MINIMUM MSE RELAYING IN CODED NETWORKS

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ABSTRACT

In order to combine the advantages of Amplify-Forward (AF) and Decode-Forward (DF) in relay networks, several strategies have been developed making use of reliability information after decoding at the relay. The use of soft-output channel decoders enables forwarding reliability information for the decoded bits. This soft information can be forwarded in different ways, e.g. by transmission of the Log Likelihood Ratios (LLRs) normalized to the power constraint. In this paper transmitting expectation values after decoding, the so-called Decode-Estimate-Forward (DEF) scheme, will be shown to be the best choice in terms of the mean squared uncorrelated error at the receiver. Additionally, it will be shown that LLR combining is superior to maximum ratio combining at the receiver as the overall disturbance of the received signal is not Gaussian in the case of DEF. Furthermore, in the uncoded case LLR combing also improves the performance in terms of effective signal-to-noise ratio and bit error rate.

1. INTRODUCTION

Soft information relaying is of increasing interest in relay networks. This approach combines the advantages of the classical relay protocols Amplify-Forward (AF) and Decode-Forward (DF) [1]. DF makes use of the discrete alphabet and of the coding gain in a coded system, but suffers from error propagation in the case of decoder failure at the relay. AF ignores the benefits of channel coding and discrete alphabets, but avoids error propagation and preserves reliability information.

The basic idea of soft information relaying is to benefit from the coding gain while still transmitting reliability information. When a soft output decoder is applied at the relay, the resulting soft values provide both, coding gain and reliability information and are therefore used for forwarding. Now the question arises in which way this information should be transmitted to achieve the best performance at the receiver when considering power constraints at the relay. For the uncoded case the optimal way of transmitting soft information in terms of the mean squared error (MSE) was derived analytically in [2] and was called Estimate-Forward (EF). The first approach for coded systems to transmit soft information after decoding at the relay was to transmit the log likelihood ratio (LLR) of each code bit normalized to the power constraint. This approach called Decode-Amplify-Forward (DAF) was applied e.g. in [3, 4, 5]. In contrast to this, in [6] so-called soft bits representing the expectation values of the code bits are used, but without a motivation or comparative analysis. The overall disturbance of the signal forwarded by the relay was assumed to be Gaussian distributed and the Log-Likelihood Ratios (LLRs) at the receiver were calculated based on this assumption. However, the exact distribution of these noisy soft bits was independently derived in [7] and [8] and used for calculation of LLRs at the receiver.

In this paper the optimality of transmitting the expectation values in terms of MSE will be extended to the coded case when assuming a-posteriori probability (APP) decoding at the relay. The resulting relay function is called Decode-Estimate-Forward (DEF) and will be compared to classical Decode-Forward (DF) and Decode-Amplify-Forward (DAF) in terms of MSE and bit error rate (BER) performance. It will be shown that transmitting the conditioned expectation values of the code bits is best in terms of receiver mean squared error also in the coded case. Furthermore, for EF and DEF the benefit of calculating LLRs based on the exact distribution of the received signal will be elaborated.

The paper is organized as follows: The system model of the relay network is introduced in Section 2. In Section 3 the basic idea of soft relaying is explained and specific relay functions are introduced. The distribution of the received signal in the case of Decode-Estimate-Forward is derived analytically in Section 4 and its Gaussian approximation is described. Simulation results for different system setups are shown and discussed in Section 5 before Section 6 gives a conclusion of the presented work.

2. SYSTEM MODEL

In this paper a general (hybrid) relay network is considered as shown in Figure 1. We restrict ourselves to a system with one source S and one destination D. Between these nodes several

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relays in serial and/or in parallel exist. It is assumed that there is no direct transmission from the source to the destination. The number of serial relays in one path is denoted as N_s and the number of these paths in parallel as N_p. $R_{p,s}$ denotes the s-th relay in the p-th hop.

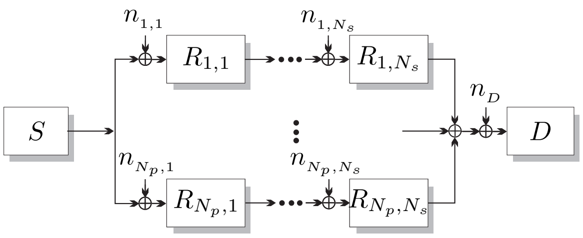

Fig. 1. Block diagram of a relay network

The source encodes the information bit vector[1]

$$\mathbf{b} = (b_1, b_2, ..., b_{N_u}) \tag{1}$$

with a channel code $\mathcal{C}$ and broadcasts

$$\mathbf{x}_s = \sqrt{P_s} \cdot \mathbf{c} \tag{2}$$

containing the BPSK-modulated code bit sequence $\mathbf{c}$ normalized to the transmit power P_s. This signal is transmitted to the N_p relays in the first hop ($s = 1$). To simplify the derivations, the channels between all nodes are assumed to be AWGN channels with noise variance $\sigma^2_{p,s}$. The transmit power of relay $R_{p,s}$ is denoted as $P_{p,s}$. The received signal at relay $R_{p,s+1}$ is then given by

$$\mathbf{y}_{p,s+1} = \mathbf{x}_{p,s} + \mathbf{n}_{p,s+1} = \frac{\sqrt{P_{p,s}}}{\sqrt{\mathrm{E}\left\{|\tilde{c}_{p,s}|^2\right\}}} \tilde{\mathbf{c}}_{p,s} + \mathbf{n}_{p,s+1} \ , \tag{3}$$

with the transmitted signal $\mathbf{x}_{p,s}$ containing the estimate $\tilde{\mathbf{c}}_{p,s}$ of the code bits normalized to the power constraint. It is assumed that the links in this network do not disturb each other which can for example be ensured by a TDMA structure. At the destination all signals transmitted from the relays in the last hop are combined, e.g. by a maximum ratio combiner (MRC) before channel decoding is applied. Links are only considered between successive relays in one path, i.e., form $R_{p,s}$ to $R_{p,s+1}$. The relaying schemes are not mixed in a network, i.e., all relays use the same relay function.

3. SOFT RELAYING

In the case of classical Decode-Forward (DF), the received signal is decoded at the relay and the estimate is equal to the hard decision at the output of the channel decoder. This estimates $\tilde{c}^{DF}_{p,s} = \hat{c}_{p,s}$ are forwarded to the destination

$$x^{DF}_{p,s} = \sqrt{P_{p,s}} \cdot \hat{c}_{p,s} \ . \tag{4}$$

[1] Throughout the paper vectors are denoted as bold letters and elements as italic letters, e.g. $\mathbf{b}$ and b. Estimates of bits are identified with a tilde $\tilde{b}$ and hard estimates by $\hat{b}$.

If a soft-input-soft-output channel decoder is applied at the relay, reliability information about the information and the code bits are available. If the decoder is an APP decoder, the resulting soft information can be described by LLRs

$$L(c|\mathbf{y}_{p,s}, \mathcal{C}) \triangleq L_{p,s}(c) = \log \left(\frac{p(c = +1|\mathbf{y}_{p,s}, \mathcal{C})}{p(c = -1|\mathbf{y}_{p,s}, \mathcal{C})} \right) \ . \tag{5}$$

In several publications (e.g. [4, 5, 3]) these LLRs are simply scaled to the transmit power constraint and then forwarded. This relay function is called Decode-Amplify-Forward (DAF) [3] and the transmitted signal from $R_{p,s}$ to the next relay in this path $R_{p,s+1}$ can be written as

$$\begin{aligned} x^{DAF}_{p,s} &= \sqrt{P_{p,s}} \cdot \frac{\tilde{c}^{DAF}_{p,s}}{\sqrt{\mathrm{E}\left\{|\tilde{c}^{DAF}_{p,s}|^2\right\}}} \\ &= \sqrt{\frac{P_{p,s}}{\mathrm{E}\left\{|L_{p,s}(c)|^2\right\}}} \cdot L_{p,s}(c) \ . \end{aligned} \tag{6}$$

DAF corresponds to AF in an uncoded system as the forwarded signal is a linear function of the LLRs. Therefore, the output of DAF is similar to AF for a signal received over a channel with increased effective SNR due to decoding. Likewise, DF is similar to demodulate-and-forward in the uncoded case except the additional decoding. Another approach was derived in [2] for an uncoded system with the aim of minimizing the residual error at the destination. For this purpose the source-relay channel and the relay function itself are modeled as one superchannel as depicted in Figure 2. The equivalent noise η on this superchannel is defined to be the uncorrelated error between the code bits and the corresponding estimates at the relay. By minimizing this noise variance of the over-

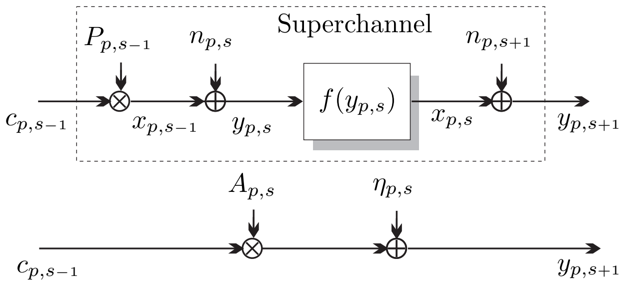

Fig. 2. Definition of a superchannel

all channel including the relay-destination channel we get the minimum mean squared uncorrelated error (MSUE) and the corresponding relay function

$$x_{p,s} = \sqrt{P_{p,s}} \cdot \frac{\tilde{c}_{p,s}}{\sqrt{\mathrm{E}\left\{|\tilde{c}_{p,s}|^2\right\}}} = f\left(\mathbf{y}_{p,s}\right) = f\left(y_{p,s}\right) \tag{7}$$

is optimal in terms of MSUE at the receiver. The last equality is only valid in the case of uncoded AWGN (and therefore memoryless) channels because one bit $x_{p,s}$ of the transmit signal only depends on one element of the input signal. In a coded system the relay can make use of the channel code and the relay function can be extended to

$$x_{p,s} = f\left(\mathbf{y}_{p,s}, \mathcal{C}\right) \ . \tag{8}$$

With channel code (e.g. convolutional code) the channel is no longer memoryless and the whole receive vector has to be considered. As shown for uncoded transmission in [2], the conditional expectation $\mathrm{E}\left\{c|\mathbf{y}\right\}$ of the transmitted bits minimizes the MSUE at the destination. The only difference between minimizing MSE and MSUE is a scaling factor and therefore we will focus on MSE due to simpler derivations. A detailed discussion of the relation between MSE and MSUE can be found in [2]. In a coded system the knowledge of the code can be incorporated in the estimation as an additional constraint

$$\mathrm{MSE} = \mathrm{E}\left\{\left(\tilde{c} - c\right)^2 |\mathbf{y}, \mathcal{C}\right\} \tag{9}$$

and the function yielding the minimum MSE can be found by setting its derivation to zero

$$\frac{\partial \mathrm{MSE}}{\partial \tilde{c}} = 2\mathrm{E}\left\{\left(\tilde{c} - c\right)|\mathbf{y}, \mathcal{C}\right\} \stackrel{!}{=} 0 \tag{10}$$

leading to

$$\tilde{c} = \mathrm{E}\left\{c|\mathbf{y}, \mathcal{C}\right\} \ . \tag{11}$$

This conditional expectation in the special case of decoding at the relay can be expressed in terms of LLRs as

$$\tilde{c}_{p,s}^{DEF} = \mathrm{E}\left\{c|\mathbf{y}, \mathcal{C}\right\} = \tanh\left(L_{p,s}(c)/2\right) \tag{12}$$

and is called soft bit. The transmit signal for this scheme called Decode-Estimate-Forward (DEF) becomes the normalized expectation value

$$\begin{aligned} x_{p,s}^{DEF} &= \sqrt{P_{p,s}}\,\frac{\tilde{c}_{p,s}^{DEF}}{\sqrt{\mathrm{E}\left\{|\tilde{c}_{p,s}^{DEF}|^2\right\}}} \\ &= \sqrt{\frac{P_{p,s}}{\mathrm{E}\left\{|\tanh\left(L_{p,s}(c)/2\right)|^2\right\}}} \cdot \tanh\left(L_{p,s}(c)/2\right) \ . \end{aligned} \tag{13}$$

This result is very similar for coded and uncoded systems and justifies the usage of soft bits for relaying in [6, 7, 8]. Although the bit and frame error rates (BER/FER) are the most interesting parameters in relay systems, the MSE is considered here due to the convenient analysis. In case of memoryless Gaussian disturbance of the superchannel the BER is proportional to the MSE. But the overall error is influenced or even caused by the relay function and therefore may be arbitrarily distributed. The BER optimal relay function for the uncoded case was derived in [9] and came out to be a $LambertW$ function. This relay function is more complicated because it depends not only on the parameters of the source-relay channel but also on those of the following channel. These parameters are in general not known to the relay so additional signaling would be necessary. Furthermore the normalization to the power constraint is quite complex as the scaling variable is within the argument of the $LambertW$ function and cannot be calculated separately. The MSE minimizing function used for DEF is quite simple and it will be shown later that MSE nevertheless seems to be a suitable parameter for performance.

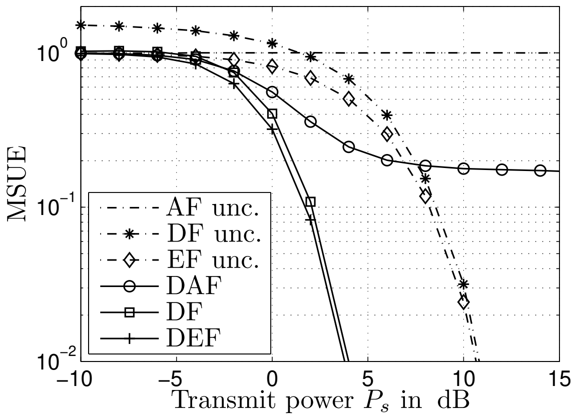

Fig. 3. Mean squared uncorrelated error at the output of the relay over transmit power for uncoded and convolutionally coded system, ($d_f = 5,\ L_c = 3$)

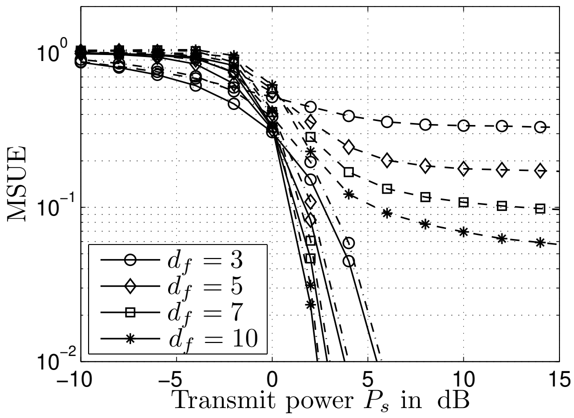

Fig. 4. Mean squared uncorrelated error at the output of the relay over transmit power for coded DF (-·-), DEF (–) and DAF (- -) and convolutional codes with different free distances d_f

In Figure 3 the simulated mean squared uncorrelated error (MSUE) at the output of one relay in the first hop of different relay schemes are shown. As a reference the MSUE curves for uncoded transmission are also depicted. All simulation results shown in this paper assume equal transmit power and equal noise variances for all links. It is interesting to see that

MSUE of AF as well as for DAF does not tend to zero for increasing transmit power. For AF this is due to linear amplification of not only the desired signal but also the received noise which is independent of the input power. For DAF the noise level is only lowered at high transmit powers by a constant due to the asymptotic coding gain exploited by the decoder. For DF and small values of P_s, wrong decisions are very likely which increase the error variance. On the other hand, DF eliminates the noise if it is smaller than the decision threshold and therefore the MSUE tends to zero for increasing transmit power. EF and DEF also result in a fast decreasing MSUE and outperform the two other schemes as well for uncoded and coded systems, respectively, for all values of P_s.
In Figure 4 the MSUE for DF, DAF and DEF is shown for convolutional codes with different free distances. It can be seen that the asymptotic value of MSUE for DAF depends on the free distance of the code which illustrates the asymptotic SNR gain of convolutional codes. Additionally, the slope of the MSUE curves for DEF and DF also depends on the free distance.

4. CHANNEL LLRS FOR DEF

In order to achieve a proper input signal for soft-input decoders, LLRs have to be calculated based on the received signal. As the source sends BPSK signals, the corresponding LLRs at the input of the relays in the first hop are easily calculated by

$$L_{ch}(c|y_{p,s}) = \frac{2\sqrt{P_{p,s-1}}}{\sigma_{p,s}^2} \cdot y_{p,s} \ . \tag{14}$$

In the case of EF and DEF, the signal transmitted by the relay is not BPSK modulated but continuously distributed in the range $[-1, +1]$. Although the equivalent noise of the superchannel is uncorrelated to the transmit signal, it is not Gaussian distributed anymore. The equivalent error of the superchannel denoted as η can be written as

$$\begin{aligned} \eta_{p,s} &= y_{p,s} - A_{p,s}c = x_{p,s-1} + n_{p,s} - A_{p,s}c \\ &= n_{p,s} + (x_{p,s-1} - A_{p,s}c) \\ &= n_{p,s} - c\underbrace{(A_{p,s} - c \cdot x_{p,s-1})}_{\bar{n}} \end{aligned} \tag{15}$$

As the second part $c\bar{n}$ of (15) is not Gaussian distributed, the total error η is not Gaussian distributed as well. Consequently, the simple calculation of channel LLRs similar to (14) is not valid. To calculate true log-likelihood values for these signals, the distribution of the received signal, i.e. the noisy soft bits $y_{p,s}$, is required. The LLRs at the decoder output of the former relay $R_{p,s-1}$ are assumed to be Gaussian distributed [10]

$$L_{p,s-1}(c) = \frac{\sigma_a^2}{2} \cdot c + n_a \tag{16}$$

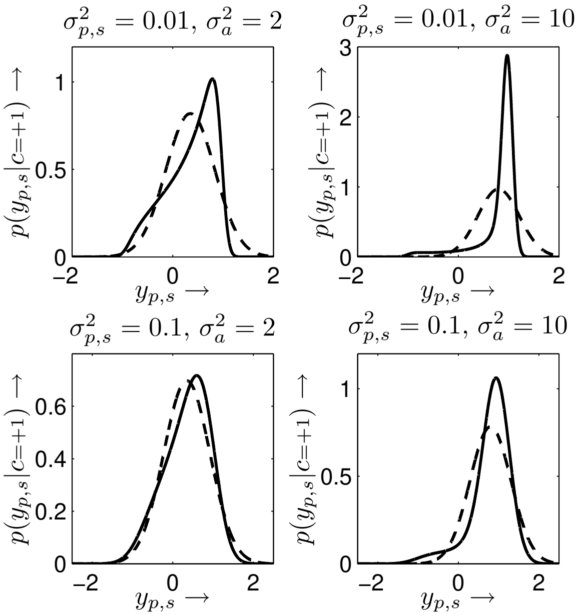

Fig. 5. Conditional PDF of received signal $p(y_{p,s}|c = +1)$ with (- -) and without (–) Gaussian approximation of $\bar{n}$, different values of $\sigma_{p,s}^2$ and σ_a^2

with n_a denoting a Gaussian random variable with zero mean and variance σ_a^2. With this model for the LLRs the conditional distribution of the soft bits becomes

$$\begin{aligned} p(x_{p,s-1}|c = \pm 1) \propto p(\tilde{c}_{p,s-1}|c = \pm 1) = \frac{1}{\sqrt{2\pi\sigma_a^2}} \cdot \\ \exp\left(-\frac{\left|2\text{atanh}(\tilde{c}_{p,s-1}) \mp \sigma_a^2/2\right|^2}{2\sigma_a^2}\right) \cdot \frac{2}{1 - \tilde{c}_{p,s-1}^2} \ , \end{aligned} \tag{17}$$

which is the transformation of the Gaussian distribution of the LLRs (16) with the $\tanh(x/2)$ function [11]. To determine the desired distribution of the noisy soft bits $y_{p,s}$, the Gaussian distribution of the channel noise

$$p(n_{p,s}) = \frac{1}{\sqrt{2\pi}\sigma_{p,s}} \exp\left(-\frac{n_{p,s}^2}{2\sigma_{p,s}^2}\right) \tag{18}$$

and the distribution of the soft bits have to be convolved [7]

$$p(y_{p,s}|c = \pm 1) = p(x_{p,s-1}|c = \pm 1) * p(n_{p,s}) \ . \tag{19}$$

This convolution cannot be solved in closed form and therefore has to be done numerically. Onother approach to calculate this distribution was derived in [8] yielding the same result. Using (19), the LLRs can be calculated at the destination

$$\begin{aligned} L_{ch}(c|y_{p,s}) &= \ln\left(\frac{p(c = +1|y_{p,s})}{p(c = \text{-}1|y_{p,s})}\right) \\ &= \ln\left(\frac{p(y_{p,s}|c = +1)}{p(y_{p,s}|c = \text{-}1)}\right) + \ln\left(\frac{p(c = +1)}{p(c = \text{-}1)}\right) \ , \end{aligned} \tag{20}$$

where the second part represents a-priori information. As equally likely symbols are assumed this equation simplfies to

$$L_{ch}(c|y_{p,s}) = \ln\left(\frac{p(y_{p,s}|c = +1)}{p(y_{p,s}|c = -1)}\right) . \qquad (21)$$

In contrast to this derivation, a Gaussian distribution of the total error $\eta_{p,s}$ was assumed in [6]. The code bits are assumed to be transmitted over an AWGN channel with channel coefficient A and noise variance η which consists of the error introduced by the relay function and the noise $n_{p,s}$ added at the receiver. With this notation the receiver signal can be described by

$$y_{p,s} = x_{p,s-1} + n_{p,s} = A \cdot c + \eta \qquad (22)$$

where $A = |\mathrm{E}\{x_{p,s-1}|c = \pm 1\}|$ and

$$\sigma_\eta^2 = \sigma_{p,s}^2 + \mathrm{E}\left\{x_{p,s-1}^2\right\} - A^2 = \sigma_{p,s}^2 + 1 - A^2 . \qquad (23)$$

Under this assumption the LLRs are approximated similar to (14)

$$L_{ch,\text{approx.}}(c|y_{p,s}) = \frac{2A}{\sigma_{p,s}^2 + 1 - A^2}\, y_{p,s} \qquad (24)$$

Fig. 5 shows the exact (19) and the Gaussian approximated conditional distributions of the noisy soft bits used for the two approaches (21) and (24), respectively. The difference between these distributions becomes obvious especially for low noise variances. In [7] and [8] the impact of the Gaussian approximation (24) was already investigated for specific systems. In the next section this impact will be evaluated to be quite small for most system setups.

5. SIMULATION RESULTS

First we consider degenerated systems were only one link with $N_s + 1$ hops (serial system) or two hops but several links (parallel system) with different number of relays before we extend the system to a general setup as in Figure 1.

5.1. Coded Serial System ($N_p = 1$)

The effective signal-to-noise ratios (SNR) at the destination for a serial system with $N_p = 1$ and $N_s = 2$, 4 and 6 are shown in Figure 6. This effective SNR is closely related to the MSUE at the relay[2]. The results verify the assumption that the conditioned expectation is better than hard decision or normalized LLRs in terms of effective SNR. At low SNR the performance of DEF is similar to DAF while at high SNR it is similar to DF both known to be the best choice for the corresponding SNR region. At medium SNR ($P \approx 0\,\mathrm{dB}$) the gain of DEF gets obvious as it clearly outperforms DAF and DF. The drawback of DAF can be seen when the signal is relayed several times because normalized LLRs lead to considerable performance loss especially when the number

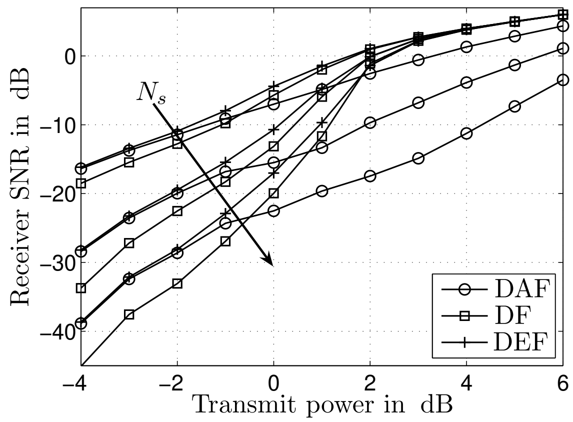

Fig. 6. Effective SNR at the destination for a serial system with $N_p = 1$ and $N_s = 2$, 4 and 6, rate 1/2 convolutional code, ($L_c = 5,\ d_f = 7$)

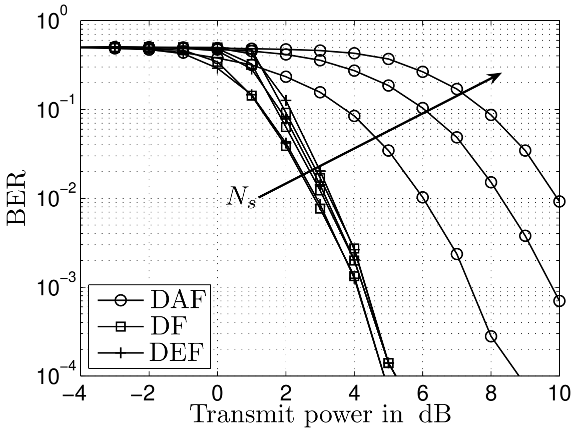

Fig. 7. Bit error rate at the destination for a serial system with $N_p = 1$ and $N_s = 2$, 4 and 6, rate 1/2 convolutional code, ($L_c = 5,\ d_f = 7$)

of hops increases. This effect is not surprising as LLRs are approximately Gaussian distributed with variance depending on the mean value. Even for a very high reliability, the variance of the LLRs is in the same order as their mean value, so that the MSE at the output of the relay for high SNRs at the input saturates, which could already be seen in Figure 3. The approaches with hard decision (DF) and expectation value (DEF) perform quite well also for an increasing number of hops. For these two approaches the MSE at the output of the relay tends to zero for increasing SNR at the input. In Figure 7 the corresponding bit error rates for these systems are depicted. The difference in terms of BER between DF and DEF is quite small for this system, but the loss of DAF in comparison to DF and DEF is significant. As mentioned before, although minimum MSE does not implicitly lead to minimum BER it nevertheless seems to be a good hint towards good BER performance. It was already noticed in

[2] that the BER optimal function for uncoded system is very similar to the MSE optimal tanh-function. For the results in Figure 7 the Gaussian approximation was applied (24). For a serial system this approximation has nearly no influence on the performance and therefore the results with calculation of LLR according to (21) are omitted for the serial system.

5.2. Coded Parallel System ($N_s = 1$)

The effect of LLR mismatch gets significant if several signals are combined at the destination. In the case of Gaussian disturbance of the superchannel, the signals are combined in terms of Maximum Ratio Combining (MRC). This MRC can

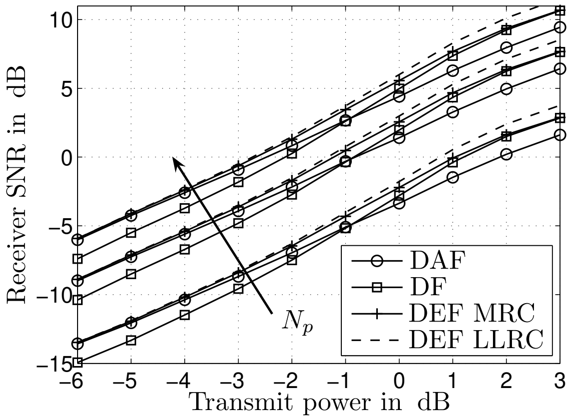

Fig. 8. Effective SNR at the destination for a parallel system with $N_p = 1$, 3 and 6, and $N_s = 1$, rate 1/2 convolutional code, ($L_c = 5,\ d_f = 7$)

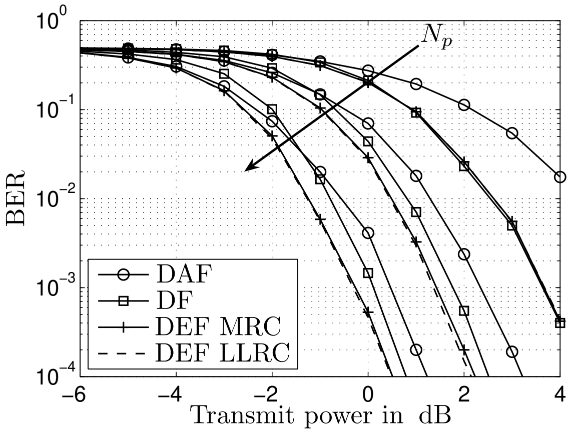

Fig. 9. Bit error rate at the destination for a parallel system with $N_p = 1$, 3 and 6, and $N_s = 1$, rate 1/2 convolutional code, ($L_c = 5,\ d_f = 7$)

be realized by summation over all input signals weighted by the corresponding SNR which coincides to the summation of the receiver LLRs for BPSK signals over AWGN channels.

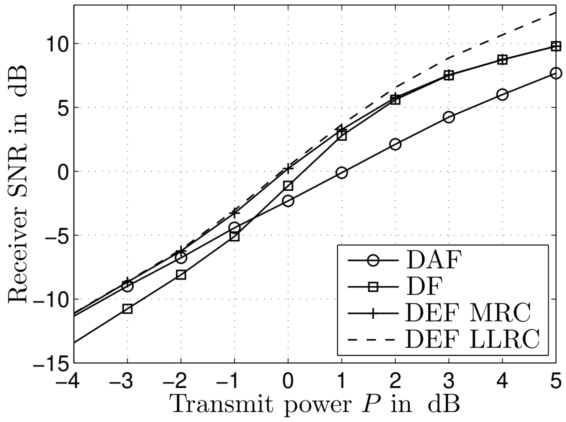

Fig. 10. Effective SNR at the destination for a hybrid system with $N_p = 3$ and $N_s = 2$, rate 1/2 convolutional code, ($L_c = 5,\ d_f = 7$)

As mentioned before, in the case of DEF the Gaussian assumption is not valid and therefore the LLRs are mismatched. This effect increases if several LLRs are summed up as is the case of a parallel system. The two combination approaches are denoted as DEF MRC if the Gaussian assumption was used and as DEF LLRC (LLR Combining) if the LLRs are calculated according to (21) and combined afterwards.
In Figure 8 and 9 the effective receiver SNR and the corresponding BER for a parallel system ($N_s = 1$) and different number of parallel relays can be seen. If the equivalent superchannel is assumed to be an AWGN channel, Maximum Ratio Combining (MRC) is applied. The effective SNR of DEF with MRC is better than that of DF and DAF also in this case. The SNR improves as the number of parallel hops increases. In contrast to the serial system the gain due to DEF over DF is obvious in terms of BER especially for increasing N_p. On the other hand the loss of DAF is smaller. The additional gain due to LLRC for DEF is significant only in terms of effective SNR at the destination but the BER performance is quite similar to DEF MRC. As mentioned before the improved MSE does not always correspond to an improved BER as can be seen here.

5.3. Coded and Uncoded Hybrid System

In the sequel a more general system setup will be considered according to the structure in Figure 1 containing several relays in parallel and serial. The different effects for DF, DAF and DEF described in the last two subsections are combined here in a hybrid system. In Figure 10 the effective SNR of a system with $N_p = 3$ and $N_s = 2$ is depicted. In accordance to the previously observed results, DEF outperforms DF and DAF also in this hybrid system. This effect is confirmed by the BER simulation results in Figure 11. The loss of DAF in comparison to DEF is similar as for the serial case and the

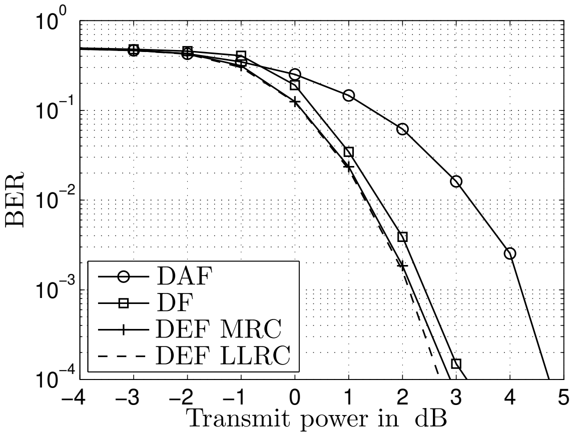

Fig. 11. Bit error rate at the destination for a system with $N_p = 3$ and $N_s = 2$, rate 1/2 convolutional code, ($L_c = 5,\ d_f = 7$)

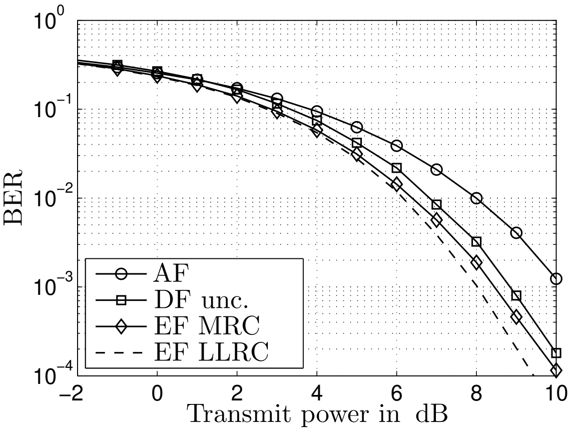

Fig. 12. Bit error rate without decoding at the relay for a hybrid system with $N_p = 3$ and $N_s = 2$, no channel code

gain of DEF over DF is nearly the same as for the parallel setup. So DEF seems to be superior for all network topologies. Furthermore the benefit of LLR combining according to (21) in terms of BER is more significant in this hybrid system. The combined impacts on the overall distortion due to serial and parallel relays increase the LLR mismatch caused by the Gaussian assumption.
Interestingly, the loss concerning BER due to this LLR mismatch is even more significant in uncoded system as can be seen in Figure 12. In contrast to the SNR in Figure 13 where the loss of EF MRC is in the range as for DEF MRC in Figure 10, the BER significantly differs. The BER gain of DEF LLRC may get more obvious at higher values of P i.e. smaller values of BER.

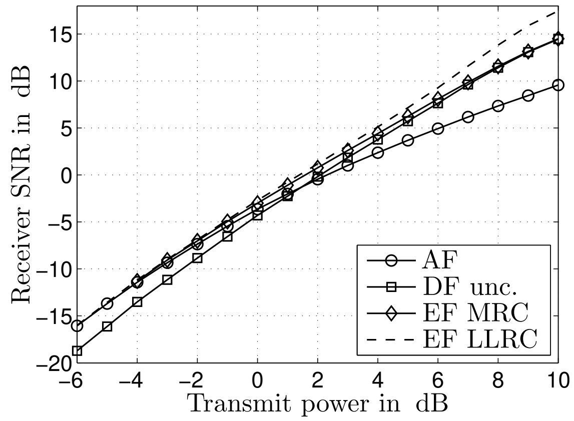

Fig. 13. Effective SNR without decoding at the relay for a system with $N_p = 3$ and $N_s = 2$, no channel code

6. CONCLUSION

In this paper an MSE optimal relaying function for coded systems was derived and discussed. The result is a scaled expectation value which is very similar to the MSE optimal function for the uncoded case. This scheme was compared to the classical relaying schemes DF and the scheme suggested in literature corresponding to AF in the coded case called DAF. Transmitting the conditioned expectation value of the code bits was shown to be the best relaying function in terms of MSE and also superior to other schemes in terms of BER. MSE and BER performance for different system setups were considered and analyzed in detail. Furthermore, the impact of the assumption of Gaussian distributed effective noise was investigated. This assumption is not valid for the expectation value used for EF and DEF but is nevertheless often used. It came out that this assumption slightly degrades the overall system performance for all considered system setups. In other words when taking the actual distribution of the noisy soft bits into account additional performance improvement is possible which may be worth the quite complex computations of the LLRs.

7. REFERENCES

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