Power Optimization of IDMA Systems with Different Target BER Constraints

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***Abstract*— Interleave Division Multiple Access (IDMA) is a promising air interface for future wireless networks. Optimization of access schemes regarding cross user aspects is also a topic of great interest for increasing efficiency. This paper introduces an optimization of the power profile for iterative parallel and successive interference cancellation (PIC/SIC) in IDMA systems. The basic approach is an optimization algorithm called *Differential Evolution* (DE). Optimized power allocation is an important means to increase the supportable load as it enhances the convergence behavior of the detector. Furthermore any additional constraint regarding the realistic modeling of communication systems can be implemented easily. In this paper different bit error rate constraints are considered and also the maximum number of iterations is taken into account. Computational complexity is reduced while the required power is barely increased. The precondition for this optimization is the analysis of the iterative detection scheme. This is done by variance transfer charts (VTC).**

I. Introduction

Iterative multiuser detection is widely used in Code Division Multiple Access (CDMA) systems and has been improved during the last years. IDMA can be regarded as a special case of CDMA where the separation of the users is done by an interleaver and not by a spreading code [1]. To achieve a reliable transmission, low rate channel codes are implemented. IDMA has many advantages known from CDMA but some interesting differences. Interleaving is done on chip-level and therefore, the interleaver length is the spreading factor times higher than for CDMA. Turbo-like detection is applied exploiting the soft information at the channel decoder output to improve interference cancellation in an iterative manner. The channel decoder and the multi-user detector are a serially concatenated system. Variance analysis of PIC in CDMA systems has been done e.g. in [2] and can be easily applied to IDMA. Regarding successive interference cancellation this is more difficult because the statistics of the users differ from each other. A generalization of the analysis to SIC and the basics of optimized power allocation were presented in [3]. In this paper the uplink of a multiuser IDMA system will be investigated focusing on some additional aspects as e.g. individual BER constraints due to different applications like voice or data traffic. In order to limit computational complexity the power profile is optimized under the additional constraint of a maximum number of iterations. These aspects modeling a more realistic scenario have not been satisfactorily considered so far in literature. The paper is organized as follows: Section II introduces the system model of the considered IDMA system. In Section III the analysis based on variance transfer charts is described and applied to parallel and successive interference cancellation. Power profile optimization is introduced in Section IV for the two detection schemes and corresponding simulation results are shown in Section V. A conclusion is given in Section VI.

II. System Model

We consider the uplink of an IDMA system with U synchronous users transmitting over an AWGN channel with real valued noise $n(k)$ of variance σ_n^2 as shown in Figure 1.

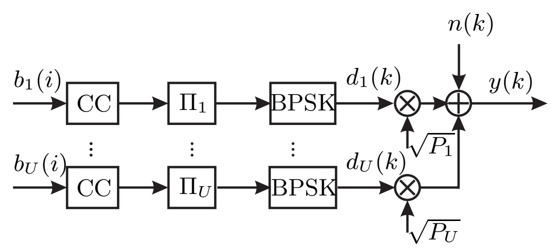

Fig. 1. Transmitter structure of an IDMA uplink system

The information bits $b_u(i)$ of user u are channel encoded, interleaved by a user-specific random interleaver Π_u and BPSK-modulated. The BPSK symbol of user u at time instant k is denoted by $d_u(k)$, so the received signal can be written as

$$y(k) = \sum_{u=1}^{U} \sqrt{P_u} \cdot d_u(k) + n(k) \qquad (1)$$

where P_u is the power allocated to the u-th user and the total power is $\sum_u P_u = P_{\text{tot}}$. The interleaver has to be different for each user to enable separation at the receiver. A low rate channel code is considered in order to support a large number of users. In this paper first a simple repetition code of rate R_c is considered. To show the possible coding gain of more powerful channel codes also a convolutional code of rate $1/2$ followed by a repetition code is investigated. The code rate of the repetition code in the latter case is chosen as $2 \cdot R_c$ to ensure a fair comparison. The analysis described in this paper can be easily applied to more complex codes like low rate convolutional codes (e.g. [4]).

Each user is corrupted by noise and the interference of all other users. This so-called multi-user interference (MUI) significantly degrades the performance even at low system loads

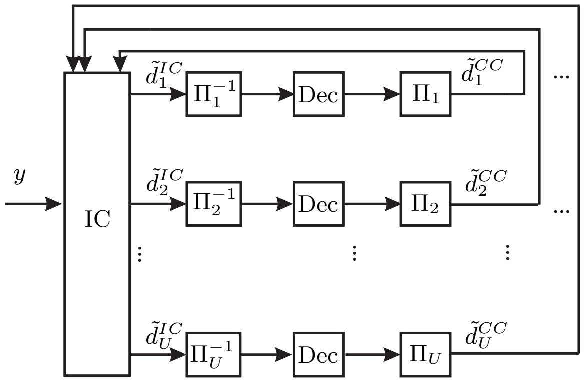

Fig. 2. Receiver structure of iterative parallel interference cancellation of an IDMA system

$\beta = U \cdot R_c$. As depicted in Figure 2 iterative soft interference cancellation is applied at the receiver [5] estimating the multiuser interference and canceling it before detection. The soft bit estimates after interference cancellation are denoted as $\tilde{d}_u^{IC}$. After soft output channel decoding the extrinsic soft bit estimates $\tilde{d}_u^{CC}$ are used as a-priori-information for the multiuser detector (MUD) in the next iteration. In this way, the reliability of the estimates can be improved in each iteration.

III. Analysis of MUD

Iterative detection schemes for serially concatenated systems can be analyzed by variance transfer charts (VTC). The relation between input and output error variance of each component is called transfer function. For the channel decoder Monte Carlo simulations may be necessary, but the transfer function of the MUD can be easily calculated. The effective variance σ^2_{eff} of the u-th user after interference cancellation is given by

$$\sigma^2_{\text{eff},u} = \text{E}\{|\sqrt{P_u} d_u - \tilde{d}_u^{IC}|^2\} = \sigma_n^2 + \sum_{v \neq u} P_v \, \sigma^2_{d,v} \qquad (2)$$

and the error variance at the output of the channel decoder is defined as

$$\sigma^2_{d,u} = \text{E}\{|d_u - \tilde{d}_u^{CC}|^2\} = f\left(\frac{\sigma^2_{\text{eff},u}}{P_u}\right) \qquad (3)$$

which is a function of the SINR at the channel decoders input denoted as $f(.)$. If all users have equal power $P_u = P$ the variances are the same for all users and (2) becomes

$$\sigma^2_{\text{eff}} = \sigma_n^2 + (U-1) \; P \; \sigma_d^2 \quad . \qquad (4)$$

Our aim is to find a definition of average variances to describe a system with unequal powers in a similar way. The average error variance at the output of the interference canceler equals

$$\begin{aligned} \bar{\sigma}^2_{\text{eff}} &= \frac{1}{U} \sum_u \sigma^2_{\text{eff},u} = \sigma_n^2 + \frac{U-1}{U} \sum_u P_u \; \sigma^2_{d,u} \\ &= \sigma_n^2 + (U-1) \; \bar{P} \; \bar{\sigma}_d^2 \end{aligned} \qquad (5)$$

with $\bar{P} = P_{\text{tot}}/U$ and

$$\bar{\sigma}_d^2 = \frac{1}{P_{\text{tot}}} \sum_u P_u \; \sigma^2_{d,u} \quad . \qquad (6)$$

This is analog to the definition for equal powers in (4). The error variance σ_d^2 at the decoder output depends only on the SINR at the input as stated in (3)

$$\begin{aligned} \sigma^2_{d,u} &= f\left(\frac{1}{\text{SINR}_u}\right) = f\left(\frac{\sigma^2_{\text{eff},u}}{P_u}\right) \\ &= f\left(\frac{\bar{\sigma}^2_{\text{eff}} + \bar{P} \; \bar{\sigma}_d^2 - P_u \; \sigma^2_{d,u}}{P_u}\right) \\ &\approx f\left(\frac{\bar{\sigma}^2_{\text{eff}}}{P_u}\right) \quad . \end{aligned} \qquad (7)$$

Assuming that $\bar{\sigma}^2_{\text{eff}} >> \bar{P} \; \bar{\sigma}_d^2 - P_u \; \sigma^2_{d,u}$ for a large number of users, the approximation in (7) is tolerable. Taking (6) and (7), $\bar{\sigma}_d^2$ can be approximately expressed in terms of $\bar{\sigma}^2_{\text{eff}}$ by

$$\begin{aligned} \bar{\sigma}_d^2 &= \frac{1}{P_{\text{tot}}} \sum_u P_u \; \sigma^2_{d,u} \\ &\approx \frac{1}{P_{\text{tot}}} \sum_u P_u \; f\left(\frac{\bar{\sigma}^2_{\text{eff}}}{P_u}\right) \quad . \end{aligned} \qquad (8)$$

The VTC of a PIC degrades to a two-dimensional problem analog to the equal power case as shown in Figure 3. The analysis of successive interference cancellation schemes was investigated for the first time in [3] for CDMA and can also be applied to IDMA. Averaging over all users at a particular iteration is no longer justified for SIC. Already the second user in the first iteration sees an improved interference situation. The variance of the remaining MUI of the u-th user in iteration m can be calculated by

$$\sigma^{2\,(m)}_{\text{eff},u} = \sigma_n^2 + \sum_{v=1}^{u-1} P_v \; \sigma^{2\,(m)}_{d,v} + \sum_{w=u+1}^{U} P_w \; \sigma^{2\,(m-1)}_{d,w} \qquad (9)$$

where the superscript (m) denotes the iteration index. For the u-th user the improved estimation of the previous users of the current iteration is already used. Therefore, each user has different variances in each iteration and the behavior can not be displayed in a two dimension diagram as for PIC. An alternative way to visualize the change of variances from iteration to iteration is to only focus on the user detected lastly. If this user reaches the SUP also all the other users are supposed to achieve SUP. Unfortunately, no reasonable transfer function can be calculated, because (9) depends on U-1 parameters of the other users and is therefore a (U-1)-dimensional function. Asymptotically SIC and PIC show the same performance if $m \to \infty$. In other words, the behavior of SIC only differs form PIC in terms of required number of iterations. In order to achieve single user performance it is necessary to have an open tunnel between the transfer functions of the MUD and the decoder like shown in Figure 3. If there is an intersection between these lines the detection will get stuck. Depending on the point of intersection this may result in a strong performance degradation. If all users have the same power, this will happen at moderate loads even in the noiseless case. To support higher loads unequal power distribution of the users is necessary, which is investigated in this paper.

IV. Optimization of power allocation

By optimizing the imbalance of the powers convergence to SUP can be achieved for much higher loads. The objective is to minimize the overall power P_{tot} and the optimized parameter is the power distribution. $\sigma^{2\,(\max)}_{\text{eff},u}$ is the effective variance of the u-th user after the maximum number of iterations. $\text{SINR}_{\min,u}$ is the minimum required SINR to satisfy the BER constraint of user u. In the case of a repetition code it can simply be calculated as $\text{SINR}_{\min,u} = 2\,R_c\,\text{erfc}^{-1}(2\,\text{BER}_{\max,u})^2$ but for e.g. convolutional codes this dependency is obtained by Monte Carlo simulations. $\text{BER}_{\max,u}$ may be different for some users due to different applications like voice or data transmission. The optimization problem can be written as

$$\min_{P_1..P_U} \sum_{u=1}^{U} P_u \quad \text{s.t.} \quad \frac{P_u}{\sigma^{2\,(\max)}_{\text{eff},u}} \geq \text{SINR}_{\text{min,u}}\;\forall\,u \quad . \tag{10}$$

A heuristic approach of using the width of the tunnel as a parameter for convergence speed was presented in [6]. Optimization was done with linear programming which is only able to consider transfer functions but not the corresponding trajectory. The number of iterations can not be calculated until the optimization run has finished.

To evaluate the constraint on the right side of (10) in each optimization step, the trajectories have to be calculated up to the maximum number of iterations. Optimization tools that only consider transfer functions are not suitable for this problem. In this paper optimization is done with a tool called differential evolution [7]. It is able to solve non-convex multi-modal problems and can handle arbitrary constraints. It has already been used for this purpose in [5], but we will use it more efficiently and make use of the advantages this tools offers: Not only PIC, but also SIC is considered with limited number of iterations in order to limit computational complexity. In this work, the constraint in (10) is based on the analysis with VTC.

A. Repetition Coded IDMA

In Figure 3 the result of this optimization for a system with PIC and 50 users is shown, which corresponds to a load of 2.5. The required BER for 15 of the users is 10^{-3} and 10^{-6} for the others. The maximum number of iterations was set to 20 for all users, which is a moderate detection complexity. In between the transfer functions the trajectory can be seen, which, in contrast to the transfer function, stems from the simulation of one block with 1000 information bits per user, where the two variances are measured in each iteration. It can be seen that with optimized power allocation the single user performance (SUP) is nearly reached. Additionally the transfer function for the same system with equally distributed powers and same average power is shown for comparison. The

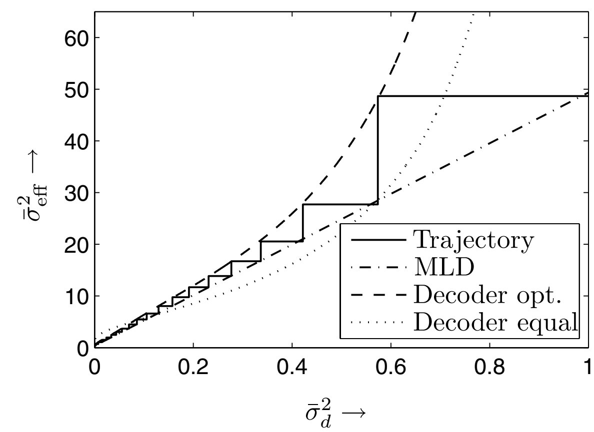

Fig. 3. Variance Transfer Chart for PIC, target BER 10^{-3} and 10^{-6} respectively, $\bar{P} \cdot R_c/\sigma_n^2 \approx 17.6$ dB, $\bar{P}$ is normalized to 1, 20 iterations

gain due to the optimized power distribution is obvious in this figure. As mentioned before SIC entails a faster convergence in terms of required iterations. In Figure 4 the trajectories for PIC and SIC are shown in one diagram using the same power profile optimized for PIC. In both cases the variances were averaged over all users for each iteration. Due to the decreasing error variances within one iteration the SIC outperforms the parallel scheme.

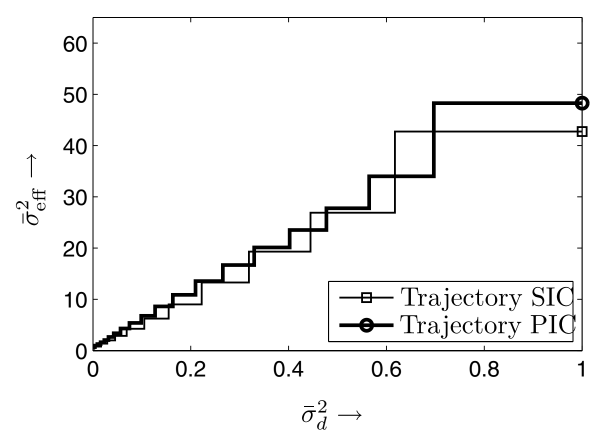

Fig. 4. Trajectories for SIC and PIC for the same system parameters

Following this result it is more suitable to optimize the SIC separately for a given number of iterations. The general description in (10) is also valid for SIC, the difference to PIC is just the evolution of $\sigma^{2\,(\max)}_{\text{eff},u}$. The resulting power profile is depicted in Figure 5 in addition to that one of PIC with same system parameters and also the mean values of these powers are shown. The saving in terms of average SNR equals approximately 1.41 dB.

B. Convolutional Coded IDMA

The waste of bandwidth by using simple repetition codes and, therefore, the benefits of exploiting coding gain were already shown in [4]. The results in Section IV-A are now extended to a convolutional coded IDMA system. A simple

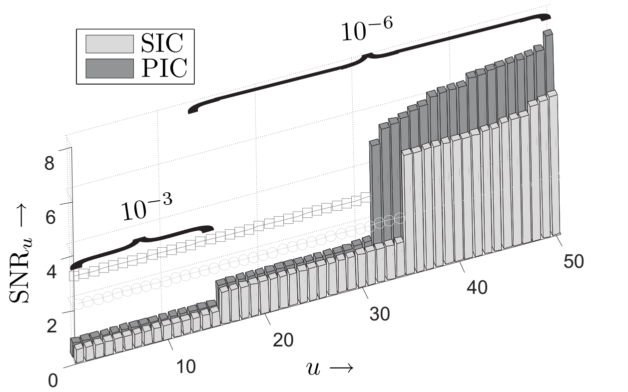

Fig. 5. SNR distributions for repetition code of rate 1/20, maximum BER= 10^{-3} and 10^{-6} respectively, 20 iterations

$[5,7]_o$ convolutional code followed by a rate $1/10$ repetition code is considered in this section. Of course there exist many stronger low rate codes than this, but we try to keep the system simple also in terms of decoding complexity. For example a low rate turbo code may be infeasible for an iterative MUD.

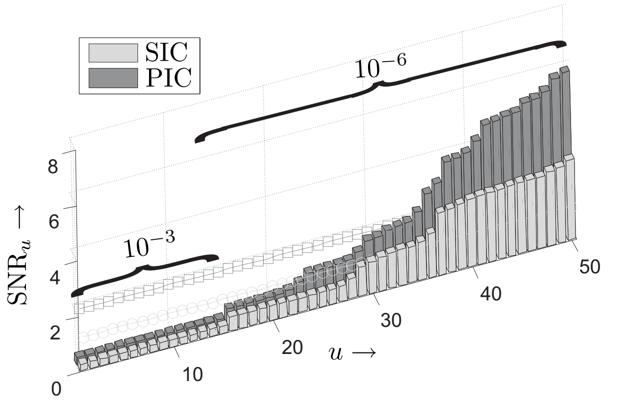

Fig. 6. SNR distributions for $[5,7]_o$ convolutional code and rate 1/10 repetition code, target BER= 10^{-3} and 10^{-6} respectively, 20 iterations

It is not surprising that this system outperforms the first one, but the intention of this Section is to shown the reliability of the proposed optimization scheme even for more complicated codes. The optimized power profiles for PIC and SIC are shown in Figure 6 and the corresponding trajectory for PIC can be seen in Figure 7. The overall required SNR is significantly decreased by $\approx 2\,\text{dB}$ only due to convolutional coding instead of repetition coding. This is due to the improved error performance of this code in high S(I)NR regions.

V. Verification by BER Simulations

The analysis derived in Sections III and IV will be verified here in terms of bit error rates obtained by Monte Carlo simulations. In Figure 8 the BER for PIC with optimized power profile and rate $1/20$ repetition code is shown. Due to the different power levels of all users also the bit error rates differ significantly. As mentioned before, convergence of all users is assumed if the weakest user reaches the SUP. Therefore this case is plotted in Figure 8.

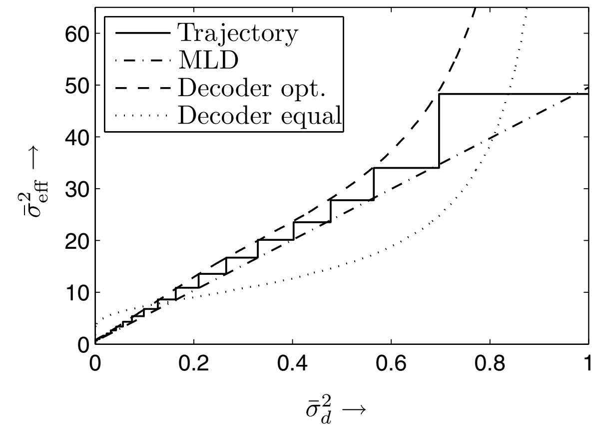

Fig. 7. Variance Transfer Chart for PIC with $[5,7]_o$ convolutional code and rate 1/10 repetition code, target BER 10^{-3} and 10^{-6} respectively, $\bar{P} \cdot R_c/\sigma_n^2 \approx 15.7$ dB, $\bar{P}$ is normalized to 1, 20 iterations

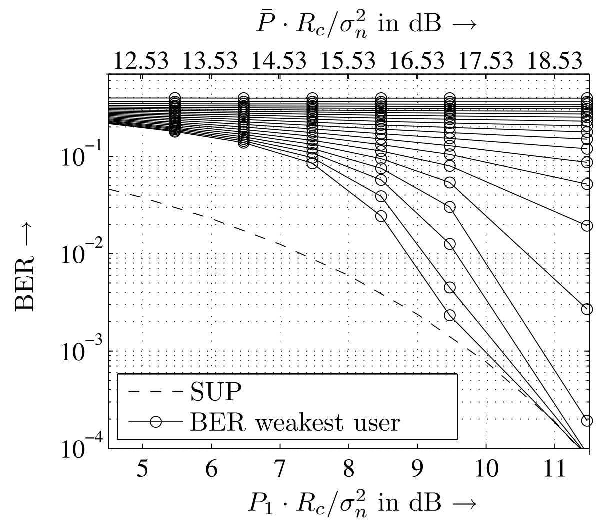

Fig. 8. Bit error rate of the weakest user for PIC using rate 1/20 repetition code

The lower axis labels the corresponding SNR $P_1 \cdot R_c/\sigma_n^2$ for the weakest user. The upper axis describes the mean value $\bar{P} \cdot R_c/\sigma_n^2$ over all users to be able to emphasize the power efficiency of the whole system. The optimization run delivered a SNR profile with $\bar{P}/\sigma_n^2 \approx 2.9$. This result can be interpreted as follows: At this particular value all users will satisfy the BER constraint of 10^{-3} for the first 15 users and 10^{-6} for the others. This can also be found in this plot. The weakest user nearly reaches the SUP at this value of $\bar{P} \cdot R_c/\sigma_n^2 \approx 17.6\,\text{dB}$ which corresponds to an individual $P_1 \cdot R_c/\sigma_n^2$ of $\approx 10\,\text{dB}$. All other users also reach the SUP at this point and therefore have a better or at least equal error rate. The results for SIC with similar system parameters and constraints is shown in Figure 9. Although the individual SNR is of course the same as for PIC as it is limited by the single-user performance, the upper axis points out the SNR gain of SIC with predefined number of iterations. In both cases a maximum number of 20 iterations are depicted as it was defined for the optimization. Figure 10

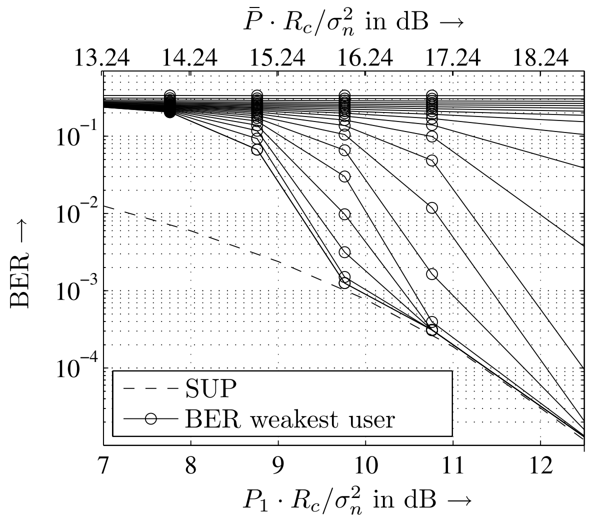

Fig. 9. Bit error rate of the weakest user for SIC using rate 1/20 repetition code

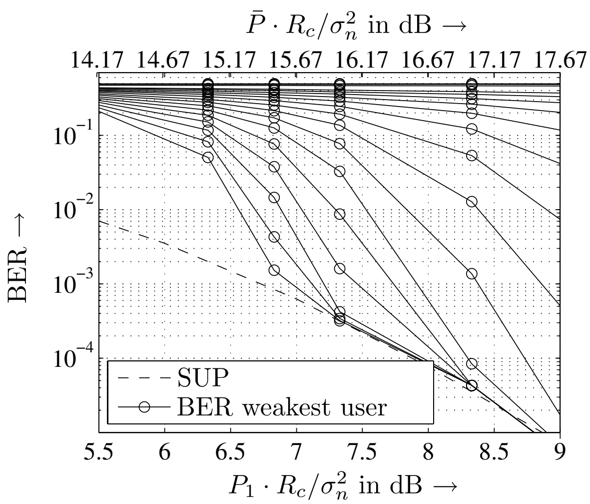

Fig. 10. Bit error rate of the weakest user for PIC using $[5,7]_o$ convolutional and rate 1/10 repetition code

confirms the coding gain of the system described in Section IV-B for PIC. All other system and optimization parameters are unchanged. In this case the target bit error rate of the weakest user of 10^{-3} is reached at an individual SNR $P_1 \cdot R_c/\sigma_n^2$ of $7\,\mathrm{dB}$ and an average SNR $\bar{P} \cdot R_c/\sigma_n^2$ of $\approx 15.7\,\mathrm{dB}$.

VI. Conclusions

In this paper a power profile optimization was applied to an IDMA system. The performance gain for PIC and SIC was shown with the help of variance transfer charts, which were also used as a basis for the optimization. Differential evolution was used to be able to take additional constraints like limited number of iterations into account. This corresponds to a direct control on the receiver complexity. To emulate heterogeneous applications in a wireless network different target BER constraints for the users were considered. Furthermore, the analysis was generalized to successive interference cancellation and corresponding trajectories for PIC and SIC were compared. When fixing the maximum number of iterations optimized SIC needs significantly reduced average power. The quality of analysis and optimization based on variance transfer charts were validated by Monte Carlo simulations of the bit error rate. The application of convolutional codes gave a hint to use stronger codes in IDMA systems to use the occupied bandwidth as good as possible.

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