Efficient Algorithm for Detecting Layered Space-Time Codes

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Abstract— **Layered space-time codes have been designed to exploit the capacity advantage of multiple antenna systems in Rayleigh fading environments. In this paper, we present a new efficient detection algorithm based on a sorted QR decomposition. It only needs a fraction of computational effort compared to the standard detection algorithm requiring multiple calculations of the pseudo inverse of the channel matrix. The derived algorithm is not restricted to layered space-time architectures, but can generally be used for detection in vector channel systems.**

Keywords— **Layered space-time codes, diversity, MIMO system, wireless communication.**

I. Introduction

In a Rayleigh fading environment, multiple antenna systems provide an enormous increase in capacity compared to single antenna systems [1]. Consequently, multiple-input multiple-output (MIMO) systems are predestined for high data rate wireless communications. Space-Time codes are designed to exploit this high capacity by using space as a second dimension of coding [2, 3].

Layered space-time (LST) codes are a special kind of space-time codes with the advantage of a feasible decoding complexity. The original D-BLAST (*Diagonal Bell Labs Layered Space Time*) architecture proposed by Foschini [4] uses a diagonally layered coding structure in which code blocks are dispersed across diagonals in space-time. Thereby, an "averaged" channel which is the same for all layers is achieved and the probability of deep fades is reduced. Due to the diagonal arrangement of the code blocks, D-BLAST is not feasible for real time implementations. A simplified version was proposed in [5] and is known as V-BLAST (*Vertical* BLAST). It associates each layer with a specific transmit antenna wich leads to an easier detection and decoding process. For detecting the layers, the multiple calculation of the pseudo inverse of the channel matrix is necessary.

In order to significantly reduce the computational effort of detection, we introduce a new and very efficient way of detecting layered space-time codes. This approach utilizes an adjusted QR decomposition (QRD) by sorting the detection sequence due to exchanging the columns of the channel matrix. This new algorithm is compared to V-BLAST by simulation results and by an estimation of the computational effort.

The remainder of this paper is organized as follows. In section II, the MIMO system and the LST architecture are described. In section III, V-BLAST and a QRD based algorithm for detecting LST architectures are reviewed. The new approach is introduced in section IV and the performance of both detection algorithms are compared in section V. In section VI forward correction coding is added in each layer and the computational effort of both detecting algorithms is compared in section VII. A summary and concluding remarks can be found in section VIII.

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II. System description

We consider the multi antenna system with n_T transmit and $n_R \geq n_T$ receive antennas shown in **Fig. 1**. The data is demultiplexed in n_T data substreams of equal length (called layers). These substreams are mapped into M-PSK or M-QAM symbols $c_1, \ldots, c_{n_T}$. Alternatively, a forward error correction (FEC) code can be used to encode the data substreams before mapping. We will investigate the application of FEC code in section VI and assume uncoded symbols until then. The substreams are organized in frames of length L and are transmitted over the n_T antennas at the same time. The system is equal to V-BLAST proposed in [4] and denoted as *Layered Space-Time* (LST) architecture in [6].

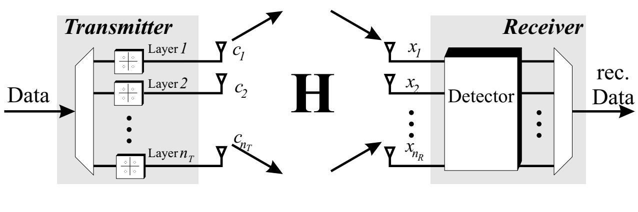

Fig. 1. Model of a MIMO system with n_T transmit and n_R receive antennas

In order to describe the MIMO system, one time slot of the time-discrete baseband model of the

MIMO system is investigated.
Let $\mathbf{c} = (c_1 c_2 \dots c_{n_T})^T$ denote the vector of transmitted symbols, then the corresponding received signal vector $\mathbf{x} = (x_1 x_2 \dots x_{n_R})^T$ is calculated by

$$\mathbf{x} = \mathbf{H} \cdot \mathbf{c} + \boldsymbol{\nu} \ . \tag{1}$$

In equation (1), $\boldsymbol{\nu} = (\nu_1 \nu_2 \dots \nu_{n_R})^T$ depicts the vector of noise terms at the n_R receiving antennas, assuming uncorrelated white gaussian noise of variance $N_0/2$ per dimension for all antennas. The transmitted symbols are normalized so that the average received energy per bit is one. The $n_R \times n_T$ channel matrix

$$\mathbf{H} = \begin{pmatrix} h_{1,1} & \dots & h_{1,n_T} \\ \vdots & \ddots & \vdots \\ h_{n_R,1} & \dots & h_{n_R,n_T} \end{pmatrix} \tag{2}$$

contains i.i.d. complex fading gains $h_{j,i}$ describing the tap gains between transmit antenna i and receive antenna j. Column i of $\mathbf{H}$ is denoted by $\mathbf{h}_i$ and represents the single-input multiple-output (SIMO) channel between transmit antenna i and the n_R receive antennas. We assume a static flat-fading environment, i.e. the channel matrix $\mathbf{H}$ is constant over a frame and changes independently from frame to frame. The distinct fading gains are assumed to be uncorrelated and are perfectly known by the receiver.

III. Detecting Layered Space-Time Codes

In this paragraph, two different detection algorithms for the LST architecture are described. First, the standard detection algorithm proposed by Bell-Labs [5] and known as V-BLAST is depicted. Shiu and Kahn utilized the QR decomposition of the channel matrix for the detection of the layers to derive error bounds for V-BLAST and D-BLAST in [6]. They presumed the knowledge of the best detection sequence, but did not discuss the problem of an efficient assorting algorithm, which is done in section IV of this paper.

A. BLAST-Algorithm

It is obvious from equation (1) that the received signals are a linear combination of the n_T transmitted signals. The optimum way of recovering the n_T signals at the receiver would be maximum-likelihood detection, which is not feasible due to the enormous complexity.

In [4] and [5], Foschini et al. proposed a successive interference cancellation technique which nulls the interferer by linearly weighting the received signal vector with a zero-forcing (ZF) nulling vector. In every detection step, all signals but one are regarded as interferer. By applying the nulling vector to interference cancellation, the influence of these signals is nulled out, the target signal is detected and subsequently subtracted from the received signal vector (*Interference Cancellation*).

For detecting signal i, the nulling vector $\mathbf{w}_i$ has to be orthogonal to columns $\mathbf{h}_l$, $l \neq i$ of the channel matrix. The condition[1]

$$\mathbf{w}_i^T \cdot \mathbf{h}_l = \begin{cases} 1 & l = i \\ 0 & l \neq i \end{cases} \tag{3}$$

is fulfilled by the i-th row of the Moore-Penrose pseudo-inverse

$$\mathbf{G} := \mathbf{H}^+ := \left(\mathbf{H}^H \mathbf{H}\right)^{-1} \mathbf{H}^H \ , \tag{4}$$

of the channel matrix $\mathbf{H}$. With $\mathbf{g}^{(i)}$ denoting row i of $\mathbf{G}$, the received signal vector $\mathbf{x}$ is linearly weighted with the nulling vector $\mathbf{w}_i^T = \mathbf{g}^{(i)}$ and the result

$$\begin{aligned} y_i &= \mathbf{w}_i^T \cdot \mathbf{x} = \mathbf{g}^{(i)} \cdot (\mathbf{H} \cdot \mathbf{c} + \boldsymbol{\nu}) \\ &= c_i + \tilde{\nu}_i \ . \end{aligned} \tag{5}$$

is used as a decision statistic for the i-th substream where $\tilde{\nu}_i = \mathbf{g}^{(i)} \cdot \boldsymbol{\nu}$ denotes the actual noise. By applying the quantization operation $\mathcal{Q}[\cdot]$ appropriate to the signal constellation, signal i can be estimated:

$$\hat{c}_i = \mathcal{Q}[y_i] \ . \tag{6}$$

The interference caused by the detected signal $\hat{c}_i$ is now subtracted from the received signal vector $\mathbf{x}_i$

$$\mathbf{x}_{i+1} = \mathbf{x}_i - \mathbf{h}_i \cdot \hat{c}_i \tag{7}$$

and the corresponding column in the channel matrix is set to zero. The indexed variables ($\mathbf{H}_i$,$\mathbf{G}_i$,$\mathbf{x}_i$) denote from now on the specific variables in detection step i, beginning with the assignment ($\mathbf{H}_1 = \mathbf{H}$, $\mathbf{G}_1 = \mathbf{G}$, $\mathbf{x}_1 = \mathbf{x}$) in the first step. Using the nomenclature introduced in [5], $\mathbf{H}_{i+1} := \mathbf{H}_i^{\bar{i}}$ describes the nulling of column i of the channel matrix $\mathbf{H}_i$ and corresponds to an equivalent system with $n_T - i$ transmit and n_R receive antennas. Thus, the pseudo inverse of this reduced channel matrix $\mathbf{H}_{i+1}$ is used to calculate the nulling vector for detecting layer $i+1$.

The order of detecting affects the error probability of the algorithm [5]. The sequence $\mathcal{S} = \{k_1, k_2, \dots, k_{n_T}\}$ is defined as a permutation of the numbers $1, 2, \dots, n_T$ to depict a specific detection sequel. Thus the values $y_{k_1}, y_{k_2}, \dots, y_{k_{n_T}}$ are filtered one by one, the transmitted signals $\hat{c}_{k_1}, \hat{c}_{k_2}, \dots, \hat{c}_{k_{n_T}}$ are estimated and the interference is cancelled step by step according to equations (5) to (7). In order to derive the minimum total error probability, it is optimal always to choose and detect the layer with the largest *post detection* signal-to-noise ratio [5]:

$$SNR_{k_i} = \frac{\mathrm{E}\left\{|c_{k_i}|^2\right\}}{\mathrm{E}\left\{|n_{k_i}|^2\right\} \|\mathbf{w}_{k_i}\|^2} \sim \frac{1}{\left\|\mathbf{g}^{(k_i)}\right\|^2} \ . \tag{8}$$

[1]The transpose and conjugate transpose (*Hermitian*) of $\mathbf{x}$ are denoted by $\mathbf{x}^T$ and $\mathbf{x}^H$, respectively.

Consequently, it is optimal to choose the row $\mathbf{g}_i^{(k_i)}$ of $\mathbf{G}_i$ with minimal norm and thus detect the associated signal c_{k_i} in detection step i. The whole detection algorithm is shown in **Fig. 2**.

$$
\begin{array}{l}
\texttt{for } i = 1, \ldots, n_T \\
\quad \mathbf{G}_i = \mathbf{H}_i^{+} \\
\quad k_i = \arg\min\limits_j \left\| \mathbf{g}_i^{(j)} \right\|^2 \\
\quad \mathbf{w}_{k_i}^T = \mathbf{g}_i^{(k_i)} \\
\quad y_{k_i} = \mathbf{w}_{k_i}^T \cdot \mathbf{x}_i \\
\quad \hat{c}_{k_i} = \mathcal{Q}[y_{k_i}] \\
\quad \mathbf{x}_{i+1} = \mathbf{x}_i - \mathbf{h}_{k_i} \cdot \hat{c}_{k_i} \\
\quad \mathbf{H}_{i+1} = \mathbf{H}_i^{\overline{k_i}} \\
\texttt{end}
\end{array}
$$

Fig. 2. V-BLAST algorithm for detecting layered space-time signals

B. *QR decomposition of the channel matrix*

In [6], Shiu and Kahn used the QR decomposition of the channel matrix $\mathbf{H}$ to derive bounds for the error probability of LST codes. Therefore, the $n_R \times n_T$ channel matrix $\mathbf{H}$

$$\mathbf{H} = \mathbf{Q} \cdot \mathbf{R}\,, \tag{9}$$

is factorized into the unitary $n_R \times n_T$ matrix $\mathbf{Q}$ and the upper triangular $n_T \times n_T$ matrix $\mathbf{R}$. By denoting the column i of $\mathbf{H}$ by $\mathbf{h}_i$ and column i of $\mathbf{Q}$ by $\mathbf{q}_i$, the decomposition in equation (9) is described columnwise by

$$(\mathbf{h}_1 \ldots \mathbf{h}_{n_T}) = (\mathbf{q}_1 \ldots \mathbf{q}_{n_T}) \cdot \begin{pmatrix} r_{1,1} & \cdots & r_{1,n_T} \\ & \ddots & \vdots \\ \mathbf{0} & & r_{n_T,n_T} \end{pmatrix} . \tag{10}$$

By multiplying equation (1) from the left with the Hermitian matrix of $\mathbf{Q}$, a $n_T \times 1$ modified received signal vector

$$\mathbf{y} = \mathbf{Q}^H \cdot \mathbf{x} = \mathbf{R} \cdot \mathbf{c} + \boldsymbol{\eta}\,. \tag{11}$$

is created from the $n_R \times 1$ received signal vector $\mathbf{x}$. Since $\mathbf{Q}$ is unitary, the statistical properties of the noise term $\boldsymbol{\eta} = \mathbf{Q}^H \cdot \boldsymbol{\nu}$ remain unchanged. Element k of vector $\mathbf{y}$ becomes

$$y_k = r_{k,k} \cdot c_k + \eta_k + d_k \tag{12}$$

with the interference term

$$d_k = \sum_{i=k+1}^{n_T} r_{k,i} \cdot c_i\,. \tag{13}$$

Thus, y_k depends on the weighted transmit signal $r_{k,k} \cdot c_k$, the noise η_k and the interference term d_k. Since $\mathbf{R}$ is upper triangular, d_k is independent of the upper layer signals $c_1, \ldots, c_{k-1}$ and hence the lowest layer (transmit signal c_{n_T}) is described by

$$y_{n_T} = r_{n_T,n_T} \cdot c_{n_T} + \eta_{n_T}\,. \tag{14}$$

Then, the decision statistic y_{n_T} is independent of the remaining transmit signals and can be used to estimate $\hat{c}_{n_T}$

$$\hat{c}_{n_T} = \mathcal{Q}\left[\frac{y_{n_T}}{r_{n_T,n_T}}\right] \tag{15}$$

by applying the quantization operation $\mathcal{Q}[\cdot]$.

For detecting layer $n_T - 1$, the interference term $r_{n_T-1,n_T} \cdot \hat{c}_{n_T}$ is eliminated in the modified received signal

$$y_{n_T-1} = r_{n_T-1,n_T-1} \cdot c_{n_T-1} + r_{n_T-1,n_T} \cdot c_{n_T} + \eta_{n_T-1}\,. \tag{16}$$

Consequently, an interference free decision statistic to estimate c_{n_T-1} is obtained under the assumption $\hat{c}_{n_T} = c_{n_T}$. Detecting layer $k = n_T - 1, \ldots, 1$ takes place in an equivalent way. With previous decisions $\hat{c}_{k+1}, \ldots, \hat{c}_{n_T}$, the interference term $\hat{d}_k$ is calculated and cancelled out in the modified received signal y_k. Assuming that all previous decisions are correct ($\hat{d}_k = d_k$), the value

$$z_k = y_k - \hat{d}_k = r_{k,k} \cdot c_k + \eta_k \tag{17}$$

is free of interference and thus it can be used to detect c_k with $\hat{c}_k = \mathcal{Q}[z_k / r_{k,k}]$.

As already stated, the order of detection is crucial for the error probability of the LST system due to the risk of error propagation [5]. When using the QR decomposition for detection, the sequence of detection is achieved by permutating the elements of $\mathbf{c}$ and the corresponding columns of $\mathbf{H}$ and thereby results in different matrices $\mathbf{Q}$ and $\mathbf{R}$. The optimum $\mathbf{R}$ maximizes

$$SNR_k = \frac{\mathrm{E}\left\{|c_k|^2\right\} \, |r_{k,k}|^2}{\mathrm{E}\left\{|n_{k_i}|^2\right\}} \sim |r_{k,k}|^2 \tag{18}$$

in each step of the detection process (corresponds to the maximization of $|r_{k,k}|$ for $k = n_T, \ldots, 1$) and can be found by performing $\mathcal{O}(n_T^2/2)$ QR decompositions of permutations of $\mathbf{H}$ [7]. In order to reduce the computational effort of finding a detection sequence, we derive a suboptimal but less complex algorithm for sorting in the next section.

IV. Sorted QR decomposition

In this section, a new and very efficient approach that comes close to the error performance of V-BLAST is introduced. It is basically an extension of the modified Gram-Schmidt algorithm [8] by ordering the columns of $\mathbf{H}$ in each orthogonalization step. In order to describe this new algorithm, we first review the modified Gram-Schmidt algorithm without sorting. In the subsequent the motivation for the sorted approach and the description of the sorted QR decomposition are presented.

A. Modified Gram-Schmidt

The Gram-Schmidt algorithm computes matrix $\mathbf{R}$ of the QR decomposition line by line from top to bottom and matrix $\mathbf{Q}$ columnwise from left to right [8]. Starting with the assignment $\mathbf{Q} := \mathbf{H} = (\mathbf{h}_1, \ldots, \mathbf{h}_{n_T})$, the following operations are executed in every step $i = 1, \ldots, n_T$:

- Assign norm of column vector $\mathbf{q}_i$ to the diagonal element $r_{i,i}$ of the upper triangular matrix $\mathbf{R}$ ($r_{i,i} = \|\mathbf{q}_i\|$) and subsequently scale $\mathbf{q}_i$ to length one ($\mathbf{q}_i = \mathbf{q}_i / r_{i,i}$).
- Orthogonalize columns $\mathbf{q}_l$, $i < l \leq n_T$, with regard to $\mathbf{q}_i$, i.e. subtract the component parallel to $\mathbf{q}_i$. The component in the direction of $\mathbf{q}_i$ is equal to the projection $r_{i,l} = \mathbf{q}_i^H \cdot \mathbf{q}_l$ and the orthogonal part is calculated by $\mathbf{q}_l = \mathbf{q}_l - r_{i,l} \cdot \mathbf{q}_i$.

In every step i the vectors $\mathbf{q}_1, \ldots, \mathbf{q}_i$ form an orthonormal basis of the vector space spanned by $\mathbf{h}_1, \ldots, \mathbf{h}_i$ and the vectors $\mathbf{q}_l$, $i < l \leq n_T$, contain the components of the corresponding $\mathbf{h}_l$ orthogonal to this vector space. The diagonal element $r_{i,i}$ denotes the length of $\mathbf{h}_i$ orthogonal to $\mathbf{q}_1, \ldots, \mathbf{q}_{i-1}$ or $\mathbf{h}_1, \ldots, \mathbf{h}_{i-1}$, respectively. Furthermore, the coefficients $r_{i,l}$ specify the component of $\mathbf{h}_l$, $i < l \leq n_T$, in the direction of $\mathbf{q}_i$.

B. Sorted Gram-Schmidt

From the explanation in section IV-A it is obvious that the modified Gram-Schmidt process calculates the diagonal elements from $r_{1,1}$ to r_{n_T,n_T}. As already stated, it would be optimal to maximize $|r_{k,k}|$ by permutating the rows of $\mathbf{Q}$ in every detection step, i.e from r_{n_T,n_T} to $r_{1,1}$. Thus, the optimal detection sequence $\mathcal{S}_{\mathcal{OPT}}$ maximizes SNR_k in every detecting step k, $k = n_T, \ldots, 1$. Unfortunately, the search for $\mathcal{S}_{\mathcal{OPT}}$ is very costly, because it requires $\mathcal{O}(n_T^2/2)$ QR decompositions.

The sorted Gram-Schmidt process (*Sorted QR Decomposition*, SQRD) proposed here searches for the detection sequence $\mathcal{S}$ that achieves small SNR_k in the upper layers. Consequently, the absolut values of the diagonal elements $r_{k,k}$ in the upper left area of the triangular matrix $\mathbf{R}$ are small. Thus, a detection fault caused by the little signal-to-noise ratio SNR_k influences only few layers $1, \ldots, k-1$ through error propagation.

In order to illustrate the functionality, one orthogonalization step i of the SQRD algorithm is explained in detail. The first $i-1$ elements of the sequence $\mathcal{S}$ are already calculated and therefore the vectors $\mathbf{q}_1, \ldots, \mathbf{q}_{i-1}$ are fixed. The ordering of the remaining columns is variable and is determined by the ordering rule in order to force small signal-to-noise ratios for the upper layers. Therefore, the column with minimal norm is chosen from the vectors $\mathbf{q}_i, \ldots, \mathbf{q}_{n_T}$ and denoted with $\mathbf{q}_{k_i}$. The corresponding $\mathbf{h}_{k_i}$ has the smallest component orthogonal to the space spanned by $\mathbf{q}_1, \ldots, \mathbf{q}_{i-1}$, wich leads to the smallest $r_{i,i}$ of the possible permutations in step i and thereby the smallest SNR_i.

The only change to the modified Gram-Schmidt algorithm is the reordering of the columns of $\mathbf{Q}$. In every decomposition step i the column $\mathbf{q}_i, \ldots, \mathbf{q}_{n_T}$ with the minimal length orthogonal to the already spanned vector space $\mathbf{q}_1, \ldots, \mathbf{q}_{i-1}$ is chosen. The whole algorithm for the signal detection is shown in **Fig. 3**. It consists of a decomposition part (line (1) to (11)) and a detection part (line (12) to (17)). In the decomposition part, the ordering is done in line (3) and (4) and provides the permutation vector $\mathcal{S}$, the orthogonal matrix $\mathbf{Q}$ and the upper triangular matrix $\mathbf{R}$. In the detection part, the received signal vector is sorted according to the permutation $\mathcal{S}$, and the modified received signal vector $\mathbf{y}$ is calculated (line (12)). The following lines (13) to (17) represent the iterative detection process described in section III-B.

`SQRD Algorithm`
(1) $\mathbf{R} = \mathbf{0}, \ \mathbf{Q} = \mathbf{H}, \ \mathcal{S} = (1, \ldots, n_T)$
(2) `for` $i = 1, \ldots, n_T$
(3) $\quad k_i = \arg \min\limits_{l=i,\ldots,n_T} \|\mathbf{q}_l\|^2$
(4) $\quad$ `exchange col.` i `and` k_i `in` $\mathbf{Q}, \mathbf{R}, \mathcal{S}$
(5) $\quad r_{i,i} = \|\mathbf{q}_i\|$
(6) $\quad \mathbf{q}_i = \mathbf{q}_i / r_{i,i}$
(7) $\quad$ `for` $l = i+1, \ldots, n_T$
(8) $\quad\quad r_{i,l} = \mathbf{q}_i^H \cdot \mathbf{q}_l$
(9) $\quad\quad \mathbf{q}_l = \mathbf{q}_l - r_{i,l} \cdot \mathbf{q}_i$
(10) $\quad$ `end`
(11) `end`
Signal Detection
(12) $\mathbf{y} = \mathbf{Q}^H \cdot \mathbf{x}$
(13) `for` $k = n_T, \ldots, 1$
(14) $\quad \hat{d}_k = \sum_{i=k+1}^{n_T} r_{k,i} \cdot \hat{c}_i$
(15) $\quad z_k = y_k - \hat{d}_k$
(16) $\quad \hat{c}_k = \mathcal{Q}[z_k / r_{k,k}]$
(17) `end`
(18) `Permutate` $\hat{c}$ `according to` $\mathcal{S}$

Fig. 3. SQRD algorithm and signal detection of layered space-time codes

V. Performance Analysis

The performance of the proposed SQRD detection algorithm and the standard LST detection algorithm (V-BLAST, [5]) was compared by means of Monte Carlo simulations for several scenarios. **Fig. 4** shows the bit error rates (BER) for an uncoded transmission of QPSK symbols in a system with $n_T = 8$ and $n_R = 12$ antennas. The iterative methods unsorted QR decomposition, SQRD and V-BLAST achieves a performance enhancement in comparison to the simple multiplication with the pseudo inverse of $\mathbf{H}$. The strong impact of ordering the QR decomposition is obvious and only a small difference of approximately 0.5 dB related to V-BLAST for a BER of 10^{-5} is visible for the SQRD algorithm.

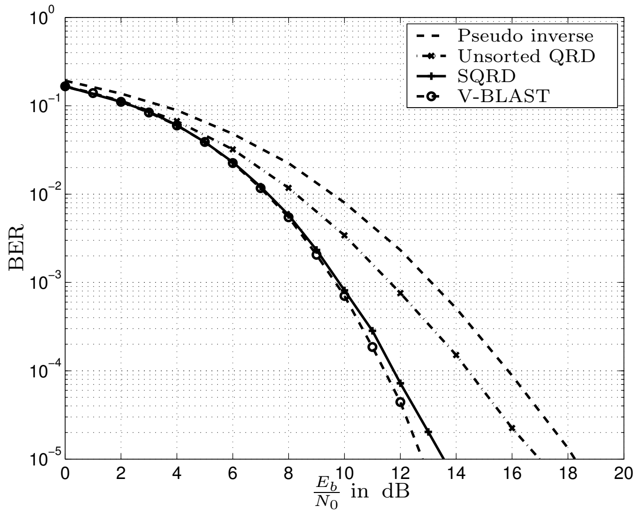

Fig. 4. Simulation with $n_T = 8$ and $n_R = 12$ antennas, uncoded QPSK symbols, spectral efficiency of 16 Bit/s/Hz

Fig. 5 shows the BER of the different detection algorithms for an uncoded system with $n_T = 4$ and $n_R = 6$ antennas. These results confirm the good performance of the SQRD algorithm with the reduced calculation complexity in mind.

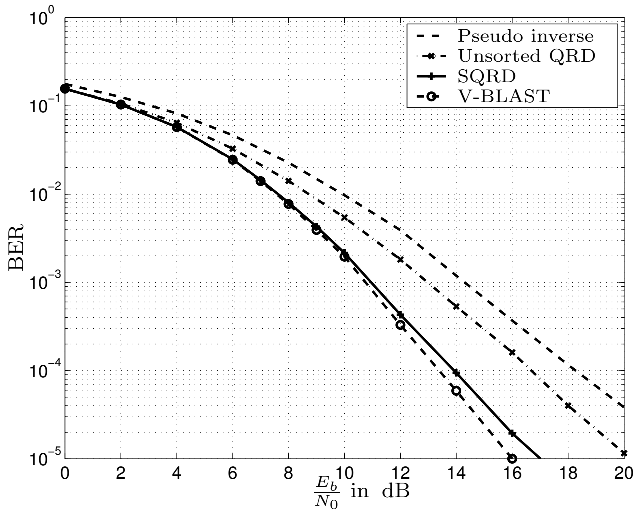

Fig. 5. Simulation with $n_T = 4$ and $n_R = 6$ antennas, uncoded QPSK symbols, spectral efficiency of 8 Bit/s/Hz

In [9] the "genie" detection process was introduced to investigate the error propagation of V-BLAST. This implies real interference suppression for each layer, but for subsequent layers ideal detection of the signals of preceding layers is assumed. Thus, only correct values are subtracted to reduce the system order and consequently no error propagation takes place.

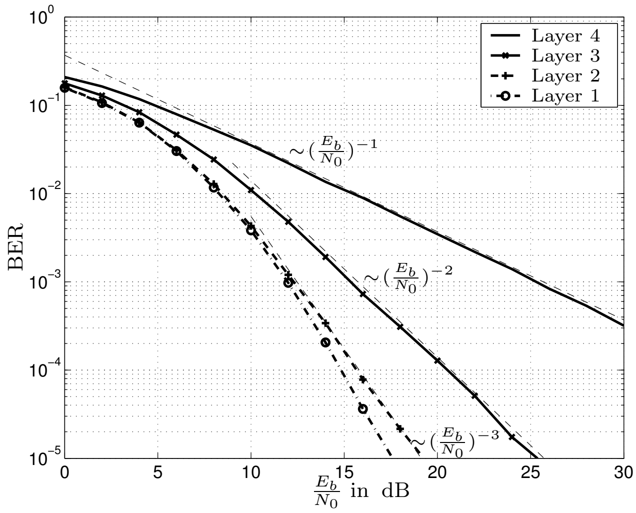

Fig. 6. "Genie" detection in a system with $n_T = 4$ and $n_R = 4$ antennas, uncoded QPSK symbols

In **Fig. 6** the BER per layer are shown for a system with $n_T = 4$ and $n_R = 4$ antennas when the "genie" detection is used. In every detection step $k = 4, \ldots, 1$ a diversity of $g_d = n_R - k + 1$ is achieved. According to the diversity levels, the BER of layer k decays with $(E_b/N_0)^{-g_d}$. Thus the BER of the upper layers decay much steeper due to the higher diversity levels in comparison to the layer detected first (layer 4).

VI. Applying Channel Coding

In order to improve the performance of the single user to user communication, each layer is now independently encoded by a channel coder. For simplicity, we used the half rate $(7,5)_{oct}$ convolutional encoder and viterbi decoding as shown in **Fig. 7**. **Fig. 8** shows the Frame Error Rate (FER) of an

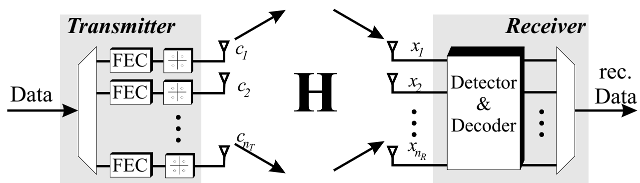

Fig. 7. Coded LST architecture with n_T transmit and n_R receive antennas, FEC in each layer

uncoded and a coded system equipped with $n_T = 8$ and $n_T = 12$ antennas using the V-BLAST or the SQRD detection algorithm, respectively. The transmitted QPSK signals are organized in frames of length $L = 100$ symbols including tail symbols for the coded case. The figure shows the expected performance enhancement for coded systems and again the SQRD detection nearly reaches the error probability of V-BLAST.

This statement is confirmed by the simulation result for a system with $n_T = 4$ and $n_R = 6$ antennas,

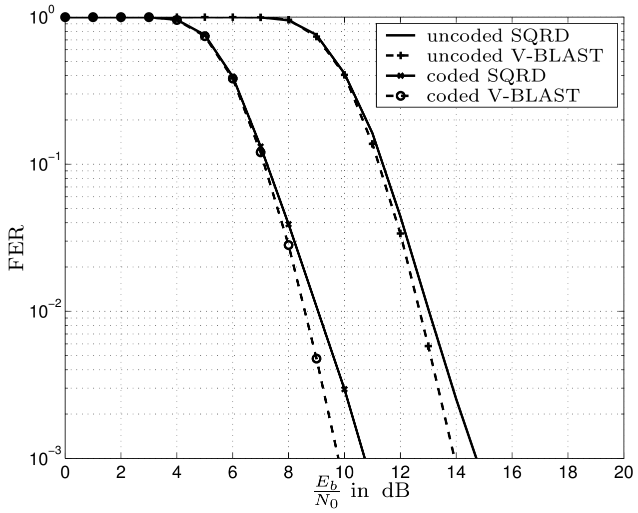

Fig. 8. FER of uncoded and convolutionally coded system with $n_T = 8$ and $n_R = 12$ antennas, frame length $L = 100$, QPSK symbols

shown in **Fig. 9**.

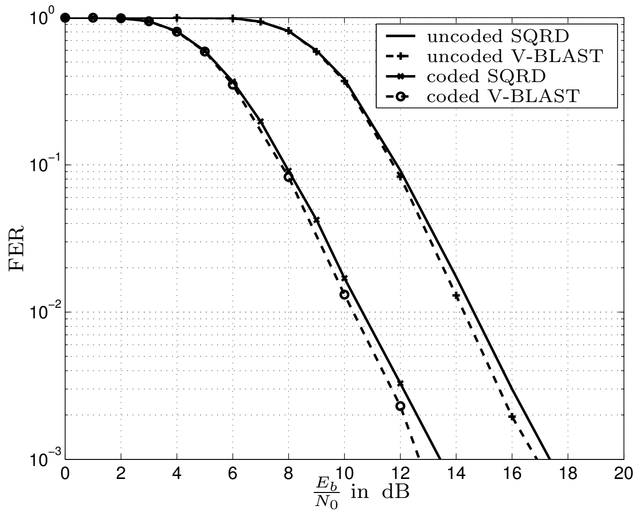

Fig. 9. FER of uncoded and convolutionally coded system with $n_T = 4$ and $n_R = 6$ antennas, frame length $L = 100$, QPSK symbols

VII. Computational Effort

The computational requirements of the proposed SQRD algorithm and V-BLAST are compared in this section. Therefore, the floating point operations of these algorithms are specified according to the system variables n_T, n_R and L, with L denoting the frame length (i.e. number of symbols transmitted within one layer). Real valued additions, multiplications and divisions are equally counted as one flop to obtain a single value for the computational effort.

With these assumptions, the V-BLAST algorithm – as stated in Fig 2 – needs

$$f_{\text{V-BLAST}} = 8\,n_T^4 + 16\,n_T^3 n_R + 8\,n_T^2 n_R + 18\,L n_T n_R \tag{19}$$

floating point operations. Using the same counting, the SQRD algorithm needs

$$\begin{aligned} f_{\text{SQRD}} \;&=\; 12\,n_T^2 n_R - 2\,n_T n_R + n_T \\ &\quad +L\,(4\,n_T^2 + 8\,n_T n_R + 2\,n_T) \end{aligned} \tag{20}$$

operations. The computational requirements of the V-BLAST and the SQRD algorithm are compared by the quotient

$$\rho = \frac{f_{\text{SQRD}}}{f_{\text{V-BLAST}}} \,. \tag{21}$$

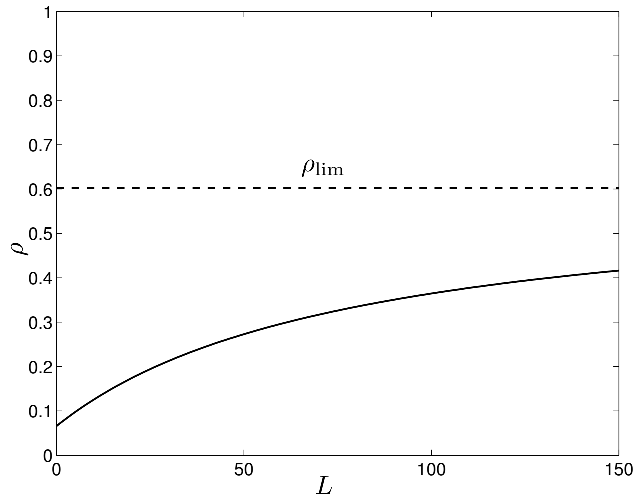

Fig. 10. Quotient ρ of required floating point operations for SQRD and V-BLAST with $n_T = 8$ and $n_R = 12$ antennas – varying frame length L

Fig. 10 depicts the computational advantage of the SQRD algorithm over the V-BLAST algorithm for a system with $n_T = 8$ and $n_R = 12$ antennas. With an increasing number L of symbols per layer the quotient ρ saturates to a limit value. This value can be calculated by

$$\rho_{\text{lim}} = \lim_{L\to\infty} \rho = \frac{4\,n_T + 8\,n_R + 2}{18\,n_R} \tag{22}$$

and is shown in Fig. 10 as a horizontal line.

Furthermore, the computational demands of V-BLAST and SQRD concerning different numbers of antennas are compared. A system with $L = 100$ symbols per layer and a varying number of transmit and receive antennas, with $n_T = n_R$, is investigated. **Fig. 11** shows the increasing computational advantage of the SQRD compared the V-BLAST algorithm for increasing number of antennas. Thus, the SQRD dramatically reduces the computational requirements for systems with larger amount of antennas.

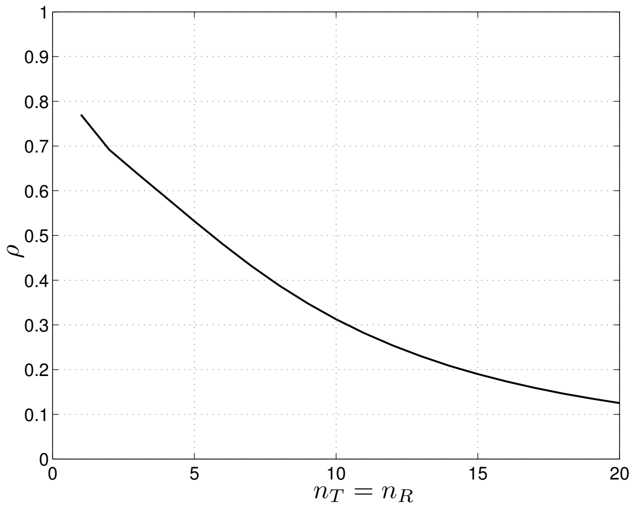

Fig. 11. Quotient ρ of required floating point operations for SQRD and V-BLAST with $L = 100$ – varying $n_T = n_R$

VIII. Summary and Conclusions

We have described a new detection algorithm for LST codes. The algorithm is based on the Gram-Schmidt algorithm for QR decomposition and requires less computational effort in comparison to the standard detection algorithm with only small degradation in error performance. We presented simulation results for several scenarios and analytically demonstrated the computational advantage of the proposed SQRD algorithm.

Since the core of our algorithm consists of a sorted kind of QR decomposition, the derived algorithm is not restricted to layered space-time architectures. It can generally be used to detect vector channel systems.

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