INTERPOLATION-BASED SUCCESSIVE INTERFERENCE CANCELLATION FOR PER-ANTENNA-CODED MIMO-OFDM SYSTEMS USING P-SQRD

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ABSTRACT

Spatial Multiplexing is a very popular approach to exploit the capacity advantage of multiple antenna systems and the combination with OFDM is a very promising approach to realize practical implementations also for wideband systems. In this paper we present a novel, computational efficient realization of successive interference cancellation for per-antenna-coded MIMO-OFDM systems with a large number of carriers. It utilizes a parallelized version of the SQRD algorithm only on a limited number of carriers in order to achieve the same detection order for all subcarriers. Afterwards, the QR decompositions for the remaining carriers are calculated by interpolation. Depending on the system configuration this leads to a significant complexity reduction in comparison to other schemes known from literature, but yielding comparable performance results.

1. INTRODUCTION

The utilization of OFDM in wideband multilayer MIMO systems can be used to significantly reduce the receiver complexity. Thereby MIMO-OFDM can be expected to be one of the first commercially used implementation of the V-BLAST multilayer architecture. In order to make use of the frequency diversity and the benefits of successive interference cancellation (SIC), forward error correction (FEC) coding has to be applied to each antenna separately (per-antenna-coding, PAC). However, this requires the same *optimized* detection order on each subcarrier. For the QR-based SIC the authors presented in [1, 2] a modified version of the SQRD algorithm [3, 4], where the so called P-SQRD (Parallel Sorted QR Decomposition) yield the Gram-Schmidt orthogonalizations for the channel matrices on all subcarriers with the same optimized detection order.

Recently, interpolation based detection algorithms have been introduced for MIMO-OFDM systems by Borgmann and Bölcskei [5] and Cescato et al. [6]. The idea of these approaches bases on the oversampling of the frequency response when applying the FFT. They propose in [6] to calculate QR decompositions only for a limited number of subcarriers n and to determine the matrices $\mathbf{Q}(n)$ and $\mathbf{R}(n)$ for the remaining carriers by interpolation, before executing SIC detection. Thereby, the computational cost for QR decompositions is reduced, yielding a reduction in overall complexity for sufficiently large number of subcarriers. Within this contribution we make use of their interpolation based QR decomposition in combination with our P-SQRD algorithm to achieve a SIC with optimized detection order.

Outline of the Paper: In Section 2 we define the transfer function of the MIMO system and introduce the system model. The fundamentals of SIC detection for MIMO-OFDM systems are reviewed in Section 3 and basics about Laurent polynomials and their interpolation are presented in Section 4. Afterwards, the interpolation based detection and the combination with the P-SQRD algorithm are explained in Section 5. The computational complexity and performance results are investigated in Section 6 and Section 7, respectively. The major results are than summarized in the final Section 8.

Notation: Matrices are represented by bold capital letters, where the element in row α and column β of a matrix $\mathbf{A}$ is indicated by $[\mathbf{A}]_{\alpha,\beta} = a_{\alpha,\beta}$. Accordingly, vectors are denoted by small capital letters, where $\mathbf{a}_\beta$ and $\mathbf{a}^{(\alpha)}$ represent the β-th column and the α-th row of $\mathbf{A}$. The matrix transpose and hermitian transpose are denoted by $(\cdot)^T$ and $(\cdot)^H$, respectively. Furthermore, $\mathbf{I}_\alpha$ represents the $\alpha \times \alpha$ identity matrix and $\mathbf{0}_{\alpha,\beta}$ denotes the $\alpha \times \beta$ all zero matrix. The Kronecker product is given by $\otimes$ and $|\mathcal{N}|$ represents the cardinality of a set $\mathcal{N}$. In order to distinguish between variables in time and in frequency domain, we indicate variables in time domain (TD) by an index TD whereas a labeling for variables in frequency domain (FD) is omitted.

2. SYSTEM DESCRIPTION

2.1. MIMO Transfer Function

We consider a multiple antenna system with N_T transmit and $N_\mathrm{R} \geq N_\mathrm{T}$ receive antennas in a frequency selective block fading environment, i.e. the channel is assumed to be constant over a frame, but changes independently between frames. The frequency selective channel of order N_H between all transmit and all receive antennas can be described by the $N_\mathrm{H}+1$ coefficient matrices $\mathbf{H}_\mathrm{TD}(\ell) \in \mathbb{C}^{N_\mathrm{R} \times N_\mathrm{T}}$, $0 \le \ell \le N_\mathrm{H}$, containing the delayed fading gains between the antennas. The corresponding transfer function of the MIMO channel is given by

$$\mathbf{H}\left(e^{j\Omega}\right) = \sum_{\ell=0}^{N_\mathrm{H}} \mathbf{H}_\mathrm{TD}(\ell)\, e^{-j\Omega\ell} \tag{1}$$

and is a polynomial matrix of degree N_H in $e^{-j\Omega}$ with the normalized frequency $0 \le \Omega < 2\pi$. By sampling this transfer function $\mathbf{H}(e^{j\Omega})$ at N_C equidistant sampling frequencies $\Omega_n = 2\pi n/N_\mathrm{C}$, $0 \le n \le N_\mathrm{C}-1$, the channel matrices

$$\mathbf{H}(n) := \mathbf{H}\left(e^{j\Omega_n}\right) = \sum_{\ell=0}^{N_\mathrm{H}} \mathbf{H}_\mathrm{TD}(\ell)\, e^{-j\Omega_n\ell} \tag{2}$$

at discrete carrier frequencies are obtained. In general $N_\mathrm{C} \gg N_\mathrm{H}$ holds and consequently the transfer function $\mathbf{H}(e^{j\Omega})$ is highly oversampled. Indeed, only $N_\mathrm{H}+1$ carriers $\mathbf{H}(n)$ are required to calculate all other $\mathbf{H}(n)$ by interpolation, because $\mathbf{H}(e^{j\Omega})$ is a Laurent polynominal matrix of degree N_H [6].

2.2. MIMO-OFDM

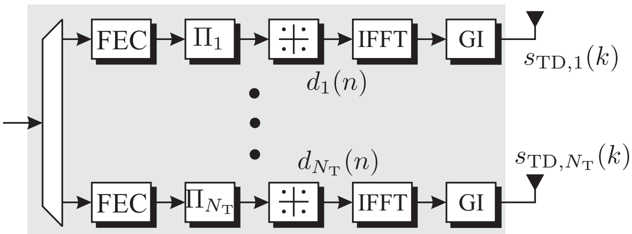

Fig. 1. MIMO-OFDM transmitter with per-antenna-coding

The transmitter of the investigated per-antenna-coded MIMO-OFDM system is shown in Fig. 1. According to this block diagram the information data is demultiplexed at the transmitter into N_T parallel data streams (*layers*), encoded by a convolutional encoder and after bitwise interleaving mapped to M-QAM or M-PSK symbols $d_i(n)$, $1 \le i \le N_\mathrm{T}$, $0 \le n \le N_\mathrm{C}-1$. After transforming the symbols to time domain by using the inverse fast fourier transform (IFFT), a guard interval (GI) of length N_G is added in form of a cyclic prefix before the sequence of $N_\mathrm{C}+N_\mathrm{G}$ signals $s_{\mathrm{TD},i}(k)$ is transmitted from each antenna i. With $\mathbf{s}_\mathrm{TD}(k) = [s_{\mathrm{TD},1}(k), \ldots, s_{\mathrm{TD},N_\mathrm{T}}(k)]^T$ denoting the N_T transmit signals at time instant k the received vector is given by

$$\mathbf{x}_\mathrm{TD}(k) = \sum_{\ell=0}^{N_\mathrm{H}} \mathbf{H}_\mathrm{TD}(\ell)\, \mathbf{s}_\mathrm{TD}(k-\ell) + \mathbf{n}_\mathrm{TD}(k)\ . \tag{3}$$

The vector $\mathbf{n}_\mathrm{TD}(k)$ denotes the additive white Gaussian noise at each receive antenna at sampling time k with covariance matrix $\mathrm{E}\{\mathbf{n}_\mathrm{TD}(k)\, \mathbf{n}_\mathrm{TD}^H(k)\} = \sigma_n^2 \mathbf{I}_{N_\mathrm{R}}$. The relation (3) expresses the superposition of transmitted symbols not only in space but also in time direction and thereby points out the two dimensional equalization problem.

At the receiver the cyclic prefix is removed and the fast fourier transform (FFT) is used to perform the transformation back into frequency domain. As long as $N_\mathrm{G} \ge N_\mathrm{H}$ holds, the application of the cyclic prefix and discrete fourier transform results in N_C orthogonal MIMO systems. With $\mathbf{d}(n) = [d_1(n), \ldots, d_{N_\mathrm{T}}(n)]^T$ denoting the $N_\mathrm{T} \times 1$ vector of modulated symbols on carrier $0 \le n \le N_\mathrm{C}-1$ and $\mathbf{H}(n)$ representing the flat MIMO channel for carrier n defined in (2), the corresponding received vector in frequency domain is given by [1]

$$\mathbf{y}(n) = \mathbf{H}(n)\, \mathbf{d}(n) + \mathbf{n}(n) \quad \text{for} \quad 0 \le n \le N_\mathrm{C}-1\ . \tag{4}$$

3. SUCCESSIVE INTERFERENCE CANCELLATION

3.1. Principle of SIC for uncoded MIMO-OFDM

Due to (4), the MIMO-OFDM scheme is separated into N_C orthogonal non-frequency selective MIMO systems and thus common detection algorithms can be used on each carrier separately in case of an *uncoded* system. In order to perform successive interference cancellation (SIC) on subcarrier n, the channel matrix $\mathbf{H}(n)$ is decomposed into the $N_\mathrm{R} \times N_\mathrm{T}$ matrix $\mathbf{Q}(n)$ with orthonormal columns and the $N_\mathrm{T} \times N_\mathrm{T}$ upper triangular matrix $\mathbf{R}(n)$ by QR decomposition. By omitting the index of the carrier n for convenience, the i-th column of $\mathbf{Q}(n)$ and the i-th row of $\mathbf{R}(n)$ are given by

$$\mathbf{q}_i = \frac{\mathbf{u}_i}{\sqrt{\mathbf{u}_i^H \cdot \mathbf{u}_i}} = \frac{\mathbf{u}_i}{r_{i,i}} \qquad \text{and} \qquad \mathbf{r}^{(i)} = \mathbf{q}_i^H \cdot \mathbf{H} \tag{5}$$

with the vector

$$\mathbf{u}_i = \mathbf{h}_i - \sum_{j=1}^{i-1} r_{j,i}\, \mathbf{q}_j \tag{6}$$

denoting the component of $\mathbf{h}_i$ orthogonal to the space spanned by $\mathbf{q}_1, \ldots, \mathbf{q}_{i-1}$. It is worth to note, that all diagonal elements $r_{i,i}$ are positive, real numbers and consequently the decomposition (5) is unique.

It is well-known, that the order of detection has a deep impact on the performance of the SIC and should therefore be optimized [7]. With respect to the QR decomposition this optimization can be achieved by permuting the columns of $\mathbf{H}(n)$ leading to different decompositions. With $\mathbf{P}(n)$ denoting an optimized permutation matrix for carrier n, the QR decomposition is than given by $\mathbf{H}(n)\,\mathbf{P}(n) = \mathbf{Q}(n)\,\mathbf{R}(n)$. An efficient algorithm to compute on optimized (not necessary the optimum) permutation is given by the Sorted QR Decomposition (SQRD) presented in [3]. After calculating the QR decomposition of $\mathbf{H}(n)\,\mathbf{P}(n)$ the received signal $\mathbf{y}(n)$ is filtered by $\mathbf{Q}^H(n)$ and due to the upper triangular form of $\mathbf{R}(n)$ the N_T-th layer of each filter output signal

$$\tilde{\mathbf{d}}(n) = \mathbf{Q}^H(n)\,\mathbf{y}(n) = \mathbf{R}(n)\,\mathbf{d}(n) + \tilde{\mathbf{n}}(n) \tag{7}$$

is free of interference. Following the quantization of this signal, the estimated interference is cancelled out from the other filter output signals and the successive interference cancellation of the remaining layers is performed [3].

The adaptation to the MMSE criterion is achieved by defining the *extended* channel matrix $\underline{\mathbf{H}}(n)$ and the *extended* receive vector $\underline{\mathbf{y}}(n)$ for carrier n through [8]

$$\underline{\mathbf{H}}(n) = \begin{bmatrix} \mathbf{H}(n) \\ \sigma_n \mathbf{I}_{N_\mathrm{T}} \end{bmatrix} \qquad \text{and} \qquad \underline{\mathbf{y}}(n) = \begin{bmatrix} \mathbf{y}(n) \\ \mathbf{0}_{N_\mathrm{T},1} \end{bmatrix} . \tag{8}$$

With the QR decomposition $\underline{\mathbf{H}}(n)\mathbf{P}(n) = \underline{\mathbf{Q}}(n)\underline{\mathbf{R}}(n)$ the filter output signal is than given by $\tilde{\mathbf{d}}(n) = \underline{\mathbf{Q}}^H(n)\,\underline{\mathbf{y}}(n)$ [4].

3.2. SIC for MIMO-OFDM with PAC

It is well-known, that encoding over frequency is necessary in OFDM systems to exploit frequency diversity. Furthermore, it is favorable to implement the FEC on each antenna separately, as the error correction capability of the code can than be used to reduce the problem of error propagation within the SIC. However, successive detection of such a PAC MIMO-OFDM system requires the same detection sequence on each carrier [1] and consequently no separate optimization is possible for each non-frequency selective MIMO system, but an optimization over all carriers has to be performed.

In [9, 10] van Zelst and Schenk proposed an optimization scheme as an extension of the V-BLAST algorithm and an approach using capacity terms was presented by Kadous in [11]. In [1, 2] the authors presented the P-SQRD algorithm, which achieves the QR decomposition $\mathbf{H}(n)\mathbf{P} = \mathbf{Q}(n)\mathbf{R}(n)$ for each permuted channel matrix $\mathbf{H}(n)\mathbf{P}$ with the *same* permutation matrix $\mathbf{P}$ for all carriers. Within this algorithm we apply the basic philosophy of SQRD to the N_C parallel QR decompositions of the matrices $\mathbf{H}(n)$ in such a way, that in each step the orthogonalization is performed with respect to that transmit antenna with minimum column norm orthogonal to the already spanned space. The complexity of the P-SQRD corresponds to the complexity of N_C QR decompositions and a minor additional overhead. In contrast to the schemes presented in [9, 10, 11] this leads to a strong reduction in complexity with only minor performance degradation [1, 2].

When the P-SQRD calculation of the PAC MIMO-OFDM system is done, the filter output signals (7) are calculated yielding an interference free signal on the N_T-th layer of $\tilde{\mathbf{d}}(n)$. After demodulation the Log-Likelihood-Ratios (LLR) are deinterleaved and fed to the corresponding channel decoder. Using the interleaved code bits for remodulation, the estimated interference is canceled out and the successive interference cancelation of the remaining layers is performed. The block diagram of the receiver is depicted in Fig. 2.

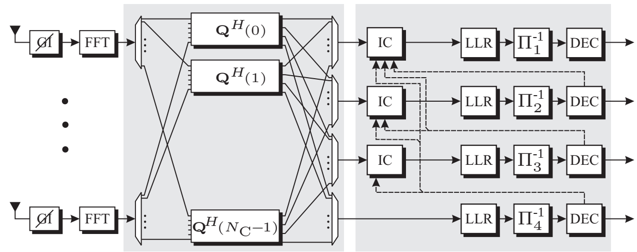

Fig. 2. Successive interference cancellation for a MIMO-OFDM scheme with $N_\mathrm{T} = 4$ transmit antennas

Instead of calculating the QR decomposition for all carriers $n \in \mathcal{N} = \{0, 1, \dots, N_\mathrm{C} - 1\}$, our aim in this paper is to perform this calculation only for $n \in \mathcal{P} \subset \mathcal{N}$ with $|\mathcal{P}| \ll |\mathcal{N}|$ subcarriers and to calculate the remaining coefficients of $\mathbf{Q}(n)$ and $\mathbf{R}(n)$ for carriers $n \in \mathcal{I} = \mathcal{N} \setminus \mathcal{P}$ by interpolation. In order understand the necessary algebraic steps, the basics about interpolation of so-called Laurent polynomials are given in the next section. Afterwards the presented results are applied to the problem at hand.

4. INTERPOLATION OF LAURENT POLYNOMIALS

In the sequel, several definitions about matrix polynomials and their interpolation are reviewed. Detailed tutorial introductions are given in [12, 13] and special results are also derived in [5, 6, 14].

4.1. Laurent Polynomial Matrix

Definition 1 (Laurent Polynomial Matrix) *Let $\mathbf{A}(z)$ denote a matrix valued function of the variable $z \in \mathcal{U}$, where $\mathcal{U}$ represents the unit circle. $\mathbf{A}(z)$ is called a Laurent polynomial (LP) matrix on $\mathcal{U}$ of degree $\deg\{\mathbf{A}(z)\} = L = L_1 + L_2$ if coefficient matrices $\mathbf{A}_\ell \in \mathbb{C}^{o \times m}$ with $\mathbf{A}_{-L_1} \neq \mathbf{0}$ and $\mathbf{A}_{L_2} \neq \mathbf{0}$ exist, so that $\mathbf{A}(z)$ can be represented by*

$$\mathbf{A}(z) = \sum_{\ell=-L_1}^{L_2} \mathbf{A}_\ell\, z^\ell . \tag{9}$$

Such a Laurent polynomial is denoted as $\mathbf{A}(z) \sim \mathrm{LP}(L_1, L_2)$.

Basically a Laurent polynomial is just an algebraic object in the sense of a common polynomial matrix, except that the indeterminant z can also have negative powers. Consequently, the *typical element* $p(z) = [\mathbf{A}]_{j,i}(z)$ is a common Laurent polynomial and can thus be described in the form

$$p(z) = p_{-L_1} z^{-L_1} + \dots + p_0 + \dots + p_{L_2} z^{L_2} , \tag{10}$$

where $p_\ell = [\mathbf{A}_\ell]_{j,i}$ is used to denote the associated elements of the coefficient matrices $\mathbf{A}_\ell$.

4.2. Interpolation

4.2.1. Basics

Following the interpolation theorem of Lagrange, the value of a polynomial of degree L at an arbitrary point z, i.e. $\mathbf{A}(z)$, is uniquely determined by the value of $\mathbf{A}(z_\ell)$ at $L+1$ unequal basis points z_ℓ. The reverse task, i.e. the values $\mathbf{A}(z_\ell)$ are given for a set of basis points z_ℓ and the polynomial has to be determined, is the fundamental problem of polynomial interpolation. In order to solve this task several approaches exist, e.g. Canonical basis, Lagrange, Newton and Trigonometic interpolation [15]. In the sequel, we shortly summarize the basics of Canonical interpolation with respect to the typical element $p(z) = [\mathbf{A}]_{j,i}(z)$ and coefficients $p_{-L_1}, \ldots, p_{L_2}$, where we also make use of fundamental properties of the Newton interpolation.

Let assume that the values $v_\ell = p(z_\ell)$ are known at $L+1$ distinct base points $z_0, \ldots, z_L \in \mathcal{U}$. Using these values a system of $L+1$ linear equations can be established

$$\begin{bmatrix} v_0 \\ v_1 \\ \vdots \\ v_L \end{bmatrix} = \begin{bmatrix} z_0^{-L_1} & \ldots & z_0^0 & \ldots & z_0^{L_2} \\ z_1^{-L_1} & \ldots & z_1^0 & \ldots & z_1^{L_2} \\ \vdots & & \vdots & & \vdots \\ z_L^{-L_1} & \ldots & z_L^0 & \ldots & z_L^{L_2} \end{bmatrix} \begin{bmatrix} p_{-L_1} \\ \vdots \\ p_0 \\ \vdots \\ p_{L_2} \end{bmatrix} . \tag{11}$$

Using matrix-vector notation (11) reads $\mathbf{v} = \mathbf{Z}\mathbf{p}$ with the Vandermonde-like matrix $\mathbf{Z}$ containing the different powers of the basis points z_i. Than, the coefficients of the polynomial $p(z)$ are given by $\mathbf{p} = \mathbf{Z}^{-1}\mathbf{v}$ and the value of $p(z)$ for an arbitrary $z \in \mathcal{U}$ can be calculated by interpolation

$$\begin{aligned} p(z) &= [z^{-L_1} \ \ldots \ z^0 \ \ldots \ z^{L_2}] \cdot \mathbf{p} \\ &= [z^{-L_1} \ \ldots \ z^0 \ \ldots \ z^{L_2}] \cdot \mathbf{Z}^{-1} \begin{bmatrix} v_0 \\ v_1 \\ \vdots \\ v_L \end{bmatrix} \end{aligned} \tag{12}$$

The direct generalization to matrix polynomials reads [14]

$$\mathbf{A}(z) = \left(\left([z^{-L_1} \ \ldots \ z^0 \ \ldots \ z^{L_2}] \cdot \mathbf{Z}^{-1} \right) \otimes \mathbf{I} \right) \cdot \begin{bmatrix} \mathbf{A}(z_0) \\ \mathbf{A}(z_1) \\ \vdots \\ \mathbf{A}(z_L) \end{bmatrix} \tag{13}$$

Hence, to interpolate from $L+1$ basis points z_ℓ the value for an arbitrary evaluation point z, the Vandermode matrix $\mathbf{Z}$ has to be inverted. Depending on the chosen basis points this matrix is almost singular and consequently numerical problems may occur during inversion. However, using some relations between the Vandermonde matrix and Newton interpolation, the value of the polynomial coefficients p_ℓ can also be calculated using the method of divided differences and without calculation of an inverse [15].

4.2.2. Evaluation of Polynomials

After the coefficients of the polynomial $p(z)$ have been calculated using the values v_ℓ at the basis points z_ℓ, the values of $p(z)$ for the remaining $z \in \mathcal{U}$ are of interest, i.e. execution of the interpolation for a number of evaluation points $z_\kappa \in \mathcal{U}$. For the problem at hand we will be interested on N_C evaluation points that are equally distributed on the unit circle. Thus, each evaluation point $z_\kappa = e^{-j\Omega_n \cdot \kappa}$ is a N_C-th root of unity and can also be described in the form $z_\kappa = z_{\kappa-1} \cdot e^{-j\Omega_n}$ with $z_0 = 1$. For a series of such equally spaced evaluation points the calculation of the polynomial can be carried out very efficiently using a shift register structure containing $L+1$ memory elements b_ℓ. For the subsequent explanation we denote the content of the ℓ-th memory element at step κ by $b_\ell^{(\kappa)}$

At the beginning ($\kappa = 0$), the memory elements are initialized by the coefficients of $p(z)$, i.e. $b_\ell^{(0)} = p_\ell$ for $-L_1 \leq \ell \leq L_2$. Consequently, the summation of all memory elements yields the result for $p(z)$ at position $z_0 = e^{-j\Omega_n \cdot 0} = 1$

$$p(z_0 = 1) = b_{-L_1}^{(0)} + \cdots + b_{L_2}^{(0)} = p_{-L_1} + \cdots + p_{L_2} . \tag{14}$$

For the next time clock, we update the memory elements by $b_\ell^{(1)} = b_\ell^{(0)} e^{-j\Omega_n \cdot \ell}$ for $-L_1 \leq \ell \leq L_2$. Summing up yields now the result for $p(z_1 = e^{-j\Omega_n \cdot 1})$

$$\begin{aligned} p(z_1 = e^{j\Omega_n}) &= b_{-L_1}^{(1)} + \cdots + b_{L_2}^{(1)} \\ &= b_{-L_1}^{(0)} e^{j\Omega_n \cdot L_1} + \cdots + b_{L_2}^{(0)} e^{-j\Omega_n \cdot L_2} \\ &= p_{-L_1} e^{j\Omega_n \cdot L_1} + \cdots + p_{L_2} e^{-j\Omega_n \cdot L_2} . \end{aligned} \tag{15}$$

Using the same update procedure for the memory elements $b_\ell^{(\kappa)} = b_\ell^{(\kappa-1)} \cdot e^{-j\Omega_n \cdot \ell}$ in each step the value of the polynomial $p(z)$ is calculated for all N_C equidistant evaluation points $z_\kappa \in \mathcal{U}$. As only the initial memory contents are effected by the current data realization and the multiplication with $e^{j\Omega_n \ell}$ corresponds to a rotation, this recursive shift register structure leads to a fast and efficient interpolation scheme.

Another efficient approach for performing the interpolation is possible using IFFT and FFT. However, to apply the Radix-2 implementations, this requires the number of basis points to be a power of 2.

5. INTERPOLATION BASED DETECTION

5.1. Problem Statement

By comparing the definition of the transfer function (1) with the formal definition of the Laurent polynomial (9) in the variable $z = e^{-j\Omega} \in \mathcal{U}$ it becomes obvious, that $\mathbf{H}(e^{j\Omega})$ is a LP matrix of degree N_H with coefficient matrices $\mathbf{H}_\mathrm{TD}(\ell)$, i.e. $\mathbf{H}(e^{j\Omega}) \sim \mathrm{LP}(N_\mathrm{H}, 0)$ [5]. Consequently, if $\mathbf{H}(n)$ is known for distinct subcarriers $n \in \mathcal{P}$ with $\mathcal{P} \subseteq \mathcal{N}$ and $N_\mathrm{P} = |\mathcal{P}| \geq N_\mathrm{H} + 1$, then all remaining coefficients $\mathbf{H}(n)$ with $n \in \mathcal{I}$ and $\mathcal{I} = \mathcal{N} \setminus \mathcal{P}$ can be calculated by interpolation.

Using this philosophy, Borgmann and Bölcskei interpolated the coefficients of the channel matrices $\mathbf{H}(n)$ for the data subcarriers from the corresponding pilot carriers [5]. Furthermore, they proposed to calculate the filter matrices for linear equalization of a MIMO-OFDM system only for a limited number of carriers and determine the remaining filter matrices by interpolation. However, as the inverse is no longer rational, the direct interpolation of the inverse matrices is not possible. But, because the determinant and the adjoint of a polynomial matrix are again polynomial and the inverse can be described with respect to these matrix functions using the rule of Cramer, the interpolation of the filter matrices can be traced back to the interpolation of the determinant and the adjoints [5, 14]. Cescato et al. extended in [6] this philosophy of interpolation based detection with respect to successive interference cancellation in terms of the QR decomposition, as described next. Later on we extend this general idea with respect to an optimized detection ordering.

5.2. Interpolation of QR Decomposition

As the matrices of the QR decomposition $\mathbf{Q}\left(e^{j\Omega}\right)$ and $\mathbf{R}\left(e^{j\Omega}\right)$ are in general rational functions of $e^{j\Omega}$, they are no LP matrices and consequently they can not be interpolated using a limited number of supporting points. However, the invertible mapping[1] $[\tilde{\mathbf{Q}}, \tilde{\mathbf{R}}] = \mathcal{M}[\mathbf{Q}, \mathbf{R}]$

$$\tilde{\mathbf{Q}} = \mathbf{Q}\boldsymbol{\Delta} \qquad \text{and} \qquad \tilde{\mathbf{R}} = \boldsymbol{\Delta}\mathbf{R} \tag{16}$$

with the $N_\mathrm{T} \times N_\mathrm{T}$ diagonal mapping matrix

$$\boldsymbol{\Delta} = \begin{bmatrix} r_{1,1} & & & \\ 0 & r_{1,1}^2 r_{2,2} & & \\ 0 & 0 & \ddots & \\ 0 & & & r_{1,1}^2 r_{2,2}^2 \cdots r_{N_\mathrm{T},N_\mathrm{T}} \end{bmatrix} \tag{17}$$

introduced in [6] yields LP matrices $\tilde{\mathbf{Q}}$ and $\tilde{\mathbf{R}}$ and, consequently, an interpolation is again possible. Of course, $\tilde{\mathbf{R}}$ is again upper triangular and $\tilde{\mathbf{Q}}$ is now an orthogonal matrix with unequal column norms. The diagonal elements $\Delta_i = [\boldsymbol{\Delta}]_{i,i}$ of (17) can be defined recursively as

$$\Delta_i = r_{i,i} \cdot \prod_{j=1}^{i-1} r_{j,j}^2 = r_{i,i} \cdot r_{i-1,i-1} \cdot \Delta_{i-1} \quad \text{for} \quad i \geq 2 \tag{18}$$

and $\Delta_1 = r_{1,1}$. Consequently, the i-th column of $\tilde{\mathbf{Q}}$ and the i-th row of $\tilde{\mathbf{R}}$ are given by

$$\tilde{\mathbf{q}}_i = \Delta_i \mathbf{q}_i \qquad \text{and} \qquad \tilde{\mathbf{r}}^{(i)} = \Delta_i \mathbf{r}^{(i)} \; . \tag{19}$$

Furthermore, we introduce the variable δ_i

$$\delta_i = \prod_{j=1}^{i-1} r_{j,j}^2 = r_{i,i}^2 \cdot \delta_{i-1} \qquad \text{for} \qquad i \geq 1 \tag{20}$$

[1]In contrast to [6] we use a slightly different notation for the mapping, i.e. an adopted definition of Δ_i. Furthermore, we omit the carrier index n for simplicity.

with $\delta_0 = 1$, so that $\Delta_i = r_{i,i}\delta_{i-1}$ and $\tilde{\mathbf{q}}_i = \delta_{i-1}\mathbf{u}_i$ hold. The corresponding demapping $[\mathbf{Q}, \mathbf{R}] = \mathcal{M}^{-1}[\tilde{\mathbf{Q}}, \tilde{\mathbf{R}}]$ is given by

$$\mathbf{Q} = \tilde{\mathbf{Q}}\boldsymbol{\Delta}^{-1} \qquad \text{and} \qquad \mathbf{R} = \boldsymbol{\Delta}^{-1}\tilde{\mathbf{R}} \tag{21}$$

or componentwise by $\mathbf{q}_i = \Delta_i^{-1}\tilde{\mathbf{q}}_i$ and $\mathbf{r}^{(i)} = \Delta_i^{-1}\tilde{\mathbf{r}}^{(i)}$, where Δ_i can be calculate using $\tilde{\mathbf{R}}$ by

$$\Delta_i = \sqrt{\tilde{r}_{i,i}\,\tilde{r}_{i-1,i-1}} \quad \text{for} \quad i \geq 2 \tag{22}$$

and $\Delta_1 = \sqrt{\tilde{r}_{1,1}}$. As stated in [6], the mapped variables are Laurent polynomials

$$\begin{aligned} \tilde{\mathbf{q}}_i &\sim \mathrm{LP}(iN_\mathrm{H}, (i-1)N_\mathrm{H}) \\ \tilde{\mathbf{r}}^{(i)} &\sim \mathrm{LP}(iN_\mathrm{H}, iN_\mathrm{H}) \\ \delta_i &\sim \mathrm{LP}(iN_\mathrm{H}, iN_\mathrm{H}) \end{aligned} \tag{23}$$

and can thus be interpolated by polynomial expressions. Due to $\tilde{\mathbf{r}}^{(N_\mathrm{T})} \sim \mathrm{LP}(N_\mathrm{T}N_\mathrm{H}, N_\mathrm{T}N_\mathrm{H})$ the maximum number of necessary basis points to interpolate $\tilde{\mathbf{Q}}(n)$ and $\tilde{\mathbf{R}}(n)$ is determined by $2N_\mathrm{T}N_\mathrm{H} + 1$.

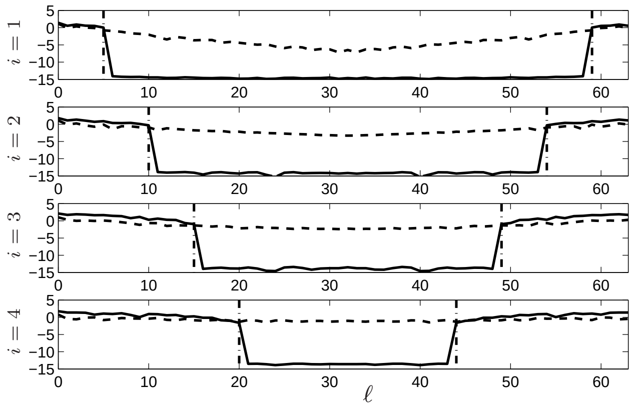

Fig. 3. Exemplary values of the diagonal elements $\log_{10}|\mathbf{R}_{\mathrm{TD},i,i}(\ell)|$ (- -) and $\log_{10}|\tilde{\mathbf{R}}_{\mathrm{TD},i,i}(\ell)|$ (–) for all time indices $0 \leq \ell \leq N_\mathrm{C} - 1$ of a system with $N_\mathrm{T} = N_\mathrm{R} = 4$ antennas, channel order $N_\mathrm{H} = 5$ and $N_\mathrm{C} = 64$ carriers

In order to visualize this basic difference between the original matrices $\mathbf{R}(n)$ and the mapped matrices $\tilde{\mathbf{R}}(n)$, Fig. 3 shows the corresponding values of the time domain representation of the diagonal elements for an arbitrary channel of order $N_\mathrm{H} = 5$ and $N_\mathrm{T} = N_\mathrm{R} = 4$. To emphasize the difference, $\mathbf{R}_{\mathrm{TD},i,i}(\ell)$ and $\tilde{\mathbf{R}}_{\mathrm{TD},i,i}(\ell)$ are given in logarithmic scale for $1 \leq i \leq N_\mathrm{T}$. Obviously, the vectors $\mathbf{R}_{\mathrm{TD},i,i}(\ell)$ contain nonzero elements for all indices ℓ and consequently can not be interpolated without an error. In contrast, $\tilde{\mathbf{R}}_{\mathrm{TD},1,1}(\ell)$ contains nonzero elements (or elements significantly larger than 10^{-15}) only for $\ell = 0, \ldots, 5$ and $\ell = 59, \ldots, 63$, or equivalently for $-5 \leq \ell \leq 5$. Thus, $\tilde{\mathbf{R}}_{1,1}(n) \sim \mathrm{LP}(5,5)$ holds and a polynomial of degree $L = 10$ with $L + 1 = 11$ evaluation points is sufficient to calculate the values of $\tilde{\mathbf{R}}_{1,1}(n)$ for all other carriers. It is furthermore obvious, that the degree of $\tilde{\mathbf{r}}^{(i)}$ increases with i and consequently a larger number of basis points are necessary for interpolation. The vertical lines in Fig. 3 indicate the largest positive and negative index of $\tilde{\mathbf{R}}_{\mathrm{TD},i,i}(\ell)$ unequal to zero.

5.3. Interpolation-based P-SQRD Detector

In order optimize the detection order for the interpolated QR decomposition, we propose to calculate the sorted QR decomposition only for the matrices $\mathbf{H}(n)$ with $n \in \mathcal{P}$ and $N_\mathrm{P} = |\mathcal{P}| \geq 2N_\mathrm{T}N_\mathrm{H}+1$ using the P-SQRD algorithm. For an unsorted QR decomposition the choice of carriers $n \in \mathcal{P}$ would be of no importance, as the polynomial is determined by an arbitrary choice of carriers. For the sorted QR decomposition an uniform distribution of carriers $n \in \mathcal{P}$ is favorable, as the determined detection order corresponds than to the sequence found on basis of all carriers $n \in \mathcal{N}$.

(1) Determine $\mathbf{H}(n)$ for subarriers $n \in \mathcal{P}$

(2) Perform P-SQRD with respect to $\mathbf{H}(n)$, $n \in \mathcal{P}$
$[\mathbf{Q}(\mathcal{P}), \mathbf{R}(\mathcal{P}), \mathbf{P}] = \mathtt{P\_SQRD}(\mathbf{H}(\mathcal{P}))$

(3) Apply mapping for each $n \in \mathcal{P}$
$[\tilde{\mathbf{Q}}(n), \tilde{\mathbf{R}}(n)] = \mathcal{M}[\mathbf{Q}(n), \mathbf{R}(n)]$

(4) Interpolate $\tilde{\mathbf{Q}}(n)$ and $\tilde{\mathbf{R}}(n)$, $n \in \mathcal{P}$, to obtain
$\tilde{\mathbf{Q}}(n)$ and $\tilde{\mathbf{R}}(n)$, $n \in \mathcal{I}$

(5) a) Apply demapping for each $n \in \mathcal{I}$
$[\mathbf{Q}(n), \mathbf{R}(n)] = \mathcal{M}^{-1}[\tilde{\mathbf{Q}}(n), \tilde{\mathbf{R}}(n)]$
b) Calculate $\mathbf{\Delta}^2(n)$ for each $n \in \mathcal{I}$

Algorithm 1: Interpolated sorted QR decomposition for a MIMO-OFDM system using P-SQRD

As listed in Algorithm 1 the corresponding matrices $\mathbf{Q}(n)$ and $\mathbf{R}(n)$ are mapped afterwards onto $\tilde{\mathbf{Q}}(n)$ and $\tilde{\mathbf{R}}(n)$, so that the remaining matrices $\tilde{\mathbf{Q}}(n)$ and $\tilde{\mathbf{R}}(n)$ with $n \in \mathcal{I}$ can be calculated by interpolation. Finally, the matrices $\mathbf{Q}(n)$ and $\mathbf{R}(n)$ for $n \in \mathcal{I}$ can be found by the demapping $\mathcal{M}^{-1}$ in Step 5 a). Thus, the ordered QR decomposition for all N_C matrices $\mathbf{H}(n)$ are achieved by calculating only N_P QR decompositions and some mapping and interpolation steps yielding a reduction in complexity if the number of subcarriers N_C significantly exceeds $2N_\mathrm{T}N_\mathrm{H}+1$.

However, we can further simplify the procedure by dropping the inverse mapping and using $\tilde{\mathbf{Q}}(n)$ directly for filtering in (7). Due to $\mathbf{H}(n) = \tilde{\mathbf{Q}}(n)\mathbf{\Delta}^{-2}(n)\tilde{\mathbf{R}}(n)$ and $\tilde{\mathbf{Q}}^H(n)\tilde{\mathbf{Q}}(n) = \mathbf{\Delta}^2(n)$, the relation $\tilde{\mathbf{Q}}^H(n)\mathbf{H}(n) = \tilde{\mathbf{R}}(n)$ follows and consequently the filter output signal is given by

$$\tilde{\mathbf{d}}(n) = \tilde{\mathbf{Q}}^H(n)\,\mathbf{y}(n) = \tilde{\mathbf{R}}\,\mathbf{d}(n) + \bar{\mathbf{n}}(n)\ , \qquad (24)$$

with $\mathrm{E}\{\bar{\mathbf{n}}(n)\,\bar{\mathbf{n}}^H(n)\} = \sigma_n^2\mathbf{\Delta}^2(n)$ denoting the corresponding noise covariance. Instead of performing the inverse mapping, we only have to compute the diagonal elements of $\mathbf{\Delta}^2(n)$ (compare Step 5 b) in Algorithm 1) by

$$\Delta_i^2(n) = \tilde{\mathbf{R}}_{i,i}(n)\,\tilde{\mathbf{R}}_{i-1,i-1}(n) \qquad (25)$$

and consider the noise variance within the LLR calculation.

The adaptation to the MMSE criterion is achieved by simply performing the QR decompositions with respect to the extended channel matrices $\underline{\mathbf{H}}(n) = [\mathbf{H}^H(n)\ \sigma_n\mathbf{I}_{N_\mathrm{T}}]^H$, which results again in LP matrices of degree N_H.

6. COMPUTATIONAL COMPLEXITY

In the sequel, we investigate the complexity of the proposed interpolation based P-SQRD detection with respect to complex floating point operations $\mathcal{F}$ and compare it to the effort required by a full P-SQRD detection and the approach by van Zelst and Schenk. In order to achieve simple terms depending only on the system configuration, we count one complex addition as one flop (floating point operation) and a complex multiplication as three flops. All other operations, e.g. addition and multiplication with respect to real numbers, division, square root, are traced back to this complexity measurement [16]. The following investigation restricts to the Zero-Forcing implementation.

The SIC following the SINR-approach by van Zelst and Schenk needs

$$\mathcal{F}_{\text{SINR-SIC}} = \mathcal{O}\left((\tfrac{1}{6}N_\mathrm{T}^4 + (2N_\mathrm{R}+\tfrac{1}{6})N_\mathrm{T}^3 + 3N_\mathrm{R}N_\mathrm{T}^2)N_\mathrm{C}\right) \qquad (26)$$

floating point operations and is less complex than the scheme by Kadous [1]. The effort for the common P-SQRD-SIC detection subdivides into the preprocessing part for sorted QR decomposition on N_C carriers, e.g. by executing the adopted Gram-Schmidt algorithm, and the subsequent SIC detection. Thus, this approach requires overall about

$$\mathcal{F}_{\text{P-SQRD-SIC},N_\mathrm{C}} = \mathcal{O}\left(((4N_\mathrm{R}+\tfrac{9}{4})N_\mathrm{T}^2 + (\tfrac{5}{2}N_\mathrm{R}-\tfrac{5}{4})N_\mathrm{T})N_\mathrm{C}\right) \qquad (27)$$

operations. For the preprocessing of the interpolated P-SQRD-SIC the common P-SQRD and the mapping are executed with respect to N_P carries and afterwards interpolation and calculation of $\mathbf{\Delta}^2(n)$ takes place on $N_\mathrm{I} = |\mathcal{I}|$ carriers. The execution of the P-SQRD algorithm on N_P carriers requires

$$\mathcal{F}_{\text{P-SQRD},N_\mathrm{P}} = \mathcal{O}\left(((4N_\mathrm{R}+\tfrac{1}{4})N_\mathrm{T}^2 - \tfrac{3}{2}N_\mathrm{R}N_\mathrm{T})N_\mathrm{P}\right) \qquad (28)$$

operations and the mapping of $\mathbf{Q}(n)$ and $\mathbf{R}(n)$ on these N_P carriers has a cost of

$$\mathcal{F}_{\mathcal{M}} = \left(N_\mathrm{T}^2 + (1+2N_\mathrm{R})N_\mathrm{T} - 3 - 2N_\mathrm{R}\right)N_\mathrm{P}\ . \qquad (29)$$

For the approximation of the interpolation complexity we follow the approach in [5, 6], where an equivalent complexity measurement is introduced, as different efficient variants for interpolation exist. They propose to approximate the cost per interpolated point by two flops and thus we achieve an overall effort for the interpolation of $\tilde{\mathbf{Q}}$ and $\tilde{\mathbf{R}}$ of

$$\mathcal{F}_{\text{Int}} = \mathcal{O}\left((\tfrac{3}{2}N_{\text{T}}^2 + (\tfrac{3}{2} + 3N_{\text{R}})N_{\text{T}})N_{\text{I}} \right) \ . \tag{30}$$

Finally, the effort for computing $\mathbf{\Delta}^2(n)$ on N_{I} carriers corresponds to

$$\mathcal{F}_{\mathbf{\Delta}^2} = \tfrac{1}{2}(N_{\text{T}} - 1)N_{\text{I}} \tag{31}$$

and the overall effort for the preprocessing part is given by

$$\mathcal{F}_{\text{Int-P-SQRD}} = \mathcal{F}_{\text{P-SQRD},N_{\text{P}}} + \mathcal{F}_{\mathcal{M}} + \mathcal{F}_{\text{Int}} + \mathcal{F}_{\mathbf{\Delta}^2} \ . \tag{32}$$

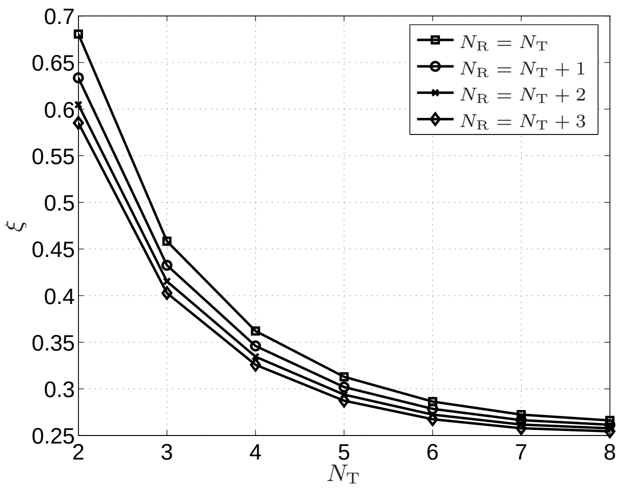

Fig. 4. Fraction of flops ξ for a MIMO-OFDM system with N_{T} transmit antennas, $N_{\text{C}} = 1024$ subcarriers and channel order $N_{\text{H}} = 8$

In order to show the fraction of saved complexity within the preprocessing-part, the quotient

$$\xi = \frac{\mathcal{F}_{\text{Int-P-SQRD}}}{\mathcal{F}_{\text{P-SQRD},N_{\text{C}}}} \tag{33}$$

is depicted for a varying number of transmit and receive antenna and $N_{\text{C}} = 1024$ subcarriers in Fig. 4. It indicates an increasing computational advantage of the new scheme for increasing number of antennas. As an example, for a system with $N_{\text{T}} = N_{\text{R}} = 4$ antennas the interpolated P-SQRD algorithm requires approximately $0.36 \cdot \mathcal{F}_{\text{P-SQRD},N_{\text{C}}}$ flops and consequently leads to a strong reduction in computational cost.

For a varying, but equal number of transmit and receive antennas $N_{\text{T}} = N_{\text{R}}$ Fig. 5 shows the required number of flops for the complete detection process with respect to the SINR-SIC, P-SQRD-SIC and interpolated P-SQRD-SIC, again for a system with $N_{\text{C}} = 1024$ carriers and channel order $N_{\text{H}} = 8$. This figure visualizes the strong decrease in computational complexity achieved by our new interpolated approach in contrast to both other approaches for the system with large number of carriers.

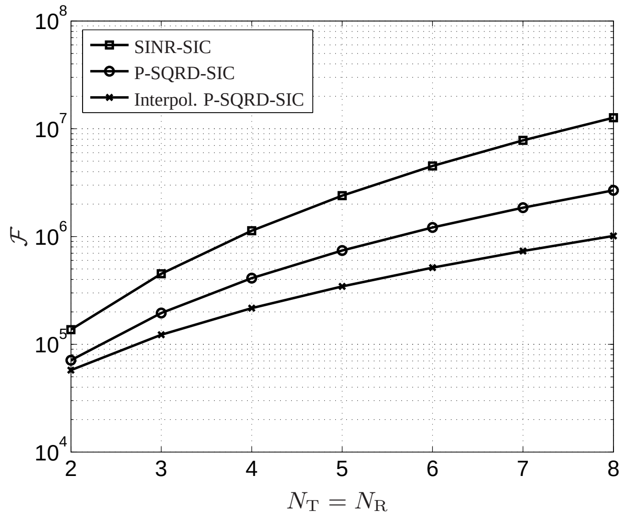

Fig. 5. Number of floating point operations $\mathcal{F}$ for SINR-SIC, P-SQRD-SIC and interpolated P-SQRD-SIC detection with respect to the ZF criterion of a MIMO-OFDM system with $N_{\text{T}} = N_{\text{R}}$ antennas, $N_{\text{C}} = 1024$ subcarriers and channel order $N_{\text{H}} = 8$

7. PERFORMANCE ANALYSIS

In this section, we investigate the bit error rates (BER) for a per-antenna-coded MIMO-OFDM system with $N_{\text{T}} = N_{\text{R}} = 4$ antennas, channel order $N_{\text{H}} = 5$ and $N_{\text{C}} = 128$ carries, where the $[7,5]_8$ convolutional code and 4-QAM modulation is applied on each substream. Fig. 6 shows the results for linear and successive detection with respect to the ZF- and the MMSE-criterion. Obviously, the sorted SIC schemes achieve a substantial performance improvement in comparison to the linear and to successive detection without ordering. As expected, the results for P-SQRD SIC with or without interpolation correspond to each other and furthermore only a small degradation with respect to the approach by Kadous is obvious. In order to avoid confusion, the BERs of the SINR-SIC are omitted, but as investigated in [1], the corresponding results match almost with the P-SQRD-SIC.

8. SUMMARY AND CONCLUSIONS

In this contribution we proposed a new detection scheme for coded MIMO-OFDM systems by applying an extended version of the SQRD algorithm only to a limited number of carriers yielding an optimized detection order for all carriers. After applying an invertible mapping it is than possible to interpolate the QR decompositions for the remaining tones. For sufficiently large number of carriers this results in a reduced receiver complexity. It was shown that the proposed receiver structure achieves comparable results to the schemes from literature with the mentioned reduction in computational complexity.

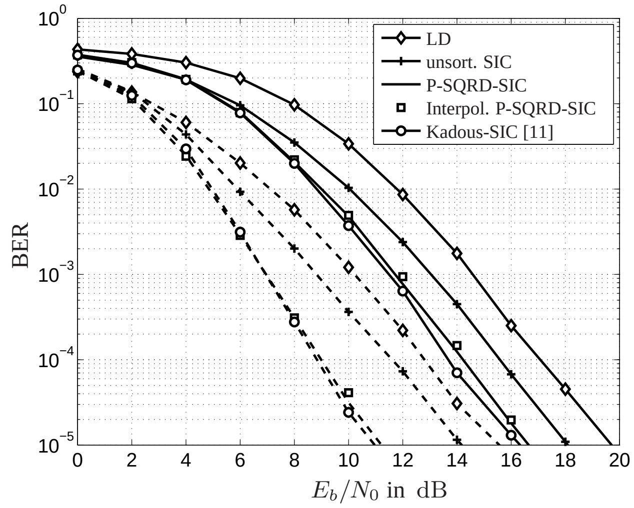

Fig. 6. BER of LD and SIC with respect to the ZF (–) or the MMSE (--) criterion for a PAC MIMO-OFDM system with $N_{\mathrm{T}} = N_{\mathrm{R}} = 4$ antennas, channel order $N_{\mathrm{H}} = 5$, $N_{\mathrm{C}} = 128$ subcarriers, guard interval of length $N_{\mathrm{G}} = 5$, 4-QAM

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