orthogonal frequency-flat channels, an adaptive allocation of resources in time and frequency is offered. Furthermore, if a subcarrier is occupied by only one user, an orthogonal frequency seperation of the users is possible [2]. If multiple transmit antennas are employed in combination with single or multiple receive antennas per user (Multiple-Input Multiple-Output, MIMO), the spatial domain can be exploited by means of Space Division Multiple Access (SDMA), i.e. users sharing the same time-frequency resource are then spatially seperated by orthogonal or semi-orthogonal beamforming techniques [3]. These techniques present suboptimal solutions with affordable complexity compared to non-linear precoding techniques but may introduce multi-user interference (MUI) [4]. Moreover, multiple receive antennas provide maximum ratio combining-like receive diversity [5]. Nevertheless, the performance of the preprocessing is highly dependent on the availability of channel state information (CSI) at the transmitter.

In cellular systems the number of users requesting system resources is usually much larger than the maximum number of possible data streams, which in general is limited by the number of transmit antennas [6]. Therefore, scheduling has to be applied at the transmitter. Appropriate user selection algorithms may operate on a per subcarrier or per chunk basis in OFDMA/SDMA systems.

As strong forward error correcting codes, i.e. Turbo or potentially LDPC codes, are favoured in the previously mentioned communication system, the achievable sum-rate of user allocation strategies is an often used information theoretical measure. In contrast, only bit or frame error rates (BER/FER) indicate the behavior of a system in terms of other than sum-rate metrics. The influence of user selection metrics and algorithms on the error rate performance of a multi-user MIMO-OFDMA system with linear beamforming will be studied in this paper. We also illustrate the effect of imperfect CSI on reconsidered allocation strategies.

The remainder of the paper is organized as follows. In Section II our system model is introduced[1]. The investigated multi-user beamforming

[1]Throughout the paper capital boldface letters denote matrices. Accordingly, small boldface letters describe column vectors. The conjugate, transpose, hermitian transpose and Moore-Penrose pseudo inverse are denoted by $(\cdot)^*$, $(\cdot)^T$, $(\cdot)^H$ and $(\cdot)^+$, respectively. Furthermore, $(\cdot)^{-1}$ is the matrix inverse and $\mathbf{I}_\alpha$ is the $\alpha \times \alpha$ identity matrix. $\mathrm{tr}\{\cdot\}$ is the trace, $\mathrm{cond}\{\cdot\}$ the 2-norm condition number of a matrix, $\|\cdot\|$ describes the vector norm and $|\cdot|$ stands for the cardinality of a set. Sets are always denoted by caligraphic letters.

techniques are described in Section III. Afterwards, possible metrics for user selection criteria as well as suboptimal grouping strategies are stated in Section IV. Simulation results for systems with perfect and imperfect CSI are investigated in Section V. Finally, we summarize our major results in Section VI.

II. System Model

We consider a multi-user MIMO-OFDMA system with N_T transmit antennas, N_C subcarriers, K users and N_R receive antennas per user terminal according to Fig. 1. For simplicity, we assume that all users have the same number of receive antennas.

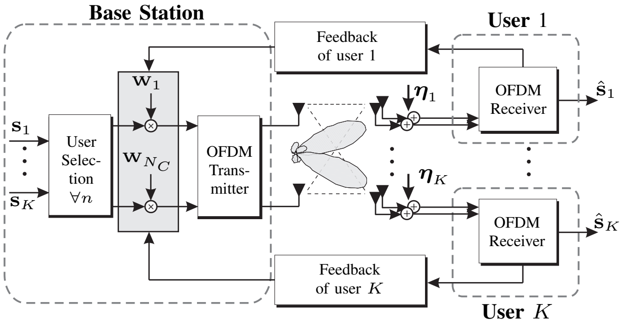

Fig. 1. Multi-user MIMO-OFDMA system model applying beamforming with N_T transmit antennas, N_C subcarriers and K users

The number of simultaneously served users on each subcarrier is usually limited by the number of transmit antennas. Thus, as we restrict ourselves to N_T definitely selected users per subcarrier, a set $\mathcal{S}_n$ with $|\mathcal{S}_n| = N_T$ active users out of $\mathcal{U} \in \{1, \ldots, K\}$ consisting of the indices of all users has to be selected for each subcarrier n. The selection process is described in Section IV. If user $i \in \mathcal{S}_n$, the downlink system in frequency domain can be described by

$$\mathbf{y}_{n,i} = \mathbf{H}_{n,i} \sum_{i \in \mathcal{S}_n} \sqrt{P_{n,i}} \mathbf{w}_{n,i} s_{n,i} + \boldsymbol{\eta}_{n,i} \ , \quad (1)$$

where $s_{n,i}$ is the corresponding transmit symbol on subcarrier n. If channel coding is applied, each user data in an OFDM symbol is independently coded (and interleaved) with a punctured 3GPP Turbo code [7]. The column vectors $\mathbf{y}_{n,i} \in \mathbb{C}^{N_R}$, $\mathbf{w}_{n,i} \in \mathbb{C}^{N_T}$ and $\boldsymbol{\eta}_{n,i} \in \mathbb{C}^{N_R}$ denote the receive vector, the precoding vector and the AWGN noise vector with variance $\sigma_\eta^2 = 1$ per element for user i, respectively. The scalar $P_{n,i}$ accounts for the power on the subcarrier. The channel $\mathbf{H}_{n,i} \in \mathbb{C}^{N_R \times N_T}$ results from the frequency-selective time-domain channel matrix $\mathbf{H}_i(\ell) \in \mathbb{C}^{N_R \times N_T}$, $0 \leq \ell \leq L_F - 1$, whose elements are i.i.d. complex Gaussian distributed. Here, L_F denotes the number of uncorrelated equal power channel taps. Hence, the channel transfer function is obtained via $\mathbf{H}_{n,i} = \frac{1}{L_F} \sum_{\ell=0}^{L_F} \mathbf{H}_i(\ell)\, \mathrm{e}^{-j\Omega_n \ell}$, where $\Omega_n = 2\pi n / N_C, 0 \leq n \leq N_C - 1$ are the equidistant sampling frequencies. $\mathbf{H}_{n,i}$ is known at the transmitter and is assumed to be constant over one OFDM symbol, but changes independently from OFDM symbol to OFDM symbol. The estimated original transmit symbols are obtained at the receiver via $\hat{s}_{n,i} = \mathbf{z}_{n,i}^H \mathbf{y}_{n,i}$, where the received signal is multiplied with the receive filter vector $\mathbf{z}_{n,i} = \frac{\mathbf{H}_{n,i} \mathbf{w}_{n,i}}{\|\mathbf{H}_{n,i} \mathbf{w}_{n,i}\|}$ to account for multiple receive antennas of the users [5].

Considering imperfect CSI, we assume a simple error model, where the channel matrix $\mathbf{H}_{n,i}$ is calculated by

$$\mathbf{H}_{n,i} = \rho \, \hat{\mathbf{H}}_{n,i} + \sqrt{1 - \rho^2} \, \boldsymbol{\Psi}_{n,i} \ , \quad (2)$$

where $\hat{\mathbf{H}}_{n,i}$ is an unbiased channel estimate obtained by a first order Minimum Mean Square Error (MMSE) channel predictor and ρ is a CSI degradation factor. $\boldsymbol{\Psi}_{n,i} \in \mathbb{C}^{N_R \times N_T}$ is an error matrix with i.i.d. complex Gaussian distributed entries according to $\mathcal{N}_C\left(\mathbf{0}_{N_R \times N_T}, L_F N_R \mathbf{I}_{N_T}\right)$. Within this model, the perfect CSI case is achieved by $\rho = 1$.

III. Linear Beamforming Strategies

In this contribution, we investigate two linear beamforming approaches, Zero-Forcing beamforming (ZF-BF) and regularized beamforming [3], [8]. Therefore, we assume projected channel matrices $\tilde{\mathbf{h}}_{n,i}^T = \mathbf{u}_{n,i}^H \mathbf{H}_{n,i}$ to account for multiple receive antennas. The vector $\mathbf{u}_{n,i}$ denotes the first left singular vector belonging to the strongest singular value of $\mathbf{H}_{n,i}$. Due to this, the beams are steered into preferred directions of the selected users. Accordingly, $\tilde{\mathbf{H}}_n(\mathcal{S}_n) = [\tilde{\mathbf{h}}_{n,1}, \ldots, \tilde{\mathbf{h}}_{n,N_T}]^T$ is the channel of subset $\mathcal{S}_n$ on subcarrier n, which is used in order to calculate the beamforming vectors. Now, according to this assumption, the beamforming vectors in ZF-BF are chosen to fulfill the orthogonality condition $\tilde{\mathbf{h}}_{n,i}^T \mathbf{w}_{n,j} = 0$ for $j \neq i$, $i, j \in \mathcal{S}_n$. This can be done via the Moore-Penrose pseudo-inverse of $\tilde{\mathbf{H}}_n(\mathcal{S}_n)$ leading to matrix $\tilde{\mathbf{W}}_n(\mathcal{S}_n) = [\tilde{\mathbf{w}}_{n,1}, \ldots, \tilde{\mathbf{w}}_{n,N_T}] = \tilde{\mathbf{H}}_n(\mathcal{S}_n)^+$. The pseudo-inverse is given by

$$\tilde{\mathbf{W}}_n(\mathcal{S}_n) = \tilde{\mathbf{H}}_n(\mathcal{S}_n)^H \left(\tilde{\mathbf{H}}_n(\mathcal{S}_n) \, \tilde{\mathbf{H}}_n(\mathcal{S}_n)^H \right)^{-1} . \quad (3)$$

In [3] it was shown, that this scheme becomes optimal in terms of sum-rate as the number of users increases.

To further optimize the system, an often encountered approach to design the preprocessing matrix $\tilde{\mathbf{W}}_n(\mathcal{S}_n)$ is to use a regularized pseudo-inverse, also referred to as MMSE beamforming, such that

$$\tilde{\mathbf{W}}_n(\mathcal{S}_n) = \tilde{\mathbf{H}}_n(\mathcal{S}_n)^H \left(\tilde{\mathbf{H}}_n(\mathcal{S}_n)\,\tilde{\mathbf{H}}_n(\mathcal{S}_n)^H + \beta \mathbf{I}_{N_T}\right)^{-1}, \quad (4)$$

where the coefficient β is typically chosen to maximize the signal-to-interference plus noise ratio (SINR). Here, it is set to $\beta = N_T/P_{n,i}$. Consequently, the aforementioned orthogonality condition of ZF-BF no longer holds and multi-user interference occurs. In return, less power loss at the transmitter is achieved. To account for the power constraint on the precoding vectors, in both cases the vectors $\tilde{\mathbf{w}}_{n,i}$ are normalized by

$$\mathbf{w}_{n,i} = \sqrt{\frac{N_T}{\operatorname{tr}\left\{\tilde{\mathbf{W}}_n(\mathcal{S}_n)\,\tilde{\mathbf{W}}_n(\mathcal{S}_n)^H\right\}}}\,\tilde{\mathbf{w}}_{n,i}\,. \quad (5)$$

IV. User Selection

In this section the applied user allocation and scheduling strategies are briefly described. To show the performance of different metrics, we evaluate ZF-BF as well as regularized BF in terms of an exhaustive search over all user combinations. In real systems this is infeasible. Hence, some additional suboptimal algorithms are also investigated. As mentioned before, we restrict ourselves to algorithms, which guarantee $|\mathcal{S}| \overset{!}{=} N_T$. It is well known that the performance of ZF-BF is increased if the number of users is less than N_T due to diminishing MUI and the better conditioning of the pseudo-inverse [9]. Algorithms like that are comparable in terms of capacity but not in terms of error rates.

Note that using our specific channel model, all algorithms considered here are inherently fair on average.

A. Exhaustive Search Metrics

In an exhaustive search $\frac{K!}{N_T!(K-N_T)!}$ possible combinations have to be considered. Despite the exponential growth in complexity the following user selection metrics are taken into account for evaluation purposes. Here, each user served gets the same amount of power on a single subcarrier, no power loading is applied.

First, the transmitter can select a SDMA group per subcarrier by choosing those users who together form a channel matrix $\tilde{\mathbf{H}}_n(\mathcal{S}_n)$ with maximum Frobenius norm as depicted in (6). Another method is using the maximum sum-rate metric as stated in (7), which is generally used in information theory [10]. A different approach is to use the 2-norm condition number of $\tilde{\mathbf{H}}_n(\mathcal{S}_n)$ as the figure of merit. As this number describes the relation of the maximum non-zero singular value to the minimum non-zero singular value, the corresponding metric in (8) avoids ill-conditioned user sets. The fourth and last metric is the well-known mean square error (MSE) criterion for the ZF solution as depicted in equation (9), which has to be minimized in order to find the optimal user grouping [11]. It corresponds to the maximization of the sum of the individual user signal-to noise ratios (SNR).

$$\mathcal{S}_n^{(\mathrm{Fro.})} = \underset{\mathcal{T}_n \in \mathcal{U}}{\operatorname{argmax}} \sqrt{\operatorname{tr}\left\{\tilde{\mathbf{H}}_n(\mathcal{T}_n)^H\,\tilde{\mathbf{H}}_n(\mathcal{T}_n)\right\}} \quad (6)$$

$$\mathcal{S}_n^{(\mathrm{Rate})} = \underset{\mathcal{T}_n \in \mathcal{U}}{\operatorname{argmax}} \log_2 \det\left(\mathbf{I}_{N_T} + \frac{P_{n,i}}{\sigma_\eta^2}\tilde{\mathbf{H}}_n(\mathcal{T}_n)\,\tilde{\mathbf{H}}_n(\mathcal{T}_n)^H\right) \quad (7)$$

$$\mathcal{S}_n^{(\mathrm{Cond.})} = \underset{\mathcal{T}_n \in \mathcal{U}}{\operatorname{argmin}} \operatorname{cond}\left\{\tilde{\mathbf{H}}_n(\mathcal{T}_n)\right\} \quad (8)$$

$$\mathcal{S}_n^{(\mathrm{MSE})} = \underset{\mathcal{T}_n \in \mathcal{U}}{\operatorname{argmin}} \operatorname{tr}\left\{\left(\left(\tilde{\mathbf{H}}_n(\mathcal{T}_n)^H\,\tilde{\mathbf{H}}_n(\mathcal{T}_n)\right)^{-1}\right)^2\right\}. \quad (9)$$

For regularized beamforming the scheduler selects user group $\mathcal{S}_n$ with the MSE metric by using $\mathcal{S}_n^{(\mathrm{MSE/Reg.})} = \underset{\mathcal{T}_n \in \mathcal{U}}{\operatorname{argmin}} \operatorname{tr}\left\{\left(\left(\tilde{\mathbf{H}}_n(\mathcal{T}_n)^H\tilde{\mathbf{H}}_n(\mathcal{T}_n)+\beta\mathbf{I}_{N_T}\right)^{-1}\right)^2\right\}$ instead of equation (9). Thus, this MSE equation corresponds to the precoding matrix $\tilde{\mathbf{W}}_n(\mathcal{S}_n)$ from equation (4). Hence, all four metrics can be applied for ZF-BF as well as for regularized beamforming.

B. Suboptimal Algorithms

As all previously mentioned exhaustive search metrics are far too complex, we also restate some existing algorithms, which decrease the user search space by employing iterative procedures. The following enumeration summarizes the analysed algorithms:

1) Greedy Correlation-based Algorithm (GCBA): In this allocation algorithm the base station initially selects the user with the highest vector norm. Other users are added based on the sum of their correlation coefficients such that

$$\mathcal{S}_n \leftarrow \mathcal{S}_n \cup \underset{k \setminus \mathcal{S}_n}{\operatorname{argmin}} \sum_{i \in \mathcal{S}_n} \rho_{ik} \quad (10)$$

with

$$\rho_{ik} = \frac{\tilde{\mathbf{h}}_{n,i}^T\tilde{\mathbf{h}}_{n,k}^*}{\left\|\tilde{\mathbf{h}}_{n,i}^T\right\|\left\|\tilde{\mathbf{h}}_{n,k}^T\right\|}\,. \quad (11)$$

This algorithm was used for initial SDMA grouping in [12].

2) Greedy ZF Dirty-Paper (ZF-DP) Algorithm: This algorithm was introduced by Tu and Blum [13] and was used e.g. in [14]. Here, it selects N_T out of K rows of the overall channel matrix, which contains all projected user vectors $\tilde{\mathbf{h}}_{n,k}^T$. The iterative allocation selects the user i with the maximum channel norm on the orthogonal complement of the subspace spanned by the channels of already selected users.

3) Greedy Condition Number-based Algorithm (GCNBA): This algorithm is similar to the algorithm in 2). In each iteration the user, whose contribution to the conditioning of the current total channel matrix is the best (small condition number required), is selected. The algorithm is briefly described in algorithm 1 and has to be accomplished for all subcarries independetly.

Algorithm 1 User selection with GCNBA

Set $i = 1$, $\mathcal{S}_0 = \emptyset$ and $\mathcal{T}_0 \in \{1, \ldots, K\}$
Find $k_1 = \underset{k}{\operatorname{argmax}}\, \tilde{\mathbf{h}}_k^T \tilde{\mathbf{h}}_k^*$
Set $\mathcal{S}_1 \leftarrow \mathcal{S}_0 \cup \{k_1\}$, $\mathcal{T}_1 \leftarrow \mathcal{T}_0 \setminus \{k_1\}$, $\tilde{\mathbf{H}}(\mathcal{S}_1) = \tilde{\mathbf{h}}_1^T$
for $i = 2 : N_T$ **do**
  for $k \in \mathcal{T}_{i-1}$ **do**
    $c_k = \operatorname{cond}\left\{ \left[\tilde{\mathbf{H}}(\mathcal{S}_{i-1})^T\, \tilde{\mathbf{h}}_k \right]^T \right\}$
  end for
  Find $k_i = \underset{k}{\operatorname{argmin}}\, c_k$
  Set $\mathcal{S}_i \leftarrow \mathcal{S}_{i-1} \cup \{k_i\}$, $\mathcal{T}_i \leftarrow \mathcal{T}_{i-1} \setminus \{k_i\}$
  Update $\tilde{\mathbf{H}}(\mathcal{S}_i) = \left[\tilde{\mathbf{H}}(\mathcal{S}_{i-1})^T\, \tilde{\mathbf{h}}_{k_i} \right]^T$
end for

V. Simulation Results

A. Perfect CSI

Average BER results for the perfect CSI case ($\rho = 1$) applying different beamforming techniques and user allocation strategies in an uncoded scenario are stated in Fig. 2. In all simulations the number of transmit antennas is $N_T = 4$, $N_C = 1024$ subcarriers are available, the channel has $L_F = 6$ taps, the gross bit rate is 2 bit/s/Hz (QPSK) for uncoded and 1 bit/s/Hz (BPSK) for coded results. Here, the number of users is set to $K = 5$, where each user terminal has $N_R = 2$ receive antennas. The allocation procedures are compared with a so-called "static" case, where the users are randomly distributed among all subcarriers regardless any metric. It can be seen that proper user allocations seriously outperform the static case. The MSE criterion is the best metric here, whereas the Frobenius norm metric gives no significant gain. For the uncoded case the rate metric becomes worse in high SNR regions, the condition number metric approaches the MSE criterion. Hence, it gives a good measure of orthogonality between the beamforming vectors. Furthermore, the ZF and MMSE beamforming schemes perform almost equal for all metrics but the MSE criterion, which considers the MMSE precoder in the search. The suboptimal algorithms ZF-DP and the GCNBA perform equal and close to the optimal MSE solution. The GCBA is the worst of the suboptimal schemes but still gives a 2.5 dB gain at $3 \cdot 10^{-3}$ BER compared to the norm metric. Consequently, a good conditioning of the overall channel matrix per subcarrier is a good criterion in terms of linear beamforming.

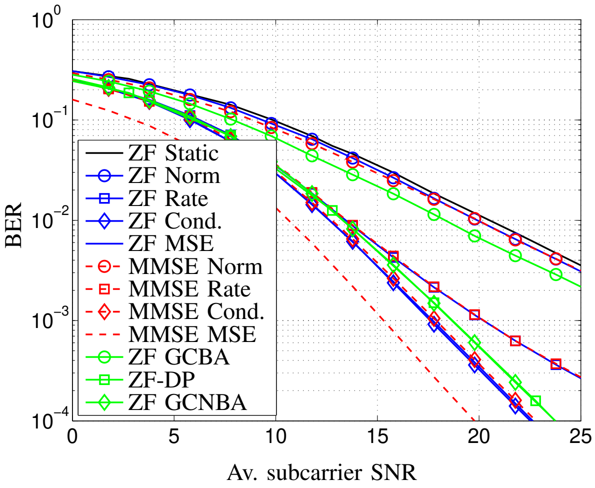

Fig. 2. Average BER comparison of allocation strategies for different BF schemes with $N_T = 4$ and $K = 5$ users with $N_R = 2$ for uncoded transmission with $N_C = 1024$ subcarriers and an average rate of 2 bit/s/Hz (QPSK); $L_F = 6$

The average FER performance curves in Fig. 3 show similar results for higher multi-user diversity ($K = 10$). The rate metric performs better with strong coding as information theoretical presumptions are more appropriate. Nevertheless, a gap of 0.25 dB at 1% FER w.r.t. the MSE criterion is still visible. A small gap of 1 dB between the condition number metric and the suboptimal results for ZF-DP and GCNBA rule out exhaustive search metrics due to complexity issues. Interestingly, the Frobenius norm metric performs worse than a random allocation in high SNR regions. Thus, this metric leads to bad conditioned channel matrices $\tilde{\mathbf{H}}_n(\mathcal{S}_n)$ even for a larger number of users. The receive diversity gain of the system is exemplarily shown for the GCBA with $N_R = 4$ receive antennas. Almost equal results are obtained for higher rates.

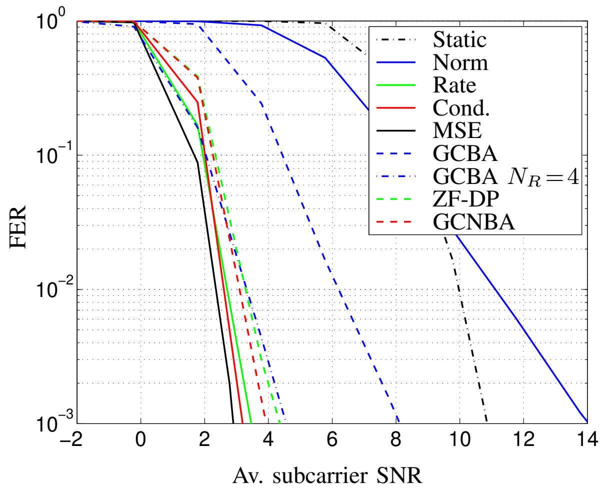

Fig. 3. Average FER comparison of allocation strategies for ZF-BF with $N_T = 4$ and $K = 10$ users with $N_R = 2$ for coded transmission using the punctured 3GPP Turbo code (code rate 1/2) with $N_C = 1024$ subcarriers and an average rate of 1 bit/s/Hz (BPSK); $L_F = 6$; perfect CSI

B. Imperfect CSI

Results for imperfect CSI are depicted in Fig. 4. Considering our model in equation (2), the degradation factor is set to $\rho = 0.9$, which corresponds to a channel predictor MSE of around 0.5. With this large error it can be seen that applying arbitrary as well as norm- or correlation-based user assignments lead to an error floor in the FER performance even for small rates. That is, only appropriate user allocation methods based on MSE, rate or condition number criteria are robust enough to cope with such large errors. E.g. a gain of 7.5 dB compared to the static case can be achieved with the suboptimal allocations ZF-DP and GCNBA at a FER of 10%.

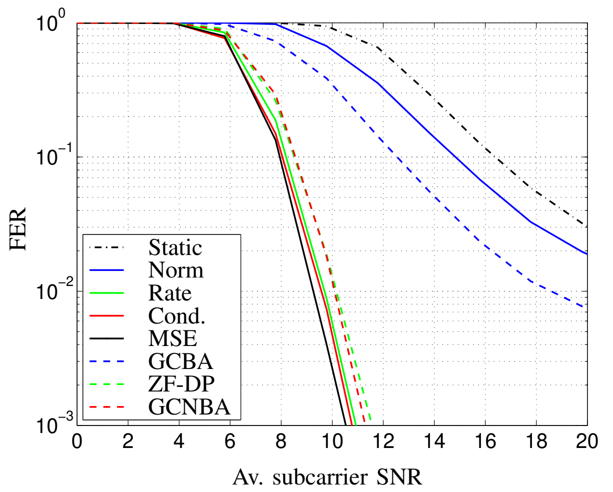

Fig. 4. Average FER comparison of allocation strategies for ZF-BF with $N_T = 4$ and $K = 5$ users with $N_R = 2$ for coded transmission using the punctured 3GPP Turbo code (code rate 1/2) with $N_C = 1024$ subcarriers and an average rate of 1 bit/s/Hz (BPSK); $L_F = 6$; imperfect CSI with $\rho = 0.9$

VI. Conclusions

In this paper, we presented results for user selection metrics and algorithms in a MIMO OFDMA system with multi-user beamforming and receive diversity. The aim is to minimize the error probability while using all transmission modes available in the system. We showed that maximizing the sum-rate is not the best metric provided and can be outperformed by considering MSE and condition number criteria. Applying strong error correcting codes can compensate for imperfect CSI if adequate user allocation is employed. This also holds for suboptimal allocation strategies if they refer to the aforementioned metrics.

References

[1] 3GPP TSG RAN: "Feasibility study for evolved Universal Terrestrial Radio Access (UTRA) and Universal Terrestrial Radio Access Network (UTRAN) (Release 7)", *Technical Report*, TR 25.912 V7.1.0, September 2006.

[2] C. Bockelmann, M. Petermann and K.D. Kammeyer: "Comparison of Receiver/Transmitter Algorithms for OFDMA with Dynamic Subcarrier Allocation", *in Proc. of 12th International OFDM-Workshop (InOWo)*, Hamburg, Germany, August 2007.

[3] T. Yoo and A. Goldsmith: "On the Optimality of Multiantenna Broadcast Scheduling using Zero-Forcing Beamforming", *IEEE Journal on Sel. Areas in Comm.*, Vol. 24, No. 3, pp. 528-541, March 2006.

[4] G. Caire and S. Shamai: "On the Achievable Throughput of a Multi-Antenna Gaussian Broadcast Channel", *IEEE Trans. on Inf. Theory*, Vol. 49, No. 7, pp. 1691-1706, July 2003.

[5] T.K.Y. Lo: "Maximum Ratio Transmission", *IEEE Trans. on Comm.*, Vol. 47, No. 10, pp. 1458-1461, October 1999.

[6] Q.H. Spencer, A.L. Swindlehurst and M. Haardt: "Zero-Forcing Methods for Downlink Spatial Multiplexing in Multiuser MIMO Channels", *IEEE Trans. on Signal Proc.*, Vol. 52, No. 2, pp. 461-471, Februar 2004.

[7] 3GPP TSG RAN: "Multiplexing and Channel Coding (FDD) (Release 7)", *Technical Specification*, TS 25.212 V7.5.0, May 2007.

[8] M. Stojnic, H. Vikalo and B. Hassibi: "Rate Maximization in Multi-Antenna Broadcast Channels with Linear Preprocessing", *IEEE Trans. on Wireless Comm.*, Vol. 5, No. 9, pp. 2338-2342, September 2006.

[9] Z. Shen, R. Chen, J.G. Andrews, R.W. Heath, Jr. and B.L. Evans: "Low Complexity User Selection Algorithms for Multiuser MIMO Systems with Block Diagonalization", *IEEE Trans. on Signal Proc.*, Vol. 53, No. 10, pp. 3658- 3663, September 2006.

[10] M. Petermann, K.D. Kammeyer and W. Xu: "Sum-Rate Evaluation of Beamforming with Limited Feedback for the Evolved UTRA Downlink", *in Proc. of ITG/IEEE Workshop on Smart Antennas*, Vienna, Austria, February 2007.

[11] M. Kountouris, D. Gesbert and L. Pittman: "Transmit Correlation-Aided Opportunistic Beamforming and Scheduling", *in Proc. of 14th European Signal Processing Conference*, Florence, Italy, September 2006.

[12] T.F. Maciel and A. Klein: "On the Performance of SDMA with Soft Dropping and SINR Balancing Power Control in the Downlink of Multi-User MIMO Systems", *in Proc. of ITG/IEEE Workshop on Smart Antennas*, Vienna, Austria, February 2007.

[13] Z. Tu and R.S. Blum: "Multiuser diversity for a dirty paper approach", *IEEE Comm. Letters*, Vol. 7, No. 8, pp. 370-372, August 2003.

[14] G. Dimic and N.D. Sidiropoulos: "On Downlink Beamforming With Greedy User Selection: Performance Analysis and a Simple New Algorithm", *IEEE Trans. on Signal Proc.*, Vol. 53, No. 10, pp. 3857-3868, October 2005.