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Layered Space-Time Algorithm for Multi User Detection

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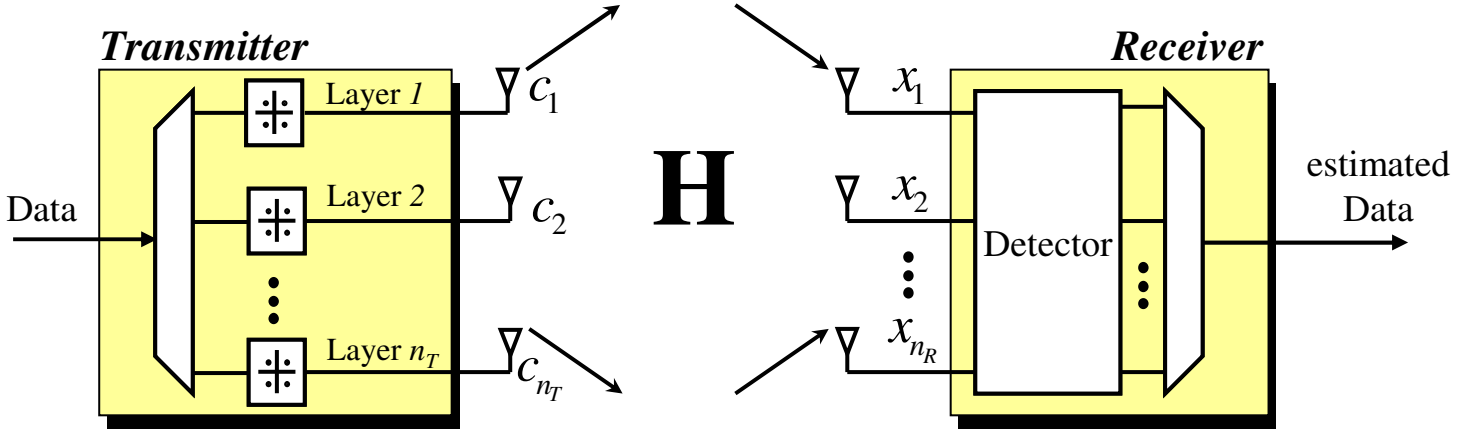
Reduce the Computational Effort of Detecting Layered Space-Time Architectures

Outline

- Layered Space-Time Codes
- V-Blast Detection
- Detection by QR Decomposition
  - Gram-Schmidt Algorithm
  - Principle Detection Scheme
  - Sorted QR Decomposition
- Performance Comparison
  - Computational Effort
  - Simulation Results
- Summary

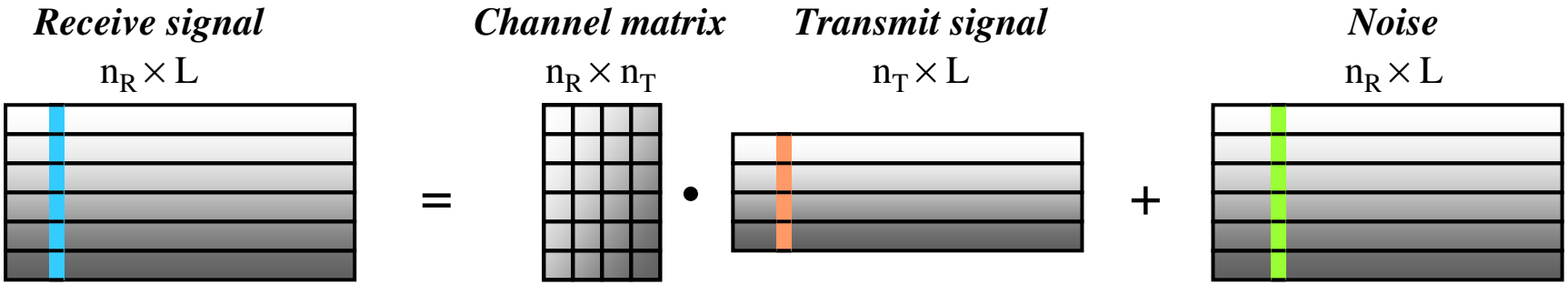
Layered Space-Time Codes (LST-Codes)



- Multi antenna system with n_T transmit and n_R receive antennas: $n_R \geq n_T$
- Flat fading channel with perfect channel state information for the receiver
- Data is separated in n_T substreams (*layers*), mapped to M-PSK or M-QAM symbols c_i and transmitted over the n_T antennas at the same time
- Each receive signal x_j is a linear combination of the transmitted signals c_i weighted by the fading gains $h_{j,i}$ and an additive noise term ν_j

System Description

- Substreams are organized in frames of length L



- Time-discrete model for one time slot

$$\begin{pmatrix} x_1 \\ \vdots \\ x_{n_R} \end{pmatrix} = \begin{pmatrix} h_{1,1} & \cdots & h_{1,n_T} \\ \vdots & \ddots & \vdots \\ h_{n_R,1} & \cdots & h_{n_R,n_T} \end{pmatrix} \cdot \begin{pmatrix} c_1 \\ \vdots \\ c_{n_T} \end{pmatrix} + \begin{pmatrix} \nu_1 \\ \vdots \\ \nu_{n_R} \end{pmatrix}$$

$$\mathbf{x} = \mathbf{H} \cdot \mathbf{c} + \boldsymbol{\nu}$$

$\mathbf{x}$: $n_R \times 1$ vector of received signals
$\mathbf{H}$: $n_R \times n_T$ channel matrix (flat fading)
$\mathbf{c}$: $n_T \times 1$ vector of transmitted signals
$\boldsymbol{\nu}$: $n_R \times 1$ noise vector

- Optimum way of recovering n_T transmitted signals: *Maximum-Likelihood*
  → not feasible due to enormous complexity

V-BLAST Detection

- Successive interference suppression and cancellation technique proposed by Bell-Labs
- Suppress interferer by linearly weighting the received signal vector with a Zero-Forcing nulling vector
  - Calculate Pseudo-Inverse $\mathbf{G}_i=\mathbf{H}_i^+$ of channel matrix
  - Find layer k_i with largest post-detection SNR
  - Specify nulling vector
  - Suppress interference
- Interference Cancellation
  - Detect target signal with index k_i
  - Subtract detected signal from received signal vector
  - Reduce channel matrix by setting column k_i to zero

for $i = 1, \ldots, n_T$

$$\mathbf{G}_i = \mathbf{H}_i^+ = \left(\mathbf{H}_i^H \cdot \mathbf{H}_i\right) \cdot \mathbf{H}_i^H$$

$$k_i = \arg\min_j \left\| \mathbf{g}_i^{(j)} \right\|^2$$

$$\mathbf{w}_{k_i}^T = \mathbf{g}_i^{(k_i)}$$

$$y_{k_i} = \mathbf{w}_{k_i}^T \cdot \mathbf{x}_i$$

$$\hat{c}_{k_i} = Q\left[y_{k_i}\right]$$

$$\mathbf{x}_{i+1} = \mathbf{x}_i - \mathbf{h}_{k_i} \cdot \hat{c}_{k_i}$$

$$\mathbf{H}_{i+1} = \mathbf{H}_i^{\overline{k_i}}$$

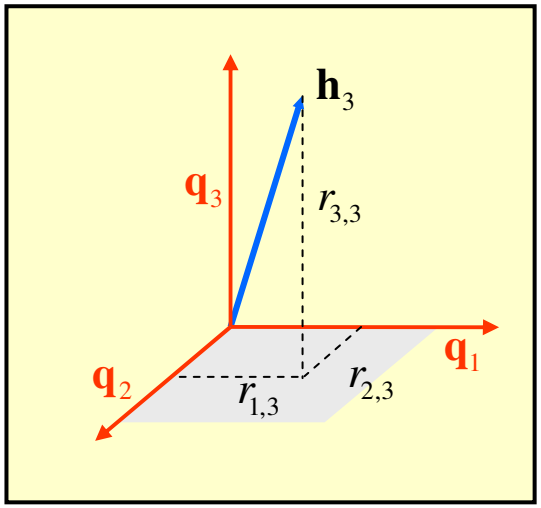
end

- *Order of detection* is crucial due to error propagation!
- Expensive calculation of pseudo-inverse in each iteration!

QR Decomposition by Gram-Schmidt

- QR decomposition of channel matrix $\mathbf{H} = \mathbf{Q} \cdot \mathbf{R}$
- Gram-Schmidt

$$\left(\mathbf{h}_1 \;\; \mathbf{h}_2 \;\; \mathbf{h}_3 \ldots \mathbf{h}_{n_T}\right) = \left(\mathbf{q}_1 \;\; \mathbf{q}_2 \;\; \mathbf{q}_3 \ldots \mathbf{q}_{n_T}\right) \cdot \begin{pmatrix} r_{1,1} & r_{1,2} & r_{1,3} & \cdots & r_{1,n_T} \\ & r_{2,2} & r_{2,3} & \cdots & r_{2,n_T} \\ & & r_{3,3} & \cdots & r_{3,n_T} \\ & \mathbf{0} & & \ddots & \vdots \\ & & & & r_{n_T,n_T} \end{pmatrix}$$



- Example: Decomposition of $\mathbf{h}_3$
  - Columns ($\mathbf{q}_1$,$\mathbf{q}_2$) form an orthonormal basis of the vector space ($\mathbf{h}_1$,$\mathbf{h}_2$)
  - $r_{1,3}$ and $r_{2,3}$ describe the component of $\mathbf{h}_3$ in the direction of $\mathbf{q}_1$ and $\mathbf{q}_2$
  - $\mathbf{q}_3$ denotes the direction of $\mathbf{h}_3$ perpendicular to the base ($\mathbf{q}_1$,$\mathbf{q}_2$)
  - $r_{3,3}$ describes the component of $\mathbf{h}_3$ in the direction of $\mathbf{q}_3$ → $\mathbf{h}_3 = \mathbf{q}_1 \cdot r_{1,3} + \mathbf{q}_2 \cdot r_{2,3} + \mathbf{q}_3 \cdot r_{3,3}$

→ Diagonal element $r_{k,k}$ denotes component of $\mathbf{h}_k$ perpendicular to base ($\mathbf{q}_1$,…,$\mathbf{q}_{k-1}$)

Principle Detection Scheme

$\mathbf{H} = \mathbf{Q} \cdot \mathbf{R}$ → Modified receive signal vector $\mathbf{y} = \mathbf{Q}^H \cdot \mathbf{x} = \mathbf{R} \cdot \mathbf{c} + \boldsymbol{\eta}$

- **R** is upper triangular → received signals are partly free from interference

$$\begin{pmatrix} y_1 \\ \vdots \\ y_k \\ y_{k+1} \\ \vdots \\ y_{n_T} \end{pmatrix} = \begin{pmatrix} r_{1,1} & \cdots & r_{1,k} & r_{1,k+1} & \cdots & r_{1,n_T} \\ & \ddots & \vdots & \vdots & & \vdots \\ & & r_{k,k} & r_{k,k+1} & \cdots & r_{k,n_T} \\ & \mathbf{0} & & r_{k+1,k+1} & \cdots & r_{k+1,n_T} \\ & & & & \ddots & \vdots \\ & & & & & r_{n_T,n_T} \end{pmatrix} \cdot \begin{pmatrix} c_1 \\ \vdots \\ c_k \\ c_{k+1} \\ \vdots \\ c_{n_T} \end{pmatrix} + \begin{pmatrix} \eta_1 \\ \vdots \\ \eta_k \\ \eta_{k+1} \\ \vdots \\ \eta_{n_T} \end{pmatrix}$$

- Layer k: $y_k = r_{k,k} \cdot c_k + \eta_k + \underbrace{\sum_{i=k+1}^{n_T} r_{k,i} \cdot c_i}$

→ $y_k = r_{k,k} \cdot c_k + \eta_k + d_k$

→ $SNR_k \sim |r_{k,k}|$

Interference Term $d_k = \sum_{i=k+1}^{n_T} r_{k,i} \cdot c_i$

→ Detection from bottom to top → Estimated interference $\hat{d}_k$

Detection with QR Decomposition

- Interference Cancellation

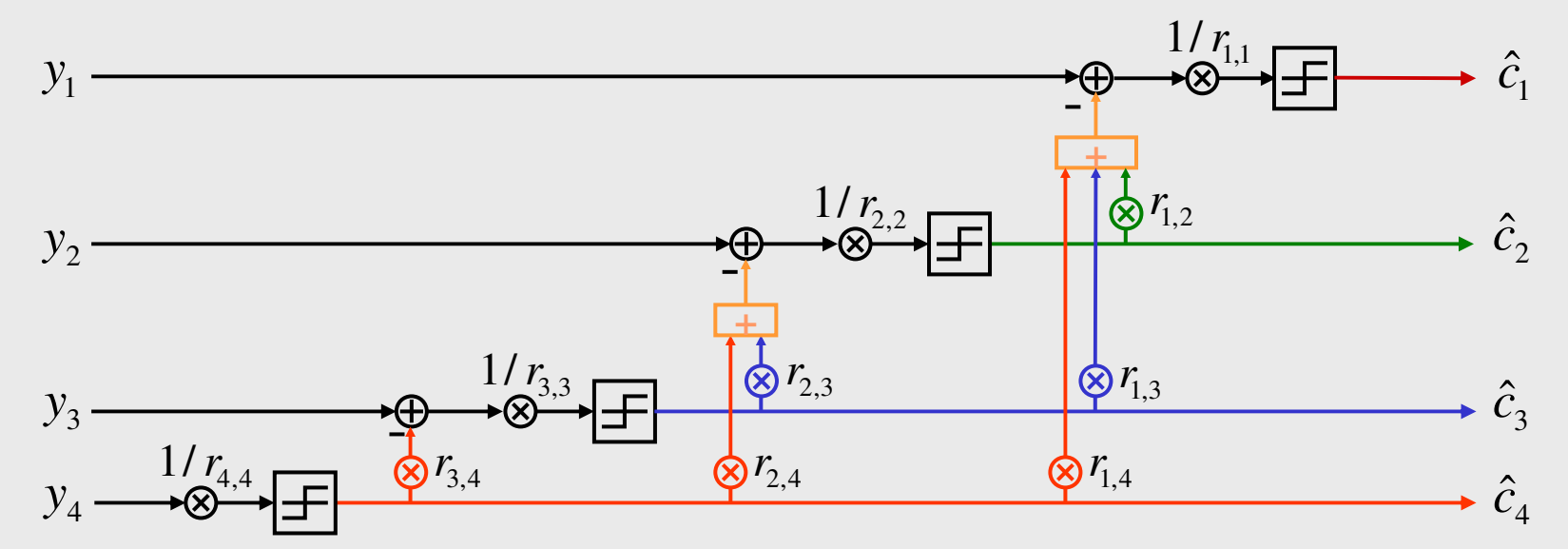
$$z_k = y_k - \hat{d}_k = r_{k,k} \cdot c_k + \eta_k$$

- Detection

$$\hat{c}_k = Q\left[z_k / r_{k,k}\right]$$

- Example ($n_T = 4$)

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} = \begin{pmatrix} r_{1,1} & r_{1,2} & r_{1,3} & r_{1,4} \\ & r_{2,2} & r_{2,3} & r_{2,4} \\ & & r_{3,3} & r_{3,4} \\ & \mathbf{0} & & r_{4,4} \end{pmatrix} \cdot \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{pmatrix} + \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \end{pmatrix}$$



Problem: Find Ordering

- Sequence of detection can be changed by permuting the columns of $\mathbf{H}$ and the corresponding elements of the transmit signal vector $\mathbf{c}$
  → different QR decompositions
- Post Detection $SNR_k \sim |r_{k,k}|$ should be maximized for each detection step

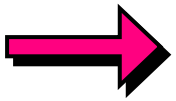
→ Exchange columns of $\mathbf{H}$ to maximize *diagonal elements* of matrix $\mathbf{R}$ for each detection step

- **Computational Conflict**
  - Optimal detection sequence maximizes $|r_{k,k}|$ in every detecting step $r_{n_T,n_T} \rightarrow r_{1,1}$
  - Modified Gram-Schmidt calculates the diagonal element from $r_{1,1} \rightarrow r_{n_T,n_T}$
- Optimal detection order can be found by calculating all different QR decompositions → very costly search

→ Suboptimal approach which needs only one decomposition?

Principle of Sorted QR Decomposition (SQRD)

- Search for the detection Sequence S that leads to small post detection SNR_k in the upper layer



Sorting Rule for each decomposition step:

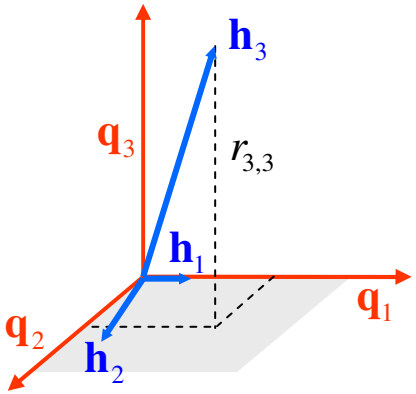
Select column $\mathbf{q}$ of the remaining columns with minimum norm as current basis vector

- Sorting prevents, that this little SNR_k is active in a lower layer, where it would effect a larger number of decisions due to error propagation

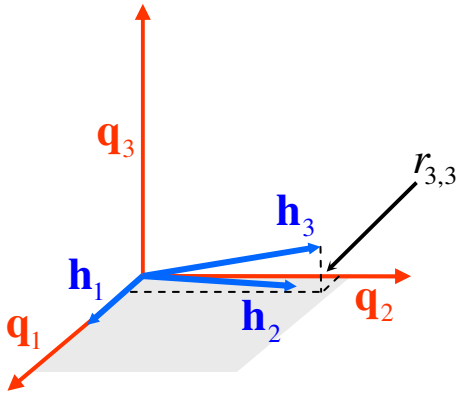
- Sorting rule intends large diagonal elements in lower right part of matrix $\mathbf{R}$
  → leads to high SNR_k for the first detection steps

Why is the sorting rule not optimal?

- Example with $n_T = 3$
  - Optimal order



  - Large part of $\mathbf{h}_3$ perpendicular to $\mathbf{h}_1$ and $\mathbf{h}_2$ leads to large coefficient $r_{3,3}$
  - Suboptimal order



  - $\mathbf{h}_2$ and $\mathbf{h}_3$ have a large norm but similar direction, which leads to a small perpendicular component $r_{3,3}$
    → small SNR

Sorted QR Decomposition (SQRD)

- Modification of Gram-Schmidt algorithm by inserting a reordering in each decomposition step
  - → Permutation vector S
- Decomposition step i
  - First i-1 elements of S and $\mathbf{q}_1, \ldots, \mathbf{q}_{i-1}$ are fixed, but order of remaining columns is variable
  - Sorting rule selects column $\mathbf{q}_{k_i}$ of remaining columns with minimum norm
  - Exchange columns of $\mathbf{Q}$, $\mathbf{R}$ and S
  - Proceed with Gram-Schmidt decomposition

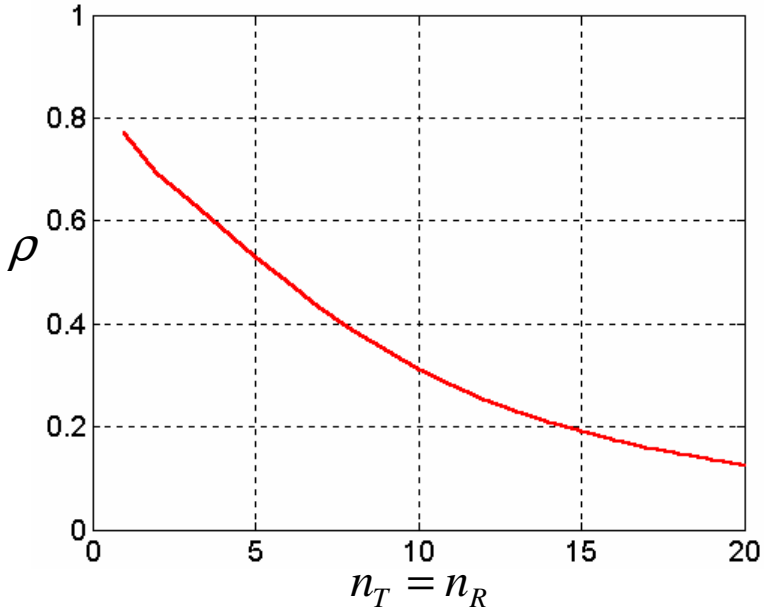
$$
\begin{aligned}
&\mathbf{R} = \mathbf{0}, \mathbf{Q} = \mathbf{H}, S = (1, \cdots, n_T) \\
&\text{for } i = 1, \ldots, n_T \\
&\quad k_i = \arg \min_{l=i,\ldots,n_T} \|\mathbf{q}_l\|^2 \\
&\quad \text{exchange col. } i \text{ and } k_i \text{ in } \mathbf{Q}, \mathbf{R} \text{ and } S \\
&\quad r_{i,i} = \|\mathbf{q}_i\| \\
&\quad \mathbf{q}_i = \mathbf{q}_i / r_{i,i} \\
&\quad \text{for } l = i+1, \ldots, n_T \\
&\qquad r_{i,l} = \mathbf{q}_i^H \cdot \mathbf{q}_l \\
&\qquad \mathbf{q}_l = \mathbf{q}_l - r_{i,l} \cdot \mathbf{q}_i \\
&\quad \text{end} \\
&\text{end}
\end{aligned}
$$

→ Only one decomposition, but *optimum* sorting is not assured!

Computational Effort

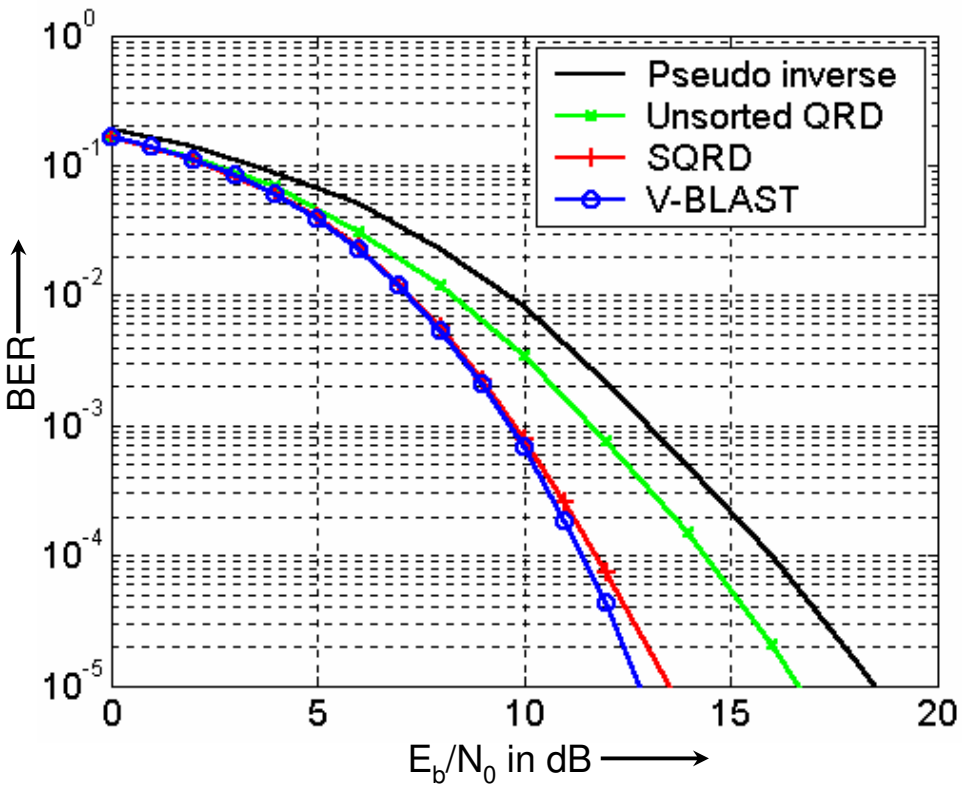
- Floating Point Operations according to n_T, n_R and frame length L
- Comparison of computational effort by quotient of flops

$$\rho = \frac{f_{SQRD}}{f_{V\text{-}BLAST}}$$



- Parameter:
  - Frame length $L = 100$
  - Varying number of antennas, with $n_T = n_R$
- Increasing computational advantage of SQRD compared to V-BLAST algorithm for increasing number of antennas
- Example: $n_T = n_R = 15$
  → only 20% of computational effort

Simulation Results

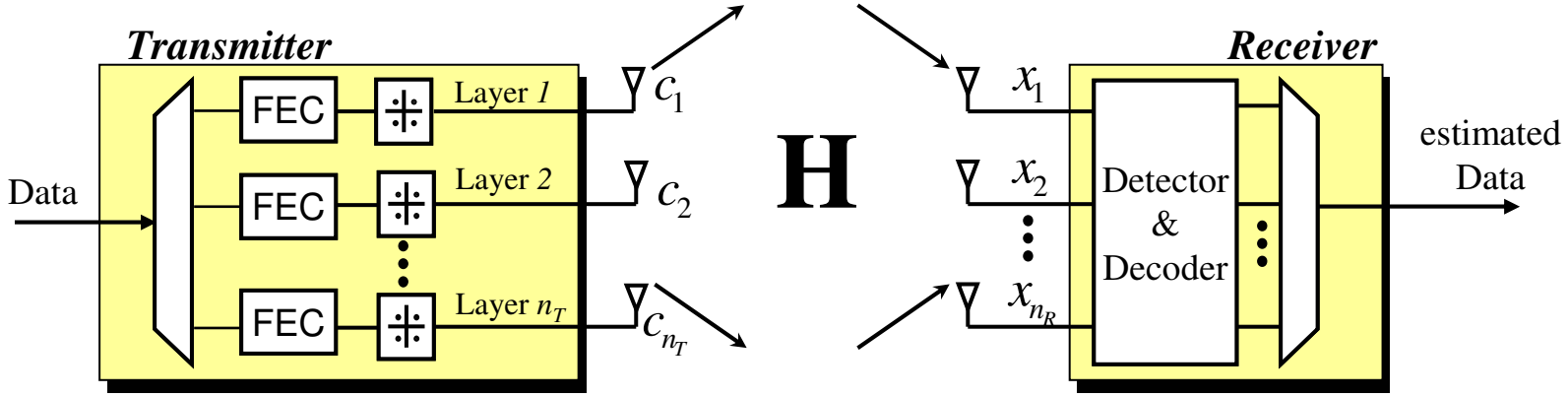


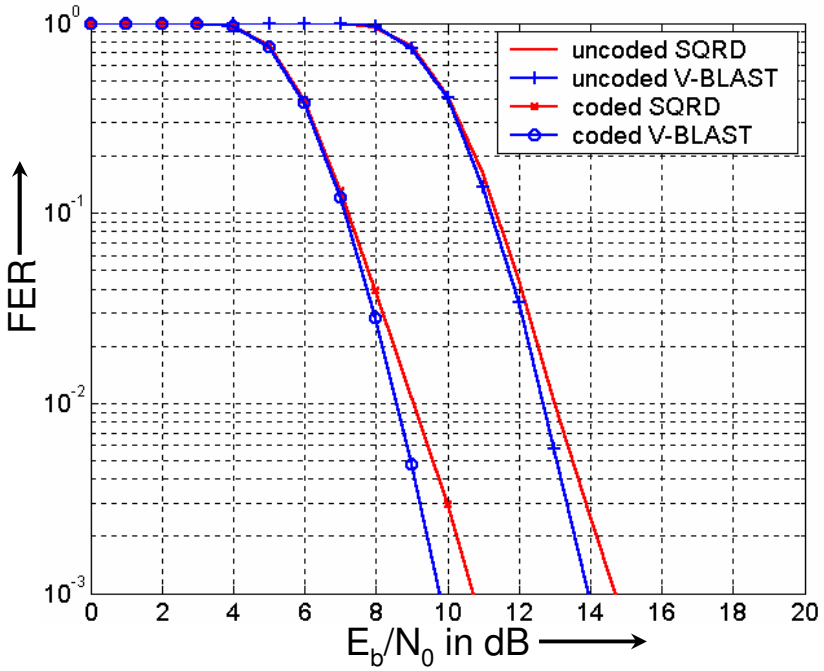
- Simulation parameters:
  - 8 transmit and 12 receive antennas
  - Uncoded QPSK symbols
  - Flat Rayleigh channel
  - Perfect channel state information
  - Monte Carlo simulations
- Strong impact of ordering is obvious
- Only a small difference of approximately 0.5 dB compared to V-BLAST for BER of 10^{-5}
- Small loss in performance of SQRD with reduced computational complexity in mind

$$\rho = \frac{f_{SQRD}}{f_{V\text{-}BLAST}} = 0.36$$

36% of computational effort leads to 0.5 dB loss for BER of 10^{-5}

Layered Space-Time with Coded Substreams





- Each layer independently encoded by terminated convolutional encoder → no block interleaving
- Decoded by viterbi in each layer
- Simulation parameters:
  - $n_T = 8$, $n_R = 12$, QPSK, L = 100, Flat Rayleigh channel,
  - $(7,5)_8$ Convolutional Coder
- Both coded schemes provide 4 dB performance gain for FER of 10^{-3}

Summary

- An alternative detection algorithm for layered space-time codes

Sorted **QR D**ecomposition, **SQRD**

- Based on sorted extension of Gram-Schmidt algorithm
- SQRD requires less computational effort with only small degradation in error performance compared to V-BLAST
- SQRD can generally be used to detect vector channels

$$\mathbf{x} = \mathbf{H} \cdot \mathbf{c} + \mathbf{v}$$

- Outlook:
  - Turbo Detection Scheme
  - Frequency Selective MIMO channels