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Space-Time Codes for Diversity Enhancement in Wireless Communication Systems

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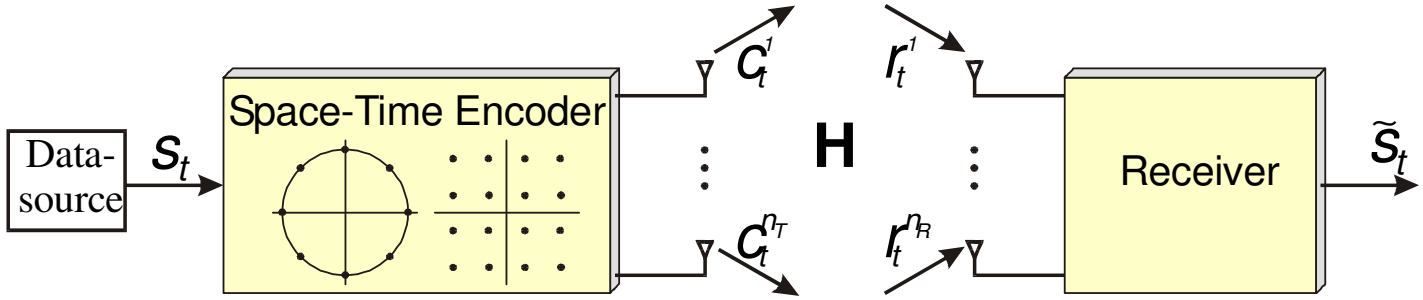
- Motivation for Space-Time Coding and Multi-Antenna Systems
- Space-Time Trellis Codes
- Space-Time Block Codes
- Summary

Motivation for Space-Time Coding

- Increasing demand of wireless communication systems
  - Higher data rates and / or reliability of communications over fading channels
  - Performance of flat fading channel can be improved by using diversity
  - An attractive way with respect to spectral efficiency is given by antenna diversity → multiple antennas in the transmitter and / or the receiver
  - Maximum Ratio Combining (MRC) uses several antennas in the receiver and achieves full diversity → optimum combining scheme
- Using more than one antenna in the transmitter to achieve also transmit diversity
  - Special kind of coding in the transmitter is necessary to achieve a diversity advantage
  - Using space as a second dimension for coding → Space-Time Codes
- Normalizing of transmitted power
  - A normalization of the signals to the number of paths (receive antennas) for MRC
  - Multi-Antenna system with n_T transmit and n_R receive antennas establishes $n_T \cdot n_R$ uncorrelated paths → normalization to the number of paths
  - PSK-Signal: $$E_s = \frac{ld(M)}{n_T \cdot n_R} \cdot E_b$$

Multi-Antenna System



- Code vector

$$\mathbf{c}_t = \left(c_t^{\,1} \quad c_t^{\,2} \quad \cdots \quad c_t^{\,n_T} \right)^T$$

- Channel matrix

$$\mathbf{H} = \begin{pmatrix} \alpha_{1,1} & \alpha_{2,1} & \cdots & \alpha_{n_T,1} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{1,n_R} & \alpha_{2,n_R} & \cdots & \alpha_{n_T,n_R} \end{pmatrix}$$

receive antenna

transmit antenna

- Noise vector

$$\boldsymbol{\eta}_t = \left(\eta_t^{\,1} \quad \eta_t^{\,2} \quad \cdots \quad \eta_t^{\,n_R} \right)^T$$

- Received signal vector

$$\mathbf{r}_t = \left(r_t^{\,1} \quad r_t^{\,2} \quad \cdots \quad r_t^{\,n_R} \right)^T$$

$$\Rightarrow \quad \mathbf{r}_t = \sqrt{E_s} \cdot \mathbf{H} \cdot \mathbf{c}_t + \boldsymbol{\eta}_t$$

Probability of Error

- Code word matrix for frame of length l

$$\mathbf{C} = (\mathbf{c}_1 \quad \mathbf{c}_2 \quad \cdots \quad \mathbf{c}_l) = \begin{pmatrix} c_1^1 & c_2^1 & \cdots & c_l^1 \\ \vdots & \vdots & \ddots & \vdots \\ c_1^{n_T} & c_2^{n_T} & \cdots & c_l^{n_T} \end{pmatrix}$$

space (transmit antennas)

time

- Difference matrix of two distinct code word matrices **C**, **E**

$$\mathbf{B}(\mathbf{C},\mathbf{E}) = \mathbf{E} - \mathbf{C} = \begin{pmatrix} e_1^1 - c_1^1 & e_2^1 - c_2^1 & \cdots & e_l^1 - c_l^1 \\ \vdots & \vdots & \ddots & \vdots \\ e_1^{n_T} - c_1^{n_T} & e_2^{n_T} - c_2^{n_T} & \cdots & e_l^{n_T} - c_l^{n_T} \end{pmatrix}$$

- Error matrix $\mathbf{A}(\mathbf{C},\mathbf{E}) = \mathbf{B}(\mathbf{C},\mathbf{E}) \cdot \mathbf{B}(\mathbf{C},\mathbf{E})^+$

- If perfect channel state information (CSI) is available at the receiver, the probability of transmitting **C** and receiving **E** is upper bounded by

$$P(\mathbf{C} \to \mathbf{E}) \le \left(\prod_{i=1}^{r} \lambda_i \right)^{-n_R} \left(\frac{E_s}{4N_0} \right)^{-r \cdot n_R}$$

$$\lambda_i = \mathrm{eig}\{\mathbf{A}(\mathbf{C},\mathbf{E})\}$$

$$r = \mathrm{rank}\{\mathbf{A}(\mathbf{C},\mathbf{E})\} = \mathrm{rank}\{\mathbf{B}(\mathbf{C},\mathbf{E})\} \le n_T$$

Design Criteria for Space-Time Codes

$$P(\mathbf{C} \to \mathbf{E}) \le \underbrace{\left(\prod_{i=1}^{r} \lambda_i \right)^{-n_R}}_{\text{Coding gain}} \left(\frac{E_s}{4N_0} \right)^{-r \cdot n_R}$$

Diversity gain $r \cdot n_R$

$$g_r = \min_{(\mathbf{C},\mathbf{E})} \left(\prod_{i=1}^{r} \lambda_i \right)^{1/r}$$

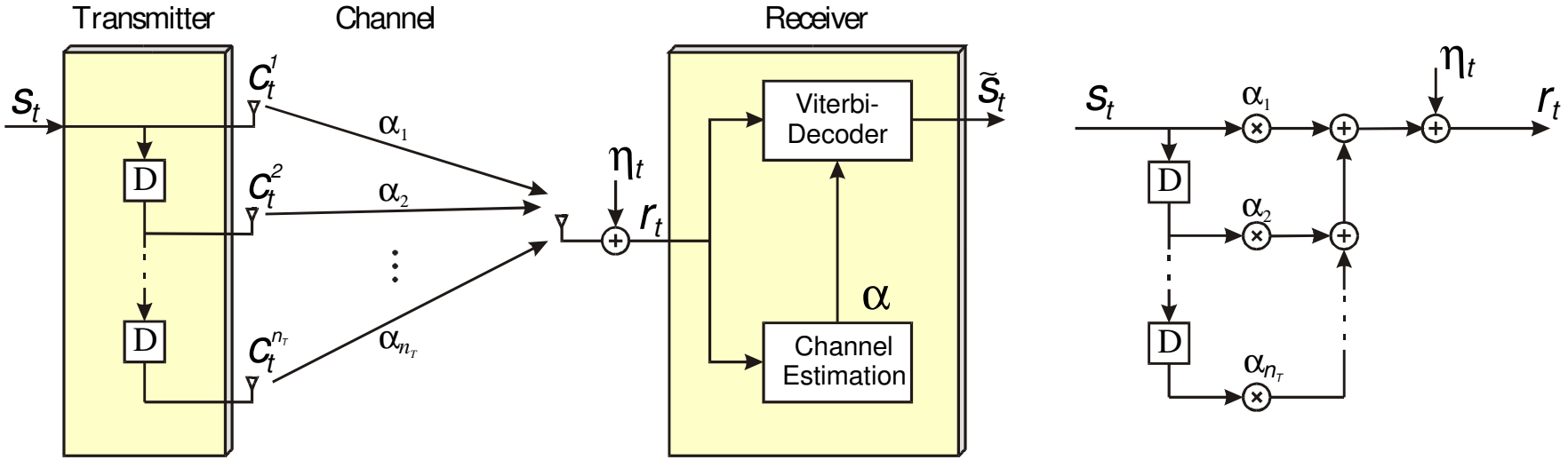
- Rank criterion:
  - To achieve the maximum diversity $n_T n_R$, the matrix $\mathbf{B}(\mathbf{C},\mathbf{E})$ has to be full rank for any two code word matrices **C** and **E**.

$$\min_{(\mathbf{C},\mathbf{E})} \operatorname{rank}\{\mathbf{B}(\mathbf{C},\mathbf{E})\} = n_T$$

- Determinant criterion:
  - To achieve maximum coding gain for a given diversity gain of $n_T \cdot n_R$, maximize the minimum product of eigenvalues for any two code word matrices **C** and **E**.

$$g_r = \min_{(\mathbf{C},\mathbf{E})} \left(\prod_{i=1}^{r=n_T} \lambda_i \right)^{1/n_T} = \min_{(\mathbf{C},\mathbf{E})} \left(\det \mathbf{A}(\mathbf{C},\mathbf{E}) \right)^{1/n_T}$$

Basic Idea of Space-Time Trellis Codes (STTC)



- Delay-Diversity
  - Sending delayed replicas of the same signal from different antennas
  - Flat fading channel is transformed into channel with random ISI
  - Optimal decoder is realized by MLSE (Viterbi-Algorithm, SOVA, MAP)
  - Maximum diversity n_T is achieved
  - Kind of repetition code! Are there better codes than the repetition code?

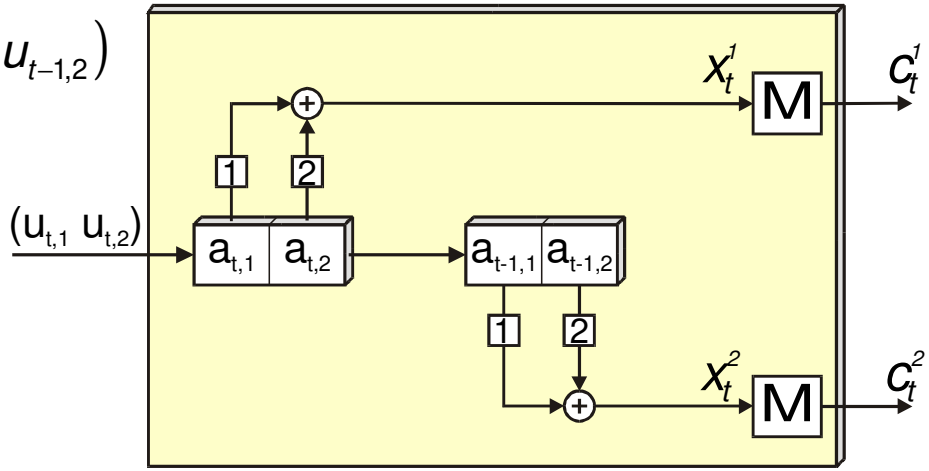
Analytical Description of Space-Time Trellis Codes

- Nonrecursive Convolutional Encoder realized by binary shift registers
- Example: Delay-Diversity for QPSK, $n_T = 2$ transmit antennas, 4 states
  - binary register elements

$$\mathbf{a}_t = (\mathbf{u}_t \quad \mathbf{u}_{t-1}) = (u_{t,1} \quad u_{t,2} \quad u_{t-1,1} \quad u_{t-1,2})$$

  - Generator matrix

$$G = \begin{pmatrix} 1 & 0 \\ 2 & 0 \\ 0 & 1 \\ 0 & 2 \end{pmatrix}$$



  - Encoder output

$$\mathbf{x}_t = (x_t^1 \quad x_t^2) = (\mathbf{a}_t \cdot \mathbf{G}) \bmod 4$$

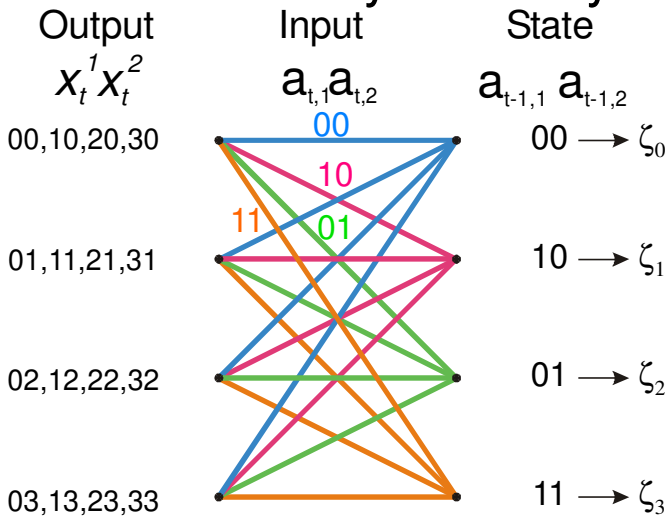
  - Code vector (transmitted QPSK-symbols with natural mapping)

$$\mathbf{c}_t = (c_t^1 \quad c_t^2) = \mathrm{M}\,(c_t \cdot \mathbf{G}) \bmod 4$$

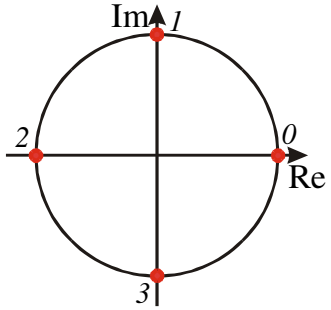
$$c_t^1 = \mathrm{M}\left(a_{t,1} + 2 \cdot a_{t,2}\right)$$
$$c_t^2 = \mathrm{M}\left(a_{t-1,1} + 2 \cdot a_{t-1,2}\right)$$

Example for Space-Time Trellis Coding

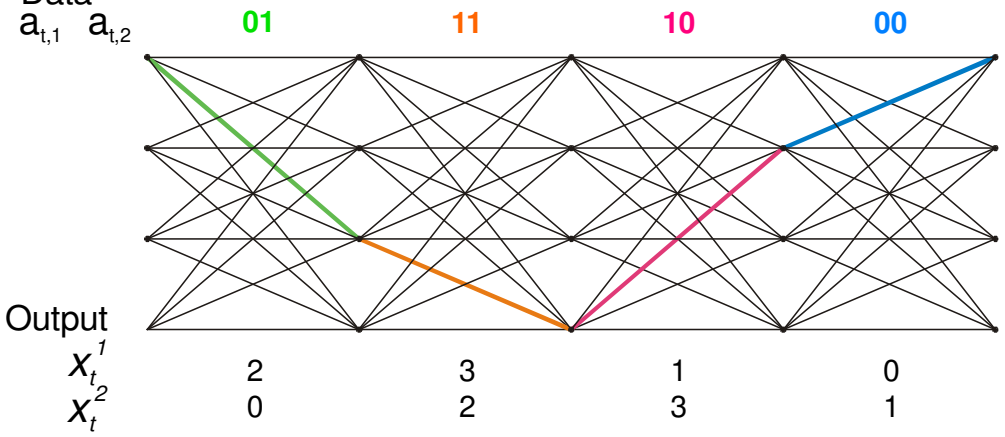
- Trellis for Delay-Diversity



- QPSK constellation



- Example
  - Input $(01 \quad 11 \quad 10 \quad 00)$
  - Output $\begin{pmatrix} 2 & 3 & 1 & 0 \\ 0 & 2 & 3 & 1 \end{pmatrix}$



  - Code word matrix

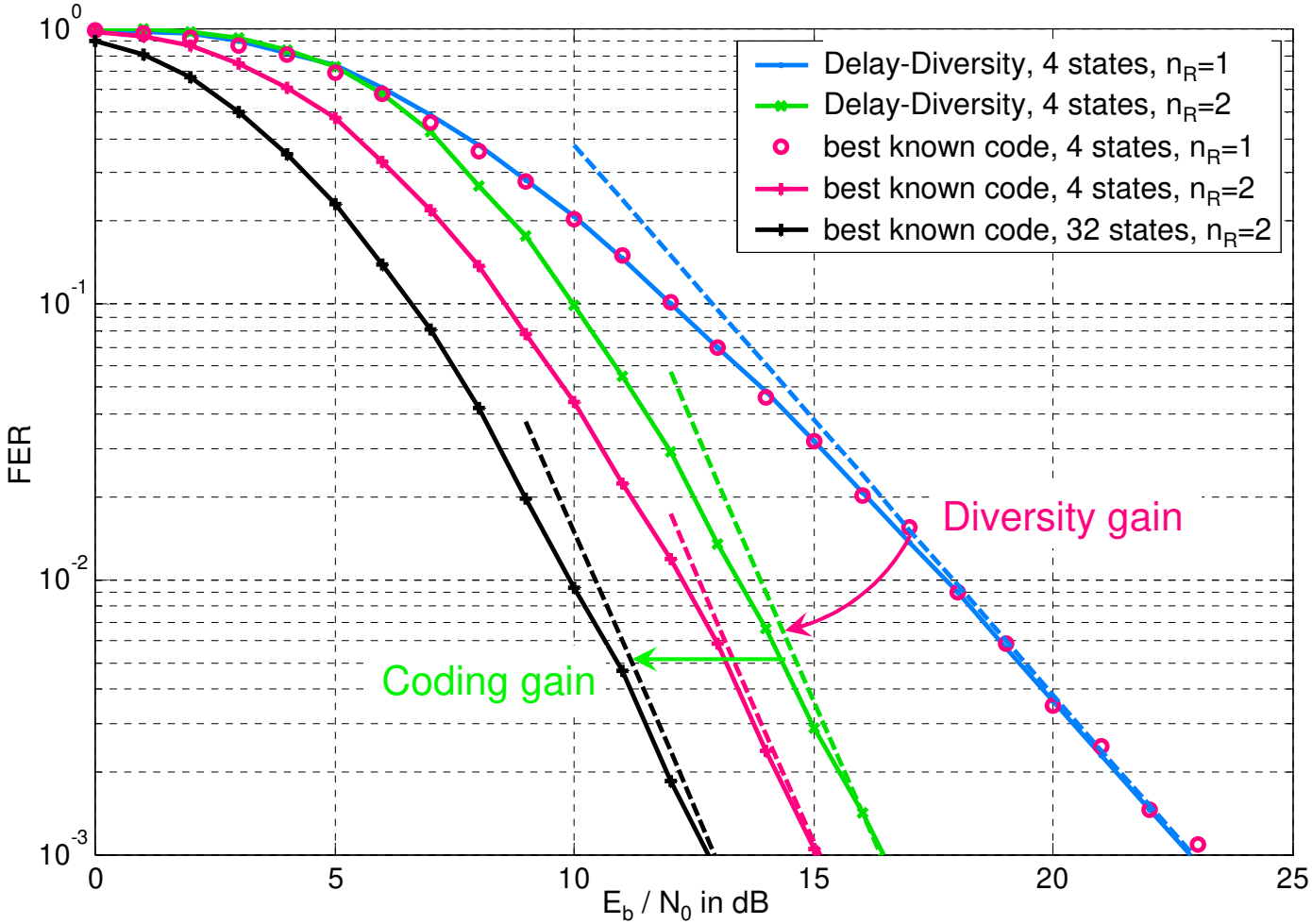
$$\xrightarrow{\text{Mapping}} \mathbf{C} = \begin{pmatrix} -1 & -j & j & 1 \\ 1 & -1 & -j & j \end{pmatrix}$$

Code Search for Space-Time Trellis Codes

- Different codes for different configurations
  - Number of transmit antennas
  - Order of modulation
  - Number of registers → number of states
- Codes for same configuration differ only in generator coefficients
- Systematic code search by calculating diversity gain and coding gain for all permutations of **G**
  - Look only for Space-Time Trellis Codes with maximum diversity
  - Choose the code with highest coding gain
  - Best (known) Space-Time Trellis Code for 2 transmit antennas, QPSK, 4 states
    found by Yan and Blum, Lehigh University

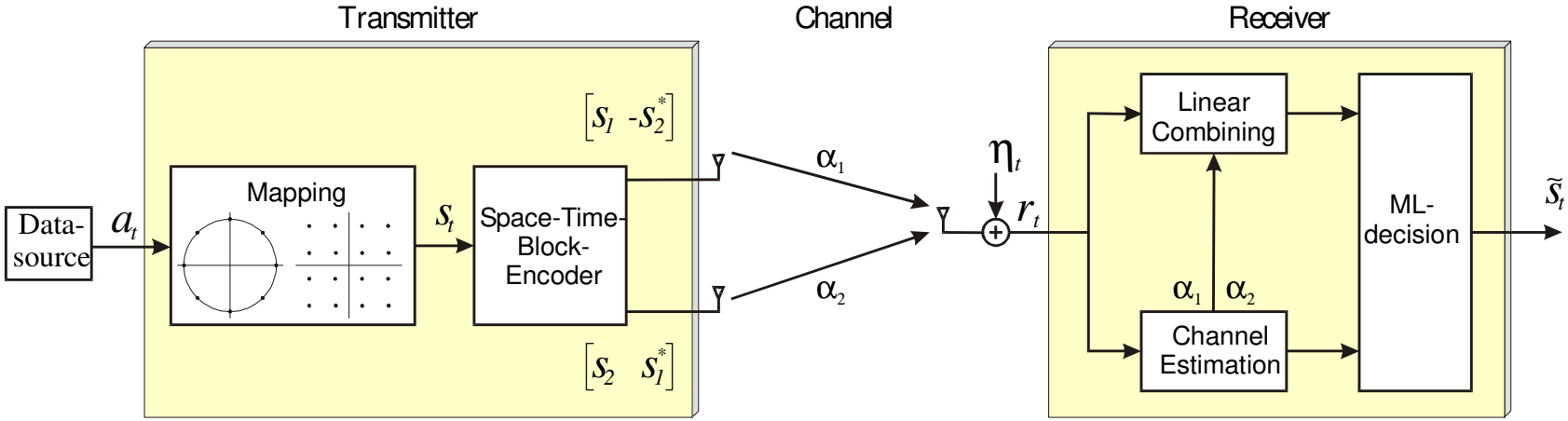
$$\mathbf{G}_{optimal} = \begin{pmatrix} 2 & 2 \\ 0 & 2 \\ 1 & 2 \\ 2 & 1 \end{pmatrix}$$

Simulation Results for STTC



- Diversity gain determines slope of FER curve
- Performance for STTC of same constellation differ only for more than one receive antenna
- Coding gain affects horizontal shift for codes of same diversity
- Increasing coding gain with number of states, but also much greater decoding effort

Space-Time Block Codes (STBC)



- Code word for two consecutive time steps $\mathbf{C} = \begin{pmatrix} s_1 & -s_2^* \\ s_2 & s_1^* \end{pmatrix}$
- Received signal vector of one block $\begin{pmatrix} r_1 \\ r_2 \end{pmatrix} = \begin{pmatrix} \alpha_1 s_1 + \alpha_2 s_2 \\ -\alpha_1 s_2^* + \alpha_2 s_1^* \end{pmatrix} + \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix}$

$$\xrightarrow{\text{conjugate } r_2} \begin{pmatrix} r_1 \\ r_2^* \end{pmatrix} = \begin{pmatrix} \alpha_1 s_1 + \alpha_2 s_2 \\ -\alpha_1^* s_2 + \alpha_2^* s_1 \end{pmatrix} + \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} = \begin{pmatrix} \alpha_1 & \alpha_2 \\ \alpha_2^* & -\alpha_1^* \end{pmatrix} \cdot \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} + \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix}$$

Decoding of STBC

- Linear combining

$$\begin{pmatrix}\tilde{r}_1\\ \tilde{r}_2\end{pmatrix}=\begin{pmatrix}\alpha_1 & \alpha_2\\ \alpha_2^* & -\alpha_1^*\end{pmatrix}^+\cdot\begin{pmatrix}r_1\\ r_2^*\end{pmatrix}=\underbrace{\begin{pmatrix}\alpha_1^* & \alpha_2\\ \alpha_2^* & -\alpha_1\end{pmatrix}\cdot\begin{pmatrix}\alpha_1 & \alpha_2\\ \alpha_2^* & -\alpha_1^*\end{pmatrix}}_{\left(|\alpha_1|^2+|\alpha_2|^2\right)\cdot\mathbf{I}_2}\cdot\begin{pmatrix}s_1\\ s_2\end{pmatrix}+\underbrace{\begin{pmatrix}\alpha_1^* & \alpha_2\\ \alpha_2^* & -\alpha_1\end{pmatrix}\cdot\begin{pmatrix}\eta_1\\ \eta_2\end{pmatrix}}_{\text{noise term is still white }\begin{pmatrix}\tilde{\eta}_1\\ \tilde{\eta}_2\end{pmatrix}}$$

  - Orthogonal code matrix in combination with linear combining yields to identity matrix
- Modified received signal vector

$$\begin{pmatrix}\tilde{r}_1\\ \tilde{r}_2\end{pmatrix}=\left(|\alpha_1|^2+|\alpha_2|^2\right)\cdot\begin{pmatrix}s_1\\ s_2\end{pmatrix}+\begin{pmatrix}\tilde{\eta}_1\\ \tilde{\eta}_2\end{pmatrix}$$

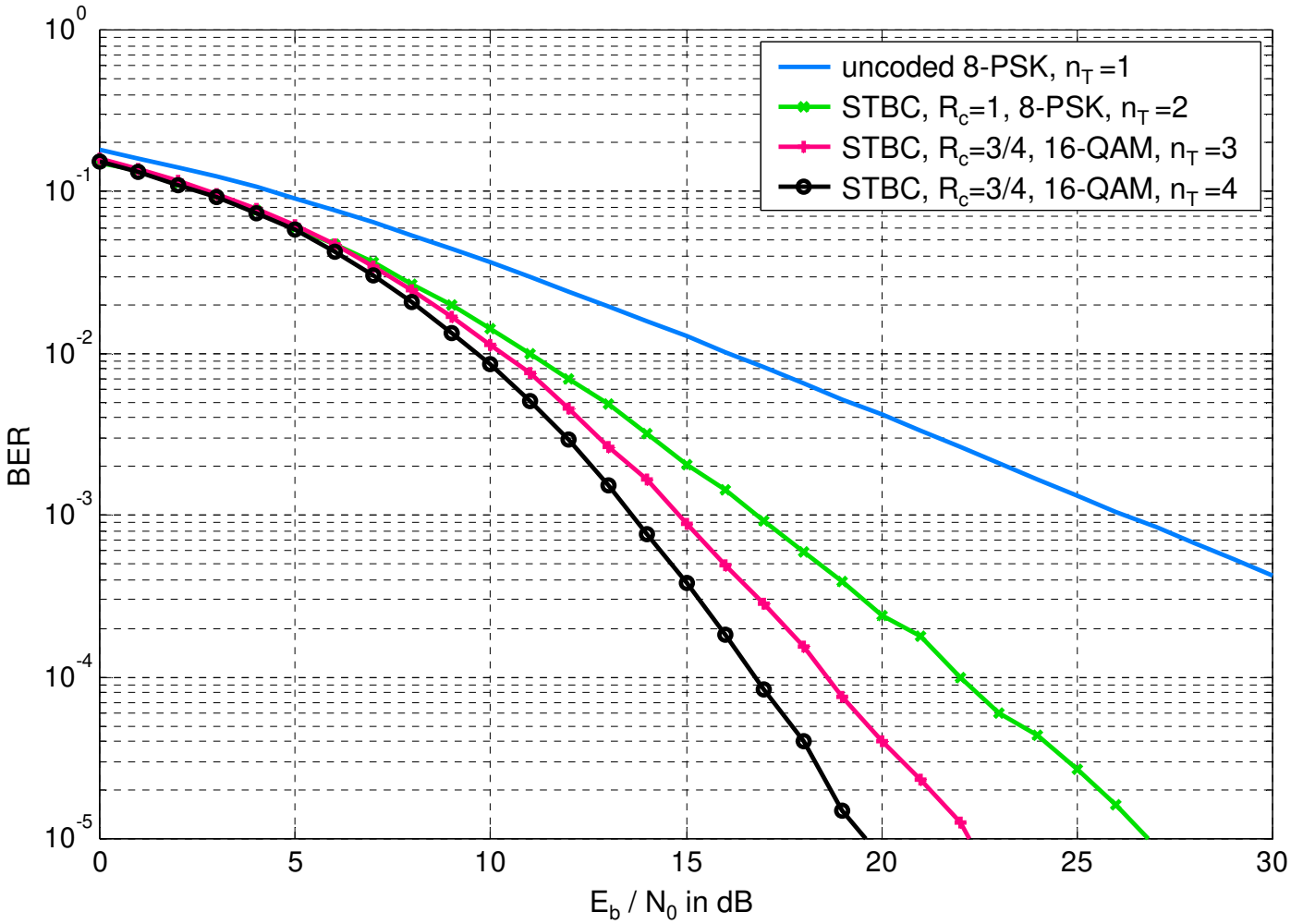
two independent signals to represent s_1 and s_2

→ Independent decoding of signals on basis of linear combining allows very easy receiver structure!

Limitations of STBC

- Code with rate 1 exists only for two transmit antennas
- Generalized orthogonal design for any arbitrary number of transmit antennas is possible with code rate ½
- Only two codes are known with code rate higher than ½ for more than two transmit antennas
- STBC achieve full diversity gain but no coding gain
  - Concatenation of STBC with trellis codes (already done in other projects)

Simulation Results



- Comparison of STBC for 3 Bit/s/Hz and one receive antenna
- Great advantage of about 10 dB in SNR for BER 10^{-3} for STBC with 2 transmit anteanns
- STBC achieve full diversity benefit
- Performance of MRC with same diversity
- Recall normalization of transmitted power

Summary

- Space-Time Codes are appropriate to improve the performance
  - Increase the spectral efficiency and thereby the data rate
- Space-Time Trellis Codes
  - Achieve full diversity gain and an additional coding gain
  - Decoding complexity increases with code rate, order of modulation, number of states and number of transmit antennas
- Space-Time Block Codes
  - Achieve full diversity gain but no additional coding gain
  - Only simple linear processing at the receiver is required for decoding
  - Only one STBC with code rate one exists
- Aspects of future investigations
  - Concatenation of ST-Codes with traditional codes
  - Realizing Space-Time Codes aspects in CDMA environments