DOPPLER-COMPENSATION FOR OFDM-TRANSMISSION BY SECTORIZED ANTENNA RECEPTION

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Abstract Rapidly time-varying channels are a major obstacle for successful data-transmission via OFDM. The resulting loss of orthogonality among neighboring subcarriers leads to intercarrier-interference, which affects channel estimation. This in turn impedes the subsequent data detection. Literature contains numerous approaches to cope with this problem working either in time- or frequency-domain. Our concern in this paper is a novel time-domain method which relies on the use of multiple directional receive antennas. Each of these antennas experiences only a fraction of the total Doppler spread of a comparable omnidirectional antenna. This not only eases channel estimation but also allows for a diversity gain due to maximum-ratio-combining. We will demonstrate that our scheme copes well with large maximum Doppler frequencies.

1. Introduction

Transmitting data over frequency-selective channels is easily accomplished by the use of OFDM. Applying a suitable cyclic prefix not only avoids intersymbol interference between successive OFDM symbols but also performs the transformation of the linear convolution with the channel's impulse response into a circular convolution. In frequency-domain the subcarriers will then be received via flat-fading channels. However, rapidly time-varying channels with large Doppler spread will introduce intercarrier interference (ICI) in frequency domain since subcarriers loose their orthogonality.
A popular approach in literature to deal with rapid channel fluctuations is the assumption of a linear model. Besides linear channel variation the authors of [1] additionally assume a block diagonal structure for the channel matrix neglecting off-diagonal elements, whose inversion demands less computional complexity but fails to capture the effect of ICI sufficiently for larger Doppler

and delay spreads. Noticing that actually the FFT introduces ICI the authors of [2] propose to estimate the time-variant channel impulse from pilot symbols in time-domain. Depending on the channel parameters this approach demands a large number of training symbols and becomes infeasible with larger delay spread. A pilot-based method was proposed in [3] based on an initial Least-Square estimate of the mean channel impulse response, followed by a reconstruction of the channel matrix by assuming again a linearly time-variant channel model. This scheme requires a rather high amount of computational cost, i.e., setting up and inverting the channel matrix, and will degrade as soon as the Doppler spread leads to channel variations which are no longer linear.
Our method relies on avoiding large Doppler spread in the first place by applying directional antennas which divide the horizontal reception space into sectors. This allows for a separation of incoming paths according to their angle of incidence and, thereby, their position in the Doppler spectrum. A suitable arrangement of the sector limits splits the Doppler spectrum into evenly spaced subspectra with reduced Doppler spread [4, 5]. Each sector/antenna (these terms will be used interchangeably in the following) is associated with a certain Doppler range corresponding with its comprised range of angles of incidence. We present a coherent receiver which exploits this structure and demonstrate that in fact channel estimation is alleviated. This eventually allows for the application of higher-level modulation at maximum Doppler frequencies which could not be handled in the case of traditional omnidirectional reception.

2. System Model

Fig. 1 depicts the assumed OFDM-transmitter in the equivalent complex baseband. Information bits $b(\xi)$ (IID) are convolutionally encoded (CC) and

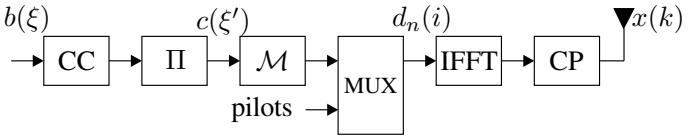

Figure 1. OFDM transmitter

randomly bit-interleaved (Π). The coded bits $c(\xi')$ are then mapped by the mapping function $\mathcal{M}$ to an M-ary QAM/PSK signal constellation. The data symbols are multiplexed (MUX) with pilot symbols which enable channel estimation at the receiver side. We denote the symbol on the n-th subcarrier for the i-th OFDM symbol by $d_n(i)$. The OFDM time-domain signal is computed by an N-point IFFT followed by prepending the cyclic prefix (CP) consisting of N_g symbols. The transmitted signal thus reads

$$x(k) = \frac{1}{\sqrt{N}} \sum_{i=-\infty}^{\infty} \sum_{\nu=0}^{N-1} d_\nu(i) \cdot \mathrm{e}^{\mathrm{j}2\pi\nu(k-i(N+N_\mathrm{g}))/N} \,. \tag{1}$$

2.1 Sectorized Receive Antenna

The impulse response according to the widely accepted WSSUS channel model [6] reads

$$h_\ell(k) = \frac{1}{\sqrt{N_\mathrm{e}}} \sum_{\mu=0}^{N_\mathrm{e}-1} a(\mu) \mathrm{e}^{\mathrm{j}2\pi f(\mu) T k} \delta(\ell - \ell(\mu)) \tag{2}$$

with delay index $\ell = 0, \ldots, L-1$, time index k, sampling period t, and the length of the channel impulse response L. The N_e path components are characterized by their path amplitude $a(\mu)$, Doppler frequency $f(\mu)$ and delay $\ell(\mu)$. Jakes' Doppler spectrum is generated by choosing $f(\mu) = f_{\mathrm{D,max}} \cos(\theta(\mu))$ with maximum Doppler frequency $f_{\mathrm{D,max}}$ and uniformly distributed phases $\theta(\mu)$ in the interval $[0, 2\pi]$. As an example Fig. 2 illustrates a sectorized antenna and its effect on Jakes' Doppler spectrum

$$P(f) = \left(\pi \sqrt{f_{\mathrm{D,max}}^2 - f^2} \right)^{-1}, \quad -f_{\mathrm{D,max}} < f < f_{\mathrm{D,max}} \, . \tag{3}$$

Fig. 2a shows its division into $S = 4$ sectors. The angles are chosen such

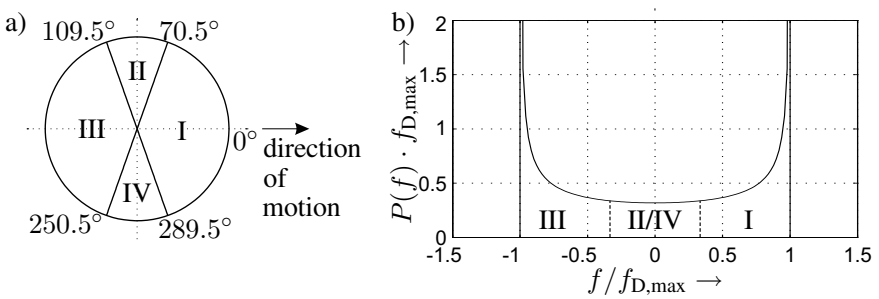

Figure 2. Correspondence between antenna sectors and Jakes' spectrum, a) receive antenna with $S = 4$ sectors, b) division of Jakes' spectrum into $S/2 + 1 = 3$ subspectra, each with reduced maximal Doppler spread

that the corresponding Doppler subspectra are evenly spaced (Fig. 2b). In Tab. 1 we have summarized those angles which lead to equisized subspectra for up to eight antennas. In the current case of $S = 4$ antennas the original Doppler spread has been reduced by $1/3$ per sector. Each sector $s = 1, \ldots, S$ corresponds with a time-variant channel impulse response $h_{\ell,s}(k)$. The key point here is that the impulse responses are accompanied by a reduced Doppler spread. We assume perfect sectorization and find the impulse responses $h_{\ell,s}(k)$ by generating (2) and by summation of those paths whose angle of incidence fall into the angle range comprised by the s-th antenna (cf. Tab.1).

S	ϕ_1	ϕ_2	ϕ_3	ϕ_4	ϕ_5	ϕ_6	ϕ_7	ϕ_8
2	90	270						
4	70.5	109.5	250.5	289.5				
6	60	90	120	240	270	300		
8	53.1	78.5	101.5	126.9	233.1	258.5	281.5	306.9

Table 1. Sector angles for equal Doppler partitioning (values in degree)

2.2 Receiver

Fig. 3 depicts our coherent receiver. The receive signal at the s-th antenna

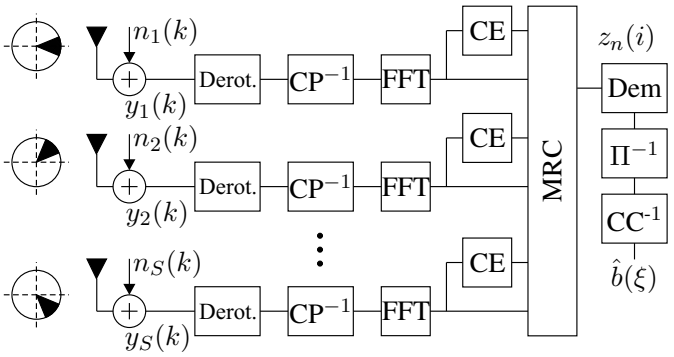

Figure 3. OFDM-receiver with maximum-ratio-combining

reads

$$y_s(k) = \sum_{\ell=0}^{L-1} h_{\ell,s}(k) \cdot x(k-\ell) + n_s(k), \;\; 1 \le s \le S \tag{4}$$

where $n_s(k)$ denotes additive white Gaussian noise (AWGN). The Doppler spectrum is divided into a set of subspectra. These exhibit no longer a pure Doppler spread but additionally a frequency shift (cf. Fig. 2) which is compensated in the block "Derot.". In [4] we find the approximation for the compensating frequency to be

$$f_{\mathrm{c}}(s) = \begin{cases} f_{\mathrm{D,max}} \cos(\phi_s/2)\,, & s = 1 \\ -f_{\mathrm{D,max}} \sin(\phi_s/2)\,, & s = S/2+1 \\ f_{\mathrm{D,max}} \cos((\phi_s + \phi_{s-1})/2)\,, & \text{else}\,. \end{cases} \tag{5}$$

Frequency compensation is simply accomplished by

$$\tilde{y}_s(k) = \mathrm{e}^{-\mathrm{j}2\pi f_{\mathrm{c}}(s) T k} y_s(k)\,. \tag{6}$$

Please note that $f_{\mathrm{c}}(s)$ is not the center frequency of the s-th subspectrum. It is an approximation for the center of gravity of the corresponding Doppler

frequencies, i.e., $f_{\mathrm{c}}(s)$ considers a bias towards large Doppler frequencies which occur with higher probability than small Doppler frequencies. Subsequently, after removing the cyclic prefix (CP^{-1}) the frequency-domain signal is computed by the DFT. The receive signal, $r_{n,s}(i)$, on the n-th subcarrier for the i-th OFDM symbol and the s-th sector thereby reads

$$r_{n,s}(i) = \frac{1}{\sqrt{N}} \sum_{\mu=0}^{N-1} \tilde{y}_s(\mu + i(N + N_{\mathrm{g}}))\mathrm{e}^{-\mathrm{j}2\pi\mu n/N} \,. \tag{7}$$

2.2.1 Channel Estimation. We consider two channel estimation approaches. The first follows [7] where the authors describe a two-dimensional MMSE filter. Pilot symbols are multiplexed in the OFDM time-frequency grid. For simplicity we assume a rectangular scattered pilot grid throughout the paper. Here pilot symbols are spaced at a distance of Δ_f symbols in frequency direction and at a distance of Δ_t symbols in time direction. It is crucial that the pilot symbol spacing adheres to the sampling theorem. To capture the maximal Doppler frequency the pilot spacing in time direction should fulfil

$$\Delta_t \leq \left\lfloor \frac{N}{2\gamma(N + N_{\mathrm{g}})} \right\rfloor \tag{8}$$

with the normalized maximum Doppler frequency $\gamma = f_{\mathrm{D,max}}/\Delta f$ and subcarrier spacing Δf. To sample the channel transfer function correctly the pilot spacing in frequency domain should follow

$$\Delta_f \leq \lfloor N/L \rfloor \,. \tag{9}$$

We find an estimate for the channel coefficients at all pilot symbol positions through division by the respective pilot symbol

$$\hat{H}^{\mathsf{LS}}_{n\Delta_f}(i) = \tilde{r}_{n\Delta_f}(i)/d_{n\Delta_f}(i) \,. \tag{10}$$

Based on (10) MMSE filtering allows for computation of all remaining channel coefficients associated with data carrying subcarriers denoted by $\hat{H}^{\mathsf{MMSE}}_{n}(i)$. In [7] omnidirectional reception is assumed. Hence, time correlation of neighboring receive samples is governed by the Bessel function of the first kind, $\mathrm{J}_0(\cdot)$. Time correlation in each branch of our sectorized receiver is different from the omnidirectional case. For the latter the Bessel function holds, for the former one can assume that a sufficient number of sectors S leads to correlations described by a Sinc function. Although this is not strictly true, we found from simulations that the resulting performance penalty is negligible. Furthermore, this assumption allows the use of the same estimator for all sectors.

Our second CE algorithm performs linear interpolation in both directions, frequency and time, based on the same rectangular grid as the MMSE approach. In a first step we determine the remaining channel coefficients for the i-th OFDM symbol, i.e., in frequency direction. The linearly interpolated channel coefficients between the $(n\Delta_f)$-th and the $((n+1)\Delta_f)$-th pilot carrying subcarrier is given by

$$\hat{H}^{\mathsf{LIN}}_{n\Delta_f+n'}(i) = \frac{\hat{H}^{\mathsf{LS}}_{(n+1)\Delta_f}(i) - \hat{H}^{\mathsf{LS}}_{n\Delta_f}(i)}{\Delta_f} n' + \hat{H}^{\mathsf{LS}}_{n\Delta_f}(i), \;\; 1 \leq n' < \Delta_f \quad (11)$$

Upon interpolation in frequency domain we perform interpolation in time direction, i.e.,

$$\hat{H}^{\mathsf{LIN}}_{n}(i\Delta_t + i') = \frac{\hat{H}^{\mathsf{LS}}_{n}((i+1)\Delta_t) - \hat{H}^{\mathsf{LS}}_{n}(i\Delta_t)}{\Delta_t} i' + \hat{H}^{\mathsf{LS}}_{n}(i\Delta_t), \;\; 1 \leq i' < \Delta_t. \quad (12)$$

This scheme follows no optimality criterion like the MMSE approach. Nevertheless, its simplicity is appealing since no knowledge about channel correlation or noise variance is required. But usually linear interpolation can handle only a limited Doppler range since rapid channel fluctuations between OFDM symbols will violate the necessity of linear channel variations. However, dividing the Doppler spectrum into subspectra avoids large Doppler spread in the first place. Hence, linear interpolation becomes possible again even for large Doppler spread, if the number of sectors is chosen suitably large.

To improve the performance of linear interpolation, especially for channels with long impulse responses, i.e., which exhibit strong frequency selectivity, we incorporate the findings of [9] into our subsequent simulation results. The respective authors observe that as a side effect convolution with a long impulse response introduces a timing offset which acts as a frequency offset in frequency domain. In [9] it is proposed to remove this frequency offset prior to linear interpolation and attaching it afterwards.

The final steps of our receiver after channel estimation are maximum-ratio-combining

$$z_n(i) = \frac{\sum_{s=1}^{S} \left(\hat{H}^{\mathsf{LIN/MMSE}}_{n,s}(i)\right)^* r_{n,s}(i)}{\sum_{s=1}^{S} \left|\hat{H}^{\mathsf{LIN/MMSE}}_{n,s}(i)\right|^2} \quad (13)$$

followed by APP demodulation ("Dem"), deinterleaving (Π^{-1}) and Viterbi decoding (CC^{-1}).

3. Simulation Results

We present results for an OFDM system with $N = 64$ subcarriers and $N_{\mathrm{g}} = 16$ guards taps. The channel is characterised by an uniformly distributed

power-delay profile with either $L = 3$ or $L = 10$ taps corresponding to the weak and strong frequency-selective case, respectively. We apply the non-systematic convolutional code $(133, 171)_8$ and QPSK as well as 16-QAM. MMSE-CE is performed according to [7]. The MMSE filters had 20 coefficients. Perfect knowledge about channel length, maximum Doppler frequency and direction of motion at the receiver side was assumed.

3.1 Single Antenna Performance

We start to investigate the performance limits for single antenna reception, i.e., $S = 1$. Fig. 4 depicts simulation results for a QPSK transmission over

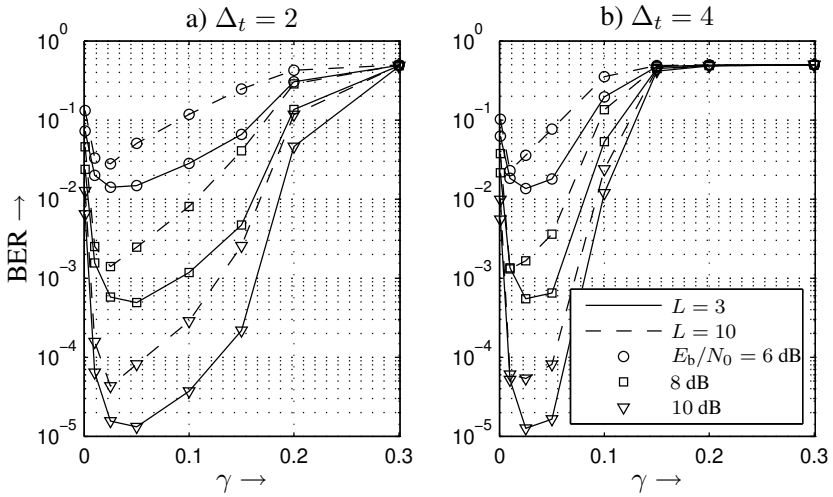

Figure 4. Single antenna bit error performance vs. normalized Doppler frequency γ for different signal-to-noise power ratios, QPSK, $S = 1$, $\Delta_f = 5$, MMSE channel estimation

weakly and strongly frequency-selective channels for different signal-to-noise ratios. In Fig. 4a the pilot spacing in time direction was $\Delta_t = 2$, in Fig. 4b $\Delta_t = 4$. According to (8) the maximal Doppler frequency which can be captured in a critical sense is $\gamma \approx 0.19$ for $\Delta_t = 2$ and $\gamma \approx 0.094$ for $\Delta_t = 4$. However, we see that the BER deteriorates already before the normalized Doppler frequency closes in on these critical values. If the Doppler frequency surpasses the critical Doppler frequencies the sampling theorem is evidently violated and channel estimation fails.

3.2 Multiple Antenna Performance

To facilitate channel estimation for large Doppler environments we could choose to decrease the pilot spacing to $\Delta_t = 1$. But since then every OFDM

symbol carries pilot data, bandwidth effciency is decreased. Besides, the largest Doppler frequency which we could capture according to (8) for this case amounts to 0.375. The channel fluctuations accompanied by such large Doppler spreads lead to intercarrier interference which further impedes channel estimation. Instead we apply our sectorization approach to decrease the time-selectivity of the channel. This allows us to keep a scattered pilot scheme with reasonable bandwith efficiency.

3.2.1 Sectorized vs. Single Antenna Performance. In Fig. 5 we have fixed the maximal Doppler frequency to $\gamma = 0.2$ which is critical for the pilot spacing $\Delta_t = 2$. Again results are given for weak and strong frequency selectivity. The number of sectors ranges from $S = 1, \cdots, 8$. Fig. 5a depicts

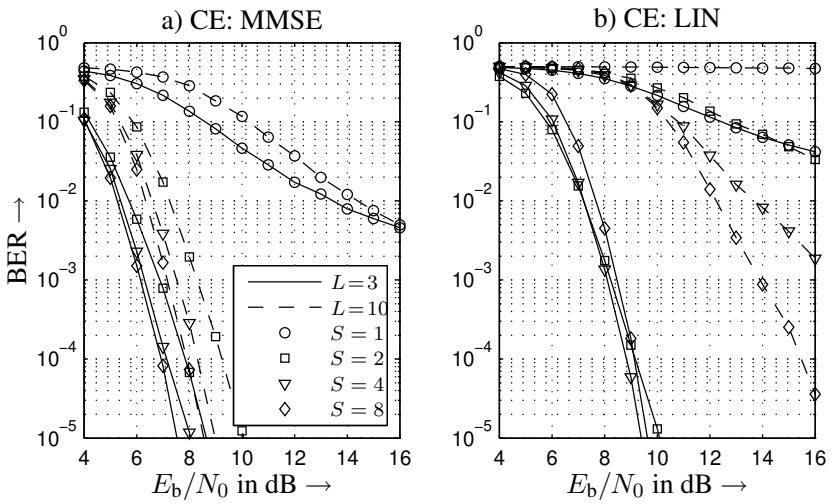

Figure 5. $\gamma = 0.2$, QPSK, $\Delta_f = 5$, $\Delta_t = 2$

results for MMSE channel estimation and Fig. 5b for linear interpolation. Single antenna reception ($S = 1$) leads to an error-floor in all cases due to critical sampling of the channel. Usage of two sectors ($S = 2$) already avoids an error-floor.

We see that MMSE channel estimation copes better with the strongly frequency selctive channel than linear interpolation does. This problem was mentioned at the end of Sec. 2.2.1 and is rooted in the timing offset accompanied by long channel impulse responses. For the results in Fig. 6 we perform phase compensation prior to linear interpolation as proposed in [9]. The performance improvement is impressive. However, the receiver has to have knowledge about the channel length.

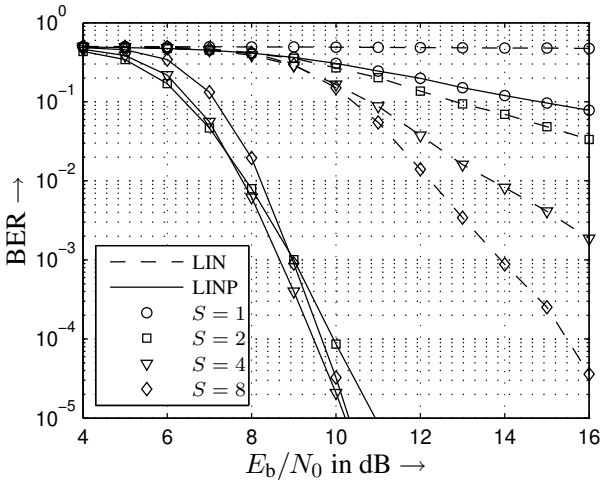

Figure 6. $\gamma = 0.2$, QPSK, $\Delta_f = 5$, $\Delta_t = 2$; LIN=linear interpolation, LINP=linear interpolation with phase correction

3.2.2 Diversity vs. Sectorization. In the previous section we have seen the benefits of sectorization. Reducing the time-selectivity of the channel greatly alleviates channel estimation allowing for reliable data detection even for severe Doppler conditions. We compare now the performance of sectorization against diversity reception. The latter means we apply the same number of *omnidirectional* antennas which are spaced far enough to yield uncorrelated signals. However, each of these signals experiences the full Doppler spread. Fig. 7 shows that diversity reception is superior to sectorization for small Doppler frequencies. The sectorized receiver suffers from a lack of time diversity since the already slowly fluctuating channel becomes quasi-static after sectorized reception. It is however evident that large Doppler frequencies can not be handled by diversity reception. Here, sectorized reception performs much more robust.

4. Conclusions

We have described a coherent OFDM-receiver with Doppler compensation based on directional antennas. These devide the Doppler spectrum into a set of subspectra with smaller Doppler spread. Since these subspectra correspond with slowly time-varying channels, channel estimation is alleviated. We have presented simulation results for two CE algorithms based on the MMSE criterion and on linear interpolation. We have shown that diversity reception is superior over sectorization for small Doppler frequencies, however, rapidly fluctuating channels are successfully coped with only by sectorization.

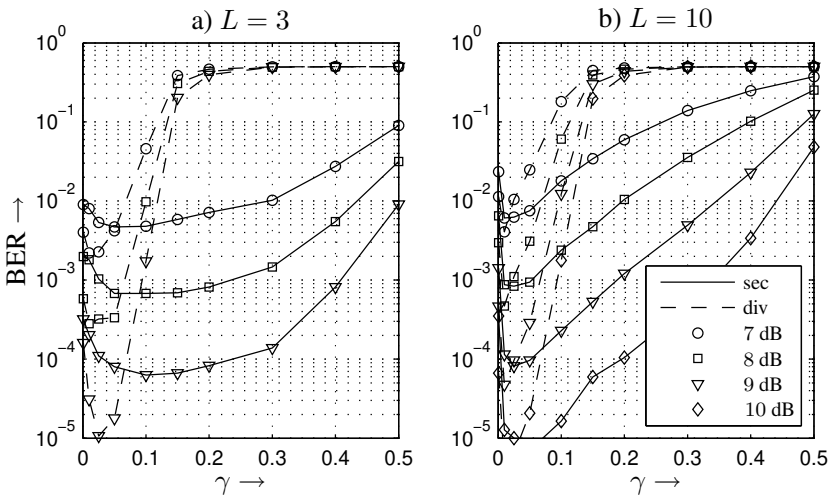

Figure 7. $S = 8$, $\Delta_f = 5$, $\Delta_t = 4$, 16QAM, MMSE channel estimation

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