Optimization of Interference Cancellation in Coded CDMA Systems by Means of Differential Evolution

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Abstract

This paper introduces an optimization of the received power profile for iterative parallel and successive interference cancellation (PIC/SIC) in coded CDMA systems. The basic approach is an optimization algorithm called *Differential Evolution* (DE). An optimized power allocation of the users at the transmitter is an important means for enhancing the convergence behavior of the detector. Either the supportable load can be significantly increased or the required transmit power is decreased by several dB. Furthermore any additional constraint regarding the modelling of realistic communication systems can be implemented very easily. In this paper the number of needed iterations is dramatically decreased. Computational complexity is reduced while the needed power is barely increased. Another constraint which is also considered is the near-far-effect that degenerates system performance. The maximum received power is limited for some users that are assumed to be at the cell border or who suffer from fading. The precondition for this optimization is the analysis of the iterative detection scheme. This is done by a parameter called multi-user efficiency (MUE).

1 Introduction

The turbo principle discovered in 1993 has been applied to nearly any concatenated system like channel estimation and equalization, coding and modulation. It was also employed for parallel and successive interference cancellation in a coded CDMA system. Both schemes exploit the soft information at the channel decoder output for improving interference cancellation in an iterative manner. The channel decoder and the multiuser detection can be regarded as serially concatenated systems. Analysis of the PIC scheme has been done by different approaches [1-3] and the possibility of power profile optimization has been shown in [4]. For successive interference cancellation the analysis is more difficult because the statistics of the users differ from each other. The generalization of the analysis to SIC and the basics of power profile optimization were presented in [5]. This tool is applied here under more realistic assumptions. To limit computational complexity for implementation aspects the power profile is optimized under the constraint of maximum number of iterations. Another problem that was not considered so far is the individual power constraint for particular users. In a cellular network system performance suffers from near-far-effects so it may be impossible to provide the allocated receive power for some users.

The paper is organized as follows: Section 2 introduces the system model of the considered CDMA system. In Section 3 the analysis based on multi-user efficiency (MUE) [2] is described and applied to the parallel interference cancellation. The difference between SIC and PIC with respect to MUE is investigated and the MUE analysis of SIC is given in Section 4. Power profile optimization is introduced in Section 5 for both detection schemes. The implementation of the additional constraints is discussed in Section 6. A conclusion is given in Section 7.

2 System Model

In order to simplify derivations and notation we assume a synchronous single carrier- (SC-) CDMA system with a complex AWGN channel and pseudo-noise spreading sequences [6]. The number of active users is denoted by U. The information bit vector of the u-th user is denoted by $\mathbf{d}_u$, which is encoded by a convolutional code of rate $R_c = 1/2$ that is identical for all users. The coded bit sequence is BPSK-modulated and interleaved by a user-specific random interleaver Π_u of length L. Finally the signals are spread with random spreading codes $s_u(k) \in \{-1/\sqrt{N}, +1/\sqrt{N}\}$. k denotes the chip and l the symbol index. The length N of the sequences $s_u(k)$ is called spreading factor and $\beta = U/N$ denotes the system load. $\mathbf{b}$ is assumed to be the vector comprising BPSK symbols of all users at a particular time instance and $\mathbf{C}$ is the $N \times U$ spreading matrix. It contains the vectors of spreading sequences as columns each multiplied with an individual phase term of the channel. The received vector containing the superposition of the spread signals of all users and the noise can be described in vector-matrix notation, yielding

$$\mathbf{y} = \mathbf{C}\mathbf{b} + \mathbf{n} \,. \tag{1}$$

For notational simplicity time indices have been dropped. The vector $\mathbf{n}$ represents the complex additive white Gaussian noise with covariance matrix $\sigma_n^2 \mathbf{I}$. At the receiver a bank of matched filters (MF) is applied for despreading and the real-valued matched filter output can be written as

$$\mathbf{r} = \mathrm{Re}\left\{\mathbf{C}^H\mathbf{y}\right\} = \underbrace{\mathrm{Re}\left\{\mathbf{C}^H\mathbf{C}\right\}}_{\mathbf{R}}\mathbf{b} + \underbrace{\mathrm{Re}\left\{\mathbf{C}^H\mathbf{n}\right\}}_{\tilde{\mathbf{n}}} \,. \tag{2}$$

The elements of $\mathbf{R}$ contain the real part of the correlation coefficients $\varrho_{ij} = \mathrm{Re}\{\rho_{ij}\}$ between the i-th and the j-th user's signature sequence with $\mathrm{E}\{|\rho_{ij}|\}_{i\neq j} = 1/N$. The multi-user interference characterized by these correlation coefficients degrades the performance significantly even for moderate system loads if individual decoding and hard decision is applied to these matched filter outputs. Interference cancellation techniques are able to improve performance significantly by estimating the multi-user interference and cancelling it before detection. Additional gain is achieved by iterative structures.

3 Multi-User Efficiency

Figure 1 shows the structure of a parallel interference canceller. In order to get at least approximated extrinsic

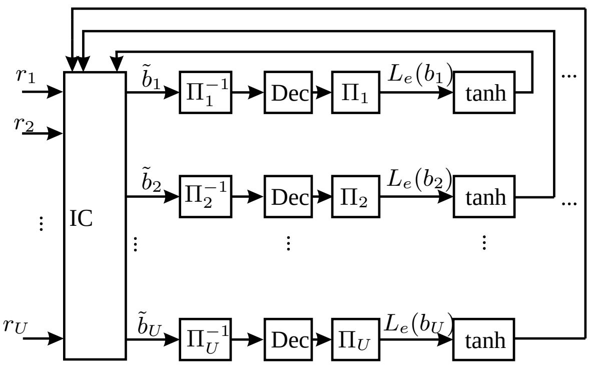

Fig. 1. Receiver structure of PIC

log-likelihood-ratios $L_e(b_u)$ at low computational cost, the Max-Log-MAP channel decoder is applied. The soft estimates of the coded symbols $\bar{b}$ are calculated as $\bar{b} = \tanh(L_e/2)$. The signal-to-interference-plus-noise-ratio (SINR) of each branch is a parameter indicating the quality of the interference cancellation and is defined as $\mathrm{SINR} = 2\sigma_d^2/(\sigma_n^2 + \sigma_{\mathrm{MUI}}^2)$. It is equal to the $\mathrm{SNR} = 2E_s/N_0$ in the case of perfect interference cancellation which is equivalent to the single-user bound (SUB). σ_d^2 and σ_{MUI}^2 describe the variance of the desired signal and of the remaining multi-user interference after cancellation respectively. The latter can be calculated as $\sigma_{\mathrm{MUI}}^2 = \sigma_d^2 \cdot \mu(U-1)/N$. The remaining mean squared error of the estimated symbols after decoding is denoted as $\mu = \mathrm{E}\{|\bar{b} - b|^2\}$ which is approximately the same for each user in the case of PIC. The ratio of SINR and SNR is called multi-user efficiency (MUE) and is denoted by η [2]. Perfect interference cancellation is indicated by $\eta = 1$ and therefore describes the case with no loss compared to the SUB. For the large system limit $(N, U \to \infty)$ $(U-1)/N \approx U/N = \beta$, η can be written as

$$\eta = \frac{\mathrm{SINR}}{\mathrm{SNR}} = \frac{2\sigma_d^2/(\sigma_n^2 + \sigma_{\mathrm{MUI}}^2)}{2\sigma_d^2/\sigma_n^2} = \frac{1}{1 + \beta\mu E_s/N_0} \,. \tag{3}$$

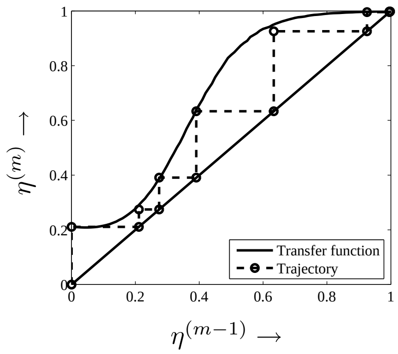

Fig. 2. Predicted transfer function and simulated trajectory for PIC, $N = 8$, $U = 16$, $E_b/N_0 = 6$ dB

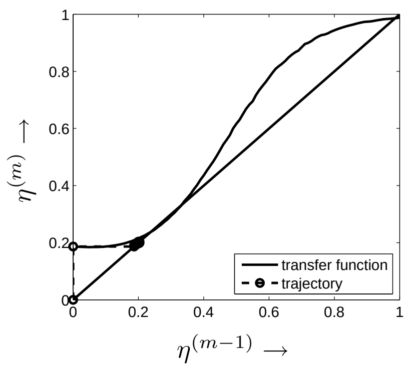

Fig. 3. Predicted transfer function and simulated trajectory for PIC, $N = 8$, $U = 24$, $E_b/N_0 = 5$ dB

The parameter η can be used to visualize and predict the behavior of an iterative detection scheme. In the initial iteration there is no a-priori information available for interference cancellation. The soft bits are initialized with zero, the variance μ is therefore equal to 1 and the MUE becomes $\eta^{(1)} = 1/(1 + \beta E_s/N_0)$. After simultaneously decoding all users, soft estimates $\bar{b}$ of the transmitted symbols are obtained which are used in the next iteration for interference cancellation. Since channel decoding is generally a nonlinear process μ cannot be calculated analytically, but has to be predetermined. The output error $\mu^{(m)}$ of the decoder in the m-th iteration depends on the SINR at the input

$$\mu^{(m)} = g\,(\mathrm{SINR}) = g\left(\eta^{(m-1)}\mathrm{SNR}\right) \tag{4}$$

and therefore on the MUE of the previous iteration $\eta^{(m-1)}$. Because $\eta^{(m)}$ itself depends on $\mu^{(m)}$ the behavior of the PIC at iteration m can be described by $\eta^{(m)} = f\left(\eta^{(m-1)}\right)$. This function is depicted in a two-dimensional plot in Figure 2. The transfer function describes the theoretical behavior and the trajectory gives the measured values during simulation. The detection starts in the lower left corner and tends to the upper right corner. This point corresponding to $\eta = 1$ describes perfect interference cancellation. It can be seen that the behavior is predicted very precisely. This plot corresponds to a system with a spreading factor $N = 8$, $U = 16$ users, an E_b/N_0 of 6 dB and a convolutional code with generator polynomials $[7, 5]_8$. This system will converge to the SUB within 6 iterations. In Figure 3 a system with load of 3 and $E_b/N_0 = 5\,\mathrm{dB}$ is depicted. There is an intersection between the transfer function and the bisecting line so the detection gets stuck at $\eta \approx 0.2$.

4 Analysis of Successive Interference Cancellation

The structure of successive interference cancellation assumed in this paper is shown in Figure 4. The prediction in the same manner as for PIC is not possible. While

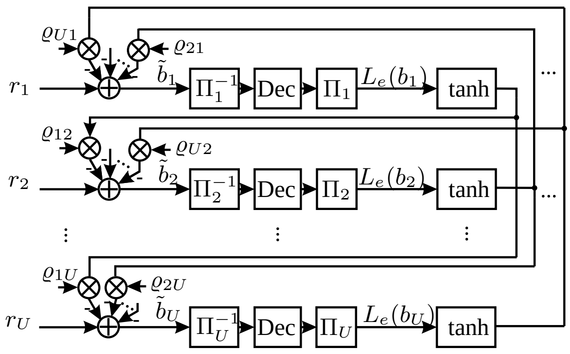

Fig. 4. Receiver structure of SIC

for PIC the error variance μ is the same for all users in the large system limit, this is not the case for SIC. The U users have different variances $\mu_u^{(m)}$ at each iteration m. The remaining errors of the users are assumed to be independent. So a simple addition of their variances weighted with the corresponding correlation coefficient can be applied for calculating the resulting interference on the desired user signal. For that reason the MUE

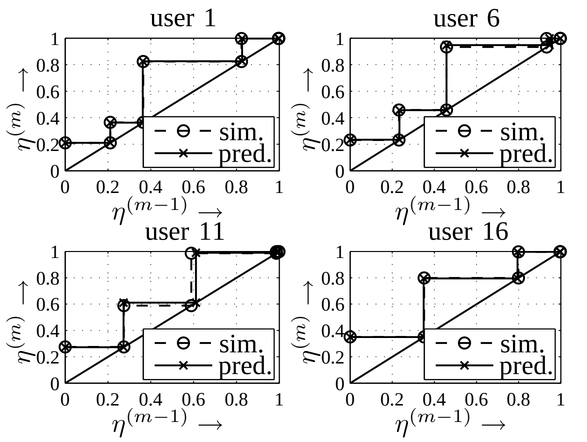

Fig. 5. Predicted and simulated trajectories for SIC, $N = 8, U = 16$ and $E_b/N_0 = 6$ dB

can be calculated by

$$\eta_u^{(m)} = \frac{1}{1 + \frac{1}{N}\left(\sum_{i=1}^{u-1}\mu_i^{(m)} + \sum_{i=u+1}^{U}\mu_i^{(m-1)}\right)\frac{E_s}{N_0}}\,. \tag{5}$$

To show how good the prediction works, Figure 5 depicts the predicted and the simulated trajectories for the same system parameters as in Figure 2 in one diagram per user. It can be seen that the prediction works well also in the successive interference cancellation case. A simple transfer function as shown in Figure 2 cannot be drawn in these plots since the transfer function differs for each user and each iteration due to being conditioned on the current state of all the other users. In order to avoid a very complex diagram only the trajectories are depicted in Figure 5.

5 Power Profile Optimization

5.1 Formulation of the Optimization Problem

Up to now the analysis was based on uniformly distributed powers of the users. To describe an unequal power distribution by multi-user efficiency, a way to calculate a kind of average efficiency is necessary. So far the transfer characteristic was calculated by simply averaging over all users. For an average MUE in the case of different powers the individual μ's have to be combined. The error variance μ is defined as the residual symbol interference independent of the received power. But the impact of this error on other users' detection is indeed dependent on the receive power which is denoted as P_u. This fact is taken into account by weighting μ_u with this user's received power. Thus the resulting multi-user efficiency can be calculated more generally as

$$\eta_u = \frac{1}{1+\frac{\bar{E}_s}{N_0}\frac{1}{N}\sum\limits_{v\neq u}\mu_v \cdot P_v} \quad , \quad \sum_v P_v = U \quad . \tag{6}$$

$\bar{E}_s/N_0$ is defined as an average value over all users in order to get an appropriate criterion for fair comparison with the equal power case. μ_u depends on the SINR at the input of the decoder and is for that reason itself dependent on P_u :

$$\mu_u = g(\text{SINR}) = g\left(\eta \frac{\bar{E}_s}{N_0} P_u\right) \quad . \tag{7}$$

The criterion for convergence is still reaching the point of $\eta = 1$ after a finite number of iterations. For the PIC this is fulfilled if $f(\eta) \geqslant \eta$, $\eta\in[0,1]$. The number of iterations needed depends on the width of the tunnel. Whether the tunnel is open or not depends also on the power distribution. For PIC it turns out that equal power for all users is not the best choice, as presented e.g. in [1] and [4].

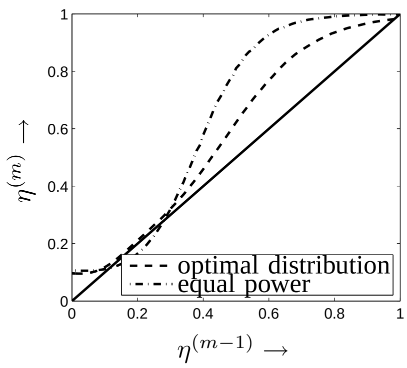

Fig. 6. Transfer characteristic of PIC at $E_b/N_0 = 7$ dB for equal and optimized power profile, $N = 4$ and $U = 16$

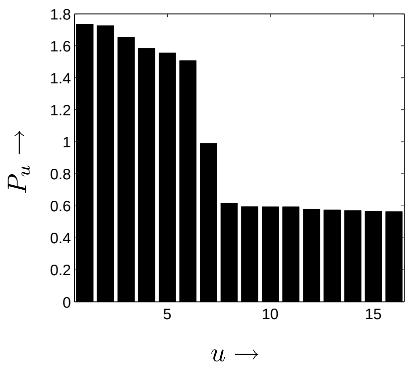

Fig. 7. Optimized power profile for PIC at $E_b/N_0 = 7$ dB, $N = 4$ and $U = 16$

The optimization problem for PIC can be described by [4]

$$\min_{P_1,..,P_U} \quad \sum_u P_u \quad \text{s. t.} \quad \begin{cases} f(\eta) \geqslant \eta + \varepsilon \ , \ \eta\in[0;1] \\ P_u > 0 \quad , \quad u = 1, ...U \end{cases} \tag{8}$$

The width of the tunnel can be influenced by ε in order to decrease the number of iterations at the cost of higher transmit power.
For the SIC a similar expression for the first condition cannot be given. A more general condition for convergence used for SIC is to reach $\eta = 1$ after an finite number of iterations.

5.2 Differential Evolution

The cost functions of the examined optimization problems contain many local optima (multimodal functions), especially for SIC. Furthermore, the search space is highly constrained. Therefore a starting point in the vicinity of the global optimum would be required for the employment of local optimization techniques. As the global optimum is not known in advance global optimization methods have to be used. In this paper the Differential Evolution (DE) algorithm is chosen that belongs to the class of evolutionary algorithms [7]. As the name indicates evolutionary algorithms are motivated by the natural evolution process. In contrast to local search algorithms not only one but several points (population) in the search space are regarded at a given time instance. The advantage is that evolutionary algorithms do not need any knowledge of the search space. Differential Evolution furthermore possesses the property of fast convergence due to an adaptive step size. Additionally, it is easy to use because of the few control parameters.

The starting population is generated randomly and should be sufficiently large for a high diversity. In this case a population size of $NP = 80$ is used. Standard settings are used for other control parameters [8]. Population members of the current population are combined using the evolutionary operators mutation, recombination and selection to generate the next generation until a stopping condition is fulfilled. In this case the distribution of population members is monitored and the algorithm terminates when the maximum distance of every individual to the best population member is beneath a threshold [9]. In Section 6 the number of generations is restricted to 1000 due to limited computational resources.

5.3 Optimization Results

The performance improvement by power optimization is illustrated in Figure 6 where the transfer functions for a system with load $\beta = 4$ at $E_b/N_0 = 7$ dB with equal and optimized power distribution are shown. For equal powers it will get stuck at $\eta \approx 0.1$ which is a loss of about 10 dB compared to the SUB whereas optimized power levels enable the detection to converge to the SUB. With equal powers only a load of ≈ 3 is possible. In Figure 7 it is shown that the weakest user has only a loss of 2.5 dB compared to the equal power case, so the remaining gain is 7.5 dB.
In Figure 8 a trajectory for optimized SIC is depicted;

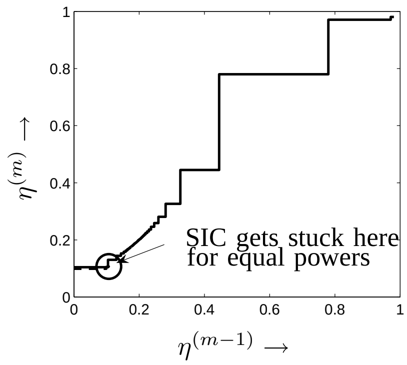

Fig. 8. Trajectory of SIC at $E_b/N_0 = 6.8\,\mathrm{dB}$ for equal and optimized power profile, $N = 4$ and $U = 16$

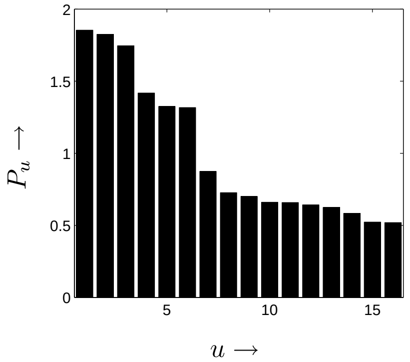

Fig. 9. Optimized power profile for SIC at $E_b/N_0 = 6.8\,\mathrm{dB}$, $N = 4$ and $U = 16$

Figure 9 shows the corresponding power profile. For the same system the results are very similar. If the tunnel of the transfer function is very narrow, the differences of the users are small and the SIC behaves more and more like the PIC. For that reason the optimized power levels are nearly the same, but the convergence speed is higher for SIC. To achieve an efficiency of $\eta = 0.98$ the PIC needs 53 iterations for this example, while the SIC requires only 35 iterations.

6 Additional Constraints

6.1 Limiting Maximum Number of Iterations

Considering latency, the number of iterations should be kept as small as possible. The channel decoding remains the most complex part although the efficient Max-Log-MAP is already applied. Up to now, the number of iterations needed for convergence was not taken into account for power profile optimization. If the receiver terminates the detection after an insufficient number of iterations, the detection did not yet converge and the loss compared to the SUB may be intolerably large. If the maximum number of iterations is introduced as an additional constraint, the minimized overall power is expected to be larger than without this constraint. In Figure 10 the transfer functions with optimized power distribution under different constraints

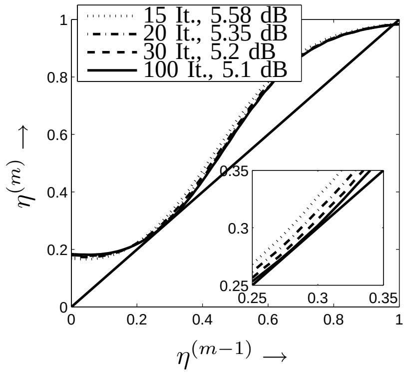

Fig. 10. Transfer function for PIC with different number of iterations, $\beta = 3$

concerning the maximum number of iterations are depicted and the corresponding average $\bar{E}_b/N_0$ are given in the legend. It can be seen that the transfer functions do not change significantly. The zoomed part in the lower right hand corner of this plot shows that the tunnel is getting wider and the corresponding $\bar{E}_b/N_0$ increases for a decreasing number of iterations. The interesting effect is that the transmit power has to be increased by only $\approx 0.5\,\mathrm{dB}$ to save about 85% of computational complexity if the power distribution is optimzed with adequate constraints. These results are obtained for a system with $N = 4$, $U = 12$ and a value of multi-user efficiency to be reached of $\eta_{max} = 0.98$. This value is chosen because the additional effort to reach $\eta = 1$ seems to be unjustified. In Figure 11

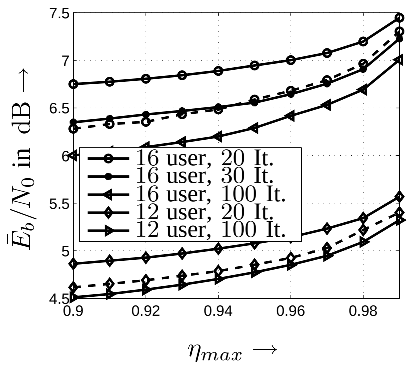

Fig. 11. E_b/N_0 versus maximum value for η for different system configurations of PIC (-) and SIC (- -)

the minimum $\bar{E}_b/N_0$ is plotted over η_{max} for different system configurations. The point corresponding to $\eta = 1$ is no longer depicted for the reason mentioned above. As expected the required power is increased for a higher load. Also the maximum number of iterations influences this parameter as already noted. It can be seen that SIC with optimized power profile outperforms the PIC with respect to the overall receive power.

6.2 Individual Power Constraint

Up to this point no individual power constraint was considered. All users are assumed to be able to transmit any desired power. In a cellular network this is not

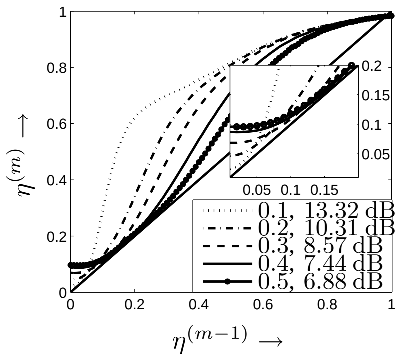

Fig. 12. Transfer functions for PIC with different individual power constraints, $\beta = 4$, 30 iterations

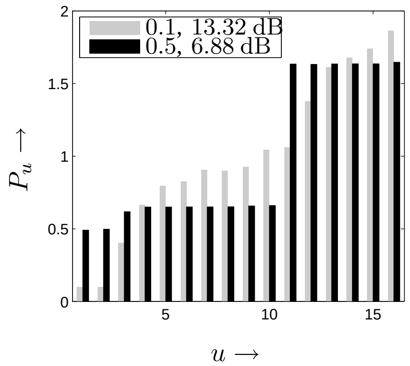

Fig. 13. Normalized power profiles with individual power constraints, $\beta = 4$, 30 iterations

realistic. Users at the cellborder may not be able to provide the transmit power to fulfill the requirements at the receiver. To handle this problem with the optimization tool, individual power constraints were implemented. To model this problem in a simple way two users were assumed to have a limited transmit power, which is prescribed in relation to the mean power of all users. For example the parameter $a = 0.1$ describes a system where two users are only able to provide 10% of the average receive power. These parameters can be adapted to any realistic scenario but for comparison this simple model is used. The results for different constraints are depicted in Figure 12. For $a = 0.5$ there is nearly no loss compared to the unconstrained case. As a decreases, the required overall power increases significantly. That means the receive powers of two users decrease but the other users have to increase power disproportionately high in order to be still able to detect all users. This behavior is depicted in Figure 13 for the case $a = 0.1$ and $a = 0.5$. The power profiles are normalized to emphasize the relative behavior of the users. To balance the weak users, mainly the moderate powers are increased.

7 Conclusion

In this paper power profile optimization is applied to iterative parallel and successive interference cancellation in a coded CDMA uplink system. Optimization is based on the prediction of convergence by multi-user efficiency and is carried out by Differential Evolution. It is shown that either the system load can be significantly increased or the required overall receive power is decreased by the use of an optimized power profile. To adapt this tool to a realistic scenario with limited computational capacity and limited transmit power of the mobile stations some additional constraints are introduced limiting the maximum number of iterations and the receive power of specific users assumed to be at the cell border. These additional constraints are at the cost of maximum system load or overall received power. But the cost is low compared with the savings of computational complexity. By an increase of the required overall power of only $0.5\,\mathrm{dB}$ the number of iterations can be changed from 100 to 15 which leads to a 85% reduction of the complexity of the detection scheme.

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