SUM-RATE EVALUATION OF BEAMFORMING WITH LIMITED FEEDBACK FOR THE EVOLVED UTRA DOWNLINK

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ABSTRACT

We consider a multi-user Multiple-Input Single-Output downlink broadcast channel with Gaussian inputs using Orthogonal Frequency Division Multiplexing for the air interface. By applying linear preprocessing techniques such as beamforming at the Base Station, spatial information of the users in a radio cell can be exploited and multi-user interference can be reduced. The performance of this preprocessing depends on the degree of channel state information at the transmitter. To cope with the limited capacity for uplink signalling, codebook-based feedback in conjunction with chunk processing is suggested in the literature. In this paper, the information theoretic impact of limited feedback on the sum-rate of such a system concerning single- and multi-user beamforming with 3GPP Evolved UTRA parameters is investigated.

1. INTRODUCTION

The upcoming standardization phase of the Third Generation Partnership Project (3GPP) towards enhancements of the existing 3G mobile telecommunications system, termed Evolved Universal Terrestrial Radio Access (E-UTRA) or Long Term Evolution (LTE), uses Orthogonal Frequency Division Multiplexing (OFDM) for the downlink air interface [1]. By separation of the frequency-selective channel into several orthogonal frequency-flat channels, an adaptive allocation of resources in time and frequency is offered [2]. The main goals of LTE are to provide higher system capacity and coverage. To achieve this improved performance, Multiple-Input Multiple-Output (MIMO) systems are a well integrated part of the specification process and preprocessing techniques at the Base Station are explicitly considered [3]. These techniques exploit the spatial information of the users and can decrease the complexity of the mobile terminals. In previous works Dirty Paper Coding (DPC) approaches have been proposed for multi-user MIMO systems, as they achieve the full capacity region of the MIMO broadcast channel (BC) [4,5]. Due to the high implementation complexity of such non-linear precoding schemes, sub-optimal solutions have been presented to null or at least minimize the multi-user interference [6]. Single-user beamforming (SU-BF) [7] and Zero-Forcing beamforming (ZF-BF) [8] are two common implementations.

However, an inherent disadvantage of these preprocessing techniques is the need for accurate Channel State Information (CSI) per user at the transmitter, where insufficient CSI at the transmitter can lead to inadequate adaptation of the resources to the existing channel conditions. A possible solution to cope with the limited amount of feedback in the uplink is given by selecting precoding vectors or matrices out of a predetermined codebook [3]. The question arises whether a unitary or non-unitary codebook should be used. Employing unitary precoding for multi-user scenarios was selected for E-UTRA standardization in [9], whereas in this paper still both options are adressed for comparison reasons. To additionally reduce the signalling overhead, groups of sub-carriers and OFDM symbols, referred to as chunks, may use the same allocated resources and precoding vectors if they experience nearly the same channel fading conditions. This paper deals with an information theoretic evaluation of codebook-based beamforming techniques in conjunction with chunk-based processing for a multi-user Multiple-Input Single-Output (MISO) OFDM downlink system.

The paper is organized as follows. At first, the system model is introduced in Section 2. The applied beamforming techniques in chunk-based processing are described in Section 3, while the requirements for limited feedback including codebook construction for unitary and non-unitary precoding as well as beamforming vector interpolation are presented in Section 4. In Section 5 the information theoretic evaluation based on the sum-rate is explained and the simulation results are shown. Finally, conclusions are provided in Section 6.

2. SYSTEM MODEL

We assume a MISO broadcast channel with N_T antennas at the Base Station (BS) and K non-cooperative users with a single receive antenna applying OFDM with N_c active subcarriers according to Fig. 1. The transmitter intends to send

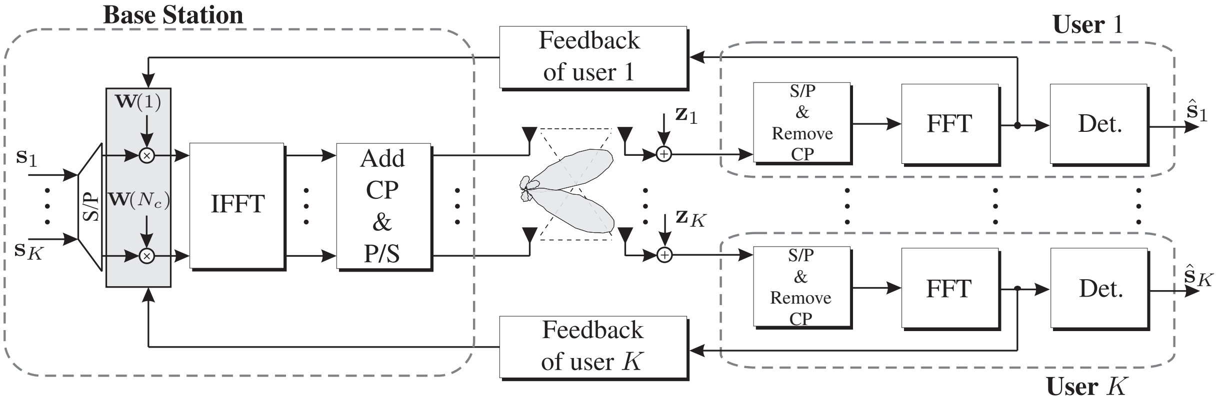

Fig. 1. Block diagram of a multi-user MISO-OFDM system with N_T transmit antennas, K users and N_c sub-carriers with linear precoding

the unit variance signals $s_k(n)$ on sub-carrier n, $1 \leq n \leq N_c$, to user k, $1 \leq k \leq K$. In order to reduce the multi-user interference, a linear superposition of all $s_k(n)$ weighted by different linear unit norm preprocessing vectors $\mathbf{w}_k(n)$ is transmitted on sub-carrier n. To transform the data into time domain the Inverse Fast Fourier Transform (IFFT) is applied and a guard interval in form of a cyclic prefix (CP) is introduced before transmission. After removing the CP and application of the Fast Fourier Transform (FFT) at the receivers, the receive signal in frequency domain of user k on sub-carrier n can be expressed as[1]

$$y_k(n) = \mathbf{h}_k^T(n) \sum_{i=1}^{K} \sqrt{P_{i,n}} \mathbf{w}_i(n)\, s_i(n) + z_k(n) \ , \quad (1)$$

where $z_k(n)$ is a white Gaussian noise term with variance σ_z^2. $\sqrt{P_{k,n}}$ is the transmission power for user k on sub-carrier n. The complex channel between the N_T transmit antennas and the receive antenna of user k on sub-carrier n can be described by the vector $\mathbf{h}_k(n) \in \mathbb{C}^{N_T \times 1}$, where the $N_T \times N_T$ channel matrix is defined by $\mathbf{H}(n) = [\mathbf{h}_1(n) \ldots \mathbf{h}_K(n)]^T$. Introducing $\mathbf{v}_k(n) = \sqrt{P_{k,n}} \mathbf{w}_k(n)\, s_k(n)$ yields

$$y_k(n) = \mathbf{h}_k^T(n) \sum_{i=1}^{K} \mathbf{v}_i(n) + z_k(n) \ . \quad (2)$$

Consequently, the $N_T \times N_T$ transmit covariance matrix for user k on sub-carrier n is

$$\begin{aligned} \mathbf{Q}_k(n) &= \mathrm{E}\left\{\mathbf{v}_k(n)\, \mathbf{v}_k^H(n)\right\} \\ &= P_{k,n} \mathbf{w}_k(n)\, \mathrm{E}\left\{s_k(n)\, s_k^*(n)\right\} \mathbf{w}_k^H(n) \\ &= P_{k,n} \mathbf{w}_k(n)\, \mathbf{w}_k^H(n) \end{aligned} \quad (3)$$

[1] Throughout the paper capital boldface letters denote matrices and small boldface letters describe column vectors. The conjugate, transpose, hermitian transpose and Moore-Penrose pseudo inverse are denoted by $(\cdot)^*$, $(\cdot)^T$, $(\cdot)^H$ and $(\cdot)^+$, respectively. Furthermore, $(\cdot)^{-1}$ is the matrix inverse and $\mathbf{I}_\alpha$ is the $\alpha \times \alpha$ identity matrix. $\|\cdot\|$ describes the vector norm, $|\cdot|$ stands for the absolute value, $\mathrm{tr}\{\cdot\}$ is the trace of a matrix and $\mathrm{diag}\{\cdot\}$ is the diag-operator.

Per definition, $\mathrm{tr}\{\mathbf{Q}_k(n)\} = P_{k,n}$ is the distributed power to user k on sub-carrier n with the total transmit power $\sum_{k=1}^{K} \sum_{n=1}^{N_c} P_{k,n} = P$. By defining the $K \times K$ diagonal matrix $\mathbf{P}^{\frac{1}{2}}(n) = \mathrm{diag}\{\sqrt{P_{1,n}} \ldots \sqrt{P_{K,n}}\}$ and the filter matrix $\mathbf{W}(n) = [\mathbf{w}_1(n) \ldots \mathbf{w}_K(n)]$, additionally stacking the received signals of all users to a vector $\mathbf{y}(n) \in \mathbb{C}^{K \times 1}$ and the noise terms into the vector $\mathbf{z}(n) \in \mathbb{C}^{K \times 1}$, equation (2) can be restated to a compact matrix-vector notation for each sub-carrier

$$\mathbf{y}(n) = \mathbf{H}(n)\, \mathbf{x}(n) + \mathbf{z}(n) \qquad 1 \leq n \leq N_c \ , \quad (4)$$

with transmit vector $\mathbf{x}(n) = \mathbf{W}(n)\, \mathbf{P}^{\frac{1}{2}}(n)\, \mathbf{s}(n)$. $\mathbf{s}(n) \in \mathbb{C}^{K \times 1}$ contains the transmit data on sub-carrier n. In order to investigate the sum-rate, an optimal channel code and perfect data detection at each receiver is assumed.

3. SPACE DIVISION IN CHUNK-BASED PROCESSING

3.1. Beamforming Aspects

It is well-known that the spectral efficiency of a multi-user MISO-OFDM system can be increased by Space Division Multiple Access (SDMA). For this purpose, users assigned to the same sub-carrier at the same time instance are divided by means of beamforming, where the number of users separable by beamforming is generally limited to $K \leq N_T$ on each sub-carrier. If SU-BF is performed, thus only a single-user is assigned to each sub-carrier, the data is transmitted into the direction of this user to obtain optimal performance in terms of channel gain. With perfect channel knowledge at the BS this can be achieved by the beamforming vector $\mathbf{w}_k^{(\mathrm{SU-BF})}(n) = \frac{\mathbf{h}_k^*(n)}{\|\mathbf{h}_k(n)\|}$, which corresponds to the dominant right singular vector of $\mathbf{h}_k^T(n)$ [10].

In contrast, with ZF-BF all K users can be served on each sub-carrier as by ZF-BF the multi-user interference is nulled and the received signal power is maximized. Thus, if all of the K users sharing the same time-frequency resource are scheduled by the system on each sub-carrier, the weighting vectors $\mathbf{w}_k(n)$ for sub-carrier n are obtained by the pseudo-inverse of the channel matrix $\mathbf{H}(n)$ such that

$$\mathbf{F}(n) = \mathbf{H}^{+}(n) = \mathbf{H}^{H}(n)\left(\mathbf{H}(n)\,\mathbf{H}^{H}(n)\right)^{-1} \;, \tag{5}$$

where $\mathbf{F}(n) = [\mathbf{f}_1(n) \dots \mathbf{f}_K(n)]$ denotes the matrix of beamforming vectors. To preserve the unit norm of the beamforming vectors, the k-th column of $\mathbf{F}\,(n)$ is chosen such that $\mathbf{w}_k^{(\mathrm{ZF-BF})}(n) = \frac{\mathbf{f}_k(n)}{\|\mathbf{f}_k(n)\|}$. Thereby, the direction of the projection of the channel vector $\mathbf{h}_k\,(n)$ on the left null space of the subspace spanned by all other coallocated users is maintained and the unit norm power constraint is achieved [7]. Thus, the beamforming vectors on sub-carrier n are chosen to fulfill the orthogonality criterion

$$\mathbf{h}_i^T(n)\,\mathbf{w}_k(n) = 0 \qquad \forall\, k, k \neq i \;, \tag{6}$$

with the condition $\mathbf{w}_k^H(n)\,\mathbf{w}_k(n) = 1$.

3.2. Chunk-based Transmission

In the E-UTRA concept sub-carriers and OFDM symbols are organized in terms of so-called chunks. In a localized mapping of these chunks [3], they consist of N_f consecutive sub-carriers for a number of N_s OFDM symbols, therefore including a set $\mathcal{M}$ of $N_f \cdot N_s$ data symbols. Usually the number of consecutive OFDM symbols is equal to the number of OFDM symbols in a subframe. Hence, only adjacent sub-carriers and OFDM symbols are combined in a chunk and the total number of chunks per subframe is N_c/N_f. Applying chunk-based beamforming means that a selected beamforming vector for user k is equal for all sub-carriers and OFDM symbols in a chunk ν, $1 \leq \nu \leq N_c/N_f$. Thus, if ZF-BF is applied per chunk (ZF-BF PC), the corresponding precoding vector of user k is obtained using the pseudo-inverse of the average channel matrix of a chunk $\underline{\mathbf{H}}\,(\nu)$ with

$$\underline{\mathbf{H}}(\nu) = \frac{1}{N_f \cdot N_s} \sum_{(m,\tau)\in\mathcal{M}} \mathbf{H}(m,\tau) \;, \tag{7}$$

where m, $1 \leq m \leq N_f$, is the frequency index and τ, $1 \leq \tau \leq N_s$, is the time index. Accordingly, $\mathbf{H}(m,\tau)$ is the corresponding channel matrix on sub-carrier m at time τ in chunk ν.

4. LIMITED FEEDBACK PREPROCESSING

In the sequel, the CSI at the transmitter is no longer perfect. One applicable method in E-UTRA is to use the frequency division duplex (FDD) scheme. In FDD systems, since different uncorrelated frequency bands are assumed, it is not trivial to exploit channel reciprocity for up- and downlink transmission [11]. In contrast, a feedback channel, which usually is rate limited, is available to exchange CSI between users and the Base Station. One solution suggested in [3] is to use a codebook based approach for quantizing the feedback information. With such a codebook, the selectable beamforming vectors are limited to a set of vectors in a predetermined vector codebook $\mathcal{C}$. A codebook consists of $N_b = 2^L$ complex unit vectors of length N_T, where L is the number of bits required for labelling any entry of the codebook. In this section, we show our design and operation criteria for two different static codebooks, which are also referred to as one-shot or non-tracking codebooks.

4.1. Non-Unitary Precoding

For the design of a non-unitary codebook the method of Hochwald et al. is chosen [12]. In order to minimize the impact of quantization on the channel gain, a maximum correlation δ of two unit norm vectors $\mathbf{c}_\ell$ and $\mathbf{c}_{\ell'}$ contained in the codebook $\mathcal{C}$ is defined. This is a simple performance measure, as it is not depending on the SNR and the number of receive antennas and was given in [12] to

$$\delta = \max_{1 \leq \ell \leq \ell' \leq N_b} |\mathbf{c}_\ell^H \mathbf{c}_{\ell'}| \qquad \mathbf{c}_\ell, \mathbf{c}_{\ell'} \in \mathcal{C} \;, \tag{8}$$

where we have to minimize δ for a randomly generated codebook. This measure is related to the definition of distance between subspaces. The column space of a vector is a line. A set of N_T lines in a N_T-dimensional vector space is called the Grassmann manifold $\mathcal{G}(N_T, 1)$. As we deal with complex elements of $\mathbf{w}_k$, the problem in finding optimally distributed codebook vectors can be solved by optimally distributing these lines in $\mathbb{C}^{N_T}$ with respect to their angles. Hence, minimizing δ means to pack the lines of all $\mathbf{c}_\ell$ and thereby maximizing their minimum distance [13], which is usually expressed in terms of the chordal distance [14, 15]

$$d\,(\mathbf{c}_\ell, \mathbf{c}_{\ell'}) = \sqrt{1 - |\mathbf{c}_\ell^H \mathbf{c}_{\ell'}|^2} \quad \forall \quad \mathbf{c}_\ell, \mathbf{c}_{\ell'} \in \mathbb{C}^{N_T} \;. \tag{9}$$

To construct our non-unitary codebook, $\mathbf{c}_1$ is an arbitrary column vector from a $N_T{\times}N_T$ DFT matrix. The remaining N_b–1 vectors of the codebook are then calculated via

$$\mathbf{c}_\ell = \mathbf{\Theta}_{\ell-1}\mathbf{c}_1 \tag{10}$$

where $\Theta_\ell \in \mathbb{C}^{N_T \times N_T}$ is diagonal unitary rotation matrix

$$\Theta_\ell = \begin{bmatrix} \mathrm{e}^{j\frac{2\pi}{N_b}u_1} & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \mathrm{e}^{j\frac{2\pi}{N_b}u_{N_T}} \end{bmatrix} \;, \tag{11}$$

with vector $\mathbf{u} = [u_1 \dots u_{N_T}]^T = [1\;\mathbf{u}']^T$. Θ_ℓ can be interpreted as the ℓ-th root of unity and the N_b-th rotation is $\Theta_{N_b} = \mathbf{I}_{N_T}$. Here the vector $\mathbf{u}'$ has the length $N_T - 1$ with elements $u_i \in \{1, 2, \dots, N_b\}$. The optimal elements of $\mathbf{u}'$ have to be found, as they are responsible for the rotation of the vectors in the complex plane and hence for the distribution of these vectors. The resulting codebook is not unique, since a vector $\mathbf{c}_\ell \cdot \mathrm{e}^{j\phi}$ lies on the same line as $\mathbf{c}_\ell$ in $\mathbb{C}^{N_T}$ for all $\phi \in [0, 2\pi)$ and therefore can not be distinguished. Hence, exhaustive computer search must be done to find a non-unitary codebook with good correlation properties. Fig. 2 shows the correlation as a function of $\ell - \ell'$ for a) a unitary and for b) a non-unitary codebook. It can be seen that the correlation for a DFT-based codebook behaves similar to an absolute value of a sinc-function, where the maximum correlation δ is quite high. By searching for a non-unitary codebook, one can obtain much better correlation properties considering all entries in the codebook while preserving a circulant correlation structure [12]. A consequence of the packing problem is that δ automatically becomes larger if the codebook size increases. As a consequence of non-interacting

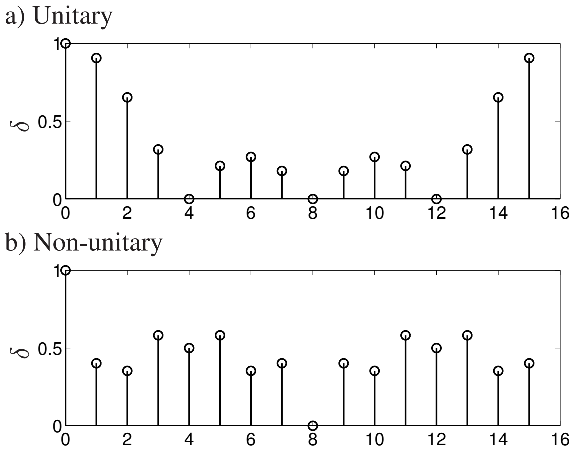

Fig. 2. Correlation δ as a function of $\ell - \ell'$ for a) a unitary and b) a non-unitary precoding codebook (cf. [12]) for codebook sizes $N_b = 16$ and $N_T = 4$.

receivers, each user k selects his best beamforming vector for each chunk ν based on

$$\mathbf{w}_k(\nu) = \arg \max_{\|\mathbf{c}_\ell\|=1} |\underline{\mathbf{h}}(\nu)\,\mathbf{c}_\ell|^2 \;, \tag{12}$$

where $\underline{\mathbf{h}}_k(\nu)$ is the k-th column of $\underline{\mathbf{H}}(\nu)$ and $\mathbf{c}_\ell$ is part of the utilized codebook. The vectors in the codebook are not orthogonal. This leads to additional interference as equation (6) is not fulfilled. This interference is assumed to be Gaussian for simplicity. Now, the restriction at the transmitter is that a codebook vector can be assigned only once during a chunk and multiple users are served only if the correlation of their selected vectors is small (here $\delta \leq 0.1$ was chosen).

4.2. Unitary Precoding

A unitary codebook is designed to have orthogonal properties between two selectable beamforming vectors. Therefore, the codebook should only consist of orthogonal vectors. It can be seen that such a property of the codebook is not possible to construct for an arbitrary number of feedback bits, since this is again related to the line packing problem in the Grassmannian manifold. To cope with this, a pseudo-unitary codebook is constructed by choosing the first N_T rows from a $N_b \times N_b$ DFT matrix and normalizing this rows to unit norm by a factor of $1/\sqrt{N_T}$. Due to the orthogonal design of the DFT matrix, no neighboured columns are orthogonal for $N_b > 4$, $N_b \in 2^L$, after truncating the last $N_b - N_T$ rows. Hence, one can divide the resulting column vectors into N_b/N_T groups, which in themselves contain only orthogonal vectors. A codebook with size $N_b = 4$ is the only codebook with full orthogonal properties between all column vectors as we can use a $N_T \times N_T$ DFT matrix for construction. The gap between orthogonal column vectors is always N_b/N_T. Equation (13) shows an example how to obtain a codebook with $N_b/N_T = 2$ different groups with orthogonal vectors of length $N_T = 4$ from a DFT matrix of size $N_b = 8$.

$$\mathcal{C} = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & \mathrm{e}^{-j2\pi/8} & \mathrm{e}^{-j\pi/2} & \mathrm{e}^{-j3\pi/4} \\ 1 & \mathrm{e}^{-j\pi/2} & \mathrm{e}^{-j\pi} & \mathrm{e}^{-j3\pi/2} \\ 1 & \mathrm{e}^{-j3\pi/4} & \mathrm{e}^{-j3\pi/2} & \mathrm{e}^{-j2\pi} \\ 1 & \mathrm{e}^{-j\pi} & \mathrm{e}^{-j2\pi} & \mathrm{e}^{-j5\pi/2} \\ 1 & \mathrm{e}^{-j5\pi/4} & \mathrm{e}^{-j5\pi/2} & \mathrm{e}^{-j3\pi} \\ 1 & \mathrm{e}^{-j3\pi/2} & \mathrm{e}^{-j3\pi} & \mathrm{e}^{-j7\pi/2} \\ 1 & \mathrm{e}^{-j2\pi} & \mathrm{e}^{-j7\pi/2} & \mathrm{e}^{-j4\pi} \end{bmatrix}^T \tag{13}$$

The odd rows in (13) belong to the first group, while the grey shaded even rows belong to the second group in this example. Thus, two groups with completely orthogonal vectors are achieved. To ensure that orthogonal vectors are selected on a sub-carrier, the beamforming vector selection operation at the transmitter has to be modified compared to the non-unitary case. The Base Station groups the users according to their feedback precoding vector. The choice of the precoding vector in chunk ν is again done via (12). If the corresponding vectors belong to a group and therefore are orthogonal, the assignment in this chunk is permitted and the power per sub-carrier is shared by the active users. Otherwise the pair is not allowed and only a single user can be served to enforce the orthogonality constraint for unitary precoding in the worst case. Due to this set-up, a minor SINR loss for served users is expected.

4.3. Beamforming Vector Interpolation

In [16], the authors propose a method to exploit the correlation properties of neighbouring sub-carriers by dividing them into several clusters. Based on the quantized beamforming vectors of the center sub-carriers in each cluster, the beamforming vectors of all other sub-carriers in-between are obtained via linear interpolation (IP) using beamforming vectors of center sub-carriers of neighbouring clusters. Here, the cluster size matches the chunk size in frequency direction N_f. As the construction of the beamforming vectors between center sub-carriers is done via interpolation at the transmitter, only the codebook index for each center sub-carrier has to be fed back to the BS and thus the feedback rate can be decreased by a factor of $1/N_f$ according to the N_c/N_f clusters. Defining the beamforming vectors of the center sub-carriers $\mathbf{a}_k(\nu) = \mathbf{w}_k((\nu-1)\,N_f+1)$ with $1 \leq \nu \leq N_c/N_f$ for notational convenience and assuming them to be given at the BS for all users, the beamforming vector for sub-carrier $(\nu-1)\,N_f + \eta$ with $1 \leq \eta \leq N_f$ can be computed at the transmitter via [16]

$$\mathbf{w}_k((\nu-1)\,N_f+\eta) = \frac{(1-b_\eta)\,\mathbf{a}_k(\nu-1) + b_\eta \mathrm{e}^{j\theta_{k,\nu-1}}\mathbf{a}_k(\nu)}{\|(1-b_\eta)\,\mathbf{a}_k(\nu-1) + b_\eta \mathrm{e}^{j\theta_{k,\nu-1}}\mathbf{a}_k(\nu)\|}\ , \quad (14)$$

where $b_\eta = (\eta-1)\,/N_f$ is the linear weight value and $\theta_{k,\nu}$ is a phase rotation parameter described later. The denominator in (14) again forces the unit norm constraint for the interpolated vectors. Due to the periodicity of the spectrum after sampling in the time domain, $\mathbf{a}_k(N_c/N_f+1) = \mathbf{a}_k(1)$ holds. Hence, the sub-carriers $\mathbf{w}_k(\eta)$ with $N_c - N_f + 1 \leq \eta \leq N_c$ are obtained by the two beamforming vectors $\mathbf{a}_k(N_c/N_f)$ and $\mathrm{e}^{j\theta_{k,N_c/N_f}}\mathbf{a}_k(N_c/N_f+1)$. The above mentioned phase rotation parameter $\theta_{k,\nu} = \theta_k((\nu-1)\,N_f+1)$ should be chosen such that the channel gain of user k in a cluster is maximized. To avoid setting up a cost function for $\theta_{k,\nu}$, the authors in [16] also suggest an optimization of this parameter via a finite grid search by uniformly quantizing the phase. Then, the best phase $\theta_{k,\nu}$ can be found by

$$\theta_{k,\nu} = \arg\max_{\theta\in\Theta} \min_{(\nu-1)N_f+1\leq\eta\leq\nu N_f} \left\|\mathbf{h}_k^T(\eta)\,\mathbf{w}_k(\eta)\right\|^2\ , \quad (15)$$

where $\Theta = \{0, (2\pi/R), (4\pi/R), \ldots, (2\,(R-1)\,\pi/R)\}$ and R is the number of presumed quantization levels. Note that the phase parameter indices of each user $\theta_{k,\nu}$ have to be fed back to the transmitter along with the corresponding beamforming vector index of the center sub-carrier in a cluster. However, the interpolation process at the transmitter is done only for active users in a cluster. As we interpolate the vectors for different sub-carriers in the frequency direction, we violate the orthogonality of two beamforming vectors and thus also term this scheme non-unitary precoding.

4.4. Feedback Analysis

In this section, a brief comparison in terms of feedback requirements of the schemes described in previous sections is given. An overview of the number of feedback bits necessary for one user in a subframe can be found in Table 1. The so-called ideal quantized beamforming describes the option that each sub-carrier at each OFDM symbol has its own beamforming vector from codebook $\mathcal{C}$ and thus requiring an enormous amount of capacity in the uplink. If the same vector per sub-carrier for all OFDM symbols in a subframe is utilized, the number of feedback bits is reduced by a factor of N_s. By applying processing per chunk according to Section 3.2, only one vector for a chunk of size $N_f{\times}N_s$ is necessary. Hence, the amount of feedback can be additionally reduced by a factor of N_f, the number of sub-carriers in a chunk. The number of feedback bits does not differ in unitary and non-unitary precoding PC, if the same codebook size is employed. But, in unitary precoding some bits are more significant than others as they are responsible for the grouping at the transmitter. If the described BF vector interpolation is performed, also the phase reference of the center sub-carrier in the chunk must be fed back, thus leading to $N_c/N_f{\cdot}\log_2(R)$ additional feedback bits.

Table 1. Required number of feedback bits for one user per subframe or Transmission Time Interval (TTI) with the compared schemes

Scheme	Feedback bits per TTI
Ideal Quantized BF	$N_s \cdot N_c \cdot \log_2(N_b)$
Quantized BF $N_f = 1$	$N_c \cdot \log_2(N_b)$
Per Chunk Processing	$\frac{N_c}{N_f} \cdot \log_2(N_b)$
IP & PC	$\frac{N_c}{N_f} \cdot (\log_2(N_b) + \log_2(R))$

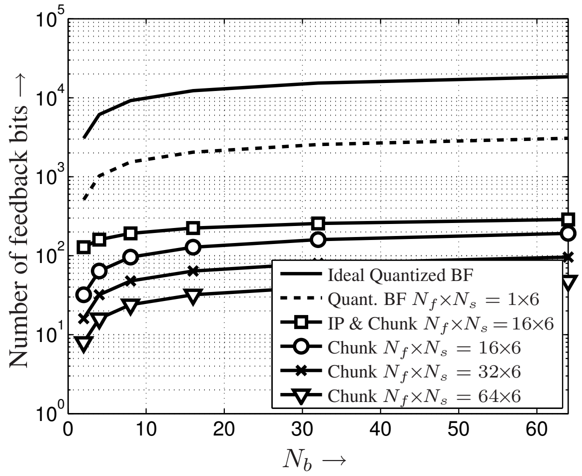

Fig. 3. Number of feedback bits vs. codebook size for the compared schemes with $N_c = 512$ and $R = 8$

To visualize the difference in feedback requirements, the number of necessary feedback bits for different codebook sizes is shown in Fig. 3. Exemplary $N_c = 512$ sub-carriers and

$$C_{\mathrm{BC}} = \max_{\mathbf{Q}_k(n)} \frac{1}{N_c} \sum_{n=1}^{N_c} \left[\sum_{k=1}^{K} \left(\log_2 \left(\sigma_z^2 + \sum_{i=1}^{K} \mathbf{h}_k^H(n)\, \mathbf{Q}_i(n)\, \mathbf{h}_k(n) \right) - \log_2 \left(\sigma_z^2 + \sum_{\substack{i=1 \\ i \neq k}}^{K} \mathbf{h}_k^H(n)\, \mathbf{Q}_i(n)\, \mathbf{h}_k(n) \right) \right) \right] \tag{16}$$

$R = 8$ phase quantization levels are chosen. The number of OFDM symbols in a subframe was set to $N_s = 6$. Chunk-based processing without interpolation has always the lowest feedback requirements as the codebook size increases. As expected, the larger the chunk size in frequency direction the less the necessary number of feedback bits. The number of feedback bits does not depend on the number of transmit antennas as mentioned in Section 4.1, whereas the number of complex multiplications and additions at both the transmitter and receiver side are, as depicted in [16].

5. SIMULATION RESULTS

To illustrate the information theoretic performance of the limited feedback schemes, the average ergodic sum-rate is considered as performance measure. If Gaussian inputs for all users are assumed at the BS, the sum-rate can be calculated according to equation (16), which describes the capacity of the multi-user MISO-OFDM BC if DPC is applied [17, 18]. Finding the optimal solution for (16) is almost intractable since it neither is a concave nor a convex function. By transforming this problem into a dual Gaussian multiple access channel (MAC) optimization problem with concave $\mathbf{Q}_k(n)$, we can determine the sum-rate capacity of the multi-user MISO-OFDM BC as the Sato upper bound for our system via [4]

$$C_{\mathrm{MAC}} = \max_{\mathbf{Q}_k(n)} \frac{1}{N_\mathrm{c}} \sum_{n=1}^{N_\mathrm{c}} \log_2 \left(\sigma_\mathrm{z}^2 + \sum_{k=1}^{K} \mathbf{h}_k^H(n) \mathbf{Q}_k(n) \mathbf{h}_k(n) \right) \tag{17}$$

with a total power constraint $\frac{1}{N_c}\sum_{k=1}^{K}\sum_{n=1}^{N_c}\mathrm{tr}\{\mathbf{Q}_k(n)\} = P$. Several algorithms have been proposed to calculate the BC capacity [19, 20].

In our simulations the extended 3GPP Spatial Channel Model (SCMe) was used [21], where K randomly distributed users are moving with 3 kph in a 500 m cell radius around a Base Station equipped with $N_\mathrm{T} = 4$ antennas. The user equipments have one receive antenna each and perfect channel knowledge at the receivers is assumed. The carrier frequency is 5 GHz and the LTE 5 MHz bandwidth parameters are used for all users to ignore all multiple bandwidth cell search procedures [3]. Thus, we evaluate the performance of beamforming only in terms of sum-rate and not in terms of scheduling. Correspondingly, no fairness aspects are considered likewise. The power P is equally distributed among all sub-carriers and hence all users have to share the transmit power per sub-carrier. Thus, additional feedback necessary for power distribution among sub-carriers is ignored. An overview of the selected parameters is given in Table 2. Fig. 4 shows the results of the average ergodic sum-rate for

Table 2. System Parameters [3]

<table>
<tr><th colspan="2">Parameter</th><th>Value</th></tr>
<tr><td colspan="2">Carrier frequency</td><td>5 GHz</td></tr>
<tr><td colspan="2">System bandwidth</td><td>5 MHz</td></tr>
<tr><td colspan="2">Antenna configuration</td><td>4×1 MU-MISO</td></tr>
<tr><td colspan="2">Channel Model</td><td>SCMe</td></tr>
<tr><td colspan="2">FFT size</td><td>512</td></tr>
<tr><td colspan="2">Number of sub-carriers</td><td>300</td></tr>
<tr><td colspan="2">Sub-carrier spacing</td><td>15 KHz</td></tr>
<tr><td colspan="2">Chunk bandwidth</td><td>240 KHz</td></tr>
<tr><td rowspan="2">Symbol duration</td><td>Effective Data</td><td>66.67μs</td></tr>
<tr><td>Cyclic Prefix</td><td>16.67μs</td></tr>
<tr><td colspan="2">TTI length</td><td>0.5 ms (6 sym.)</td></tr>
</table>

the investigated schemes with non-unitary and unitary codebooks versus the overall SNR. Two different codebook sizes with $L = 2$ ($N_b = 4$) and $L = 6$ ($N_b = 64$) are selected for comparison for all schemes. A chunk size of $N_f \times N_s = 16 \times 6$ is assumed in the following. The results for $K = 2$ users in Fig. 4a indicate a superior behavior for larger codebooks. It can be seen that ensuring orthogonality in unitary precoding and keeping the correlation of users small in non-unitary precoding results in an almost negligible performance difference for a large codebook. For a small codebook size the slope of unitary precoding curve flattens at the high SNR region. If the two users are close together in the cell, they both might choose the same beamforming vector out of the codebook and one user is not served in this chunk. For both codebook sizes

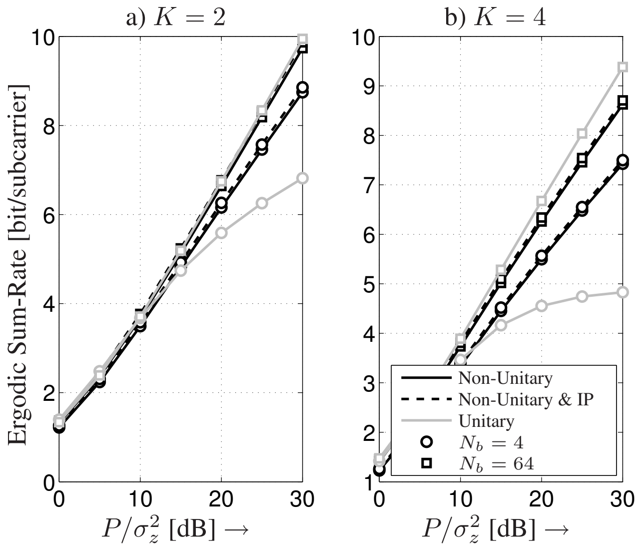

Fig. 4. Average ergodic sum-rate vs. SNR for different BF schemes and codebook sizes N_b for a chunk size of $N_f \times N_s = 16 \times 6$

precoding with interpolation performs slightly better than the pure chunk processing at the expense of additional bits for phase signalling. There, the correlation of neighbouring sub-carriers is exploited and the arising interference from IP of both users seems to be not dominating. As the phase rotation for interpolated sub-carriers in the clusters may be rotated with the same angles $\theta_{k,\nu}$, the same correlation properties of the user vectors are maintained on neighbouring sub-carriers. As the gain of IP is very small, it is not reasonable to apply this scheme. The same observation holds for the case with four users in Fig. 4b. However, a superior performance of unitary precoding is apparent. If the number of users increases, the probability that up to four orthogonal users can be served in one chunk also increases.

The results for the average ergodic sum-rates for different precoding schemes in comparison with different optimal and sub-optimal schemes with perfect channel knowledge at the BS for a system with $K = 2$ users are depicted in Fig. 5. There, the case with no CSI at the transmitter defines our lower bound, while the previously described sum-rate capacity based on (16) is the upper bound. If ZF-BF is applied for two users, the sum-rate capacity of the system is almost reached. This result can be achieved as two independent parallel channels for the users after precoding emerge. The remaining gap to the capacity is due to the non-existing power allocation in our system. If ZF-BF PC is utilized, a degradation due to the loss in channel gain can be observed [7]. The optimal beamforming in the single-user case (SU-BF) performs worse as only one user per sub-carrier is active. For the limited feedback beamforming schemes with codebook size $N_b = 64$ we get sum-rates, which strongly differ from the ZF-BF solution, especially in the high SNR region. This is due to the infrequent pairing of both users, which results in an equal slope compared to the SU-BF case. The advantage of the pairing is more efficient, if the number of users is increased. For $K = 4$ users (Fig. 6) the difference to ZF-BF is less than for two users. Since a fully loaded system is present on each sub-carrier, a quadratic channel matrix $\underline{\mathbf{H}}(\nu)$ of size $N_T \times N_T$, which is ill-conditioned with at least one small eigenvalue, describes the channel. Consequently, the zero-forcing solution performs bad in high SNR regions if all spatial modes are utilized. Hence, less degradation of the limited feedback schemes compared to ZF-BF is obtained, whereas in low SNR regions a better performance for the limited feedback schemes is observed, which is again due to shutdown of users, which go unserved and therefore are not interfering with served users. As the gap to the sum-rate capacity is still huge, we concede that linear beamforming schemes with limited feedback are far away from being the optimal choice but are straightforward to implement in real systems like LTE in terms of complexity issues.

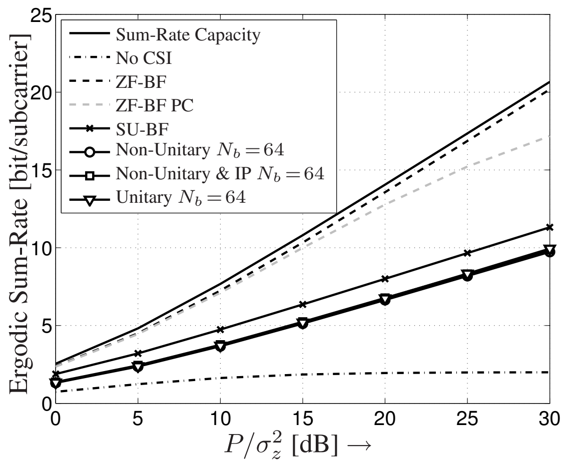

Fig. 5. Average ergodic sum-rate vs. SNR for different schemes with $K=2$ users and a chunk size of $N_f \times N_s = 16 \times 6$

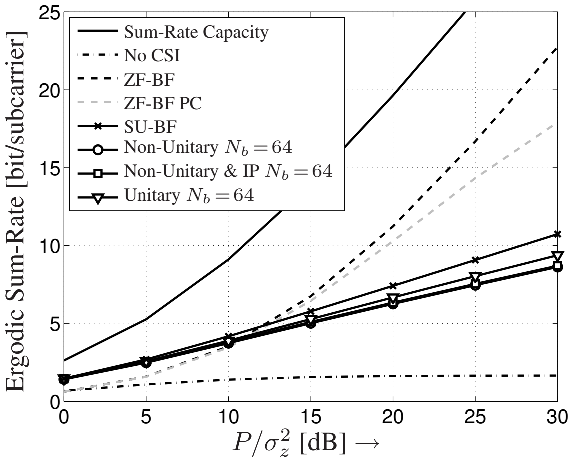

Fig. 6. Average ergodic sum-rate vs. SNR for different schemes with $K=4$ users and a chunk size of $N_f \times N_s = 16 \times 6$

6. CONCLUSIONS

In this contribution we the compared the ergodic sum-rates of a downlink multi-user MISO-OFDM system with different beamforming schemes based on limited feedback. The evaluation was done according to parameters specified for the new E-UTRA enhancement of the 3G mobile communications systems. A superiority of unitary beamforming can be determined if a codebook-based approach is utilized in a multi-user scenario. This was due to the grouping of the users in the Base Station, which enforces orthogonality of the beams. In contrast, applying non-unitary precoding leads to a SINR loss, which increases with the number of users in the system. Furthermore, as the gap with regard to ZF-BF is still large at high SNR ratios a precise comparison between linear and non-linear precoding schemes as well as an increase in the number of receive antennas per user is necessary to exceedingly exploit the advantages of MIMO in E-UTRA. This is part of our future work.

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