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MMSE Extension of V-BLAST based on Sorted QR Decomposition

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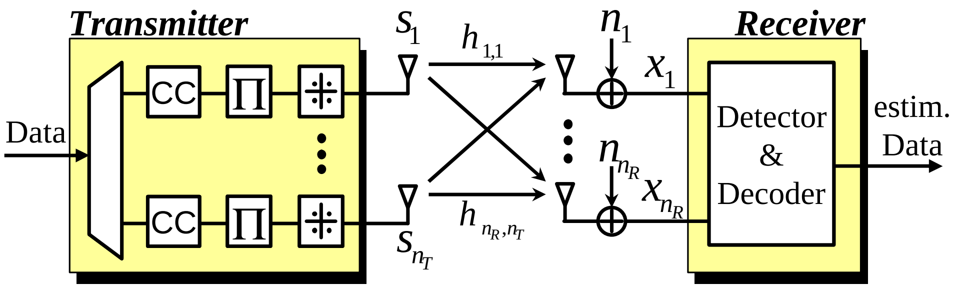
Outline:

- System model
- MMSE detector utilizing QR decomposition of the channel matrix
- Criterion for optimum detection ordering
    - Sorted QR Decomposition
    - Post-Sorting-Algorithm
- Investigation of computational effort and performance comparison
- Summary

V-BLAST Architecture

- MIMO system with n_T transmit and $n_R \geq n_T$ receive antennas



$$\begin{bmatrix} x_1 \\ \vdots \\ x_{n_R} \end{bmatrix} = \begin{bmatrix} h_{1,1} & \cdots & h_{1,n_T} \\ \vdots & \ddots & \vdots \\ h_{n_R,1} & \cdots & h_{n_R,n_T} \end{bmatrix} \cdot \begin{bmatrix} s_1 \\ \vdots \\ s_{n_T} \end{bmatrix} + \begin{bmatrix} n_1 \\ \vdots \\ n_{n_R} \end{bmatrix}$$

➡ $\mathbf{x} = \mathbf{H}\mathbf{s} + \mathbf{n}$

- **O**rdered **S**uccessive **I**nterference **C**ancellation (O-SIC)
  - V-BLAST requires multiple calculation of pseudo-inverse → high comput. effort
  - QR based SIC utilizes decomposition $\mathbf{H}=\mathbf{QR}$ and computes $\tilde{\mathbf{s}} = \mathbf{Q}^H \mathbf{x} = \mathbf{R}\mathbf{s} + \tilde{\mathbf{n}}$
  - Due to upper triangular structure of $\mathbf{R}$, filter output signal $\tilde{\mathbf{s}}$ is partially free of interference and can be detected layer by layer in the sequence: $n_T, n_T\text{-}1, \ldots, 1$
- General Problems:
  - ZF leads to noise enhancement → detector with MMSE criterion is favorable
  - Efficient algorithm to incorporate ordering into QR-based SIC

Relation between Linear MMSE and Linear ZF

- No *obvious* way to extend QR-based SIC to MMSE criterion
- Relation between linear MMSE detector and linear ZF detector
  - MMSE minimizes the mean squared error between the transmit vector $\mathbf{s}$ and the output of a linear detector $\mathbf{G}$
  - Layers are detected by threshold decision of filter output signals

$$\tilde{\mathbf{s}} = \mathbf{G}\mathbf{x} = \left(\mathbf{H}^H\mathbf{H} + \sigma_n^2 \mathbf{I}_{n_T}\right)^{-1}\mathbf{H}^H\mathbf{x} \quad \Rightarrow \quad \tilde{\mathbf{s}} = \left(\underline{\mathbf{H}}^H\underline{\mathbf{H}}\right)^{-1}\underline{\mathbf{H}}^H\underline{\mathbf{x}} = \underline{\mathbf{H}}^+\underline{\mathbf{x}}$$

  - Error covariance matrix

$$\mathbf{\Phi} = \mathrm{E}\left\{(\tilde{\mathbf{s}}-\mathbf{s})(\tilde{\mathbf{s}}-\mathbf{s})^H\right\} = \sigma_n^2\left(\mathbf{H}^H\mathbf{H} + \sigma_n^2\mathbf{I}_{n_T}\right)^{-1} \quad \Rightarrow \quad \mathbf{\Phi} = \sigma_n^2\left(\underline{\mathbf{H}}^H\underline{\mathbf{H}}\right)^{-1}$$

  - Definition of (n_T+n_R) x n_T extended channel matrix and (n_T+n_R) x 1 extended receive vector

$$\underline{\mathbf{H}} = \begin{bmatrix} \mathbf{H} \\ \sigma_n \mathbf{I}_{n_T} \end{bmatrix} \qquad \underline{\mathbf{x}} = \begin{bmatrix} \mathbf{x} \\ \mathbf{0}_{n_T,1} \end{bmatrix} = \underline{\mathbf{H}}\mathbf{s} + \underline{\mathbf{n}}$$

$\Rightarrow$

- Expressions correspond to ZF solution!
- MMSE = ZF with respect to $\underline{\mathbf{H}}$

MMSE Detection with QR Decomposition

- QR decomposition of extended channel matrix

$$\underline{\mathbf{H}} = \begin{bmatrix} \mathbf{H} \\ \sigma_n \mathbf{I}_{n_T} \end{bmatrix} = \underline{\mathbf{Q}}\,\underline{\mathbf{R}} = \begin{bmatrix} \mathbf{Q}_1 \\ \mathbf{Q}_2 \end{bmatrix} \underline{\mathbf{R}} = \begin{bmatrix} \mathbf{Q}_1 \underline{\mathbf{R}} \\ \mathbf{Q}_2 \underline{\mathbf{R}} \end{bmatrix}$$

$\underline{\mathbf{R}}$: $n_T \times n_T$ upper triangular matrix
$\underline{\mathbf{Q}}$: $(n_T + n_R) \times n_T$ matrix with $\underline{\mathbf{Q}}^H \underline{\mathbf{Q}} = \mathbf{I}_{n_T}$
$\mathbf{Q_1}$: $n_R \times n_T$ matrix
$\mathbf{Q_2}$: $n_T \times n_T$ matrix

  - Relation 1:

$$\sigma_n \mathbf{I}_{n_T} = \mathbf{Q}_2 \underline{\mathbf{R}} \;\Rightarrow\; \underline{\mathbf{R}}^{-1} = \sigma_n^{-1} \mathbf{Q}_2 \;\Rightarrow$$

  - $\underline{\mathbf{R}}^{-1}$ is given by QR decomposition of $\underline{\mathbf{H}}$
    → useful for post-sorting algorithm
  - $\mathbf{Q}_2$ is an upper triangular matrix

  - Relation 2:

$$\underline{\mathbf{Q}}^H \underline{\mathbf{H}} = \underline{\mathbf{R}} = \mathbf{Q}_1^H \mathbf{H} + \sigma_n \mathbf{Q}_2^H \;\Rightarrow\; \mathbf{Q}_1^H \mathbf{H} = \underline{\mathbf{R}} - \sigma_n \mathbf{Q}_2^H$$

- Filtered receive vector

$$\tilde{\mathbf{s}} = \underline{\mathbf{Q}}^H \underline{\mathbf{x}} = \mathbf{Q}_1^H \mathbf{x} = \underline{\mathbf{R}}\mathbf{s} + \mathbf{Q}_1^H \mathbf{n} - \sigma_n \mathbf{Q}_2^H \mathbf{s}$$

  - Last term constitutes the remaining interference that can not be removed
  - Points out the trade-off between noise amplification and interference suppression

Optimum Detection Order

- Within QR-based O-SIC the order of detection can be arranged by exchanging the elements in $\mathbf{s}$ and the corresponding columns of $\underline{\mathbf{H}}$
  → leads to different QR decompositions
- Investigation of error covariance matrix using $\underline{\mathbf{H}} = \underline{\mathbf{Q}}\underline{\mathbf{R}}$

$$\mathbf{F} = s_n^2 \left(\underline{\mathbf{H}}^H \underline{\mathbf{H}} \right)^{-1} = s_n^2 \left(\underline{\mathbf{R}}^H \underline{\mathbf{R}} \right)^{-1} \quad \Rightarrow \quad \mathbf{F}(k,k) = s_n^2 \left| \underline{r}_{k,k} \right|^{-2}$$

  - $\mathrm{F}(k,k)$ gives estimation error for s_k after perfect interference cancellation and is proportional to the inverse of the diagonal element $\underline{r}_{k,k}$
  - Optimum sorting requires, that the SNR_k is maximized ($\mathrm{F}(k,k)$ is minimized) in order of detection (n_T, n_T-1, …, 1)
    → The optimum column permutation of $\underline{\mathbf{H}}$ maximizes the diagonal elements $\underline{r}_{k,k}$ in this decreasing sequence (n_T, n_T-1, …, 1)
  - Brute Force approach calculates $n_T(n_T+1)/2$-1 different QR decompositions!

Sorted QR Decomposition (SQRD)

- Modified Gram-Schmidt algorithm for QR decomposition
  - Triangular matrix $\underline{\mathbf{R}}$ is computed row by row from top to bottom $(1, \ldots, n_T)$
  - Order of decomposition is just contrary to the ordering criterion of SIC → impossible to find always the optimum order within one QR Decomposition
- **S**orted **QR D**ecomposition
  - The determinant of the Gram matrix $\underline{\mathbf{H}}^H\underline{\mathbf{H}}$ is invariant to column exchanges in $\underline{\mathbf{H}}$

    $\det\left(\underline{\mathbf{H}}^H\underline{\mathbf{H}}\right)=\left[\det\left(\underline{\mathbf{R}}\right)\right]^2=\prod_{k=1}^{n_T}\underline{r}_{k,k}^2=const.$ ➡ product of $\underline{r}_{k,k}$ is constant
  - Basic idea: Exchange the columns to *minimize* the diagonal elements $\underline{r}_{k,k}$ in order of their calculation
  - As the product is constant, *small* $\underline{r}_{k,k}$ in the upper left part lead to *large* elements in the lower right part of $\underline{\mathbf{R}}$
  - Only very small computational effort additional to unsorted QRD
  - SQRD optimizes detection sequence within the QR decomposition, but does not always lead to the perfect detection sequence → Post-Sorting-Algorithm

Post-Sorting-Algorithm (PSA)

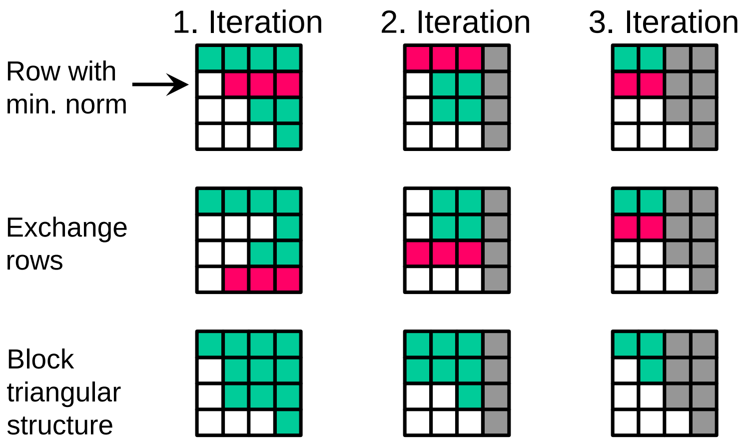
- Investigation of error covariance matrix in case of optimal sorting

$$\mathbf{F} = \sigma_n^2 \left(\underline{\mathbf{R}}^H \underline{\mathbf{R}}\right)^{-1} = \mathbf{Q}_2 \mathbf{Q}_2^H$$

→ $\mathrm{F}(k,k)$ is proportional to k-th row norm of $\mathbf{Q}_2$

  - Due to detection order, last row of $\mathbf{Q}_2$ must have minimum norm of all rows
  - If this condition is fulfilled, the last row of the upper left $(n_T\text{-}1)$x$(n_T\text{-}1)$ submatrix of $\mathbf{Q}_2$ must have minimum norm of all rows in this submatrix, ...

- Now assume, that this condition is not fulfilled for $\mathbf{Q}_2$

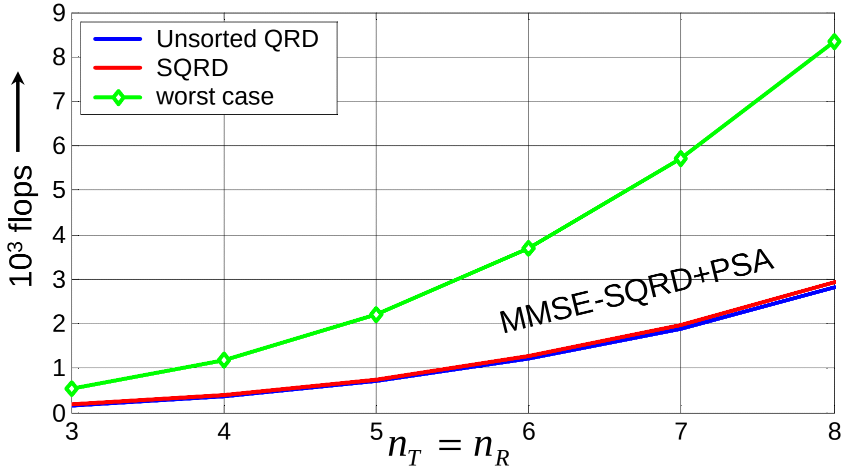


  - Exchange row with minimum norm & last row → destroys triangular structure
  - Block triangular structure is achieved by right multiplication with unitary Housholder matrix Q (also calculate $\mathbf{Q}_1\mathrm{Q}$)
  - Iterate this ordering and reflection steps for upper left $(n_T\text{-}1)$x$(n_T\text{-}1)$ submatrix of $\mathbf{Q}_2$, …

→ **Achieves optimally ordered QRD of $\underline{\mathbf{H}}$!**

Computational Effort for Sorting Algorithms

- Count each addition as one flop and each multiplication as three flops
  - Overhead of SQRD in comparison to QRD is marginal: $f_{\text{SQRD}} = f_{QRD} + 2\left(n_T^2 - n_T\right)$
  - The effort of PSA depends on number of required permutations
    → upper bounded by ignoring the upper triangular structure of $\mathbf{Q}_2$: f_{PSA}
  - Hassibi's Square-Root approach utilizes a common QRD, explicitly computes $\underline{\mathbf{R}}^{-1}$ and ignores structure of $\mathbf{Q}_2$: $f_{\text{Hassibi}} \approx f_{\text{SQRD}} + f_{\text{PSA}}$



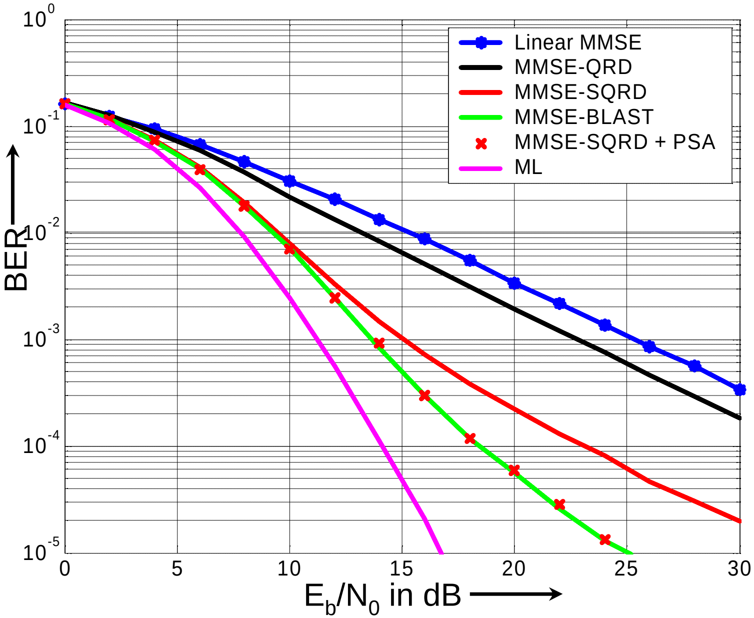
- MMSE-SQRD + PSA:
  SQRD already achieves an optimized sorting, thus PSA is only required in fraction of all transmissions (≈20%)

  → worst case effort is given by f_{Hassibi}

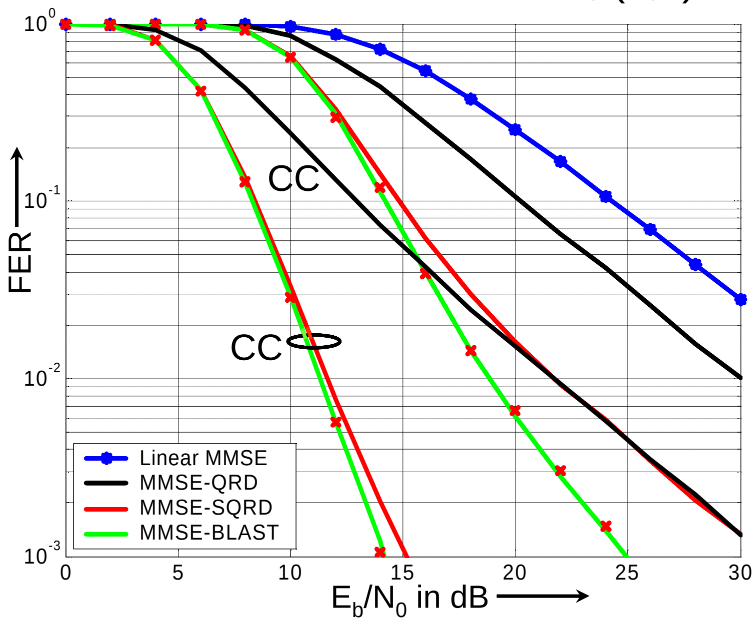
→ **A very efficient and adjustable algorithm is achieved!**

Simulation Results for $n_T = n_R = 4$ and QPSK

BER of uncoded transmission



FER of encoded transmission, (7,5)-CC



- Order of detection has strong impact
- Up to 10 dB only small performance loss for MMSE-SQRD
- Combination of MMSE-SQRD and PSA exactly achieves V-BLAST performance

- Coded schemes provide large gain
- Only small performance loss for coded MMSE-SQRD (1 dB for FER of 10^{-3})
- For coded schemes enormous gain of MMSE-SQRD to unsorted MMSE-QRD

Summary

- Unified framework for ZF and MMSE detection of V-BLAST
  - Ordered SIC based on QR decomposition
  - Equivalence of MMSE and ZF solutions by introducing extended system model
  - QR-based MMSE detection scheme is achieved due to this equivalence
- **S**orted **QR D**ecomposition (**SQRD**)
  - Sorted extension of Gram-Schmidt algorithm
  - Achieves enormous gain in comparison to unsorted QRD with small computational overhead
  - Requires less computational effort with only small degradation in error performance compared to V-BLAST
  - For coded transmission, performance degradation is marginal
- **P**ost-**S**orting-**A**lgorithm (**PSA**)
  - Perfectly closes the performance gap with minimum computational complexity