EXACT OUTAGE PROBABILITY OF V-BLAST WITH ORDERED MMSE-SIC DETECTION

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ABSTRACT

A new method for determining the layer-wise SINR distribution as well as the total outage probability of V-BLAST with successive interference cancellation at the receiver is presented. In contrast to previous publications, we do not restrict to zero-forcing, but also consider minimum mean square error interference suppression. It is shown that optimizing the detection sequence is even more important in this case, and that ordered MMSE-SIC can achieve full receive diversity if the data rate per layer is not larger than one.

1. INTRODUCTION

Multiple antenna systems can be used to achieve very high spectral efficiencies [1]. The layered V-BLAST architecture is a practical way to realize unprecedented data rates [2]. A detailed performance analysis for simple linear as well as optimal maximum-likelihood receivers can be found in [3]. However, for the ordered successive interference cancellation (SIC) proposed in [2], analytical results are more difficult to obtain. In [4], it was shown that without sorting the diversity order of the k-th layer is given by $N_R - N_T + k$, where N_T and N_R denote the number of transmit and receive antennas. The importance of an optimized detection order for the information outage probability was highlighted in [5], where a uniform power and rate allocation among a subset of transmit antennas was conjectured to be optimal, but the required distribution of the layer-wise signal to noise ratio (SNR) with optimal ordering was only approximated by Monte-Carlo simulations. The exact expression for the case of two transmit antennas was first determined in [6] using the distribution of the angle between two complex Gaussian vectors, which was also employed to find loose bounds for $N_T > 2$ in [7]. An alternative approach based on the inverted complex Wishart distribution was recently presented in [8].

The above mentioned publications assume perfect interference suppression by a linear zero-forcing (ZF) filter. The method described in this paper is somewhat more intuitive than previous ones. Even more important, it can be extended to analyze the signal to interference and noise ratio (SINR) when using a minimum mean square error (MMSE) filter. This will be used to derive the exact outage probability of various SIC receiver structures.

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2. SYSTEM MODEL

Consider the equivalent baseband model of a single-user multiple antenna system with N_T transmit and $N_R \geq N_T$ receive antennas. The channels are uncorrelated and flat Rayleigh fading. Hence, the $N_R \times N_T$ channel matrix $\mathbf{H}$ consists of independent circularly symmetric complex Gaussian entries with zero mean and unit variance. The receive vector is given by

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \,, \tag{1}$$

where the vector $\mathbf{x} = [x_1, \ldots, x_{N_T}]^T$ with covariance matrix $\mathrm{E}\left\{\mathbf{x}\mathbf{x}^H\right\} = \mathbf{I}_{N_T}$ contains independent transmit symbols, and $\mathbf{n}$ represents circularly symmetric and white complex Gaussian noise with variance σ_n^2. Perfect channel state information is assumed at the receiver.

3. ZF-SIC WITHOUT ORDERING

For each layer, the interference caused by already detected layers is subtracted from the receive signal, and the remaining interference is suppressed by a linear filter. The required filter matrices follow from the QL decomposition of the channel matrix $\mathbf{H} = \mathbf{QL}$, where the $N_R \times N_T$ matrix $\mathbf{Q}$ has orthogonal columns with unit norm and $\mathbf{L}$ is lower triangular with real-valued and nonnegative diagonal elements [9]. Assuming correct decisions in all previous detection steps $1, \ldots, k-1$, the estimate $\hat{x}_k$ of x_k can be obtained from the filtered receive signal $\mathbf{z} = \mathbf{Q}^H\mathbf{y}$ via

$$\tilde{x}_k = z_k - \sum_{m=1}^{k-1} l_{km}\,\hat{x}_m = l_{kk}\,x_k + \tilde{n}_k \,, \tag{2}$$

where the noise $\tilde{n}_k$ is still white with variance σ_n^2. Thus, the SNR of the k-th layer is given by

$$\mathrm{SNR}_k = l_{kk}^2/\sigma_n^2 \,. \tag{3}$$

3.1. SNR Distribution

From the rotational invariance of the multivariate Gaussian distribution of $\mathbf{h}_k$ it can be deduced that the elements of $\mathbf{L}$ are independent and l_{mk} is complex Gaussian for $m > k$. Furthermore, as the squared column norm $\|\mathbf{h}_k\|^2$ has a χ^2 distribution with $2N_R$ degrees of freedom [10], the squared diagonal element l_{kk}^2 is also χ^2-distributed, but with only $2(N_R - N_T + k)$ degrees of freedom.

Hence, we get the pdf's

$$p_{l_{kk}^2}(t) = \frac{t^{N_R-N_T+k-1}e^{-t}}{\Gamma(N_R-N_T+k)}\ , \tag{4}$$

$$p_{|l_{mk}|^2}(t) = e^{-t}\ , \quad m>k \tag{5}$$

that are zero for $t<0$. Using (3), the cdf of the SNR on the k-th layer can be obtained by integrating over (4)

$$P_{\mathrm{SNR}_k}(\vartheta) = P_{l_{kk}^2}(\sigma_n^2\vartheta) = \tilde{\gamma}(N_R-N_T+k, \sigma_n^2\vartheta)\ , \tag{6}$$

where $\tilde{\gamma}(n,x) = \gamma(n,x)/\Gamma(n)$ is a normalized version of the incomplete gamma function [11].

3.2. Outage Probability

An outage occurs if the channel capacity of at least one layer is too small to support the chosen data rate R. For Gaussian transmit symbols, error-free transmission is only possible if

$$C_k = \log_2(1+\mathrm{SNR}_k) > R \quad \Rightarrow \quad \mathrm{SNR}_k > 2^R-1 \tag{7}$$

holds for all k. Since the layers are statistically independent, the total outage probability becomes

$$P_{\mathrm{out}} = 1 - \prod_{k=1}^{N_T} \tilde{\Gamma}(N_R-N_T+k, \sigma_n^2[2^R-1]) \tag{8}$$

with the regularized complementary incomplete gamma function $\tilde{\Gamma}(n,x) = \Gamma(n,x)/\Gamma(n) = 1-\tilde{\gamma}(n,x)$.

4. ZF-SIC WITH OPTIMIZED ORDERING

The order of detection can be optimized by exchanging elements of the transmit vector $\mathbf{x}$ and the corresponding columns of the channel matrix $\mathbf{H}$ before the QL decomposition [9]. Let us define $\check{\mathbf{H}} = \mathbf{H}\mathbf{\Pi} = \check{\mathbf{Q}}\check{\mathbf{L}}$ for some permutation matrix $\mathbf{\Pi}$ and restrict to the case of $N_T = 2$ transmit antennas in the following, for simplicity. In order to maximize the minimum SNR, we need to choose the detection sequence such that

$$\check{l}_{22}^2 = \min\left\{\|\mathbf{h}_1\|^2, |\mathbf{h}_2\|^2\right\} \quad \Rightarrow \quad \check{l}_{22}^2 \leq \check{l}_{11}^2 + |\check{l}_{21}|^2\ , \tag{9}$$

i.e. the layer with the weaker channel is detected last and does not have to suppress interference by a linear filter.

4.1. SNR Distribution

With (9), the cdf of the squared second diagonal element can easily be found using order statistics [10]

$$\begin{aligned} P_{\check{l}_{22}^2}(\xi) =& 1 - \left[\Pr\left\{\|\mathbf{h}_k\|^2 \geq \xi\right\}\right]^2 = 1 - \left[\tilde{\Gamma}(N_R,\xi)\right]^2 \\ =& \tilde{\gamma}(N_R,\xi)\cdot[2-\tilde{\gamma}(N_R,\xi)]\ . \end{aligned} \tag{10}$$

Then, similar to (6), the distribution of SNR_2 for the optimal permutation is given by $P_{\mathrm{SNR}_2}(\vartheta) = P_{\check{l}_{22}^2}(\sigma_n^2\vartheta)$. As already observed in [6], it is approximately doubled due to sorting.

Unfortunately, the diagonal elements of $\check{\mathbf{L}}$ are not independent anymore. However, conditioned on $\check{l}_{22}^2 = \xi$, the natural ordering is optimal whenever $l_{11}^2 + |l_{21}|^2 \geq \xi$ is fulfilled according to (9). As both detection orders are equiprobable, this leads to the conditional cdf

$$P_{\mathrm{SNR}_1}\left(\vartheta \,\middle|\, \check{l}_{22}^2 = \xi\right) = P_{l_{11}^2}\left(\sigma_n^2\vartheta \,\middle|\, l_{11}^2 + |l_{21}|^2 \geq \xi\right)\ . \tag{11}$$

Exploiting the statistical independence of l_{11} and l_{21}, we first calculate the joint probability

$$\begin{aligned} &\Pr\left\{l_{11}^2 < \sigma_n^2\vartheta\,,\, l_{11}^2 + |l_{21}|^2 \geq \xi\right\} \\ &= \int_0^{\sigma_n^2\vartheta}\int_{(\xi-t)^+}^{\infty} p_{l_{11}^2}(t)\cdot p_{|l_{21}|^2}(u)\,du\,dt \qquad (12) \\ &= \begin{cases} [\sigma_n^2\vartheta]^{N_R-1}e^{-\xi}/\Gamma(N_R) & ,\ \sigma_n^2\vartheta \leq \xi \\ \tilde{\gamma}(N_R-1,\sigma_n^2\vartheta) - \tilde{\gamma}(N_R,\xi) & ,\ \sigma_n^2\vartheta > \xi \end{cases} \qquad (13) \end{aligned}$$

from the pdf's (4) and (5), where the notation $(x)^+ = \max\{x,0\}$ was introduced. As expected, (13) corresponds to (6) for $\xi = 0$, while for $\sigma_n^2\vartheta \to \infty$ we get the complementary cdf of $\|\mathbf{h}_k\|^2$

$$\Pr\left\{l_{11}^2 + |l_{21}|^2 \geq \xi\right\} = 1 - \tilde{\gamma}(N_R,\xi) = \tilde{\Gamma}(N_R,\xi)\ . \tag{14}$$

Hence, (11) is given by

$$\begin{aligned} P_{\mathrm{SNR}_1}\left(\vartheta \,\middle|\, \check{l}_{22}^2 = \xi\right) &= \frac{\Pr\left\{l_{11}^2 < \sigma_n^2\vartheta\,,\, l_{11}^2 + |l_{21}|^2 \geq \xi\right\}}{\Pr\left\{l_{11}^2 + |l_{21}|^2 \geq \xi\right\}} \\ &= \begin{cases} [\sigma_n^2\vartheta]^{N_R-1}e^{-\xi}/\Gamma(N_R,\xi) & ,\ \sigma_n^2\vartheta \leq \xi \\ 1 - \tilde{\Gamma}(N_R-1,\sigma_n^2\vartheta)/\tilde{\Gamma}(N_R,\xi) & ,\ \sigma_n^2\vartheta > \xi\ . \end{cases} \end{aligned} \tag{15}$$

The unconditional cdf of SNR_1 can be computed by averaging (15) over $\check{l}_{22}^2$. The required pdf of $\check{l}_{22}^2$

$$p_{\check{l}_{22}^2}(\xi) = \frac{2\xi^{N_R-1}e^{-\xi}}{\Gamma(N_R)}\tilde{\Gamma}(N_R,\xi) \tag{16}$$

is obtained by taking the derivative of (10). With this, we finally arrive at

$$\begin{aligned} P_{\mathrm{SNR}_1}(\vartheta) &= \int_0^{\infty} P_{\mathrm{SNR}_1}\left(\vartheta \,\middle|\, \check{l}_{22}^2 = \xi\right)\cdot p_{\check{l}_{22}^2}(\xi)\,d\xi \qquad (17) \\ &= \frac{[\sigma_n^2\vartheta]^{N_R-1}\tilde{\Gamma}(N_R,2\sigma_n^2\vartheta)}{2^{N_R-1}\Gamma(N_R)} + 1 - \left[\tilde{\Gamma}(N_R,\sigma_n^2\vartheta)\right]^2 \\ &\quad - 2\tilde{\Gamma}(N_R-1,\sigma_n^2\vartheta)\cdot\tilde{\gamma}(N_R,\sigma_n^2\vartheta)\ . \qquad (18) \end{aligned}$$

Note that the integral in (17) must be split up into two parts, and the first term in (18) belongs to the case $\sigma_n^2\vartheta \leq \xi$ in (15).

4.2. Outage Probability

In contrast to Section 3.2, the joint SNR distribution can not be factorized anymore. However, letting $\vartheta = 2^R-1$ and keeping in mind that $\mathrm{SNR}_2 \geq \vartheta$ is equivalent to $\check{l}_{22}^2 \geq \sigma_n^2\vartheta$, the exact outage probability of ordered ZF-SIC

$$\begin{aligned} P_{\mathrm{out}} &= P_{\mathrm{SNR}_2}(\vartheta) + \Pr\left\{\mathrm{SNR}_1 < \vartheta\,,\, \mathrm{SNR}_2 \geq \vartheta\right\} \qquad (19) \\ &= 1 - \left[\tilde{\Gamma}(N_R,\sigma_n^2\vartheta)\right]^2 + \frac{[\sigma_n^2\vartheta]^{N_R-1}\tilde{\Gamma}(N_R,2\sigma_n^2\vartheta)}{2^{N_R-1}\Gamma(N_R)} \qquad (20) \end{aligned}$$

directly follows from (10) and (18).

5. MMSE-SIC WITHOUT ORDERING

In [12], it was demonstrated that SIC with MMSE filtering corresponds to the ZF-SIC described in Section 3 if the matrices $\mathbf{Q}$ and $\mathbf{L}$ are replaced by $\mathbf{Q}_1$ and $\underline{\mathbf{L}}$ obtained from the QL decomposition of the extended channel matrix

$$\underline{\mathbf{H}} = \begin{pmatrix} \mathbf{H} \\ \sigma_n \mathbf{I}_{N_T} \end{pmatrix} = \begin{pmatrix} \mathbf{Q}_1 \\ \mathbf{Q}_2 \end{pmatrix} \underline{\mathbf{L}} \,. \tag{21}$$

We will focus on SINR_1 in the following, because the results for the second layer remain unaffected as no interference needs to be suppressed. Using the relations $\|\underline{\mathbf{h}}_k\|^2 = \|\mathbf{h}_k\|^2 + \sigma_n^2$ and $|\underline{\mathbf{h}}_2^H \underline{\mathbf{h}}_1|^2 = l_{22}^2 |l_{21}|^2$, it can be shown that

$$\mathrm{SINR}_1 = \frac{\underline{l}_{11}^2}{\sigma_n^2} - 1 = \frac{l_{11}^2}{\sigma_n^2} + \frac{|l_{21}|^2}{l_{22}^2 + \sigma_n^2} \,. \tag{22}$$

5.1. SINR Distribution

Similar to (12), the SINR distribution of the first layer conditioned on $l_{22}^2 = \xi$ can be calculated by integrating over the joint pdf of l_{11}^2 and $|l_{21}|^2$

$$\begin{aligned} P_{\mathrm{SINR}_1}\left(\vartheta \,\middle|\, l_{22}^2 = \xi\right) &= \Pr\left\{\frac{l_{11}^2}{\sigma_n^2} + \frac{|l_{21}|^2}{\xi + \sigma_n^2} < \vartheta\right\} \\ &= \int_0^{\sigma_n^2 \vartheta} \int_0^{[\xi+\sigma_n^2]\vartheta - \frac{\xi+\sigma_n^2}{\sigma_n^2} t} p_{l_{11}^2}(t) \cdot p_{|l_{21}|^2}(u)\, du\, dt \qquad (23) \\ &= \tilde{\gamma}(N_R - 1, \sigma_n^2 \vartheta) - \frac{e^{-[\xi+\sigma_n^2]\vartheta} \tilde{\gamma}(N_R - 1, -\vartheta\xi)}{[-\xi/\sigma_n^2]^{N_R - 1}} \,. \qquad (24) \end{aligned}$$

For $\xi/\sigma_n^2 \to \infty$, the second term vanishes and we obtain the cdf of SNR_1 with ZF filtering. On the other hand, (24) tends to $\tilde{\gamma}(N_R, \sigma_n^2 \vartheta)$ for $\xi/\sigma_n^2 \to 0$, which is identical to the SNR distribution after maximum ratio combining. Hence, the MMSE filter benefits from small SNR's on the second layer. The unconditional cdf can then be determined from

$$\begin{aligned} P_{\mathrm{SINR}_1}(\vartheta) &= \int_0^\infty P_{\mathrm{SINR}_1}(\vartheta \,|\, l_{22}^2 = \xi) \cdot p_{l_{22}^2}(\xi)\, d\xi \qquad (25) \\ &= \tilde{\gamma}(N_R - 1, \sigma_n^2 \vartheta) - \frac{[\sigma_n^2 \vartheta]^{N_R - 1} e^{-\sigma_n^2 \vartheta}}{\Gamma(N_R) \cdot [\vartheta + 1]} \,. \qquad (26) \end{aligned}$$

5.2. Outage Probability

The outage probability corresponds to (19) with SNR_1 substituted by SINR_1. The joint probability therein is equal to (25) if the lower limit of the integral is replaced by $\sigma_n^2 \vartheta$, which yields after some manipulations

$$\begin{aligned} P_{\mathrm{out}} =& 1 - \tilde{\Gamma}(N_R - 1, \sigma_n^2 \vartheta) \cdot \tilde{\Gamma}(N_R, \sigma_n^2 \vartheta) \\ &- \frac{[\sigma_n^2 \vartheta]^{N_R - 1} e^{-\sigma_n^2 \vartheta}}{\Gamma(N_R) \cdot [\vartheta + 1]} \Bigg[\tilde{\Gamma}(N_R - 1, \sigma_n^2 \vartheta) \\ &+ \frac{e^{-[\vartheta+1]\sigma_n^2 \vartheta}}{[-\vartheta]^{N_R - 1}} \tilde{\gamma}(N_R - 1, -\sigma_n^2 \vartheta^2) \Bigg] \,. \end{aligned} \tag{27}$$

6. MMSE-SIC WITH OPTIMIZED ORDERING

We now finally turn to the case of ordered MMSE-SIC. The detection is based on the QL decomposition of $\check{\underline{\mathbf{H}}} = \underline{\mathbf{H}}\mathbf{\Pi}$, and the ordering criterion (9) is still optimal, so we can simply combine the approaches from the previous sections.

6.1. SINR Distribution

Along the lines of (12) and (23), we have to calculate the probability $\Pr\left\{\frac{l_{11}^2}{\sigma_n^2} + \frac{|l_{21}|^2}{\xi + \sigma_n^2} < \vartheta \,,\, l_{11}^2 + |l_{21}|^2 \geq \xi\right\}$ from the joint pdf of l_{11}^2 and $|l_{21}|^2$, and then divide it by $\tilde{\Gamma}(N_R, \xi)$ as in (15) to obtain the conditional cdf $P_{\mathrm{SINR}_1}\left(\vartheta \,\middle|\, \check{l}_{22}^2 = \xi\right)$. Then, taking the expectation over $\check{l}_{22}^2$ and performing some simplifications finally leads to

$$\begin{aligned} P_{\mathrm{SINR}_1}(\vartheta) =& 1 - \left[\tilde{\Gamma}(N_R, \sigma_n^2 \vartheta)\right]^2 \\ &- 2\tilde{\Gamma}(N_R - 1, \sigma_n^2 \vartheta) \cdot \tilde{\gamma}(N_R, \sigma_n^2 \vartheta) \\ &+ \frac{(-\sigma_n^2)^{N_R - 1}}{\Gamma(N_R)} \Bigg[\frac{(-\sigma_n^2 \vartheta^2)^{N_R - 1}}{\Gamma(N_R)} e^{-2\sigma_n^2 \vartheta} \\ &- \frac{2(-\vartheta)^{N_R - 1} e^{-\sigma_n^2 \vartheta}}{\vartheta + 1} \tilde{\gamma}(N_R - 1, \sigma_n^2 \vartheta) \\ &+ \frac{(1 - \vartheta)^{N_R} e^{\frac{2\sigma_n^2 \vartheta}{\vartheta - 1}}}{2^{N_R - 1}(\vartheta + 1)} \Upsilon\left(N_R - 1, \frac{2\sigma_n^2 \vartheta^2}{\vartheta - 1}\right) \Bigg] \end{aligned} \tag{28}$$

$$\text{with } \Upsilon(n, x) = \begin{cases} \tilde{\gamma}(n, x) & , \ x \leq 0 \\ -\tilde{\Gamma}(n, x) & , \ x > 0 \,. \end{cases} \tag{29}$$

6.2. Outage Probability

As for the case of ordered ZF-SIC, the expression for the outage probability turns out to be an intermediate result during the calculation of (28). It is given by

$$\begin{aligned} P_{\mathrm{out}} =& 1 - \left[\tilde{\Gamma}(N_R, \sigma_n^2 \vartheta)\right]^2 \\ &+ \frac{(-\sigma_n^2)^{N_R - 1}}{\Gamma(N_R)} \Bigg[\frac{(-\sigma_n^2 \vartheta^2)^{N_R - 1}}{\Gamma(N_R)} e^{-2\sigma_n^2 \vartheta} \\ &- \frac{2e^{-[\vartheta+2]\sigma_n^2 \vartheta}}{\vartheta + 1} \tilde{\gamma}(N_R - 1, -\sigma_n^2 \vartheta^2) \\ &+ \frac{(1 - \vartheta)^{N_R} e^{\frac{2\sigma_n^2 \vartheta}{\vartheta - 1}}}{2^{N_R - 1}(\vartheta + 1)} \Upsilon\left(N_R - 1, \frac{2\sigma_n^2 \vartheta^2}{\vartheta - 1}\right) \Bigg] \,. \end{aligned} \tag{30}$$

7. NUMERICAL RESULTS

In this section, we present numerical results for a system with two transmit and receive antennas. Fig. 1 depicts the SINR distributions per layer for fixed noise variance σ_n^2 and varying rate $\vartheta = 2^R - 1$. As already noted before, optimal sorting approximately doubles the cdf of the second layer, while the curve of the first layer is shifted to the right by $3\,\mathrm{dB}$ for ZF-SIC. The impact of MMSE filtering is most pronounced for strong noise. Using the optimum detection order, a cliff at $R \approx 1$ can be seen, and the cdf of the first layer rapidly converges to that of the second one with the inverted optimal ordering $\check{l}_{22}^2 = \max\{\|\mathbf{h}_1\|^2, \|\mathbf{h}_2\|^2\}$. Without sorting, a constant gap remains. From Fig. 2, we find that the performance of ordered MMSE-SIC is strictly dominated by the second layer as long as $R \leq 1$, thus it achieves full receive diversity. However, for larger data rates the cdf of SINR_1 flattens, and the diversity degree is equal to that of the other receiver variants. Interestingly, this behavior can also be observed for uncoded bit error rates [12]. Finally, Fig. 3 compares the outage probabilities

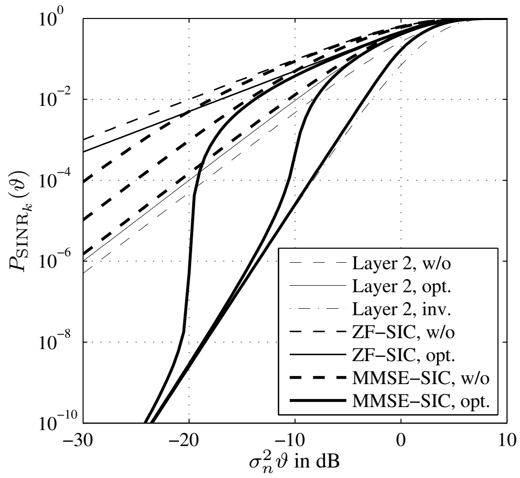

Fig. 1. SINR distributions for $\sigma_n^2 \in \{0.01, 0.1, 1\}$ (left to right).

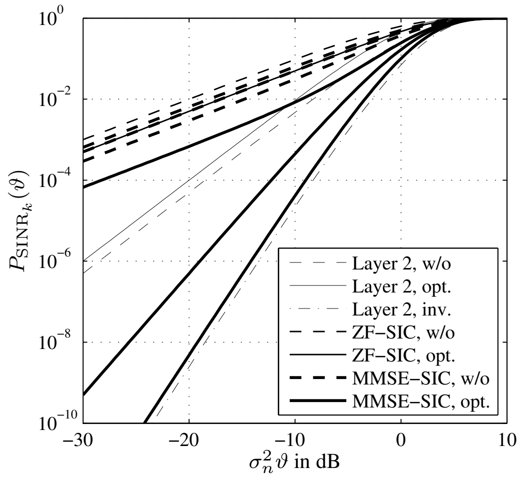

Fig. 2. SINR distributions for $R \in \{0.5, 1, 1.5\}$ (right to left).

to the corresponding union bounds. Although the approximation is very tight for sufficiently small P_{out}, it is usually even easier to compute the exact solution than the sum of the layer-wise SINR distributions.

8. CONCLUSION

A unified approach to the SINR and outage analysis of V-BLAST with different SIC detection schemes has been presented. We restricted to the case of two transmit antennas, for simplicity, because the integrals become quite involved for $N_T > 2$. It was demonstrated that the conditional cdf of the SINR on the first layer can be calculated from the QL decomposition of the channel matrix for all considered receiver structures. The unconditional cdf was then found by averaging over the second layer. With this, it could be shown that ordered MMSE-SIC achieves full receive diversity if the data rate per layer is not larger than one. As a byproduct, we also obtained the joint SINR distribution, which enabled us to compute the exact outage probabilities.

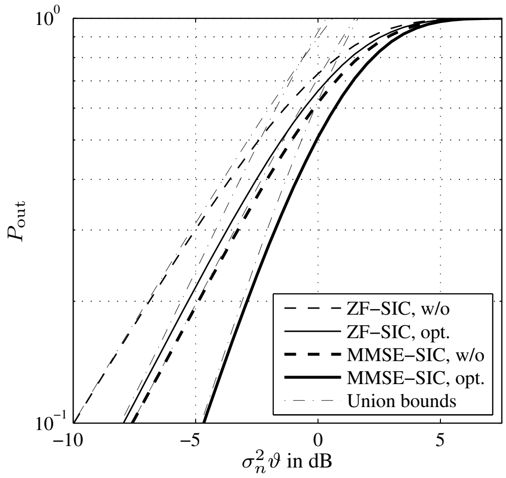

Fig. 3. Exact outage probabilities and union bounds for $R = 1$.

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