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Low complexity successive interference cancellation for MIMO–OFDM systems†

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SUMMARY

Layered architectures like the V-BLAST scheme are a promising candidate to exploit the capacity advantages of multiple antenna systems leading to practical wireless communication schemes with very high data rates. The combination with orthogonal frequency division multiplexing (OFDM), called MIMO–OFDM, with per-antenna-coding is one of the most likely implementations of these multilayer architectures in frequency selective environments. In this paper, we present a novel, computational efficient implementation of successive interference cancellation (SIC) for coded MIMO–OFDM. It utilises a parallelised version of the Sorted QR Decomposition (SQRD) to achieve the same optimised detection order for all subcarriers, in order to exploit the error correction capability of the forward error correction code within the SIC. In comparison to other schemes known from literature, our approach requires only a fraction of computational complexity with almost the same performance. Copyright © 2007 John Wiley & Sons, Ltd.

1. INTRODUCTION

In order to realise mobile communication systems with high spectral efficiency, the application of multiple antennas at the transmitter and at the receiver side is a very promising approach. One popular candidate for future practical implementation is given by the spatial multiplexing architecture V-BLAST (Vertical Bell Labs Layered Space-Time) proposed in References [1, 2]. It uses a vertically layered coding structure, where independent blocks (called layers) are transmitted in parallel from the antennas and, consequently, a superposition of these layers arrives at the receiver. For the purpose of estimating the transmitted information, several receiver implementations have been investigated in the past. One popular approach is given by successive interference cancellation (SIC), where the layers are detected step-by-step and the estimated interference of already detected layers is successively subtracted from the received signals. For this purpose the well-known V-BLAST detection algorithm requires the repeated calculation of filter matrices and, thus, a relatively high computational effort [1, 2]. In order to reduce this complexity, several algorithms have been presented in the literature, for example [3–5]. Schemes applying the QR decomposition of the channel matrix have been investigated in References [6–10] and their adaptation to MIMO–OFDM is in the scope of this paper.

In frequency selective environments, the intersymbol interference (ISI) leads to a temporal superposition of the signals resulting in a two-dimensional equalisation problem. In order to implement efficient detection algorithms, the application of orthogonal frequency division multiplexing (OFDM) seems to be a promising approach, as the two-dimensional task is parallelised into N_C one-dimensional spatial equalisations, where N_C denotes the number of subcarriers [11, 12]. Due to this benefit, MIMO–OFDM gained a lot of interest in the research

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domain [13–21] and is currently under investigation as the physical layer scheme for future wireless communication systems like the IEEE 802.16 standard, also known as WiMAX (Worldwide Interoperability for Microwave Access), and the 3GPP Long Term Evolution (LTE) [22].

In order to gain from frequency diversity, OFDM schemes have to be used in combination with forward error correction (FEC) coding. In the investigated MIMO scheme, coding is applied to each antenna separately to make use of the error correction capability within the successive interference cancellation. As this per-antenna-coding (PAC) requires the same detection order on each subcarrier, an adopted version of the V-BLAST detection algorithm was proposed by van Zelst and Schenk [15, 16] requiring the repeated calculation of pseudo-inverses per subcarrier. Another approach for defining the detection order based on capacity terms was given by Kadous [14]. In this contribution, we present a novel detection scheme with comparable performance but clearly less computational complexity. For this purpose, the basic idea of Sorted QR Decomposition (SQRD) [7–10] is extended to N_C parallel MIMO channels resulting in the same detection order on each subcarrier.

1.1. Outline of the paper

The remainder of this paper is organised as follows. In Section 2 the system model is introduced. In order to simplify the derivation of the SIC, we recall linear equalisation with respect to the zero-forcing (ZF) and the minimum-mean-square-error (MMSE) criterion for MIMO–OFDM in Section 3. The two different approaches for ordered successive interference cancellation known from literature are presented in Section 4 and our new detection scheme based on the so-called Parallel-SQRD (P-SQRD) algorithm is given in Section 5. The computational effort and the performance analysis are given in Section 6 and 7, respectively. Concluding remarks can be found in Section 8.

1.2. Notation

Matrices are represented by bold capital letters, where the element in row α and column β of a matrix $\mathbf{A}$ is indicated by $[\mathbf{A}]_{\alpha,\beta} = a_{\alpha,\beta}$. Accordingly, vectors are denoted by bold lower case letters. The matrix transpose, Hermitian transpose and Moore-Penrose pseudo-inverse are denoted by $(\cdot)^{\mathrm{T}}$, $(\cdot)^{\mathrm{H}}$ and $(\cdot)^{+}$, respectively. Furthermore, $\mathbf{I}_\alpha$ represents the $\alpha \times \alpha$ identity matrix and $\mathbf{0}_{\alpha,\beta}$ denotes the $\alpha \times \beta$ all zero matrix. In order to distinguish between variables in time and in frequency domain, we indicate variables in time domain (TD) by the subscript TD, whereas a labelling for variables in frequency domain (FD) is generally omitted.

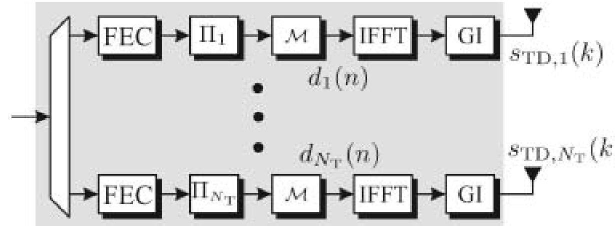

Figure 1. MIMO–OFDM transmitter with per-antenna-coding.

2. SYSTEM DESCRIPTION

We consider a multiple antenna system with N_T transmit and N_R receive antennas and apply spatial multiplexing in a frequency selective environment. The channel is assumed to be constant over each frame but changes independently between frames (*frequency selective block fading channel*). Throughout the paper we assume perfect knowledge of the channel state information at the receiver, but no knowledge at the transmitter.

According to the block diagram in Figure 1, the information data are demultiplexed at the transmitter into N_T parallel data streams (*layers*), encoded by a terminated convolutional encoder and after bitwise interleaving Π_i mapped to M-QAM or M-PSK symbols $d_i(n)$, $1 \leqslant i \leqslant N_\mathrm{T}$, $1 \leqslant n \leqslant N_\mathrm{C}$ of unit variance using the mapping function $\mathcal{M}$. After transforming the symbols to time domain by using the inverse fast Fourier transformation (IFFT), a guard interval (GI) of length N_G is added to form a cyclic prefix, before the sequence of $N_\mathrm{C} + N_\mathrm{G}$ signals $\mathbf{s}_{\mathrm{TD},i}(k)$ is transmitted from each antenna i. With $\mathbf{s}_{\mathrm{TD}}(k) = [s_{\mathrm{TD},1}(k), \ldots, s_{\mathrm{TD},N_\mathrm{T}}(k)]^{\mathrm{T}}$ denoting the N_T transmit signals at time k and $\mathbf{H}_{\mathrm{TD}}(\kappa)$, $0 \leqslant \kappa \leqslant N_\mathrm{H}$, representing the $N_\mathrm{R} \times N_\mathrm{T}$ channel matrix taps of the frequency selective channel of order N_H, the $N_\mathrm{R} \times 1$ receive vector at time k is given by

$$\mathbf{x}_{\mathrm{TD}}(k) = \sum_{\kappa=0}^{N_\mathrm{H}} \mathbf{H}_{\mathrm{TD}}(\kappa)\, \mathbf{s}_{\mathrm{TD}}(k-\kappa) + \mathbf{n}_{\mathrm{TD}}(k). \tag{1}$$

Here $\mathbf{n}_{\mathrm{TD}}(k)$ denotes the vector of additive white Gaussian noise at each receive antenna with covariance matrix $\mathrm{E}\left\{\mathbf{n}_{\mathrm{TD}}(k)\, \mathbf{n}_{\mathrm{TD}}^{\mathrm{H}}(k')\right\} = \sigma_n^2 \mathbf{I}_{N_\mathrm{R}}\, \delta(k-k')$ and δ denoting the Kronecker delta function. Equation (1) expresses the superposition of transmitted symbols not only in space but also in time and, thereby, points out the two-dimensional equalisation problem.

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At the receiver, the cyclic prefix is removed and the fast Fourier transform (FFT) is used to perform the transformation back into frequency domain. As long as $N_G \geqslant N_H$ holds, the application of the cyclic prefix and discrete Fourier transform results in N_C orthogonal MIMO systems. With $\mathbf{d}(n)$ denoting the $N_T \times 1$ vector of modulated symbols on subcarrier $1 \leqslant n \leqslant N_C$, the corresponding received vector in frequency domain is given by

$$\mathbf{y}(n) = \mathbf{H}(n)\mathbf{d}(n) + \mathbf{n}(n) \quad \text{for} \quad 1 \leqslant n \leqslant N_C \tag{2}$$

with the flat MIMO channel matrix for subcarrier n [17]

$$\mathbf{H}(n) = \sum_{\kappa=0}^{N_H} \mathbf{H}_{TD}(\kappa)\, e^{-j\frac{2\pi}{N_C}(n-1)\kappa}. \tag{3}$$

Due to this separation in N_C non-frequency selective parallel MIMO systems, common detection algorithms using linear equalisation or successive interference cancellation can be used for each subcarrier without any modification in case of an uncoded MIMO–OFDM scheme. For per-antenna-coded schemes, this is only true for linear equalisation, as described next.

3. LINEAR EQUALISATION

For zero-forcing linear equalisation (LE) the received vector $\mathbf{y}(n)$ of each subcarrier $1 \leqslant n \leqslant N_C$ is multiplied by the related ZF filter matrix accomplished by the Moore–Penrose pseudo-inverse of $\mathbf{H}(n)$

$$\mathbf{G}_{ZF}(n) = \mathbf{H}^{+}(n) = \left(\mathbf{H}^{H}(n)\mathbf{H}(n)\right)^{-1}\mathbf{H}^{H}(n). \tag{4}$$

The filter output on subcarrier n is then given by

$$\tilde{\mathbf{d}}_{ZF}(n) = \mathbf{G}_{ZF}(n)\,\mathbf{y}(n) = \mathbf{d}(n) + \mathbf{H}^{+}(n)\mathbf{n}(n) \tag{5}$$

with error-covariance matrix

$$\begin{aligned}\mathbf{\Phi}_{\mathbf{ee},ZF}(n) &= \mathrm{E}\left\{(\tilde{\mathbf{d}}_{ZF}(n) - \mathbf{d}(n))(\tilde{\mathbf{d}}_{ZF}(n) - \mathbf{d}(n))^{H}\right\}\\ &= \sigma_n^2\left(\mathbf{H}^{H}(n)\mathbf{H}(n)\right)^{-1}.\end{aligned} \tag{6}$$

Consequently, the signal-to-noise ratio of layer i on subcarrier n is given by $\mathrm{SNR}_i(n) = 1/[\mathbf{\Phi}_{\mathbf{ee},ZF}(n)]_{i,i}$. After calculating the log-likelihood-ratios (L-values) for each layer by an adequate demodulation $\mathcal{D}$ and after deinterleaving Π_i^{-1}, these L-values are fed to the corresponding decoder and the decoding per layer takes place as shown in Figure 2.

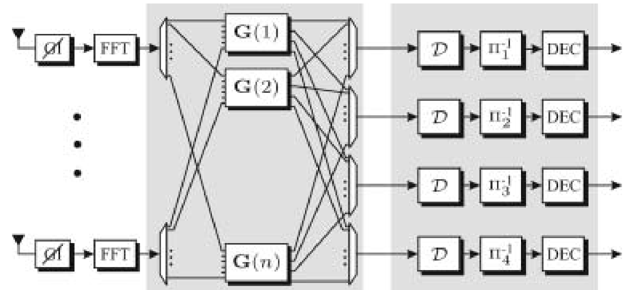

Figure 2. Linear equalisation for a MIMO–OFDM scheme with $N_T = 4$ transmit antennas and subsequent decoding per layer.

On the other hand, the MMSE approach minimises the mean squared error between the transmit vector $\mathbf{d}(n)$ and the output of the linear filter and leads to the filter matrix

$$\mathbf{G}_{MMSE}(n) = \left(\mathbf{H}^{H}(n)\mathbf{H}(n) + \sigma_n^2\mathbf{I}_{N_T}\right)^{-1}\mathbf{H}^{H}(n). \tag{7}$$

The estimation error of the different layers corresponds to the main diagonal elements of the error-covariance matrix

$$\mathbf{\Phi}_{\mathbf{ee},MMSE}(n) = \sigma_n^2\left(\mathbf{H}^{H}(n)\mathbf{H}(n) + \sigma_n^2\mathbf{I}_{N_T}\right)^{-1} \tag{8}$$

and results in the signal-to-interference-and-noise-ratio $\mathrm{SINR}_i(n) = 1/[\mathbf{\Phi}_{\mathbf{ee},MMSE}(n)]_{i,i} - 1$, where the term -1 is caused by the bias. Within the calculation of the L-values, this bias can be considered by regarding the equivalent channel $[\mathbf{G}_{MMSE}(n)\mathbf{H}(n)]_{i,i}$ for layer i and subcarrier n.

For the description of the SIC schemes in Section 4, it will be useful to represent the MMSE filter matrix (7) by $\mathbf{G}_{MMSE}(n) = \tilde{\mathbf{\Phi}}_{\mathbf{ee},MMSE}(n)\mathbf{H}^{H}(n)$ with $\tilde{\mathbf{\Phi}}_{\mathbf{ee},MMSE}(n) = \sigma_n^{-2}\mathbf{\Phi}_{\mathbf{ee},MMSE}(n)$ denoting the scaled error-covariance matrix for the MMSE criterion. Likewise, the ZF filter matrix (4) can be rewritten as $\mathbf{G}_{ZF}(n) = \tilde{\mathbf{\Phi}}_{\mathbf{ee},ZF}(n)\mathbf{H}^{H}(n)$ with $\tilde{\mathbf{\Phi}}_{\mathbf{ee},ZF}(n) = \sigma_n^{-2}\mathbf{\Phi}_{\mathbf{ee},ZF}(n)$.

Notice, that in principle the solution for the linear equalisation of a coded MIMO–OFDM scheme coincides to the linear equalisation of an uncoded MIMO–OFDM system. The only difference consists within the detection step, as the simple hard decision has to be replaced by the calculation of the L-values, deinterleaving and decoding.

4. SUCCESSIVE INTERFERENCE CANCELLATION

It is generally known, that linear equalisation is outperformed by successive interference cancellation,

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which is often called decision-feedback equalisation (DFE) in the literature. Thereby, the transmitted symbols of one subcarrier are not estimated in parallel but one after another. However, as already detected symbols directly influence subsequent symbol decisions, the problem of error propagation arises and it is well known that an optimised detection order can significantly reduce this effect [9].

Thus, one approach to implement SIC for per-antenna-coded MIMO–OFDM is given by performing an independent successive detection with optimised order on each subcarrier and feeding the corresponding L-values or hard decisions to the channel decoder afterwards [19, 20]. However, this approach would not exploit the error correction capability of FEC within the detection process and, furthermore, the occurring decision errors lead to L-values of minor quality [23].

To avoid these drawbacks, it is wholesome to utilise the forward error correction code before removing the estimated interference within the SIC. Due to the encoding structure of the PAC MIMO–OFDM system, this requires the same order of detection on each subcarrier [14–18]. Thus, no separate optimisation is possible for each non-frequency selective MIMO system, but an optimisation over all subcarriers has to be performed. Next, the two approaches published by van Zelst and Schenk [15, 16] and by Kadous [14] are reviewed. Afterwards, the SIC detection based on the QR decomposition and the new P-SQRD algorithm for determining an optimised detection order are presented in Section 5.

4.1. SINR-optimisation

In order to optimise the detection sequence, van Zelst and Schenk proposed to run N_C parallel V-BLAST algorithms with an adopted ordering criterion. In the first detection step, the ZF or MMSE error-covariance matrices $\mathbf{\Phi}_{\mathbf{ee}}(n)$ are calculated for each subcarrier, whereas the ν-th diagonal element $[\mathbf{\Phi}_{\mathbf{ee}}(n)]_{\nu,\nu}$ denotes the estimation error on the ν-th layer of subcarrier n. Afterwards, the diagonal elements of the error-covariance matrices are summed up

$$\mathrm{error}(\nu) = \frac{1}{N_\mathrm{C}} \sum_{n=1}^{N_\mathrm{C}} [\mathbf{\Phi}_{\mathbf{ee}}(n)]_{\nu,\nu} \quad \text{for} \quad 1 \leqslant \nu \leqslant N_\mathrm{T} \tag{9}$$

and the layer μ with the smallest overall error, that is with the largest SINR, is selected as the current target layer for all subcarriers. Subsequently, the received signals $\mathbf{y}(n)$ are filtered with the $1 \times N_\mathrm{R}$ filter vectors $\mathbf{w}_1(n) = \tilde{\mathbf{\Phi}}_{\mathbf{ee}}^{(\mu)}(n)\mathbf{H}^\mathrm{H}(n)$, where $\tilde{\mathbf{\Phi}}_{\mathbf{ee}}^{(\mu)}(n)$ denotes the μ-row of the scaled error-covariance matrix $\tilde{\mathbf{\Phi}}_{\mathbf{ee}}(n)$. After parallel to serial conversion of the filter output signals, the adequate demodulation is performed. Based on the calculated L-values, channel decoding is carried out by Viterbi or BCJR algorithm including the calculation of the corresponding code bits [15, 16]. Then, these estimated code bits are mapped to QAM/PSK symbols, the estimated interference is subtracted on each subcarrier and the columns of the target layer are set to zero in the N_C channel matrices $\mathbf{H}(n)$. The detection of the remaining layers is performed in the same way following the V-BLAST philosophy. Thus, overall $N_\mathrm{C}(N_\mathrm{T}-1)$ inverses have to be calculated for determining the error-covariance matrices leading to a considerable computational complexity.

(1) Init: $\mathbf{A}_n = \mathbf{H}_n^\mathrm{H}\mathbf{H}_n + \sigma_n^2\mathbf{I}_{N_\mathrm{T}}$ for $n = 1, \ldots, N_\mathrm{C}$
$\quad\mathbf{p} = \mathbf{0}_{1,N_\mathrm{T}}, \mathcal{I} = [1 \ldots N_\mathrm{T}]$
(2) for $i = 1, \ldots, N_\mathrm{T}$
(3) $\quad$ for $n = 1, \ldots, N_\mathrm{C}$
(4) $\quad\quad$ $\tilde{\mathbf{\Phi}}_{\mathbf{ee},n} = \mathbf{A}_n^{-1}$
(5) $\quad$ end
(6) $\quad$ for $\nu = 1, \ldots, N_\mathrm{T} - i + 1$
(7) $\quad\quad$ $\mathrm{error}(\nu) = \frac{\sigma_n^2}{N_\mathrm{C}} \sum_{n=1}^{N_\mathrm{C}} [\tilde{\mathbf{\Phi}}_{\mathbf{ee},n}]_{\nu,\nu}$
(8) $\quad$ end
(9) $\quad$ $\mu = \mathrm{argmin}_{\nu=1,\ldots,N_\mathrm{T}-i+1}\, \mathrm{error}(\nu)$
(10) $\quad$ $p_i = \mathcal{I}(\mu)$ and clear element μ in $\mathcal{I}$
(11) $\quad$ for $n = 1, \ldots, N_\mathrm{C}$
(12) $\quad\quad$ $\mathbf{w}_i(n) = \tilde{\mathbf{\Phi}}_{\mathbf{ee},n}(\mu, :)\,\mathbf{H}_n^\mathrm{H}$
(13) $\quad\quad$ clear col. μ and row μ in $\mathbf{A}_n$ and col. μ in $\mathbf{H}_n$
(14) $\quad$ end
(15) end

Algorithm 1: SINR-Algorithm for determining the filter vectors $\mathbf{w}_i(n)$ for a system with N_T transmit, N_R receive antennas, and N_C subcarriers

The pseudo-code for a convenient implementation of the SINR approach is given in Algorithm 1, where the grey highlighted entry in line (1) is only required for the MMSE implementation. In order to simplify the description, we make use of the Matlab notation for indicating matrix elements. To further avoid three-dimensional matrices, the n-th channel matrix $\mathbf{H}(n)$ is denoted by $\mathbf{H}_n$ and corresponding definitions are also used for other matrices.

In order to reduce the computational complexity, the symmetric matrices $\mathbf{A}(n) = \mathbf{H}^\mathrm{H}(n)\mathbf{H}(n) + \sigma_n^2\mathbf{I}_{N_\mathrm{T}}$ or $\mathbf{A}(n) = \mathbf{H}^\mathrm{H}(n)\mathbf{H}(n)$ are computed for each subcarrier once. After calculating $\tilde{\mathbf{\Phi}}_{\mathbf{ee}}(n)$ by inverting $\mathbf{A}(n)$ in line (4), the layer μ with the smallest cumulative estimation error is determined and, finally, the N_C filter vectors for that layer μ are computed in line (12). Afterwards, the μ-th column and the μ-th row of $\mathbf{A}(n)$ as well as the μ-th column of $\mathbf{H}(n)$ are cancelled on each subcarrier. The further steps for calculating

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the ordered list of filter vectors take place in the same way.[‡]

4.2. CMOS-optimisation

Another approach to optimise the detection order has been proposed by Kadous in Reference [14]. Within his capacity mapping ordering scheme (CMOS) algorithm the averaged capacity of layer $1 \leqslant \nu \leqslant N_{\mathrm{T}}$ after ZF filtering is calculated

$$C(\nu) = \frac{1}{N_{\mathrm{C}}} \sum_{n=1}^{N_{\mathrm{C}}} \log_2 \left(1 + \mathrm{SNR}_{\nu}(n)\right) \tag{10}$$

and the layer with the maximum $C(\nu)$ is selected as the current layer of interest. The actual detection process corresponds to the SINR approach and for applying the MMSE criterion, the SINR is used in the expression for the capacity in Equation (10) instead. Due to the mapping of the SNR/SINR to the capacity term, this approach requires slightly more operations than the SINR approach. However, as demonstrated in Section 7, this approach outperforms the SINR approach with respect to bit error rates due to the more meaningful optimisation criterion.

5. QR-BASED SUCCESSIVE INTERFERENCE CANCELLATION

Instead of performing the SIC by repeatedly calculating filter matrices, we propose to use the QR decompositions of the channel matrices $\mathbf{H}(n)$ instead. For that purpose, we recall the general approach for single-carrier systems and extend the corresponding detection process to multi-carrier systems. Afterwards, the P-SQRD algorithm and the corresponding pseudo-code are introduced.

5.1. Approach for single-carrier systems

As shown in several publications, successive interference cancellation for non-frequency selective multilayer systems can be restated in terms of the QR decomposition of the channel matrix $\mathbf{H} = \mathbf{QR}$, where the $N_{\mathrm{R}} \times N_{\mathrm{T}}$ matrix $\mathbf{Q}$ has orthogonal columns with unit norm and the $N_{\mathrm{T}} \times N_{\mathrm{T}}$ matrix $\mathbf{R}$ is upper triangular. The adaptation of the detection order is achieved by permuting the columns of $\mathbf{H}$, that is by factorising the matrix $\mathbf{H}_{\mathrm{perm}} = \mathbf{HP} = \mathbf{QR}$ with $\mathbf{P}$ denoting the corresponding permutation matrix[§] [6–10]. An efficient solution for calculating this factorisation with an optimised detection sequence is given by the Sorted QR Decomposition [7, 8], which determines the permutation matrix *within* the QR decomposition. For the adaptation to the MMSE criterion, the *extended* channel matrix $\underline{\mathbf{H}}$ and the *extended* receive vector $\underline{\mathbf{y}}$

$$\underline{\mathbf{H}} = \begin{bmatrix} \mathbf{H} \\ \sigma_n \mathbf{I}_{N_{\mathrm{T}}} \end{bmatrix} \quad \text{and} \quad \underline{\mathbf{y}} = \begin{bmatrix} \mathbf{y} \\ \mathbf{0}_{N_{\mathrm{T}},1} \end{bmatrix} \tag{11}$$

are defined [6] and the sorted decomposition

$$\underline{\mathbf{H}}_{\mathrm{perm}} = \begin{bmatrix} \mathbf{HP} \\ \sigma_n \mathbf{I}_{N_{\mathrm{T}}} \end{bmatrix} = \underline{\mathbf{Q}}\,\underline{\mathbf{R}} \tag{12}$$

has to be calculated [9, 10].

5.2. Approach for multi-carrier systems

Adopting this idea to MIMO–OFDM, but ignoring the adaption of the detection order for the moment, the QR decompositions

$$\mathbf{H}(n) = \mathbf{Q}(n)\,\mathbf{R}(n) \quad \text{for} \quad 1 \leqslant n \leqslant N_{\mathrm{C}} \tag{13}$$

for all channel matrices $\mathbf{H}(n)$ have to be calculated. Again, the extension with respect to the MMSE criterion is achieved by the decomposition of the extended MMSE channel matrices defined in Equation (11), that is by $\underline{\mathbf{H}}(n) = \underline{\mathbf{Q}}(n)\,\underline{\mathbf{R}}(n)$. Then, each received vector $\mathbf{y}(n)$ is filtered by $\mathbf{Q}^{\mathrm{H}}(n)$ (when adopting the MMSE criterion $\underline{\mathbf{y}}(n)$ is filtered by $\underline{\mathbf{Q}}^{\mathrm{H}}(n)$ [9, 10]) and due to the upper triangular form of $\mathbf{R}(n)$ the N_{T}-th layer of each filter output signal

$$\tilde{\mathbf{d}}(n) = \mathbf{Q}^{\mathrm{H}}(n)\,\mathbf{y}(n) = \mathbf{R}(n)\mathbf{d}(n) + \tilde{\mathbf{n}}(n) \tag{14}$$

is free of interference with $\tilde{\mathbf{n}}(n) = \mathbf{Q}^{\mathrm{H}}(n)\,\mathbf{n}(n)$ denoting the noise term at the filter output. After performing the demodulation, the L-values are deinterleaved and fed to the

[‡] It is also possible to calculate the error-covariance matrices recursively using the Sherman-Morrison formula [3] or Greville's algorithm [4]. Another approach for limiting the complexity using Cholesky decomposition was presented in Reference [5]. However, the adaptation of the SINR-algorithm given in References [15, 16] with respect to these recursive schemes is not within the scope of this paper. Thus, we will focus on the published SIC schemes for multi-carrier systems for comparison reason.

[§] A square matrix $\mathbf{P}$ is called a permutation matrix, if in each row and each column of $\mathbf{P}$ exactly one element is equal to one and all remaining elements are equal to zero. The multiplication $\mathbf{AP}$ yields a permutation of the columns of $\mathbf{A}$, whereas $\mathbf{PA}$ results in a permutation of the rows.

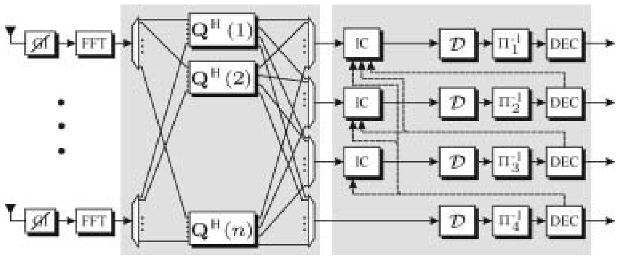

Figure 3. Successive interference cancellation for a MIMO–OFDM scheme with $N_T = 4$ transmit antennas.

channel decoder, as shown in Figure 3. Using the interleaved code bits for remodulation, the estimated interference is cancelled out within the block interference cancellation (IC) and the successive interference cancellation of the remaining layers is performed in the same way.

From Equation (14) it is obvious, that the SINR on layer i and subcarrier n is proportional to $|r_{i,i}(n)|^2$ under the assumption of perfect symbol decisions in the previous decision steps. Due to the detection sequence from layer $N_T, N_T - 1, \ldots, 1$ it would be optimal to permute the channel matrices $\mathbf{H}(n)$ in such a way, that the absolute value of the diagonal elements $|r_{i,i}(n)|$ are large in the lower part of $\mathbf{R}(n)$, that is for the first detection steps. In order to incorporate this optimisation of the detection order, we consequently have to replace Equation (13) by QR decompositions of permuted channel matrices. However, as the encoding is performed over all subcarriers within the PAC MIMO–OFDM system, the order of detection has to be the same on each subcarrier. Thus, we have to perform the decompositions on all subcarriers with respect to one *global* permutation matrix $\mathbf{P}$

$$\mathbf{H}(n)\mathbf{P} = \mathbf{Q}(n)\,\mathbf{R}(n) \quad \text{for} \quad 1 \leqslant n \leqslant N_C. \tag{15}$$

Of course the question arises, how to find this global permutation matrix $\mathbf{P}$ in an efficient way? In order to solve this problem, an extended version of our SQRD algorithm is proposed in the next subsection.

5.3. Parallel Sorted QR Decomposition

The general task is to calculate the N_C QR decompositions and to inherently find the global permutation matrix $\mathbf{P}$. In order to find $\mathbf{P}$, it is necessary to perform the N_C QR factorisations in parallel. In the first step of the new Parallel Sorted QR Decomposition (P-SQRD), the squared column norm of each layer over all subcarriers is calculated

$$\mathbf{b}(i) = \sum_{j=1}^{N_R}\sum_{n=1}^{N_C} |h_{j,i}(n)|^2 = \sum_{j=1}^{N_R}\sum_{\kappa=0}^{N_H} |h_{\text{TD},j,i}(\kappa)|^2 \tag{16}$$

and equals the squared norm of all fading coefficients $h_{\text{TD},j,i}(\kappa)$ in time domain belonging to transmit antenna i. Following the philosophy of SQRD, the layer with *minimum norm* is determined and permuted to the first position on each subcarrier. Subsequently, each $\mathbf{H}(n)$ is orthogonalised with respect to the according column vector and the norm in Equation (16) is updated, in order to denote only that part of each column vector orthogonal to the already spanned orthonormal basis. In the second step again, the layer with minimum norm is selected, the other columns are orthogonalised with respect to this layer and the norm is again updated. The decomposition of the remaining layers takes place in the same manner. Basically, the N_C parallel QR decompositions are extended by a global permutation of the channel matrices $\mathbf{H}(n)$, where the sequence optimisation is done contrary to the detection sequence, that is in the first decomposition step, the layer to be detected last is determined.

As already mentioned, the MMSE solution is found by decomposition of the corresponding extended channel matrix $\underline{\mathbf{H}}(n)$. However, instead of calculating the column norms of $\underline{\mathbf{H}}(n)$ according to Equation (16), it is less complex to just add the accumulated noise terms yielding

$$\mathbf{b}(i) = \sum_{j=1}^{N_R}\sum_{\kappa=0}^{N_H} |h_{\text{TD},j,i}(\kappa)|^2 + \sigma_n^2\, N_C. \tag{17}$$

The pseudo-code[‖] of the P-SQRD is given in Algorithm 2, where the grey highlighted entries are again only required for the MMSE implementation. By comparing the pseudo-code to the corresponding calculation of N_C unsorted QR decompositions, only few additional operations are required, leading only to a marginal computational overhead [23].

Instead of optimising the detection order with respect to the SINR per layer, it is also possible to adapt the P-SQRD algorithm with respect to the achieved capacity in the high SNR region. For $\text{SNR}_\nu(n) \gg 1$, the capacity term

[‖] The algorithm is given as an extension of the Modified Gram-Schmidt orthogonalisation. However, similar expressions can also be achieved using Householder reflexion or Givens rotation for QR decomposition [23]. Furthermore, $\mathbf{p}$ denotes a permutation vector with $\mathbf{P} = \mathbf{I}_{N_T}(:, \mathbf{p})$.

(1) Init: $\mathbf{R}_n = \mathbf{0}$, $\mathbf{Q}_n = [\mathbf{H}_n^{\mathrm{T}}\ \sigma_n \mathbf{I}_{N_{\mathrm{T}}}]^{\mathrm{T}}$ for $n = 1, \ldots, N_{\mathrm{C}}$
    $\mathbf{p} = [1 \ldots N_{\mathrm{T}}]$
(2) for $i = 1, \ldots, N_{\mathrm{T}}$
(3) $\mathbf{b}(i) = \sum_{n=1}^{N_{\mathrm{C}}} \|\mathbf{Q}_n(1 : N_{\mathrm{R}}, i)\|^2 + \sigma_n^2 N_{\mathrm{C}}$
(4) end
(5) for $i = 1, \ldots, N_{\mathrm{T}}$
(6) $\mu = \mathrm{argmin}_{\nu=i,\ldots,N_{\mathrm{T}}}\, \mathbf{b}(\nu)$
(7)   Exchange columns i and $i + \mu - 1$ in $\mathbf{p}$ and $\mathbf{b}$
(8)   for $n = 1, \ldots, N_{\mathrm{C}}$
(9)   Exchange columns i and $i + \mu - 1$ in $\mathbf{R}_n$ and in the first $N_{\mathrm{R}} + i - 1$ rows of $\mathbf{Q}_n$
(10) $\mathbf{R}_n(i, i) := \|\mathbf{Q}_n(1 : N_{\mathrm{R}} + i, i)\|$
(11) $\mathbf{Q}_n(1 : N_{\mathrm{R}} + i, i) := \mathbf{Q}_n(1 : N_{\mathrm{R}} + i, i)/\mathbf{R}_n(i, i)$
(12)   for $\nu = i + 1, \ldots, N_{\mathrm{T}}$
(13) $\mathbf{R}_n(i, \nu) := \mathbf{Q}_n^{\mathrm{H}}(1 : N_{\mathrm{R}} + i - 1, i) \cdot \mathbf{Q}_n(1 : N_{\mathrm{R}} + i - 1, \nu)$
(14) $\mathbf{Q}_n(1 : N_{\mathrm{R}} + i, \nu) := \mathbf{Q}_n(1 : N_{\mathrm{R}} + i, \nu) - \mathbf{R}_n(i, \nu)\mathbf{Q}_n(1 : N_{\mathrm{R}} + i, i)$
(15) $\mathbf{b}(\nu) := \mathbf{b}(\nu) - |\mathbf{R}_n(i, \nu)|^2$
(16)   end
(17)   end
(18) end

Algorithm 2: P-SQRD-Algorithm for a system with N_{T} transmit and N_{R} receive antennas and N_{C} subcarriers (grey labeled entries are only required for the MMSE solution)

in Equation (10) can be approximated by

$$C(\nu) \approx \frac{1}{N_{\mathrm{C}}} \sum_{n=1}^{N_{\mathrm{C}}} \log_2 (\mathrm{SNR}_\nu(n)) = \frac{1}{N_{\mathrm{C}}} \log_2 \prod_{n=1}^{N_{\mathrm{C}}} \mathrm{SNR}_\nu(n) \tag{18}$$

and, as the logarithm is a monotonic function, the layer μ that minimises Equation (18) corresponds to

$$\mu = \min_{\nu} C(\nu) = \min_{\nu} \left(\prod_{n=1}^{N_{\mathrm{C}}} |r_{\nu,\nu}(n)|^2 \right) \tag{19}$$

with $\mathrm{SNR}_\nu(n) \sim |r_{\nu,\nu}(n)|^2$. In order to apply the SQRD philosophy, we have to minimise in each decomposition step the capacity, that is we have to determine the layer that leads to the minimum product of squared diagonal elements. In contrast to Algorithm 2, the sum of the channel coefficients per subcarrier has to be replaced by the corresponding product of these coefficients. However, as shown by simulation results, this additional computational overhead does not lead to an improved performance and, thus, this second variant is not considered in the following due to the larger computational costs.

6. COMPUTATIONAL COMPLEXITY

Next, the computational complexity of the SINR, the CMOS and the P-SQRD approach with respect to complex floating point operations (flops) $\mathcal{F}$ are investigated. In order to achieve simple terms depending only on the system configuration, we count one complex addition as one flop and a complex multiplication as three flops. All other operations, for example addition and multiplication with respect to real numbers, division, square root, logarithm are traced back to this complexity measure [23].

The SINR approach with respect to the zero-forcing criterion requires the calculation of the matrices $\mathbf{A}(n)$ for each subcarrier once and in each step i, the calculation of N_{C} inverses of the symmetric matrices of dimension $(N_{\mathrm{T}} - i + 1) \times (N_{\mathrm{T}} - i + 1)$ in line (4), the summation of the errors over all subcarriers n per layer ν and finally the calculation of N_{C} filter vectors. According to Algorithm 1 this requires

$$\mathcal{F}_{\mathrm{SINR}}^{\mathrm{ZF}} = \mathcal{O}\left(\left(\frac{5}{6} N_{\mathrm{T}}^4 + \frac{23}{6} N_{\mathrm{T}}^3 + 4 N_{\mathrm{R}} N_{\mathrm{T}}^2 \right) N_{\mathrm{C}} \right) \tag{20}$$

operations. For the corresponding MMSE solution only $N_{\mathrm{C}} N_{\mathrm{T}}$ real additions of σ_n^2 on the main diagonal of $\mathbf{A}(n)$ are necessary in addition. The CMOS approach differs form the SINR approach in the fact, that capacity terms are used for optimising the detection order, requiring a mapping of the SNR/SINR values to the corresponding capacities. The overhead is given by $(\frac{5}{4} N_{\mathrm{T}}^2 + \frac{5}{4} N_{\mathrm{T}} - \frac{5}{2}) N_{\mathrm{C}}$ flops. The necessary steps for the detection process (including filtering of the received signals, subtracting the estimated interference and quantisation, but ignoring the effort for channel decoding) requires approximately

$$\mathcal{F}_{\mathrm{SIC}} = (8 N_{\mathrm{T}} - 4)\, N_{\mathrm{R}} N_{\mathrm{C}} \tag{21}$$

floating point operations leading to an overall complexity for the SINR-SIC detection $\mathcal{F}_{\mathrm{SINR-SIC}} = \mathcal{F}_{\mathrm{SINR}}^{\mathrm{ZF}} + \mathcal{F}_{\mathrm{SIC}}$.

In contrast, the P-SQRD-SIC mainly consists of N_{C} QR decompositions and some additional operations for defining the decomposition sequence and therewith the detection order. Using a detailed complexity consideration of Algorithm 2, the complexity for the P-SQRD algorithm with respect to the zero-forcing criterion requires

$$\mathcal{F}_{\mathrm{P-SQRD}}^{\mathrm{ZF}} = \mathcal{O}\left(\left(4 N_{\mathrm{R}} N_{\mathrm{T}}^2 + \frac{1}{4} N_{\mathrm{T}}^2 - \frac{3}{2} N_{\mathrm{R}} N_{\mathrm{T}} \right) N_{\mathrm{C}} \right) \tag{22}$$

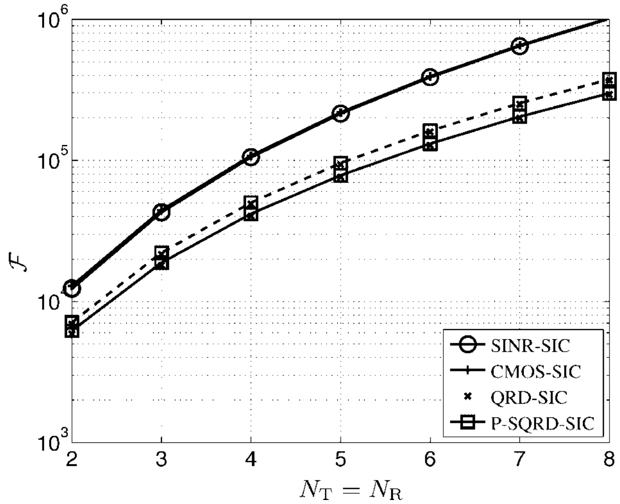

Figure 4. Number of floating point operations $\mathcal{F}$ for SINR-SIC, CMOS-SIC, P-SQRD-SIC and QRD-SIC detection with respect to the ZF (–) and the MMSE (- -) criterion of a MIMO–OFDM system with $N_\mathrm{T} = N_\mathrm{R}$ antennas and $N_\mathrm{C} = 128$ subcarriers.

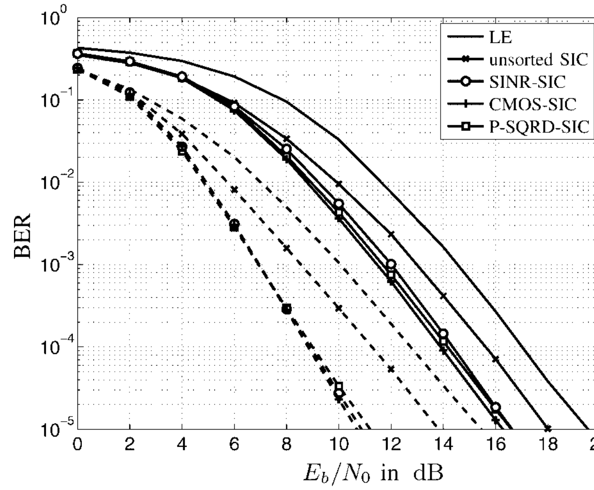

Figure 5. BER of LE and SIC with respect to ZF (–) or MMSE (- -) criterion for a MIMO–OFDM system with $N_\mathrm{T} = N_\mathrm{R} = 4$ antennas, channel order $N_\mathrm{H} = 5$, $N_\mathrm{C} = 128$ subcarriers, guard interval of length $N_\mathrm{G} = 5$, 4-QAM symbols and terminated $[7, 5]_8$ convolutional code.

flops and for the MMSE criterion the cost is given by

$$\mathcal{F}^{\mathrm{MMSE}}_{\mathrm{P-SQRD}} = \mathcal{O}\left(\left(\frac{4}{3}N_\mathrm{T}^3 + \left(4N_\mathrm{R} - \frac{5}{4}\right)N_\mathrm{T}^2 - \frac{3}{2}N_\mathrm{R}N_\mathrm{T}\right)N_\mathrm{C}\right). \quad (23)$$

The actual successive detection process requires additionally $(2N_\mathrm{T}^2 + (4N_\mathrm{R} - 2)N_\mathrm{T})N_\mathrm{C}$ operations. Comparing Equation (22) with Equation (20), a strong reduction with respect to computational cost becomes obvious.

Figure 4 shows for a varying, but equal number of transmit and receive antennas $N_\mathrm{T} = N_\mathrm{R}$, the required number of flops for SINR-SIC, CMOS-SIC, P-SQRD-SIC and a QR-based SIC without adapting the detection order in case of the ZF and the MMSE implementation with $N_\mathrm{C} = 128$ subcarriers. As the SINR and CMOS approach require almost the same operations for ZF and MMSE implementation, the corresponding lines overlay. However, the figure visualises the strong decrease in computational complexity achieved by our new approach. As an example, for a system with $N_\mathrm{T} = N_\mathrm{R} = 4$ antennas, the P-SQRD-SIC with respect to the ZF criterion requires approximately $0.39 \cdot \mathcal{F}^{\mathrm{ZF}}_{\mathrm{SINR-SIC}}$ flops and consequently, leads to a strong reduction in computational cost. Furthermore it becomes obvious, that the P-SQRD-SIC requires only a marginal overhead in comparison to an unsorted QR-based SIC detection.

7. PERFORMANCE ANALYSIS

In this section, we investigate the bit error rates (BERs) for a per-antenna-coded MIMO–OFDM system with $N_\mathrm{T} = N_\mathrm{R} = 4$ antennas. We assume uncorrelated SISO channels of order $N_\mathrm{H} = 5$ with a constant power delay profile, that is the variance of all fading coefficients is equal to $1/(N_\mathrm{H} + 1)$. Furthermore, each OFDM symbol contains a cyclic prefix of length $N_\mathrm{G} = 5$ and all $N_\mathrm{C} = 128$ subcarriers are used for signal transmission. For the simulations, perfect estimation of the channel coefficients and the noise variance is assumed. The loss due to the guard interval is considered in the E_b/N_0.

In Figure 5, the BERs for different detection schemes with respect to the ZF- and the MMSE-criterion are shown when the terminated $[7, 5]_8$ convolutional code and 4-QAM modulation is applied on each substream. Obviously, the sorted SIC schemes achieve substantial performance improvements in comparison to linear equalisation and successive detection without ordering. The results for P-SQRD-SIC, SINR-SIC and CMOS-SIC are comparable, with minor advantages for the later ones in case of the MMSE criterion. However, this small performance impairment of the P-SQRD approach comes with a strong reduction in computational complexity.

In Figure 6, the BERs for a system with 16-QAM modulation and the terminated $[133, 171]_8$ convolutional code of constraint length 7 are shown. Again, only a small

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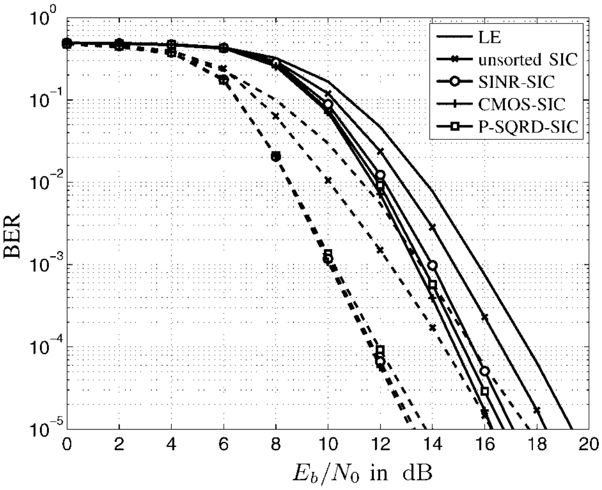

Figure 6. BER of LE and SIC with respect ZF (–) or MMSE (- -) criterion of a MIMO–OFDM system with $N_T = N_R = 4$ antennas, channel order $N_H = 5$, $N_C = 128$ subcarriers, guard interval of length $N_G = 5$, 16-QAM and terminated $[133, 171]_8$ convolutional code.

difference in performance can be observed between the different ordering criterion. These results demonstrate that it is important to optimise the detection order, whereas the choosen optimisation criterion has only a minor impact to the performance. This points out the potential of the proposed P-SQRD approach for schemes with high spectral efficiencies.

8. SUMMARY AND CONCLUSIONS

Within this paper we proposed a new detection scheme for coded MIMO–OFDM systems by introducing an extended version of the SQRD algorithm. The Parallel Sorted QR Decomposition (P-SQRD) algorithm achieves an adapted detection ordering within the QR decompositions of the N_C channel matrices and can be implemented with respect to the zero-forcing and the minimum-mean-square-error criterion. We presented simulation results for different scenarios and analytically demonstrated the computational advantage of the P-SQRD algorithm. We were able to show, that this new algorithm achieves comparable performance results to the schemes from literature, but with less computational complexity. Thus, a feasible implementation for MIMO–OFDM for future wireless local area networks is given.

In order to further reduce the complexity for systems with large number of subcarriers, the principle of P-SQRD was recently extended in Reference [24] to the interpolation based scheme presented in Reference [25]. Therefore, the P-SQRD is applied only on a number of subcarriers and afterwards, the QR decompositions for the remaining subcarriers are calculated by interpolation. Depending on the system configuration, this leads to a further significant complexity reduction, but yielding the same performance results. Finally, the concept of P-SQRD can also be used for MIMO receivers using frequency domain equalisation, leading again to an efficient detection scheme with optimised detection order [23].

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