Adaptive MIMO-OFDM for Future Mobile Radio Communications

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Abstract

This paper addresses the topic of bit and power loading in coded MIMO-OFDM systems. Due to the need of designing communication systems with high spectral efficiencies, the combination of multiple antennas at transmitter and receiver with OFDM represents a promising approach. Moreover, channel knowledge at the transmitter can be exploited to adapt the transmission to the channel. While loading strategies for the uncoded case as well as the information theoretical solution are already known, the optimum scheme is still unknown for coded systems. In this paper, we compare different loading approaches for coded systems with respect to their error rate performance.

1 Introduction

Since bandwidth is a limited and valuable resource, modern communication systems have to be designed to achieve very high spectral efficiencies. One candidate for an appropriate air interface of future mobile radio systems is the multi-carrier technique OFDM (Orthogonal Frequency Division Multiplexing) [1]. It has already proven its practicability by finding its way into various standards for broadcast as well as point-to-point communications [5,6,7]. One of the main benefits is the simple equalization of frequency selective channels. Additionally, OFDM allows an easy adaptation to the channel conditions due to its granularity in the frequency domain if channel knowledge is available at the transmitter.

A second approach leading to high spectral efficiencies is to use multiple antennas at the transmitter as well as the receiver. The high potential of multiple-input multiple-output (MIMO) systems stems from the fact that parallel data streams can be transmitted thereby multiplying the achievable data rate [12]. For frequency selective channels, a combination of OFDM and multiple antennas is obvious.

In this paper, we consider a coded MIMO-OFDM system and assume perfect channel knowledge at transmitter and receiver. This assumption allows the exploitation of the eigenmodes of the MIMO channel and the transmission of independent parallel data streams. Transmit power and signal constellation can be individually chosen for each stream with respect to an appropriate optimization criterion.

The paper is organized as follows: Section 2 describes the system model and Section 3 the investigated loading strategies. Simulation results comparing the different concepts are discussed in Section 4 and concluding remarks can be found in Section 5.

2 System Model

The structure of the MIMO-OFDM transmitter is depicted in Figure 1. The information bits are first encoded by an FEC encoder that is further specified in Section 4. The encoded stream is demultiplexed into $N = N_C N_T$ layers, where N_C denotes the number of OFDM subcarriers and N_T the number of transmit antennas. Since we assume perfect channel knowledge at transmitter and receiver, each layer is individually modulated (block 'M') according to an appropriate signal constellation. As OFDM ensures that each carrier is only affected by flat fading, the MIMO channel corresponding to carrier ν can be represented by an $N_R \times N_T$ matrix $\mathbf{H}_\nu$ with the singular value decomposition $\mathbf{H}_\nu = \mathbf{U}_\nu \mathbf{\Sigma}_\nu \mathbf{V}_\nu^H$. The unitary $N_T \times N_T$ matrices $\mathbf{V}_\nu$ are used for a linear pre-processing in order to exploit the eigenmodes of the MIMO channel, i.e. the transmitted vector becomes $\mathbf{x}_\nu = \mathbf{V}_\nu \mathbf{s}_\nu$ with $\mathbf{s}_\nu$ containing the data symbols.

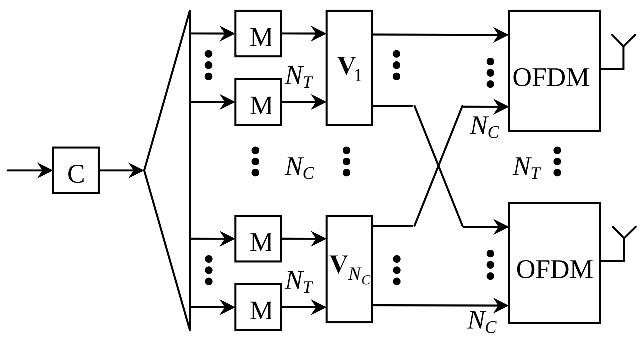

Figure 1: MIMO-OFDM transmitter

The N symbols are subsequently assigned to N_T streams each comprising N_C symbols forming one OFDM symbol per antenna.

At the receiver depicted in Figure 2, OFDM demodulation is performed at each of the N_R antennas. Next, the permutation is reversed so that N_C streams each containing the signals of N_R antennas are formed. The receive vector on the ν-th subcarrier is given by

$$\mathbf{y}_\nu = \mathbf{H}_\nu \mathbf{x}_\nu + \mathbf{n}_\nu . \tag{1}$$

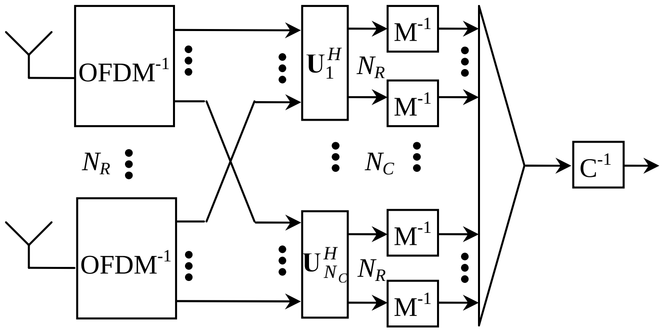

Figure 2: MIMO-OFDM receiver

After filtering with the unitary matrix $\mathbf{U}_\nu^H$ we obtain

$$\mathbf{r}_\nu = \mathbf{U}_\nu^H \mathbf{y}_\nu = \mathbf{\Sigma}_\nu \mathbf{s}_\nu + \tilde{\mathbf{n}}_\nu \tag{2}$$

Stacking all transmit and receive signals into large vectors $\mathbf{s}$ and $\mathbf{r}$ finally leads to the mathematical model

$$\mathbf{r} = \mathbf{\Sigma}\mathbf{s} + \tilde{\mathbf{n}} \tag{3}$$

Since $\mathbf{\Sigma}$ represents a diagonal matrix containing the singular values σ_μ of all submatrices $\mathbf{H}_\nu$ we obtain N parallel channels whose rates and transmit powers have to be appropriately chosen according to the strategies presented in the next section.

3 Loading Strategies

In this section we briefly review some existing bit and power loading schemes. The results for the extreme cases of ideal and no channel coding will turn out to be also meaningful for realistic coded transmission.

3.1 Minimizing the outage probability

It is well known that the mutual information between transmit and receive signal for a given channel matrix $I(\mathbf{x};\mathbf{y}\,|\,\mathbf{H})$ is maximized if the transmit powers P_μ are assigned according to the water-filling criterion

$$P_\mu = \max\left\{\theta - \frac{1}{\sigma_\mu^2}, \quad 0\right\}, \tag{4}$$

where the parameter θ must be chosen such that the total power constraint is fulfilled. Obviously, this power allocation minimizes the probability that the mutual information is smaller than the desired transmission rate R for a certain channel realization, i.e.

$$P_{\text{out}} = \Pr\{I(\mathbf{x};\mathbf{y}\,|\,\mathbf{H}) < R\}. \tag{5}$$

This outage probability corresponds to the frame error rate of an ideal code. As capacity achieving codebooks resemble independent transmit symbols, it is not necessary to use different codes with variable rates on the parallel subchannels. Instead, a single code can be applied across all frequencies and spatial eigenmodes and pure power loading is sufficient [2]. Note that this important observation does not only reduce the implementation complexity, but also improve the performance, because good random-like codes benefit from an increased codeword length. Hence, information theory gives a strong motivation for the transmitter structure depicted in Figure 1 using only one channel encoder. Moreover, from this point of view adaptive modulation is not required, so the symbol mapping could also be performed before demultiplexing the data streams.

3.2 Minimizing the uncoded BER

Turning to uncoded transmission, we are interested in minimizing the average bit error probability for a given channel realization

$$P_b = \frac{1}{R}\sum_{\mu=1}^{N} R_\mu P_{b,\mu} \tag{6}$$

by properly adjusting the rates R_μ and powers P_μ. For square QAM constellations with Gray-labelling, the error probability on the μ-th subchannel can be well approximated by

$$P_{b,\mu} \approx \frac{4}{R_\mu}\left(1 - \frac{1}{2^{R_\mu/2}}\right)\cdot \mathrm{Q}\left(\sqrt{\frac{3}{2^{R_\mu}-1}\sigma_\mu^2 P_\mu}\right) \tag{7}$$

with the Gaussian error integral

$$\mathrm{Q}(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-t^2/2}dt. \tag{8}$$

For a fixed rate allocation, the power optimization can be performed based on the Lagrange dual function of (6). However, since all feasible rate tuples must be checked, the computational complexity is extremely high for large number of subcarriers or antennas.

The problem at hand can be greatly simplified for sufficiently high signal to noise ratio (SNR). In that case, the bit error rates on all active subchannels will be approximately the same. Ignoring the term in front of the rapidly decreasing Q function, (7) can be solved for the rate

$$R_\mu \approx \log_2\left(1 + \sigma_\mu^2 \frac{P_\mu}{\Gamma}\right), \tag{9}$$

which is identical to the capacity of an AWGN channel with transmit power reduced by the constant factor Γ. Hence, the optimum rate and power distribution again follows from the water-filling criterion (4), where the SNR gap Γ ensures that both rate and power constraint can be fulfilled. Note that only discrete rates are possible for uncoded data, which is accounted for in the efficient algorithm presented in [3].

Slightly better results can be obtained if the transmit power is first minimized for the given data rate and some arbitrary target error rate without making use of any approximations, and then scaled to the desired value. For this purpose, we take the look-up table based bisection search from [9], which runs considerably faster than the greedy Hughes-Hartogs algorithm [8].

3.3 Minimizing the coded BER

For very powerful codes it may be conjectured that the results from Section 3.1 can be applied. However, little is known for suboptimal channel coding schemes.

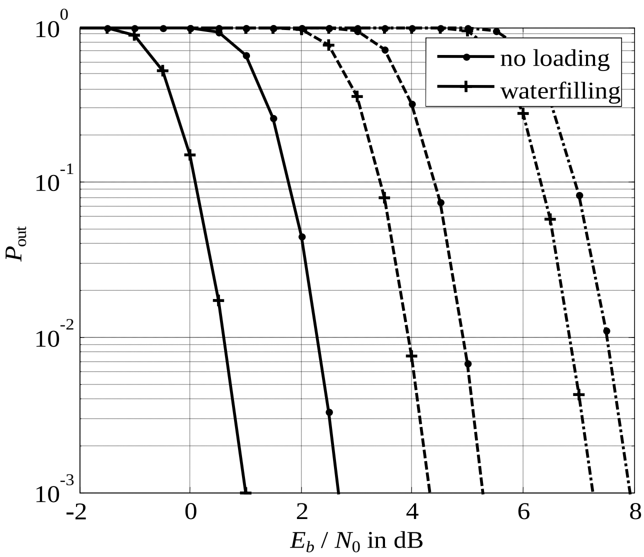

Figure 3: Outage probability with and without water-filling

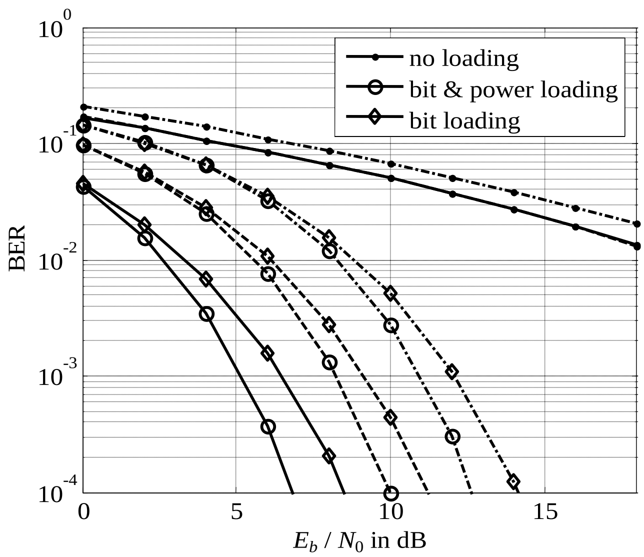

Figure 4: Performance of different loading schemes for uncoded transmission

In [10], a hard decision is assumed before the decoder. This transfers each of the parallel AWGN channels into a binary symmetric channel (BSC), whose error probability can be minimized using the methods described in the previous section. However, the reliability information about the code bits is lost due to the quantization, which leads to a severe performance degradation that may even overcompensate the gains due to the adaptive transmission.

An approximation for the coded BER with soft-input decoding is derived in [11]. Interestingly, the resulting rate allocation formula is identical to (9) with SNR gap inversely proportional to the minimum Hamming distance of the channel code. This leads to the assumption that algorithms originally developed for the uncoded case are also reasonable for coded transmission. So, all in all, both information theoretic and uncoded results appear to be suited for realistic codes. However, while rate adaptation is not required for ideal coding schemes, it is essential in the absence of channel coding. It is not intuitively clear, which approach turns out to be better.

4 Numerical Results

In order to evaluate the different loading strategies, we consider a MIMO-OFDM system with $N_C = 32$ carriers and $N_T = N_R = 4$ antennas at transmitter and receiver. The channel impulse responses consist of $L_H = 6$ uncorrelated complex Gaussian distributed taps with equal variance and the received SNR per bit is defined as

$$\frac{E_b}{N_0} = \frac{N_R P}{R}. \tag{10}$$

Figure 3 shows the outage probability of the channel. Here and in the following figures, solid, dashed, and dash-dotted lines correspond to data rates of $R = 4, 8, \text{and } 12$ bit/s/Hz, respectively. It can be observed that the gain due to water-filling diminishes with increasing rate.

In Figure 4, results for the uncoded case are depicted. Not surprisingly, non-adaptive transmission performs poorly, because no diversity is used and weak subchannels dominate the average error rate. Enormous improvements are possible with the bit and power loading algorithm described in Section 3.2, where the maximum constellation size was limited to 256 QAM. Most of the gain can already be achieved by combining the resulting bit allocation with an equal power distribution among active subchannels, so power loading seems to be less important.

The loading algorithms were then applied to coded transmission. Each codeword spans $L_F = 10$ consecutive MIMO-OFDM symbols (consisting of $L_F N_C N_T = 1280$ QAM symbols) during which the channel remains constant. Figure 5 depicts the BER for a half-rate non-recursive convolutional code with generator polynomials $(7,5)_8$ in octal representation. Looking at a spectral efficiency of $R = 4$ bit/s/Hz, we observe that pure bit loading shows the best performance and is slightly better than the combination of bit and power loading. Obviously, a power distribution enforcing equal bit error probabilities on all subchannels does not represent the optimum approach for coded systems. The gain of bit loading compared to the non-adaptive systems amounts to 5 dB at a bit error rate of 10^{-4}. The different slopes of the curves indicate that this code can not utilize the full diversity of the channel without bit loading. On the other hand, water-filling performs even worse than the non-adaptive system. The reason is that the code is highly punctured because bits assigned to channels with $P_\mu = 0$ are not transmitted. This is in contrast to the case of bit loading, where all code bits are transmitted. For growing spectral efficiency, the water filling approach becomes the best choice at low signal to noise ratios while pure bit loading performs best for medium and high SNR. For $R = 12$ bit/s/Hz, nearly all channels – even the worst ones – have to be used so that puncturing has only a minor influence. The bit loading

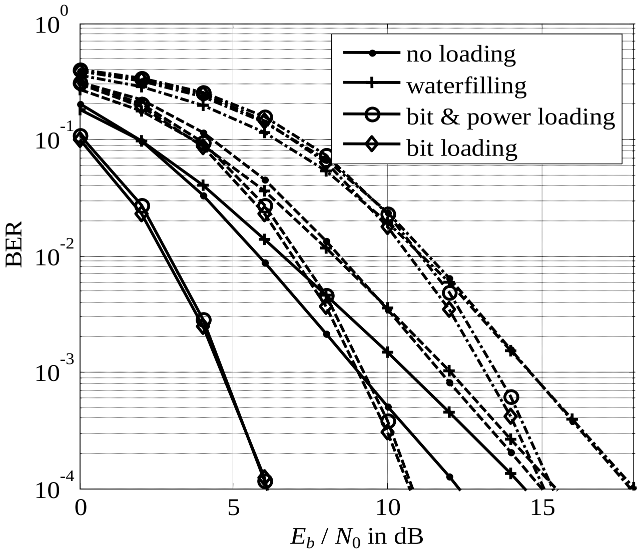

Figure 5: Performance of different loading schemes for a half-rate convolutional code

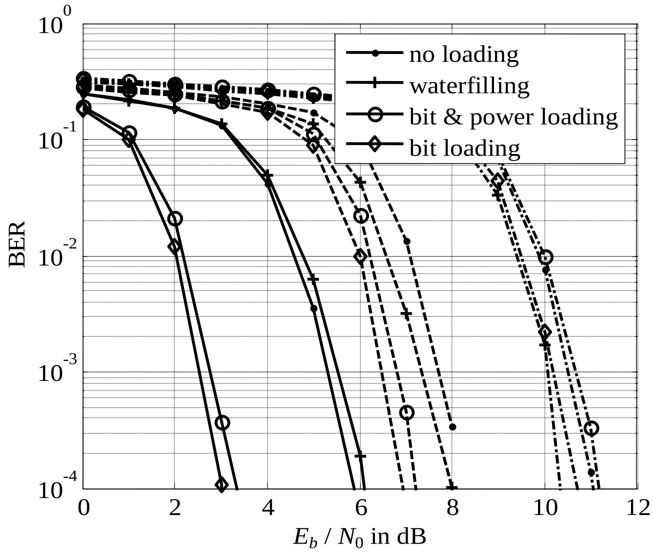

Figure 6: Performance of different loading schemes for a half-rate turbo code

scheme gains about 3 dB. A comparison with Figure 4 reveals that for this high data uncoded adaptive transmission is another 2.5 dB better, because due to the code rate $R_c = 1/2$ each QAM symbol contains on average 6 code bits while the maximum is set to 8 bits, which limits the degrees of freedom for the loading process.

Figure 6 shows the corresponding results for the turbo code consisting of two identical constituent encoders with generator polynomials $(1,17/13)_8$ and punctured to rate $R_c = 1/2$. For low and medium spectral efficiencies, bit loading still performs best closely followed by bit and power loading. The gains amount to 3 dB for $R = 4$ bit/s/Hz and 1.5 dB for $R = 8$ bit/s/Hz. For an efficiency of $R = 12$ bit/s/Hz, the relations change and water filling achieves the lowest error rate. Astonishingly, the combination of bit and power loading performs worst. The gains reduce to less than 1 dB. For this powerful turbo code and the high spectral efficiency, the information theoretical approach can be confirmed. In contrast to the convolutional code, the turbo code is able to exploit the full spatial and frequency diversity contained in the system.

5 Conclusions

It has been shown that loading strategies exploiting channel knowledge at the transmitter can improve the error rate performance significantly. For weak codes, pure bit loading seems to be an appropriate choice while the water filling approach performs best for strong codes and extremely high spectral efficiencies. However, it is still an open question how optimum bit and power loading has to be performed for a specific code.

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