Parallel-SQRD for Low Complexity Successive Interference Cancellation in Per-Antenna-Coded MIMO-OFDM Schemes

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Abstract

In recent years, communication systems with multiple antennas at the transmitter and at the receiver have gained considerable interest. Layered architectures like the V-BLAST scheme are a strong candidate to exploit the capacity advantages of multiple antenna systems leading to practical wireless communication schemes with very high data rates. The combination with orthogonal frequency division multiplexing (OFDM), called MIMO-OFDM, with per-antenna-coding is one of the most likely implementations of multilayer architectures in frequency selective environments. In this paper, we present a novel, computational efficient implementation of successive interference cancellation (SIC) for coded MIMO-OFDM. It utilizes a parallelized version of the Sorted QR Decomposition (SQRD) to achieve the same optimized detection order for all subcarriers in order to exploit the error correction capability of the code within the SIC. In comparison to other schemes known from literature our approach requires only a fraction of computational complexity with almost the same performance.

1 Introduction

The application of multiple antenna systems in rich-scattering environments may lead to a dramatic increase of the data rate. The V-BLAST (Vertical Bell Labs Layered Space-Time) architecture proposed in [1] is one popular candidate for future practical implementation. It uses a vertically layered coding structure, where independent blocks (called layers) are transmitted in parallel from the antennas and consequently a superposition of these layers arrive at the receiver. In order to estimate the transmitted information several receiver implementations have been investigated in the past. One popular approach is given by successive interference cancellation (SIC), where the layers are detected step by step and the estimated interference of already detected layers is successively subtracted from the received signals. Therefore, the well-known V-BLAST detection algorithm requires the repeated calculation of a pseudo-inverse (in case of Zero-Forcing detection) of the channel matrix and thus a relatively high computational effort [1]. Schemes on basis of the QR decomposition with reduced complexity were presented in [2]–[5].

In frequency selective environments the present intersymbol interference (ISI) leads to a temporal superposition, resulting in a two-dimensional equalization problem. In order to realize efficient detection algorithms the application of orthogonal frequency division multiplexing (OFDM) seems to be a promising approach, as the two-dimensional task is parallelized into N_C one-dimensional spatial equalizations, with N_C denoting the number of subcarriers. Due to the current research activities MIMO-OFDM can be expected to be one of the first commercially utilized implementations of the V-BLAST multilayer architecture [6]–[11].

In order to gain from frequency diversity OFDM schemes have to be used in combination with forward error correction (FEC) coding. Within the investigated MIMO scheme coding is applied to each antenna separately to make use of the error correction capability within the successive interference cancellation. As this per-antenna-coding (PAC) requires the same detection order on each subcarrier an adopted version of the V-BLAST detection algorithm was proposed by van Zelst and Schenk [8], [9] requiring the repeated calculation of the pseudo-inverse on each carrier. Another approach for defining the detection order based on capacity terms was given by Kadous [7]. Within this contribution we present a novel detection scheme with comparable performance but clearly less computational complexity. Therefore, the basic idea of Sorted QR Decomposition (SQRD) [3]–[5] is extended to N_C parallel MIMO channels resulting in the same detection order on each subcarrier.

The remainder of this paper is organized as follows. In Section 2 the system model and notation is introduced. In order to simplify the derivation of the SIC we recall linear ZF and MMSE equalization for MIMO-OFDM in Section 3. The three different approaches for ordered successive interference cancellation are presented in Section 4, where also the pseudo-code of the new Parallel-SQRD (P-SQRD) algorithm is given. The computational effort and the performance analysis are given in Section 5 and 6, respectively. Concluding marks can be found in Section 7.

2 System description

We consider a multiple antenna system with N_T transmit and $N_\mathrm{R} \geq N_\mathrm{T}$ receive antennas in a frequency selective block fading environment, i.e. the channel is assumed to be constant over a frame but changes independently between frames.

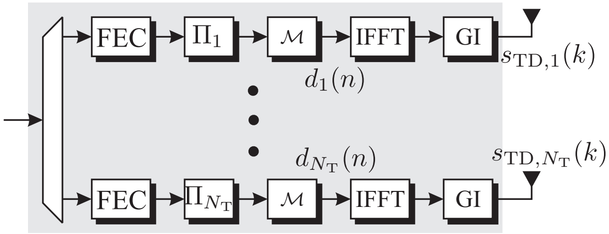

Fig. 1. MIMO-OFDM Transmitter with Per-Antenna-Coding

As shown in Fig. 1 the information data is demultiplexed at the transmitter into N_T parallel data streams, encoded by a convolutional encoder and mapped to M-QAM or M-PSK symbols using the mapper $\mathcal{M}$ after bitwise interleaving. In order to distinguish between signals defined in frequency domain (FD) or in time domain (TD) we subsequently indicate variables in time domain by an index TD, whereas an index for symbols in frequency domain is omitted. Thus, $d_i(n)$ denotes the n-th symbol of layer i and the matrix containing all modulated symbols of one frame is given by

$$\mathbf{D} = \begin{bmatrix} d_1(1) & d_1(2) & \dots & d_1(N_\mathrm{C}) \\ \vdots & \vdots & \dots & \vdots \\ d_{N_\mathrm{T}}(1) & d_{N_\mathrm{T}}(2) & \dots & d_{N_\mathrm{T}}(N_\mathrm{C}) \end{bmatrix} . \qquad (1)$$

Before transmitting these signals, they are shifted into time domain by using the Inverse Fast Fourier Transformation (IFFT) and a guard interval (GI) in form of a cyclic prefix of length N_G is added.

In order to describe the transformation from TD to FD in matrix notation the $N_\mathrm{C} \times N_\mathrm{C}$ Discrete Fourier Transformation (DFT) matrix $\mathbf{F}$ with[1]

$$[\mathbf{F}]_{n,k} = e^{-j\frac{2\pi}{N_\mathrm{C}}(n-1)(k-1)} / \sqrt{N_\mathrm{C}} \qquad (2)$$

is introduced, whereas the practical implementation is achieved again using the efficient Fast Fourier Transformation (FFT) algorithm. As $\mathbf{F}$ is unitary, the Inverse Discrete Fourier Transformation (IDFT) is given by $\mathbf{F}^H$. Furthermore the matrices

$$\tilde{\mathbf{G}}_\mathrm{T} = \begin{bmatrix} \mathbf{0}_{N_\mathrm{G},N_\mathrm{C}-N_\mathrm{G}} \; \mathbf{I}_{N_\mathrm{G}} \\ \mathbf{I}_{N_\mathrm{C}} \end{bmatrix} \text{ and } \tilde{\mathbf{G}}_\mathrm{R} = \begin{bmatrix} \mathbf{0}_{N_\mathrm{C},N_\mathrm{G}} & \mathbf{I}_{N_\mathrm{C}} \end{bmatrix} \qquad (3)$$

can be used for adding and removing the cyclic prefix on each transmit and receive antenna, respectively. By extending these matrices using the Kronecker product $\otimes$, it will be straight forward to describe the MIMO-OFDM system. Therefore, in the sequel

$$\mathbf{F}_\mathrm{T} = \mathbf{F} \otimes \mathbf{I}_{N_\mathrm{T}} \qquad \mathbf{F}_\mathrm{R} = \mathbf{F} \otimes \mathbf{I}_{N_\mathrm{R}} \qquad (4)$$

denote the $N_\mathrm{C}N_\mathrm{T} \times N_\mathrm{C}N_\mathrm{T}$ MIMO-DFT matrix at the transmitter and the $N_\mathrm{C}N_\mathrm{R} \times N_\mathrm{C}N_\mathrm{R}$ MIMO-IDFT matrix at the receiver side and

$$\mathbf{G}_\mathrm{T} = \tilde{\mathbf{G}}_\mathrm{T} \otimes \mathbf{I}_{N_\mathrm{T}} \qquad \mathbf{G}_\mathrm{R} = \tilde{\mathbf{G}}_\mathrm{R} \otimes \mathbf{I}_{N_\mathrm{R}} \qquad (5)$$

declare the matrices for adding the cyclic prefix at the transmitter and removing it at the receiver. With[2] $\mathbf{d} = \mathrm{vec}\{\mathbf{D}\}$ describing the $N_\mathrm{C}N_\mathrm{T}\times 1$ column vector of modulated symbols, the corresponding $(N_\mathrm{G}+N_\mathrm{C})N_\mathrm{T}\times 1$ vector of transmit signals is given by $\mathbf{s}_\mathrm{TD} = \mathbf{G}_\mathrm{T}\mathbf{F}_\mathrm{T}^H\mathbf{d}$. These signals are transmitted over the frequency selective block fading channel of order $N_\mathrm{H} \leq N_\mathrm{G}$. With the $N_\mathrm{R}(N_\mathrm{C}+N_\mathrm{G})\times N_\mathrm{T}(N_\mathrm{C}+N_\mathrm{G})$ block-toeplitz channel matrix consisting of all fading coefficients $\mathbf{H}_\mathrm{TD}(\kappa)$

$$\mathcal{T}\{\mathbf{H}_\mathrm{TD}\} = \begin{bmatrix} \mathbf{H}_\mathrm{TD}(0) & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & & \mathbf{0} \\ \mathbf{H}_\mathrm{TD}(1) & \mathbf{H}_\mathrm{TD}(0) & \mathbf{0} & \cdots & \vdots & & \mathbf{0} \\ \vdots & \vdots & \ddots & & \vdots & & \mathbf{0} \\ \mathbf{H}_\mathrm{TD}(N_\mathrm{H}) & \mathbf{H}_\mathrm{TD}(N_\mathrm{H}-1) & & \mathbf{H}_\mathrm{TD}(0) & \mathbf{0} & & \vdots \\ \mathbf{0} & \mathbf{H}_\mathrm{TD}(N_\mathrm{H}) & & \mathbf{H}_\mathrm{TD}(1) & \mathbf{H}_\mathrm{TD}(0) & \mathbf{0} & \vdots \\ \vdots & & \ddots & \vdots & \vdots & \ddots & \\ \mathbf{0} & \cdots & & \mathbf{H}_\mathrm{TD}(N_\mathrm{H}) & \mathbf{H}_\mathrm{TD}(N_\mathrm{H}-1) & \cdots & \mathbf{H}_\mathrm{TD}(0) \end{bmatrix}$$

the received vector becomes

$$\mathbf{x}_\mathrm{TD} = \mathcal{T}\{\mathbf{H}_\mathrm{TD}\}\mathbf{s}_\mathrm{TD} + \mathbf{n}_\mathrm{TD} \qquad (6)$$

where the vector $\mathbf{n}_\mathrm{TD}$ contains the additive white Gaussian noise of variance σ_n^2. After removing the guard interval and FFT at the receiver the received sequence in frequency domain is given by

$$\begin{aligned} \mathbf{y} &= \mathbf{F}_\mathrm{R}\mathbf{G}_\mathrm{R}\mathbf{x}_\mathrm{TD} \\ &= \mathbf{F}_\mathrm{R}\mathbf{G}_\mathrm{R}\mathcal{T}\{\mathbf{H}_\mathrm{TD}\}\mathbf{G}_\mathrm{T}\mathbf{F}_\mathrm{T}^H\mathbf{d} + \mathbf{F}_\mathrm{R}\mathbf{G}_\mathrm{R}\mathbf{n}_\mathrm{TD} \qquad (7) \\ &= \mathbf{H}\,\mathbf{d} + \mathbf{n} \,. \end{aligned}$$

As the matrix $\mathbf{G}_\mathrm{R}\mathcal{T}\{\mathbf{H}_\mathrm{TD}\}\mathbf{G}_\mathrm{T}$ is block-circulant the matrix

$$\mathbf{H} = \mathbf{F}_\mathrm{R}\mathbf{G}_\mathrm{R}\mathcal{T}\{\mathbf{H}_\mathrm{TD}\}\mathbf{G}_\mathrm{T}\mathbf{F}_\mathrm{T}^H \qquad (8)$$

has block-diagonal structure with $\mathbf{H}(n)$ denoting the $N_\mathrm{R} \times N_\mathrm{T}$ channel matrix of carrier n, i.e. $\mathbf{H} = \mathrm{blockdiag}\{\mathbf{H}(1), \dots, \mathbf{H}(N_\mathrm{C})\}$ with

$$\mathbf{H}(n) = \sum_{\kappa=0}^{N_\mathrm{H}} \mathbf{H}_\mathrm{TD}(\kappa)\, e^{-j\frac{2\pi}{N_\mathrm{C}}(n-1)\kappa} \,. \qquad (9)$$

Correspondingly, the MIMO-OFDM system separates into N_C independent non-frequency selective MIMO schemes

$$\mathbf{y}(n) = \mathbf{H}(n)\,\mathbf{d}(n) + \mathbf{n}(n) \quad \text{for } 1 \leq n \leq N_\mathrm{C} \qquad (10)$$

[1]Throughout this paper, $(\cdot)^T$ and $(\cdot)^H$ denote matrix transpose and hermitian transpose, respectively. The Moore-Penrose pseudo-inverse is indicated by $(\cdot)^+$. Furthermore, $\mathbf{I}_\alpha$ indicates the $\alpha \times \alpha$ identity matrix and $\mathbf{0}_{\alpha,\beta}$ denotes the $\alpha \times \beta$ all zero matrix. The element in row α and column β of a matrix $\mathbf{A}$ is given by $[\mathbf{A}]_{\alpha,\beta}$.

[2]With $\mathbf{a}_i$ denoting the i-th column of a $n \times m$ matrix $\mathbf{A}$ the corresponding stacked $nm \times 1$ vector is given by $\mathrm{vec}\{\mathbf{A}\} = [\mathbf{a}_1^T\ \mathbf{a}_2^T\ \dots \mathbf{a}_m^T]^T$.

with $\mathrm{E}\{\mathbf{n}(n)\,\mathbf{n}^H(n)\} = \sigma_n^2 \mathbf{I}_{N_\mathrm{R}}$. In case of uncoded transmission common detection algorithms using linear equalization or successive interference cancellation can be used for each carrier without any modification. As shown in the sequel, for per-antenna-coded schemes this is only true for linear equalization.

3 Linear Equalization

For linear Zero-Forcing (ZF) equalization the received vector $\mathbf{y}(n)$ of each carrier is multiplied by the ZF filter matrix $\mathbf{G}_\mathrm{ZF}(n) = \mathbf{H}^+(n)$ and the output is given by $\tilde{\mathbf{d}}_\mathrm{ZF}(n) = \mathbf{d}(n) + \mathbf{H}^+(n)\,\mathbf{n}(n)$ with error-covariance matrix

$$\mathbf{\Phi}_{\mathbf{ee},\mathrm{ZF}}(n) = \sigma_n^2 \left(\mathbf{H}^H(n)\,\mathbf{H}(n)\right)^{-1} . \qquad (11)$$

Consequently, the signal-to-noise-ratio of layer i on carrier n is given by $\mathrm{SNR}_i(n) = 1/[\mathbf{\Phi}_{\mathbf{ee},\mathrm{ZF}}(n)]_{i,i}$. After calculating the Log-Likelihood-Ratios (L-values) for each layer by an adequate demodulation $\mathcal{D}$, these L-values are fed to the corresponding decoder and the per layer decoding takes place as shown in Fig. 2.

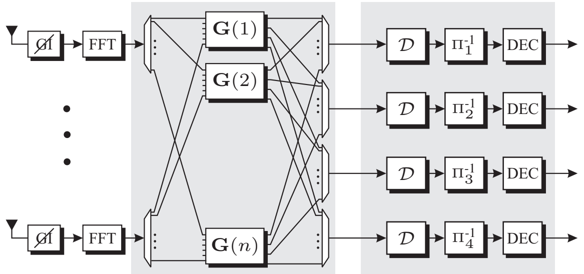

Fig. 2. Linear Equalization for a MIMO-OFDM scheme with $N_\mathrm{T} = 4$ transmit antennas

In order to perform linear equalization with respect to the MMSE criterion the filter matrix $\mathbf{G}_\mathrm{MMSE}(n) = \left(\mathbf{H}^H(n)\,\mathbf{H}(n) + \sigma_n^2 \mathbf{I}_{N_\mathrm{T}}\right)^{-1} \mathbf{H}^H(n)$ is used and the error-covariance matrix is now given by

$$\mathbf{\Phi}_{\mathbf{ee},\mathrm{MMSE}}(n) = \sigma_n^2 \left(\mathbf{H}^H(n)\,\mathbf{H}(n) + \sigma_n^2 \mathbf{I}_{N_\mathrm{T}}\right)^{-1} \qquad (12)$$

resulting in a signal-to-interference-and-noise-ratio $\mathrm{SINR}_i(n) = 1/[\mathbf{\Phi}_{\mathbf{ee},\mathrm{MMSE}}(n)]_{i,i} - 1$. The occuring biase can be considered within the L-value calculation. By introducing the extended channel matrix $\underline{\mathbf{H}}(n)$ and the extended receive vector $\underline{\mathbf{y}}(n)$ for carrier n through [2]

$$\underline{\mathbf{H}}(n) = \begin{bmatrix} \mathbf{H}(n) \\ \sigma_n \mathbf{I}_{N_\mathrm{T}} \end{bmatrix} \text{ and } \underline{\mathbf{y}}(n) = \begin{bmatrix} \mathbf{y}(n) \\ \mathbf{0}_{N_\mathrm{T},1} \end{bmatrix} \qquad (13)$$

the filter output signal is also given by $\tilde{\mathbf{s}}_\mathrm{MMSE}(n) = \underline{\mathbf{H}}^+(n)\,\underline{\mathbf{y}}(n)$ and the error covariance matrix (12) becomes [4]

$$\mathbf{\Phi}_{\mathbf{ee},\mathrm{MMSE}}(n) = \sigma_n^2 \left(\underline{\mathbf{H}}^H(n)\,\underline{\mathbf{H}}(n)\right)^{-1} . \qquad (14)$$

Thus, equalization with respect to the MMSE criterion equals ZF filtering with respect to the extended channel model (13).

4 Successive Interference Cancellation

Using the technique of successive interference cancellation (SIC) the transmit symbols of one carrier are not detected in parallel but one after another. As already detected symbols directly influence succeeding symbol decisions the problem of error propagation arises and it is well-known that an optimized detection order can significantly reduce this effect [4]. Thus, one approach to implement SIC for per-antenna-coded MIMO-OFDM is given by performing an independent successive detection with optimized order on each subcarrier and feed the corresponding L-values or hard decisions to the channel decoder afterwards [10], [11]. However, this architecture does not exploit the error correction capability of FEC within the detection process and furthermore the occuring decision errors lead to L-values of minor quality.

To avoid these drawbacks it is favorable to utilize the forward error correction code before removing the estimated interference within the SIC. Due to the encoding structure of the PAC MIMO-OFDM system, this requires the same order of detection on each carrier [7]–[9]. Thus, no separat optimization is possible for each non-frequency selective MIMO system, but an optimization over all carriers has to be performed. In the sequel, we shortly introduce the approaches proposed by van Zelst and Schenk [8], [9] and by Kadous [7]. Afterwards our new approach P-SQRD is presented.

4.1 SINR-Optimization

In order to optimize the detection sequence van Zelst and Schenk proposed to run N_C parallel V-BLAST algorithms with an adopted ordering criterion. In the first detection step, the ZF or MMSE filter matrix $\mathbf{G}(n)$ and the corresponding error covariance matrix $\mathbf{\Phi}_{\mathbf{ee}}(n)$ are calculated for each carrier, whereby the i-th diagonal element $[\mathbf{\Phi}_{\mathbf{ee}}(n)]_{i,i}$ denotes the estimation error on the i-th layer of carrier n. Thus, in case of ZF filtering N_C pseudo-inverses have to be calculated. Afterwards, the diagonal elements of the error covariance matrices are summed up

$$\overline{\mathbf{\Phi}}_{\mathbf{ee},i} = \frac{1}{N_\mathrm{C}} \sum_{n=1}^{N_\mathrm{C}} [\mathbf{\Phi}_{\mathbf{ee}}(n)]_{i,i} \qquad (15)$$

and the layer with the smallest overall error is selected as the target layer on each carrier.

Subsequently, the received signals $\mathbf{y}(n)$ are filtered with the corresponding row of $\mathbf{G}(n)$ and after parallel to serial conversion the adequate demodulation is performed. On basis of the calculated L-values channel decoding is carried out by Viterbi or BCJR algorithm including the calculation of the corresponding code bits [9]. Than these estimated code bits are mapped to QAM/PSK symbols, the estimated interference is canceled out on each carrier and the columns of the target layer are set to zero in the N_C channel matrices $\mathbf{H}(n)$. The detection of the remaining layers takes place in the same way following the V-BLAST philosophy. Thus, overall $N_\mathrm{C}(N_\mathrm{T}-1)$ pseudo-inverses have to be calculated. The corresponding MMSE detection is performed in the same way using the MMSE filter matrices and the error-covariance (12) or (14).

4.2 CMOS-Optimization

Another approach to optimize the detection order has been proposed by Kadous in [7]. Within his CMOS (Capacity Mapping Ordering Scheme) algorithm the averaged capacity of layer $1 \leq i \leq N_\mathrm{T}$ after ZF filtering is calculated

$$C(i) = \frac{1}{N_\mathrm{C}} \sum_{n=1}^{N_\mathrm{C}} \log_2\left(1 + \mathrm{SNR}_i(n)\right) \tag{16}$$

and the layer with the maximum $C(i)$ is selected as the current layer of interest. The actual detection process corresponds to the SINR-optimization and for MMSE detection the corresponding SINR is used in the expression for the capacity (16).

4.3 P-SQRD Approach

As shown in several publications, successive interference cancellation for non-frequency selective multilayer systems can be restated in terms of the QR decomposition of the channel matrix, where the order of detection is achieved by permuting its columns [2]–[5].

Adopting this idea to MIMO-OFDM, a QR decomposition $\mathbf{H}(n)\mathbf{P}(n) = \mathbf{Q}(n)\mathbf{R}(n)$ of each permuted channel matrix $\mathbf{H}(n)\mathbf{P}(n)$ with permutation matrix $\mathbf{P}(n)$ has to be calculated. For an uncoded system a factorization with optimized detection sequence is efficiently found by the Sorted QR Decomposition (SQRD) and the extension to the MMSE criterion is achieved by factorization of the extended MMSE channel matrices $\underline{\mathbf{H}}(n) = \underline{\mathbf{Q}}(n)\underline{\mathbf{R}}(n)$ [4].

As the order of detection has now to be the same on each carrier for PAC MIMO-OFDM systems, we propose an extended version of our SQRD algorithm to find a global permutation matrix. In the first step of this Parallel Sorted QR Decomposition (P-SQRD) the squared column norm of each layer over all carriers is calculated

$$\mathbf{b}(i) = \sum_{j=1}^{N_\mathrm{R}} \sum_{n=1}^{N_\mathrm{C}} |h_{j,i}(n)|^2 = \sum_{j=1}^{N_\mathrm{R}} \sum_{\kappa=0}^{N_\mathrm{H}} |h_{\mathrm{TD},j,i}(\kappa)|^2 \tag{17}$$

and equals the squared norm of all fading coefficients $h_{\mathrm{TD},j,i}(\kappa)$ in time domain belonging to transmit antenna i. Following the philosophy of SQRD, the layer with *minimum norm* is determined and permuted to the first position on each carrier. Subsequently, each $\mathbf{H}(n)$ is orthogonalized with respect to the according column vector and the norm (17) is updated in order to denote only that part of each column vector orthogonal to the spanned orthonormal basis. In the second step again the layer with minimum norm is selected, the other columns are orthogonalized with respect to this layer and the norm is again updated. The decomposition of the remaining layers takes place in the same manner and consequently, we basically extended the N_C parallel QR decomposition by a global permutation of the channel matrices $\mathbf{H}(n)$.

As already mentioned, the MMSE solution is found by decomposition of the corresponding extended channel matrix $\underline{\mathbf{H}}(n)$. The pseudo-code[3] of the P-SQRD is given in Algorithm 1, where the gray labeled entries are only required for the MMSE implementation. In order to simplify the description we make us of the Matlab notation for indicating matrix elements. To further avoid three-dimensional matrices, the n-th channel matrix $\mathbf{H}(n)$ is denoted by $\mathbf{H}_n$ and corresponding definitions are also used for other matrices.

Init: $\mathbf{R}_n = \mathbf{0}$, $\mathbf{Q}_n = [\mathbf{H}_n^T \; \sigma_n \mathbf{I}_{N_\mathrm{T}}\,]^T$ for
    $n = 1, \ldots, N_\mathrm{C}$, $\mathbf{p} = [1 \; \ldots \; N_\mathrm{T}]$
for $i = 1, \ldots, N_\mathrm{T}$
    $\mathbf{b}(i) = \sum_{n=1}^{N_\mathrm{C}} \|\mathbf{Q}_n(1\!:\!N_\mathrm{R}, i)\|^2 + \sigma_n^2 N_\mathrm{C}$
end
for $i = 1, \ldots, N_\mathrm{T}$
    $\mu = \mathrm{argmin}_{\nu=i,\ldots,N_\mathrm{T}} \mathbf{b}(\nu)$
    Exchange columns i and $i+\mu-1$ in $\mathbf{p}$ and $\mathbf{b}$
    for $n = 1, \ldots, N_\mathrm{C}$
        Exchange columns i and $i+\mu-1$ in $\mathbf{R}_n$ and in the first $N_\mathrm{R}+i-1$ rows of $\mathbf{Q}_n$
        $\mathbf{R}_n(i,i) := \|\mathbf{Q}_n(1\!:\!N_\mathrm{R}+i, i)\|$
        $\mathbf{Q}_n(1\!:\!N_\mathrm{R}+i, i) := \mathbf{Q}_n(1\!:\!N_\mathrm{R}+i, i) / \mathbf{R}_n(i,i)$
        for $\nu = i+1, \ldots, N_\mathrm{T}$
            $\mathbf{R}_n(i,\nu) := \mathbf{Q}_n^H(1\!:\!N_\mathrm{R}+i-1, i) \cdot \mathbf{Q}_n(1\!:\!N_\mathrm{R}+i-1, \nu)$
            $\mathbf{Q}_n(1\!:\!N_\mathrm{R}+i, \nu) := \mathbf{Q}_n(1\!:\!N_\mathrm{R}+i, \nu) - \mathbf{R}_n(i,\nu)\, \mathbf{Q}_n(1\!:\!N_\mathrm{R}+i, i)$
            $\mathbf{b}(\nu) := \mathbf{b}(\nu) - |\mathbf{R}_n(i,\nu)|^2$
        end
    end
end

Algorithmn 1: P-SQRD-Algorithm for a system with N_T transmit and N_R receive antennas and N_C carrier (gray labeled entries are only required for the MMSE solution)

When the P-SQRD calculation is done, each received vector $\mathbf{y}(n)$ is filtered by $\mathbf{Q}^H(n)$ (when adopting the MMSE criterion $\underline{\mathbf{y}}(n)$ is filtered by $\underline{\mathbf{Q}}^H(n)$ [4]) and due to the upper triangular form of $\mathbf{R}(n)$ the N_T-th layer

[3]The algorithm is given as an extension of the Modified Gram-Schmidt orthogonalization. However, similar expressions can also be achieved using Householder reflexion or Givens rotation for QR decomposition [12]. Furthermore, $\mathbf{p}$ denotes a permutation vector with $\mathbf{P} = \mathbf{I}_{N_\mathrm{T}}(:, \mathbf{p})$.

of each filter output signal

$$\tilde{\mathbf{d}}(n) = \mathbf{Q}^H(n)\,\mathbf{y}(n) = \mathbf{R}(n)\,\mathbf{d}(n) + \tilde{\mathbf{n}}(n) \qquad (18)$$

is free of interference with $\tilde{\mathbf{n}}(n) = \mathbf{Q}^H(n)\,\mathbf{n}(n)$ denoting the noise term at the filter output. After demodulation the L-values are deinterleaved and fed to the channel decoder, as shown in Fig. 3. Using the interleaved code bits for remodulation the estimated interference is canceled out within the block IC (interference cancellation) and the successive interference cancellation of the remaining layers is performed.

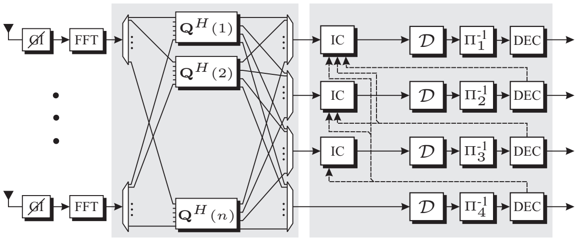

Fig. 3. Successive Interference Cancellation for a MIMO-OFDM scheme with $N_\mathrm{T} = 4$ transmit antennas

5 Computational Complexity

In this section we investigate the computational complexity of the SINR and the P-SQRD approach with respect to complex floating point operations $\mathcal{F}$. In order to achieve simple terms depending only on the system configuration, we count one complex addition as one flop (floating point operation) and a complex multiplication as three flops. Furthermore, the CMOS-approach is omitted as it requires an additional complexity in comparison to the SINR-ordering due to the calculation of the capacity term (16).

In case of ZF-filtering the SINR-approach requires the calculation of $N_\mathrm{T} - 1$ pseudo-inverses for each carrier n. Considering not only these filter calculations but also the detection process (without considering channel decoding) this approach requires approximately $\mathcal{O}\left((\frac{1}{6}N_\mathrm{T}^4 + 2N_\mathrm{R}N_\mathrm{T}^3 + \frac{1}{6}N_\mathrm{T}^3)N_\mathrm{C}\right)$ floating point operations. In contrast, the P-SQRD approach mainly consists of N_C QR decompositions. Using a detailed complexity consideration the overall complexity of this detection scheme is given by $\mathcal{O}\left((4N_\mathrm{R}N_\mathrm{T}^2 + \frac{9}{4}N_\mathrm{T}^2 + \frac{5}{2}N_\mathrm{R}N_\mathrm{T})N_\mathrm{C}\right)$ for the ZF-implementation. Thus, a strong reduction with respect to computational cost is achieved.

For a varying, but equal number of transmit and receive antennas $N_\mathrm{T} = N_\mathrm{R}$ Fig. 4 shows the required number of Flops for SINR-SIC and P-SQRD-SIC in case of ZF implementation with $N_\mathrm{C} = 32$ carriers. This figure visualizes the strong decrease in computational complexity achieved by our new approach. In order to show the fraction of saved complexity, the quotient

$$\xi = \frac{\mathcal{F}_{\text{P-SQRD-SIC}}}{\mathcal{F}_{\text{SINR-SIC}}} \qquad (19)$$

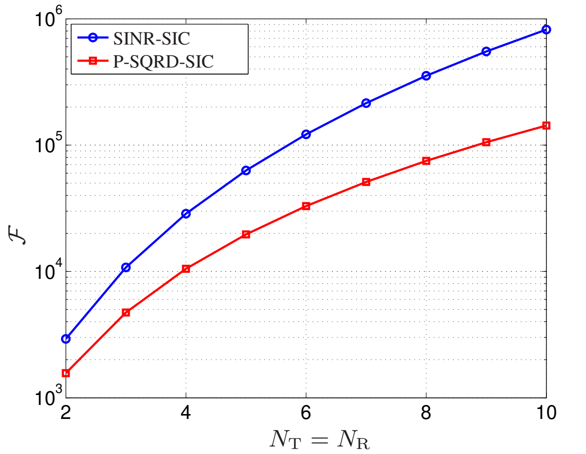

Fig. 4. Number of floating point operations $\mathcal{F}$ for SINR-SIC and P-SQRD-SIC detection with respect ZF criterion of MIMO-OFDM system with $N_\mathrm{T} = N_\mathrm{R}$ antennas and $N_\mathrm{C} = 32$ subcarriers

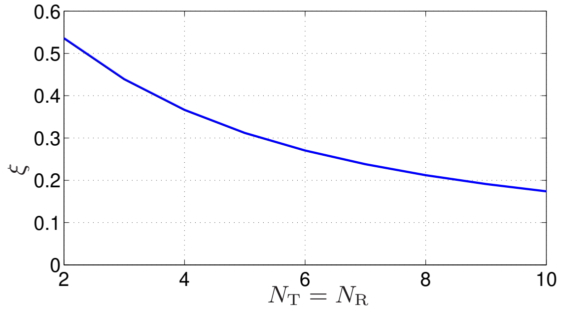

Fig. 5. Fraction of Flops $\xi = \mathcal{F}_{\text{P-SQRD-SIC}}/\mathcal{F}_{\text{SINR-SIC}}$ for a MIMO-OFDM system with $N_\mathrm{T} = N_\mathrm{R}$ antennas and $N_\mathrm{C} = 32$ subcarriers and Zero-Forcing criterion

for a varying number of antennas $N_\mathrm{T} = N_\mathrm{R}$ is depicted in Fig. 5. It indicates an increasing computational advantage of the new scheme for increasing number of antennas. As an example, for a system with $N_\mathrm{T} = N_\mathrm{R} = 4$ antennas the P-SQRD-SIC requires approximately $0.36 \cdot \mathcal{F}_{\text{SINR-SIC}}$ flops and consequently leads to a strong reduction in computational cost.

6 Performance Analysis

In this section we investigate the bit error rates (BER) for a per-antenna-coded MIMO-OFDM system with $N_\mathrm{T} = N_\mathrm{R} = 4$ antennas. We assume uncorrelated SISO channels of order $N_\mathrm{H} = 5$ with a constant power delay profile, i.e. the variance of all fading coefficients is equal to $1/(N_\mathrm{H}+1)$. Furthermore, each OFDM symbol contains a cyclic prefix of length $N_\mathrm{G} = 5$ and all $N_\mathrm{C} = 32$ subcarriers are used for signal transmission. For the simulations perfect estimation of the channel coefficients and of the noise variance is assumed.

In Fig. 6 the bit error rates for linear and successive detection with respect to the ZF- and the MMSE-criterion are shown when the $[7, 5]_8$ convolutional code and 4-QAM modulation is applied on each substream. Obviously, the sorted SIC schemes achieve substantial

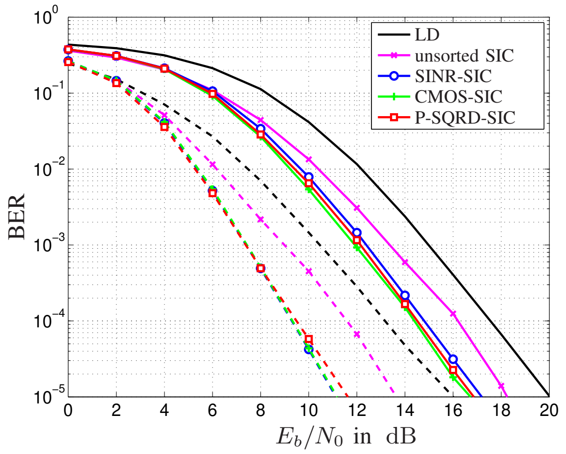

Fig. 6. BER of LD and SIC with respect to ZF (-) or MMSE (- -) criterion for a MIMO-OFDM system with $N_\mathrm{T} = N_\mathrm{R} = 4$ antennas, channel order $N_\mathrm{H} = 5$, $N_\mathrm{C} = 32$ subcarriers, guard interval of length $N_\mathrm{G} = 5$, 4-QAM symbols and $[7, 5]_8$ convolutional code

performance improvements in comparison to linear and successive detection without ordering. The results for P-SQRD-SIC, SINR-SIC and CMOS-SIC are comparable, with minor advantages for the later ones in case of MMSE detection. However, this small performance impairment comes with a strong reduction in computational complexity.

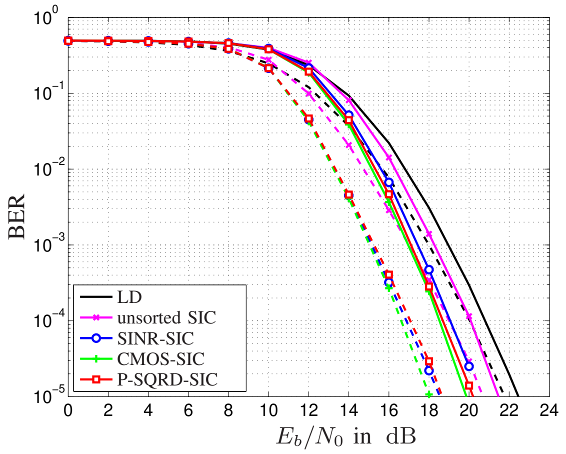

Fig. 7. BER of LD and SIC with respect ZF (-) or MMSE (- -) criterion of a MIMO-OFDM system with $N_\mathrm{T} = N_\mathrm{R} = 4$ antennas, channel order $N_\mathrm{H} = 5$, $N_\mathrm{C} = 32$ subcarriers, guard interval of length $N_\mathrm{G} = 5$, 64-QAM and $[133, 171]_8$ convolutional code

In Fig. 7 the BERs for a system with 64-QAM modulation and the $[133, 171]_8$ convolutional code of constraint length 7 are shown. Again, only a small difference in performance can be observed between the different ordering criterions. This demonstrates the potential of the proposed P-SQRD approach for schemes with high spectral efficiencies.

7 Summary and Conclusions

Within this paper we proposed a new detection scheme for coded MIMO-OFDM systems by introducing an extended version of the SQRD algorithm. The Parallel Sorted QR Decomposition (P-SQRD) algorithm achieves an adopted detection ordering within the QR decompositions of the N_C channel matrices and can be implemented with respect to the Zero-Forcing and the Minimum Mean Squared Error criterion. We presented simulation results for different scenarios and analytically demonstrated the computational advantage of the P-SQRD algorithm. We were able to show, that this new algorithm achieves comparable performance results to the schemes from literature, however requiring only a fraction of computational complexity. Thus, a feasible implementation for MIMO-OFDM for future wireless local area networks is given. As shown in [12] the concept of P-SQRD can also be used in MIMO receiver using frequency domain equalization, leading again to an efficient detection scheme with optimized detection order.

References

[1] P. Wolniansky, G. Foschini, G. Golden, and R. Valenzuela, "V-BLAST: An Architecture for Realizing Very High Data Rates Over the Rich-Scattering Wireless Channel," in *Proc. International Symposium on Signals, Systems, and Electronics (ISSSE)*, Pisa, Italy, 29. September 1998, pp. 295–300.

[2] B. Hassibi, "A Fast Square-Root Implementation for BLAST," in *Proc. Asilomar Conference on Signals, Systems and Computers*, vol. 2, Asilomar, USA, Nov. 2000, pp. 1255 – 1259.

[3] D. Wübben, J. Rinas, R. Böhnke, V. Kühn, and K.-D. Kammeyer, "Efficient Algorithm for Detecting Layered Space-Time Codes," in *Proc. International ITG Conference on Source and Channel Coding*, Berlin, Jan. 2002, pp. 399–405.

[4] D. Wübben, R. Böhnke, V. Kühn, and K.-D. Kammeyer, "MMSE Extension of V-BLAST based on Sorted QR Decomposition," in *Proc. VTC*, Orlando, FL, USA, Oct. 2003.

[5] R. Böhnke, D. Wübben, V. Kühn, and K.-D. Kammeyer, "Reduced Complexity MMSE Detection for BLAST Architectures," in *IEEE Proc. Global Communications Conference (GLOBECOM)*, San Francisco, CA, USA, Dec. 2003, pp. 2258–2262.

[6] Y. Li, J. Winters, and N. Sollenberger, "MIMO-OFDM for Wireless Communications: Signal Detection With Enhanced Channel Estimation," *IEEE Trans. on Communications*, vol. 50, no. 9, pp. 1471–1477, Sept. 2002.

[7] T. Kadous, "Ordered H-BLAST for MIMO/OFDM Systems over Multipath Channels," in *IEEE Symposium on Computers and Communication (ISCC)*, vol. 1, Kiris-Kemer, Turkey, June 2003, pp. 481–485.

[8] A. van Zelst, "Per-Antenna-Coded Schemes for MIMO OFDM," in *IEEE Proc. International Conference on Communications (ICC)*, Anchorage, AL, USA, May 2003.

[9] A. van Zelst and T. Schenk, "Implementation of a MIMO OFDM-Based Wireless LAN System," *IEEE Trans. on Signal Processing*, vol. 52, pp. 483–493, Feb. 2004.

[10] C. Michalke, H. Venkataraman, V. Sinha, W. Rave, and G. Fettweis, "Application of SQRD Algorithm for Efficient MIMO-OFDM Systems," in *Proc. European Wireless Conference*, Nicosia, Cyprus, Apr. 2005.

[11] H. Lee and I. Lee, "New Approach for Coded Layered Space-Time OFDM Systems," in *IEEE Proc. International Conference on Communications (ICC)*, Seoul, Korea, May 2005, pp. 608 – 612.

[12] D. Wübben, "Effiziente Detektionsverfahren für Multilayer-MIMO-Systeme," Ph.D. dissertation, Universität Bremen, Arbeitsbereich Nachrichtentechnik, Shaker Verlag, Feb. 2006, on german.