Two-Dimensional (Recursive) Channel Equalization for a Multicarrier System with Soft Impulse Shaping (MCSIS)

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Abstract

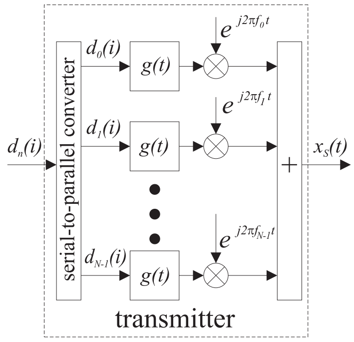
Multicarrier (MC) systems are attractive for the use in mobile radio environments, because they can handle the frequency selective effects better than single carrier (SC) systems. To be able to optimally exploit the diversity immanent in the transmission channels a multicarrier system with soft impulse shaping (MCSIS) has been introduced in previous works [**?**, **?**]. This system, that in itself carries intersymbol and adjacent channel interference (ISI and ACI), has so far been implemented with the suboptimal solution of a one-dimensional equalization in either time or frequency direction only. This paper now investigates the possibilities of using MCSIS with two-dimensional equalization structures, either implemented as a combination of two one-dimensional equalizers (pseudo two-dimensional equalization) or implemented with a two-dimensional, recursive equalizer.

Keywords: Multicarrier systems, optimal design, one- and two-dimensional Viterbi equalization

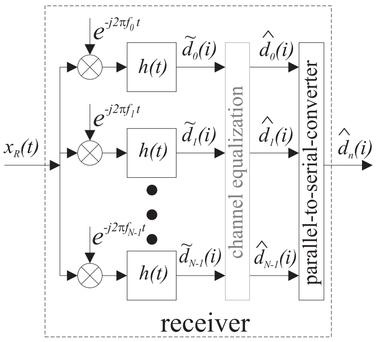
1 Introduction

In the evolving communication society mobile communication at high transmission rates has become a necessity. As ordinary single carrier systems are limited in handling multipath propagation of mobile radio channels, multicarrier systems have been drawn into the center of attention. Due to the separation of the data on N equally spaced subchannels, the symbol duration T becomes considerably longer and the channel echoes easier to correct with it. Figure 1 shows the general structure of MC-systems.

The design of MC-systems generally differs in the choice of filters $g(t)$ and $h(t)$. The criteria that in most cases backs this choice is to create an orthogonal (ISI- and ACI-free) system when transmitting over AWGN-channels (e.g. [**?**, **?**, **?**]). For the use in frequency selective environments this approach only makes sense if additional measures ensure the orthogonality. For the most well known MC-system OFDM, that uses rectangular filters for $g(t)$ and $h(t)$, the insertion of the guard interval has this purpose. The correction of the transmission channel's influence reduces for OFDM to a straight forward multiplication with a complex factor, that can be derived from the channel impulse response and the subchannel.



(a)



(b)

Fig. 1: Block diagram of a multicarrier system, (a): transmitter, (b): receiver

OFDM shows [**?**] the definite advantage of having an easy structure. But, as has been shown in [**?**], without further measures just this structure makes it impossible for OFDM to achieve a diversity gain.

When designing MCSIS the idea of creating an orthogonal system had been neglected completely. The intension was to make a system that has to be equalized in order to exploit the transmission channel's diversity. To ensure relative insensivity to distortions (like e.g. synchronization errors) the pulses with the smallest time-bandwidth-product, Gaussian impulses, were used for $g(t)$ and $h(t)$.

$$g(t) = h(t) = e^{-\alpha^2\left(\frac{t}{T}\right)^2} \text{ with } \alpha = \sqrt{\frac{2}{\ln(2)}}\,\pi f_{3dB}T \qquad (1)$$

The complexity of MCSIS can be kept small when implemented with polyphase filterbanks (see [?] and [?] for further details). Polyphase filterbanks are easy in structure and additionally allow variable subcarrier spacing[1] $\Delta F = r\,\frac{1}{T}$ with $r > 1$.

In the following Section 2 we will explain the two-dimensionality of the overall impulse response of MCSIS. In Section 3 the recursive two-dimensional and Section 4 the pseudo two-dimensional equalization structures are introduced and valued. In Section 5 we will draw the final conclusions.

2 The Two-Dimensional Problem

If we calculate for every subchannel m of MCSIS the received signal (for ideal transmission environments !), we receive the following equation.

$$\begin{aligned}\tilde{d}_m(i) &= \sum_{n=0}^{N-1} \left((d_n(t) * g(t))\, e^{j2\pi n\Delta Ft} \right) e^{-j2\pi m\Delta Ft} * h(t)\Big|_{t=iT} \\ &= \sum_{n=0}^{N-1} \sum_{\ell=-\infty}^{\infty} d_n(\ell)\, e^{j2\pi(n-m)\ell\Delta FT} \cdot \qquad (2) \\ &\qquad \underbrace{g(t-\ell T)\, e^{j2\pi(n-m)\Delta F(t-\ell T_S)} * h(t)}_{r_{n,m}(t-\ell T_S)}\Big|_{t=iT}\end{aligned}$$

with $r_{n,m}(t)$ the elementary impulse[2]:

$$r_{n,m}(t) = \begin{cases} e^{-\frac{1}{2}\frac{(\pi\Delta FT)^2}{\alpha}(n-m)^2} & \text{for } t = 0 \\ e^{-\frac{1}{2}\frac{(\pi\Delta FT)^2}{\alpha}(n-m)^2}\, e^{-\frac{\alpha^2}{2}\left(\frac{t}{T}\right)^2}\, e^{j(n-m)\pi\Delta Ft} & \text{else} \end{cases}$$

Taking that only the neighbouring subchannels and pulses have a noteworthy influence on the received signal (we used the pulses with the smallest time-bandwidth product) we can split $\tilde{d}_m(i)$ into the following influences: Wanted signal, ISI, ACI (and ISCI, intersymbol interference from adjacent subchannels; which is small enough to be neglected in the following discussion).

$$\begin{aligned}\tilde{d}_m(i) &= d_m(i)\, r_{m,m}(0) && \text{wanted signal} \\ &\quad + \sum_{\ell=-\infty,\ell\neq i}^{\infty} d_m(\ell) r_{m,m}((i-\ell)T) && \text{ISI} \\ &\quad + \sum_{n=0,n\neq m}^{N-1} d_n(i) e^{-j2\pi(n-m)\Delta FiT} r_{n,m}(0) && \text{ACI} \\ &= d_m(i)\, r_{m,m}(0) && \text{wanted signal} \\ &\quad + d_m(i-1)\, r_{m,m}(T) && \text{ISI} \\ &\quad + d_m(i+1)\, r_{m,m}(-T) && \text{ISI (3)} \\ &\quad + d_{m-1}(i)\, e^{-j2\pi\Delta FiT} r_{m-1,m}(0) && \text{ACI} \\ &\quad + d_{m+1}(i)\, e^{j2\pi\Delta FiT} r_{m+1,m}(0) && \text{ACI}\end{aligned}$$

For integer subcarrier spacing $\Delta FT = n, n\epsilon\mathbb{N}$ the rotation factor of the ACI $e^{-j2\pi\Delta FiT} = 1$ has no influence. Without loss of generality it will be neglected here. Figure 2 shows the transmission model affiliated to Equation 3.

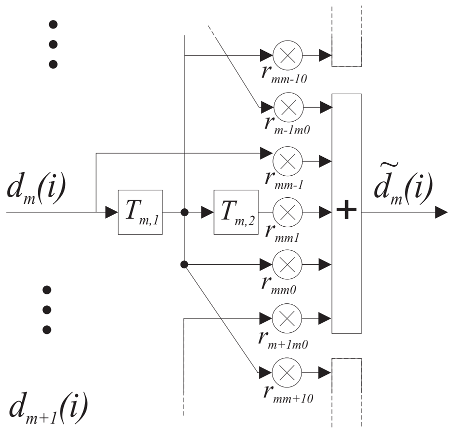


Fig. 2: Two-dimensional transmission model for MCSIS with $r_{nmi} = r_{n,m}(iT)$

The size of the interferences depends on the $f_{3dB}T$-rate chosen. As can be seen in Figure 3, we have for small $f_{3dB}T$-rates large ISI and small ACI and for large $f_{3dB}T$-rates small ISI and large ACI. With the choice of the "right" $f_{3dB}T$, the system can be implemented with a one-dimensional equalization in either time ($f_{3dB}T$ small) or frequency direction ($f_{3dB}T$ large) [?]. Even though it was shown that the interference power is not the only parameter that influences the performance of the system (another important parameter is the SNR-loss immanent in the Viterbi equalizer [?]) these solutions seem to be suboptimal, just because the overall interference power is not minimized. It therefore has to be investigated if a two-dimensional equalization for MCSIS, implemented with the $f_{3dB}T$-rate that causes the smallest interferences

$$f_{3dB}T_{opt} = \sqrt{\frac{\ln(2)}{2\pi}}\Delta FT, \qquad (4)$$

can further improve the performance.

It should be emphasized that for all MC-systems that need two-dimensional equalization in mobile radio environments (which most systems apart from OFDM do) similar functionalities are expected. The difference is that MCSIS can be very flexibly adapted to the surroundings with the variation of its

[1] For OFDM only subcarrier spacings of $\Delta F = n\,\frac{1}{T_{sub}}; n \in \mathbb{N}^+$ can be realized. The polyphase filterbank then reduces to a simple DFT [?].

[2] When transmitting over mobile radio channels the elementary impulse can be calculated including the channel impulse response $c(t)$: $r_{c,n,m}(t) = g(t)e^{j2\pi(n-m)\Delta Ft} * c(t)e^{-j2\pi m\Delta Ft} * h(t)$. For the sake of clarity $c(t)$ is not included at this point.

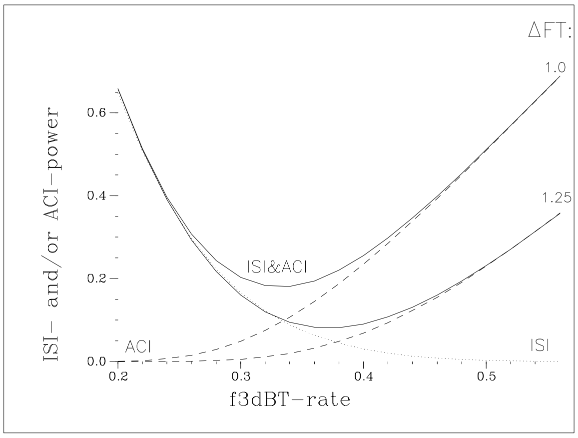


Fig. 3: ISI- and ACI-power within the MCSIS

$f_{3dB}T$-rate and the number of subcarriers N [**?**] and therefore does not *have* to be used with a two-dimensional equalization. Any two-dimensional problem can, without major changes be retransfered to a one-dimensional, if necessary.

3 Recursive Two-Dimensional Equalization

An expansion of the Viterbi algorithm to compensate two-dimensional distortions has been published in [**?**] for image restoration. The additional dimension of the MLSE-criterion (MAP-criterion in the case of [**?**]) was added to the contents of the components of the trellis (the trellis itself keeps its original two dimensions): The states contain matrices (the contents of the memories of *all* subchannels) instead of vectors and the transitions contain vectors (the possibly transmitted data on *all* subchannels) instead of single values.

states transitions states

1-dim :

$$\left[\; T_1\{\} \;\cdots\; T_{\ell_t}\text{-}1\{\} \;\right] \quad \xrightarrow{d(i)} \quad \left[\; T_1\{\} \;\cdots\; T_{\ell_t-1}\{\} \;\right]$$

2-dim :

$$\begin{bmatrix} T_{0,1}\{\}\cdots T_{0,\ell_t\text{-}1}\{\} \\ \vdots \quad \cdots \quad \vdots \\ T_{N\text{-}1,1}\{\}\cdots T_{N\text{-}1,\ell_t\text{-}1}\{\} \end{bmatrix} \xrightarrow{d_0(i)\cdots d_{N\text{-}1}(i)} \begin{bmatrix} T_{0,1}\{\}\cdots T_{0,\ell_t\text{-}1}\{\} \\ \vdots \quad \cdots \quad \vdots \\ T_{N\text{-}1,1}\{\}\cdots T_{N\text{-}1,\ell_t\text{-}1}\{\} \end{bmatrix}$$

with ℓ_t the length of the impulse response in time direction and $T_{n,i}\{\}$ the contents of the transmission memory as defined in Figure 2. For an M-ary transmission the number components consequently increases by the power of N: The number of states to $M^{(l_t-1)N}$, the number of transition from each state to M^N and the overall number of transitions to M^{l_tN}. Note that the length of the ACI ℓ_f has no impact on the layout of the Trellis. The ACI is only of interest for the calculation of the Eucledean distances of the transitions. Equation 5 shows the extended calculation of the distances $D_p(i)$ as used for MCSIS (only the neighbouring subchannels are affected).

$$\begin{aligned} D_p(i) &= D_{p'}(i\text{-}1) + \sum_{n=0}^{N} \Big|\tilde{d}_n(i) - \sum_{k=-1}^{1} d_{p'',n+k}(\ell) * r_{n+k,n}(\ell)|_i\Big|^2 \\ &= D_{p'}(i\text{-}1) + \sum_{n=0}^{N} \Big|\tilde{d}_n(i) - \\ &\qquad \big(\, d_{p'',n-1}(1)\, r_{n-1n0} \\ &\qquad +\, d_{p'',n}(i+1)\, r_{nn-1} \\ &\qquad +\, d_{p'',n}(i)\, r_{nn0} \\ &\qquad +\, d_{p'',n}(i-1)\, r_{nn1} \\ &\qquad +\, d_{p'',n+1}(i)\, r_{n+1n0}\big)\Big|^2 \end{aligned} \tag{5}$$

$p = 0\ldots M^{\ell_t N}\text{-}1,\, p' = 0\ldots M^{(\ell_t-1)N}\text{-}1,\, p'' = 0\ldots M^N\text{-}1$

In the steady state this algorithm will decide N values at once (one for every subchannel).

For in time and frequency direction causal channel impulse responses with no ISCI ($r_{n,m}(iT) = 0\ \forall\ (n \neq m$ and $i \neq 0)$ or $n > m$ or $i < 0$) a variation with a three dimensional trellis is possible. Here not only the contents of the states and transmissions of the trellis change their dimensions but also the states (dots become lines) and transitions (lines become surfaces) themselves. As this method cannot be used for MCSIS but is of interest none the less it will be introduced in the appendix of this paper. The number of states and transitions does not change with this method though.

As was shown, the complexity of the two-dimensional recursive equalization is tremendous. The following section will therefore investigate the possibilities of achieving a performance gain when using MCSIS with a combination of two one-dimensional equalizers.

4 Pseudo Two-Dimensional Equalization

An enormous reduction in complexity could be achieved if a recursive equalization was implemented in one direction (M^{ℓ_t} transitions only) and if the results were used in the other direction in a decision feed forward circuit. Meaning that to begin with data on the first subcarrier would be equalized and the thus decided data would be used to reduce the ACI on the second subcarrier. Then the data on the second subcarrier would be equalized and the decided data would be used to reduce the ACI on the third subcarrier and so on. This method, that has been investigated for a combined channel equalization and decoding in [**?**], can sensibly only be used if the ACI disturbs the neighbouring subchannels to one side only. It is therefore unusable for MCSIS as the latter is non-causal in frequency direction.

A concept that does work for non-causal impulse responses is shown in Figure 4. First an equalization in one (f.ex the frequency) direction is performed. The with these equalized data affiliated interferences (ACI) can be subtracted from the received data before it is equalized in the other direction. This concept works for systems that contain ISI and ACI only. ISCI (small for MCSIS, but maybe not for other systems) can not be compensated. The complexity of the system with $M^{\ell_f} + M^{\ell_t}$ transitions is considerably smaller than that of the recursive two-dimensional Viterbi.

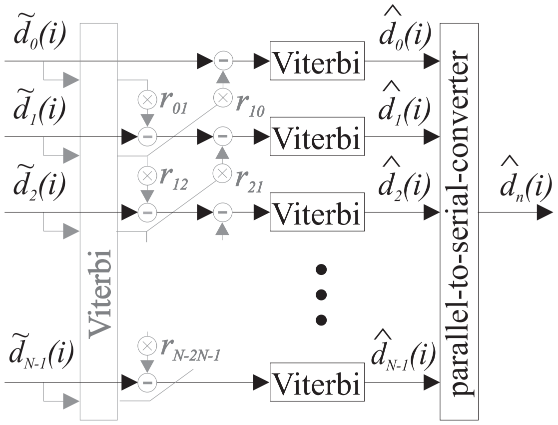


Fig. 4: Pseudo two-dimensional equalization structure; in black the time-direction equalization, in grey the frequency-direction components.

The data at the entrance of the time-direction Viterbi

$$\begin{aligned} \tilde{d}_m(i) \quad = \quad & d_m(i)\, r_{m,m}(0) \\ & + d_m(i-1)\, r_{m,m}(T) \\ & + d_m(i+1)\, r_{m,m}(-T) \\ & + \left(d_{m-1}(i) - \hat{d}_{m-1}(i)\right) r_{m-1,m}(0) \\ & + \left(d_{m+1}(i) - \hat{d}_{m+1}(i)\right) r_{m+1,m}(0) \end{aligned} \tag{6}$$

contains, a correct decision of the frequency-direction equalizer granted[3], no more ACI. This reduced interference can lead to an increase of performance. For the transmission over AWGN-channels though no additional information (in terms of diversity) is gained with this method. The performance of MCSIS in AWGN-environments therefore improves only slightly. In mobile radio envoronments the achieved gain is larger, but also here the SNR-loss due to the Viterbi has to be included twice and the gain remains fairly small. Figure 5 shows the performance results of MCSIS equalized with the pseudo two-dimensional equalization scheme.

5 Conclusion

In this paper two-dimensional equalization structures for a multicarrier with soft impulse shaping (MCSIS) were investigated. The possible methods were subdivided into pseudo two-dimensional (a combination of two one-dimensional equalizers) and recursive two-dimensional ones. A further distinction had to be made between methods that have no limitations concerning the channel impulse response and methods that can only be applied to impulse responses with certain characteristics. As the channel impulse response for MCSIS is

[3]The possibility of error propagation in the case of a wrong decision is one of the concept's major drawbacks.

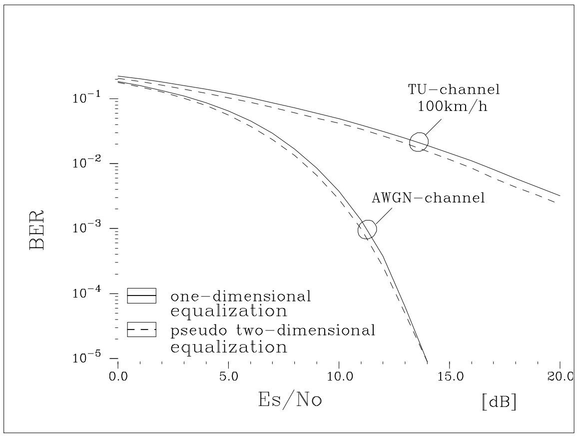


Fig. 5: Performance of MCSIS when pseudo two-dimensionally equalized.

non causal in frequency direction, only a recursive equalization adapted from [**?**] and one pseudo two-dimensional equalization were applied.

It was shown that the achievable performance gain of MCSIS with a pseudo two-dimensional equalization (Section 4) is limited. The reasons for this are that due to the possible error propagation and due to the double Viterbi loss applying this method is always a suboptimal solution. The recursive structures (Section 3) are being investigated but because of the enormous complexity simulation results could not be derived up to this point. In the final version of this paper it will be shown whether a significant gain is possible with the (optimal) recursive structure or not. It will remain questionable though, if the gain (if achieved) justifies the complexity.

6 Appendix

To illustrate this method that leads to a three-dimensional trellis the easiest form of a causal (in time and frequency direction) impulse response is used as an example channel:

$$\mathbf{r} = \left[\begin{array}{cc} r_{m,m}(0) & r_{m,m}(-T) \\ r_{m-1,m}(0) & 0 \end{array} \right]. \tag{7}$$

In this case not only memories for the ISI are used (in the transmission models marked with T) but also for the ACI. In Figure 6, that shows the transmission model for the impulse response of Equation 7, these ACI-memories are called ΔF.

Figure 7 shows the first few steps of how this trellis is used. The approach is via the diagonal axis. For the first transition surfaces only the possible distances for the first subchannel and timeslot are calculated. In the second step the two values sent in the second timeslot and first subchannel and the first timeslot and second subchannel are considered. In the steady state the contents of the state(-line)s are matrices and the contents of the transition(-surfaces)s are vectors, just like with the system explained in Section 3. Also the number of components stays the same. The routine that calculates the Eucledean distances is

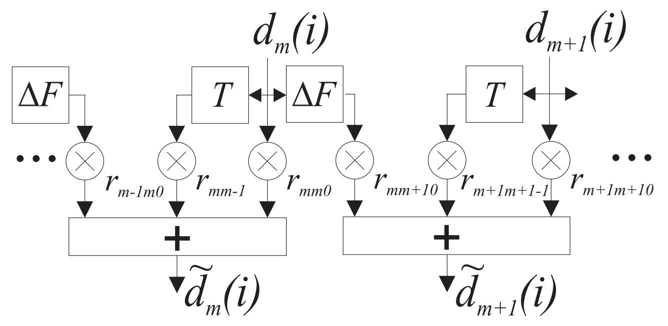


Fig. 6: Transmission model of the simple two-dimensional channel impulse response of Equation 7.

different:

$$
\begin{aligned}
D_p(i) = D_{p'}(i-1) + \sum_{n=0}^{N-1} \Big| \tilde{d}_n(i-n) - {}& \\
\big(d_{p'',n-1}(i-n+1)\, r_{n-1n0}& \\
+\, d_{p'',n}(i-n)\, r_{nn0}& \\
+\, d_{p'',n}(i-n-1)\, r_{nn1}\big)\Big|^2&
\end{aligned}
\tag{8}
$$

$$p = 0 \ldots M^{\ell_t N}\text{-}1, p' = 0 \ldots M^{(\ell_t-1)N}\text{-}1, p'' = 0 \ldots M^{N}\text{-}1$$

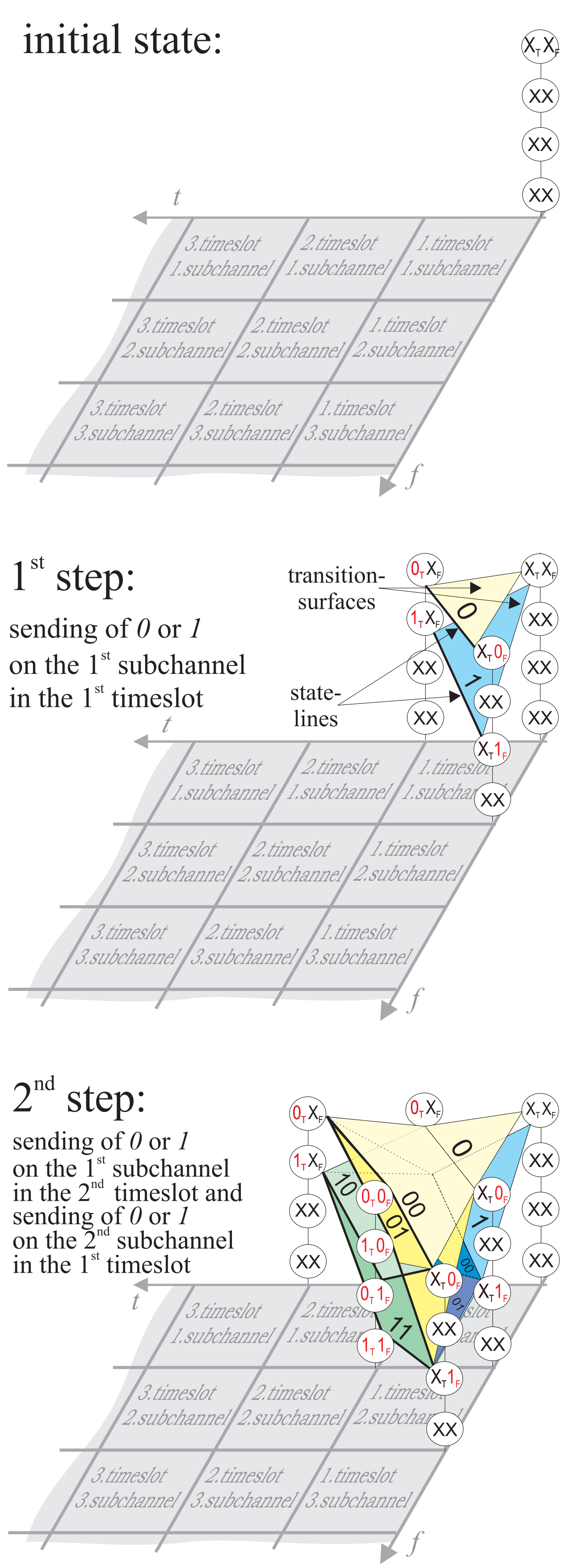


Fig. 7: First initializing steps for three-dimensional trellis.