ANALYSIS OF ITERATIVE SUCCESSIVE INTERFERENCE CANCELLATION IN SC-CDMA SYSTEMS

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Abstract This paper analyzes the convergence behavior and performance of iterative successive interference cancellation (SIC) for a CDMA system with random spreading using the so-called multi-user efficiency (MUE). The goal of such an analysis is the optimization of the detection scheme. Moreover, an optimized power allocation of the users at the transmitter is an important means for enhancing the convergence behavior of the detector and is based on the possibility of prediction. While this analysis has only been applied to parallel interference cancellation (PIC) we will generalize it in this paper also for SIC. It will be shown that the achievable system load can be significantly increased.

1. Introduction

The turbo principle discovered in 1993 has been applied to nearly any concatenated system like channel estimation and equalization, coding and modulation. It was also applied to parallel and successive interference cancellation in a coded CDMA system. Both schemes exploit the soft information at the channel decoder output for improving interference cancellation in an iterative manner. The channel decoder and the multi-user detection can be regarded as serially concatenated systems. Analysis of the PIC scheme has been done by different approaches [1-3]. In this case the analysis reduces to a two-dimensional one as will be described in Section 3. For successive interference cancellation this simplification is not possible because the statistics of the users differ from each other. In this paper the analysis derived for PIC is generalized to SIC by taking the dependencies of the users into account. The goal is to predict the behavior not only of the whole system but of any particular user during the iterations.

The paper is organized as follows: Section 2 introduces the system model of the considered CDMA system. In Section 3 the analysis based on multi-user efficiency (MUE) is described and applied to the parallel interference cancellation. The difference between SIC and PIC with respect to MUE is investigated and the MUE analysis of SIC is given in Section 4. Power optimization is done in Section 5 for both detection schemes. A conclusion is given in Section 6.

2. System Model

In order to simplify derivations and notation we assume a synchronous singel carrier- (SC-) CDMA system with a complex AWGN channel and pseudo-noise spreading sequences [4], but the analysis can be applied to MC-CDMA as well. The number of active users is denoted by U. The information bit vector of the u-th user is denoted by $\mathbf{d}_u$, which is encoded with a convolutional code of rate $R_c = 1/n$ identical for all users. The coded bit sequence is BPSK-modulated and interleaved by a user-specific interleaver Π_u of length L and then spread with random spreading codes $s_u(k) \in \{-1/\sqrt{N}, +1/\sqrt{N}\}$. k denotes the chip and l the symbol index. The length N of the sequences $s_u(k)$ is called spreading factor and the system load $\beta = U/N$ is an important parameter of the system. Assuming $\mathbf{b}(l)$ to be the vector comprising BPSK symbols of all users at time instance l and $\mathbf{C}(l)$ a $N \times U$ matrix containing the vectors of spreading sequences as columns each multiplied with an individual phase term of the channel, the received vector containing the superposition of the spread signals of all users and the noise can be described in vector-matrix notation

$$\mathbf{y}(k) = \mathbf{C}(l)\mathbf{b}(l) + \mathbf{n}(k) \; . \tag{1}$$

The vector $\mathbf{n}(k)$ represents the complex additive white Gaussian noise with covariance matrix $\sigma_n^2 \, \mathbf{I}$. At the receiver a bank of matched filters (MF) is applied for despreading and the real-valued matched filter output can be written as

$$\mathbf{r} = \mathrm{Re}\left\{\mathbf{C}^H \mathbf{y}\right\} = \underbrace{\mathrm{Re}\left\{\mathbf{C}^H \mathbf{C}\right\}}_{\mathbf{R}} \mathbf{b} + \underbrace{\mathrm{Re}\left\{\mathbf{C}^H \mathbf{n}\right\}}_{\tilde{\mathbf{n}}} \; . \tag{2}$$

For notational simplicity the time index k has been dropped. The off-diagonal elements of $\mathbf{R}$ contain the real part of the correlation coefficients $\varrho_{ij} = \mathrm{Re}\{\rho_{ij}\}$ between the i-th and the j-th user's signature sequence with $\mathrm{E}\{|\rho_{ij}|\} = 1/N$. Detection by individual decoding and hard decision will not be appropriate for systems with moderate to high loaded systems, since the multi-user interference degrades the performance significantly. Considerable improvement can be obtained by the application of interference cancellation. The idea of interference cancellation schemes is to estimate the interference and remove it from the received signal before detection.

3. Multi-User Efficiency

The structure of the parallel interference cancelers is depicted in Figure 1. The channel decoding is realized by a Max-Log-MAP decoder

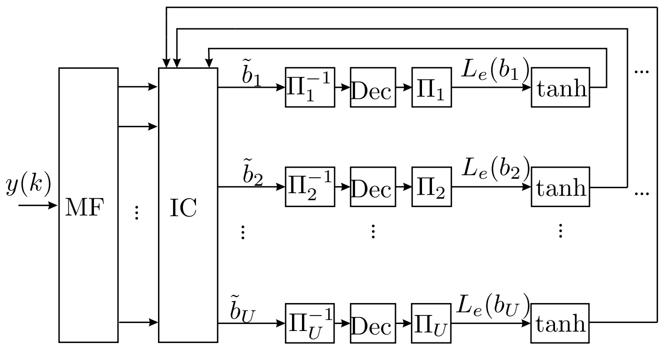
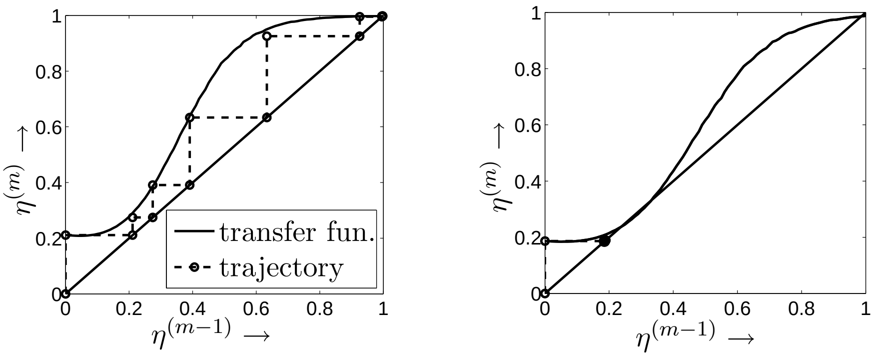


Figure 1. Receiver structure of PIC

deriving approximated extrinsic log-likelihood-ratios $L_e(b_u)$. The soft estimates of the coded symbols $\bar{b}$ are calculated as $\bar{b} = \tanh(L_e/2)$. The signal-to-interference-plus-noise-ratio (SINR) of each branch is relevant for the quality of the interference cancellation. It is defined as $\text{SINR} = 2\sigma_d^2/(\sigma_n^2 + \sigma_{\text{MUI}}^2)$ and is equal to the $\text{SNR} = 2E_s/N_0$ in the case of perfect interference cancellation which is equivalent to the single-user bound (SUB). σ_d^2 is the variance of the desired signal and σ_{MUI}^2 is the variance of the remaining multi-user interference after cancellation which can be calculated as $\sigma_{\text{MUI}}^2 = \sigma_d^2 \cdot \mu(U-1)/N$. $\mu = \text{E}\{|\bar{b} - b|^2\}$ is the remaining mean squared error of the estimated symbols after decoding which is approximately the same for each user in the case of PIC. The ratio of SINR and SNR is called multi-user efficiency (MUE) and is denoted by η [2]. If η is equal to one there is no loss compared to the SUB and therefore this case describes perfect interference cancellation. For the large system limit $(N, U \to \infty)$ $(U-1)/N \approx U/N = \beta$, η can be written as

$$\eta = \frac{\text{SINR}}{\text{SNR}} = \frac{2\sigma_d^2/(\sigma_n^2 + \sigma_{\text{MUI}}^2)}{2\sigma_d^2/\sigma_n^2} = \frac{1}{1 + \beta\mu E_s/N_0} \quad . \tag{3}$$

The parameter η can be used to visualize and predict the behavior of the



(a) (b)

Figure 2. Predicted transfer function and simulated trajectory for PIC, $N = 8$, $U = 16$, $E_b/N_0 = 6$ dB (a) and $N = 8$, $U = 24$, $E_b/N_0 = 5$ dB (b)

iterative detection. In the first step, we have the matched filter outputs without a-priori information from the channel decoder. The variance μ is therefore equal to 1 and the MUE becomes $\eta^{(1)} = 1/(1+\beta E_s/N_0)$. After simultaneously decoding all users, soft estimates $\bar{b}$ of the transmitted symbols are obtained which are used in the next iteration for interference cancellation. Since channel decoding is generally a nonlinear process μ cannot be calculated analytically, but has to be predetermined. The output error $\mu^{(m)}$ of the decoder in the m-th iteration depends on the SINR at the input

$$\mu^{(m)} = g\,(\mathrm{SINR}) = g\left(\eta^{(m-1)}\mathrm{SNR}\right) \tag{4}$$

and therefore on the MUE of the previous iteration $\eta^{(m-1)}$. Because $\eta^{(m)}$ depends itself on $\mu^{(m)}$ the behavior of the PIC at iteration m can be described by $\eta^{(m)} = f\left(\eta^{(m-1)}\right)$. This function is illustrated in a two-dimensional plot in Figure 2(a). The transfer function describes the theoretical behavior and the trajectory the measured values during simulation. The detection starts in the lower left corner and tends to the upper right corner. This point corresponding to $\eta = 1$ describes perfect interference cancellation. It can be seen that the behavior can be predicted very precisely. This plot corresponds to a system with a spreading factor $N = 8$, $U = 16$ users, an E_b/N_0 of 6 dB and a convolutional code with generator polynomials $[7\ 5]_8$ in octal representation. This system will converge to the SUB within 6 iterations. In Figure 2(b) a system with load of 3 and $E_b/N_0 = 5\,\mathrm{dB}$ is depicted. There is an intersection between the transfer function and the bisecting line so the detection gets stuck at $\eta \approx 0.2$.

4. Analysis of Successive Interference Cancellation

The structure of successive interference cancellation is shown in Figure 3. The prediction in the same manner as for PIC is not possible. While

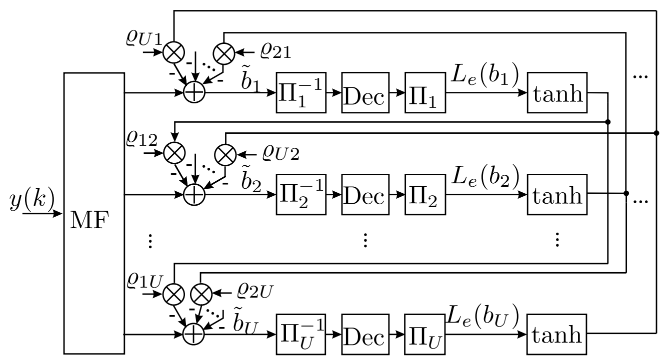


Figure 3. Receiver structure of SIC

for PIC the error variance μ is the same for all users in the large system limit, this is not the case for SIC. The U users have different variances $\mu_u^{(m)}$ at each iteration m. The remaining errors of the users are assumed to be independent. So a simple addition of their variances weighted with the corresponding correlation coefficient can be applied for calculating the resulting interference on the desired user signal. For that reason the MUE can be calculated by

$$\eta_u^{(m)} = \frac{1}{1 + \frac{1}{N}\left(\sum_{i=1}^{u-1} \mu_i^{(m)} + \sum_{i=u+1}^{U} \mu_i^{(m-1)}\right) E_s/N_0} \tag{5}$$

To show how good the prediction works, Figure 4 depicts the predicted and the simulated trajectories for the same system parameters as in Fig. 2(a) in one diagram per user. It can be seen that the prediction works well also in the successive interference cancellation case. A simple transfer function as shown in Fig. 2(a) cannot be drawn in these plots since the transfer function differs for each user and each iteration due to being conditioned on the current state of all the other users. In order to avoid a very complex diagram only the trajectories are depicted in Figure 4.

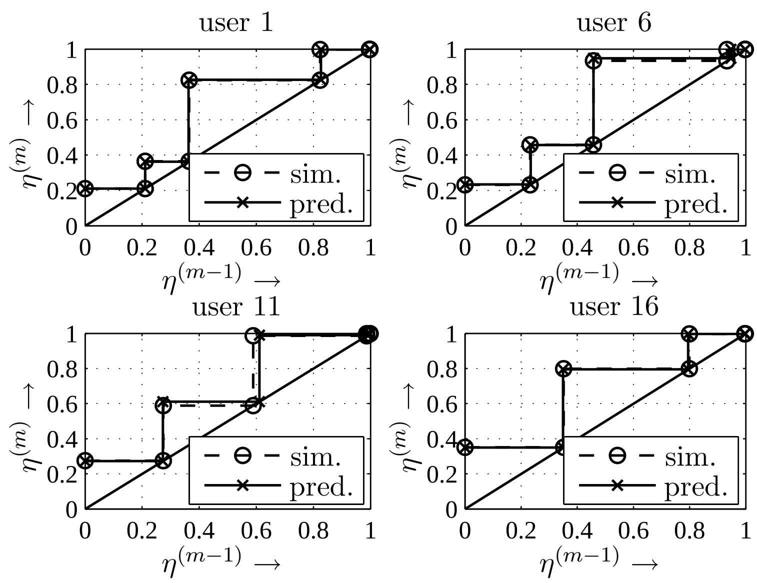


Figure 4. Predicted and simulated trajectories for SIC, $N = 8$, $U = 16$ and $E_b/N_0 =$ 6 dB

5. Power distribution optimization

Up to now analysis was based on uniformly distributed powers of the users. To describe an unequal power distribution by multi-user efficiency, a way to calculate a kind of average efficiency is necessary. So far the transfer characteristic was calculated by simply averaging over all users. For an average MUE in the case of different powers the individual μ's have to be obtained and combined. The error variance μ is defined as the residual symbol interference independent of the received power. But the impact of this error on other users' detection is indeed dependent on the receive power which is denoted as P_u. This fact is taken into account by weighting μ_u with this user's received power. Thus the resulting multi-user efficiency can be calculated more generally as

$$\eta_u = \frac{1}{1 + \frac{\bar{E}_s}{N_0} \frac{1}{N} \sum_{v \neq u} \mu_v \cdot P_v} \quad , \quad \sum_v P_v = U \quad . \tag{6}$$

$\bar{E}_s/N_0$ is now defined as an average value over all users in order to get an appropriate criterion for fair comparison with the equal power case. μ_u depends on the SINR at the input of the decoder and is for that reason itself dependent on P_u :

$$\mu_u = g(\text{SINR}) = g\left(\eta \frac{\bar{E}_s}{N_0} P_u\right) \tag{7}$$

The criterion for convergence is still reaching the point of $\eta = 1$ after an finite number of iterations. For the PIC this is fulfilled if $f(\eta) \geqslant \eta$, $\eta \in [0,1]$. The number of iterations needed depends on the width of the tunnel. Whether the tunnel is open or not depends also on the power distribution. For PIC it turns out that equal power for all users is not the best choice, as presented e.g. in [1] and [5].
In this paper optimization is solved by means of *differential evolution* [6]. Especially in the case of SIC there exist many local optima in the cost function (multimodal function). For local optimization techniques a starting point is needed, which is already near to the global optimum. A better approach is a global optimization, especially algorithms motivated by evolution do not need any knowledge of the searching area.
The main difference between local search algorithms and evolutionary techniques is that instead of a single point a set of points called population is regarded at each time instance. Evolutionary algorithms combine elements of the current population and compare the generated children with the parents. If the child has a lower cost than the parents it will replace one parent. There exist many algorithms motivated by evolutionary processes, but differential evolution has the advantage of converging faster in the neighborhood of the optimal solution due to an adaptive step size. The optimization problem for PIC can be described by

$$\underbrace{\text{minimize}}_{P_1,..,P_U} \quad \sum_u P_u \quad \text{s. t.} \quad \begin{cases} f(\eta) \geqslant \eta \quad (+\,\varepsilon) \quad , \quad \eta \in [0;1] \\ P_u > 0 \quad \forall \quad u \end{cases} \tag{8}$$

By ε the width of the tunnel can be increased in order to decrease the number of iterations at the cost of higher transmit power.
For the SIC a similar expression for the first condition cannot be given. A more general condition for convergence used for SIC is to reach $\eta = 1$ after an finite number of iteration. The starting population is generated randomly and should be sufficiently large for a high diversity. The performance improvement by power optimization is illustrated in Figure 5(a) where the transfer functions for a system with load $\beta = 4$ at $E_b/N_0 = 6.5$ dB with equal and optimized power distribution are shown. For equal powers it will get stuck at $\eta \approx 0.1$ which is a loss of about 10 dB compared to the SUB whereas optimized power levels enable the detection to converge to the SUB. With equal powers only a load of ≈ 3 is possible. In Figure 5(b) it is shown that the weakest user has only a

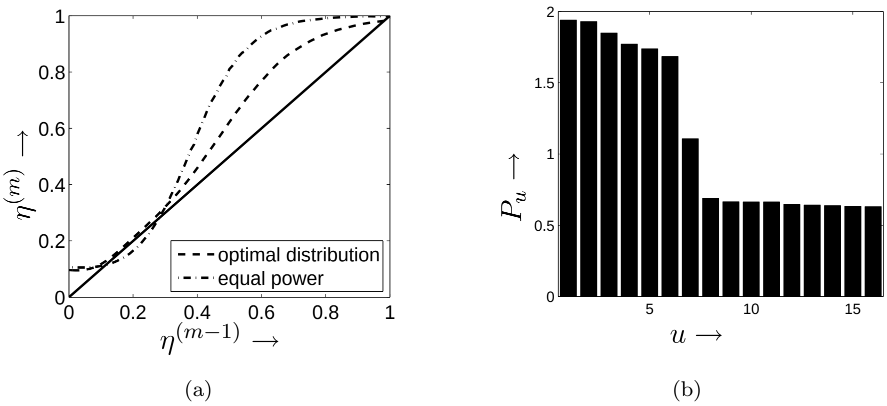


Figure 5. Transfer characteristic (a) and power profile (b) of optimized PIC at $E_b/N_0 = 6.5$ dB

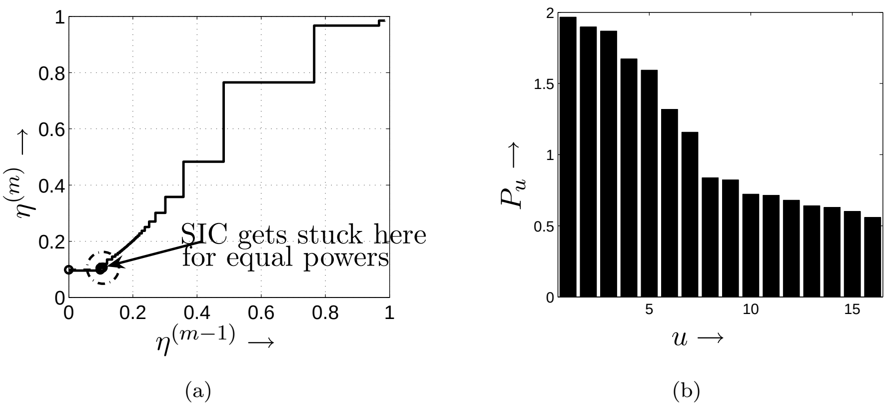


Figure 6. Trajectory (a) and power profile (b) of optimized SIC at $E_b/N_0 = 6.5$ dB

loss of 2.5 dB compared to the equal power case, so the gain is 7.5 dB. In Fig. 6(a) a trajectory for optimized SIC is depicted; Figure 6(b) shows the corresponding power profile. For the same system the results are nearly the same. If the tunnel of the transfer function is very narrow, the differences of the users are small and the SIC behaves more and more like the PIC. For that reason the optimized power levels are nearly the same, but the convergence speed is higher for SIC. To achieve an efficiency of $\eta = 0.98$ the PIC needs for this example 53 iterations, the SIC only 35.

6. Conclusion

In this paper, an analysis of iterative successive interference cancellation based on multi-user efficiency was presented. The quality of prediction is as good as for the PIC; only the graphical representation is not so simple and obvious. Unequal power distribution leads to significantly better performance. For PIC and SIC optimization of the power distribution was carried out by differential evolution. It was based on the prediction of convergence. It was shown that the system load could be significantly increased by the use of an optimized power profile.
Future work will adapt the constraints for the optimization to more realistic scenarios by e.g. individual power constraints or maximum number of iterations. This additional constraints are at the cost of maximum system load or overall received power.

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