Multi-User Detection for Coded Multirate OFDM-CDMA Systems

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Abstract — This paper analyzes the performance of multi-user detection (MUD) for a quasi-synchronous OFDM-CDMA uplink transmission. OFDM-CDMA has been chosen because it offers a great advantage over single carrier systems due to flat fading conditions on each subcarrier. This leads to much lower implementation costs of MUD techniques. In order to allow different data rates for each user, the CDMA spreading factors vary in the range of 4 up to 32. Specifically, we regard linear MUD techniques such as the MMSE approach as well as a combination with the nonlinear parallel interference cancellation (PIC) resulting in a recursive receiver structure.

Assuming perfectly known channel impulse responses for each user and a rough synchronization it turns out that decreasing the spreading factors and keeping the number of sub-carriers constant leads to a performance loss due to a smaller diversity gain of each user signal.

We will show that the effects of multi-user interference can be mitigated by applying a combination of MMSE and PIC. Thus the performance of the system can be improved.

Keywords — OFDM-CDMA, multi user detection, interference cancellation, multirate system

I. INTRODUCTION

Code Division Multiple Access (CDMA) has been chosen as multiple access technique for third generation mobile radio systems [1], [2], [3]. In this paper, the uplink of a multi-carrier CDMA (MC-CDMA) system [4], [5] is considered using OFDM (Orthogonal Frequency Division Multiplex) to combat frequency selectivity of the mobile radio channel. Therefore, each subcarrier is affected by flat fading and a one tap equalizer suffices for eliminating channel distortion.

In contrast to a synchronous downlink transmission where orthogonal spreading sequences with appropriate equalization techniques suppress multi-user interference (MUI) efficiently, this orthogonality would be destroyed in the asynchronous uplink. Therefore, pseudo-noise (PN) sequences are used and multi-user interference is the limiting factor concerning system capacity.

In order to achieve a large flexibility concerning data rates, variable spreading factors can be chosen by each user. This leads to a system with different processing gains and different levels of multi-user interference. In contrast to the UMTS (Universal Mobile Telecommunication System) standard where orthogonality is maintained for variable spreading factors by using OVSF (Orthogonal Variable Spreading Factor) codes we employ simple pseudo noise (PN) sequences and apply multi-user detection (MUD) techniques at the receiver.

In the last years, plenty of work has been spent on multi-user detection [6], [7], [8], [9], [10], [11]. Capacity bounds have been analytically derived for different MUD techniques indicating the maximum system load that should be reachable theoretically [8], [12], [13]. Furthermore, a lot of simulations have been carried out for single carrier systems operating in frequency nonselective and even frequency selective environments [14].

The aim of this paper is to analyze the performance of linear and nonlinear MUD for a coded multirate OFDM-CDMA system. One specific characteristic of OFDM-CDMA is the one-tap equalization due to flat fading on each subcarrier. This enables us to apply MUD algorithms developed for frequency nonselective channels saving valuable implementation costs when compared to frequency selective fading and single carrier systems. Specifically, the influence of different spreading factors is investigated and interference cancellation strategies are proposed.

The paper is structured as follows: Section 2 describes the OFDM-CDMA system with FEC coding, variable processing gains and single-user detection (SUD). Next, section 3 presents the considered MUD techniques, their application in a multirate OFDM-CDMA environment and discusses the obtained simulation results. Finally, section 4 gives some conclusions.

II. MULTIRATE SYSTEM

Figure 1 depicts the structure of a multirate OFDM-CDMA transmitter. In order to achieve a certain flexibility for user specific data rates $R_i^{(j)}$, the spreading factor $N_{SP}^{(j)}$ is adapted to the bandwidth demand of different users. As a constraint, the chip rate ($1/T_c$) and the number of carriers per OFDM symbol ($N_c = 64$) are kept constant for all users allowing an efficient realization of the receiver. Therefore, higher data rates correspond to lower processing gains and more information bits
per OFDM symbol and vice versa.

![OFDM transmitter diagram](image)

Fig. 1. Typical structure of an OFDM-CDMA transmitter

In general, user $j$ maps $w_j$ information bits onto one OFDM symbol. This leads to different bit rates at the input of the FEC encoders (CC) requiring different time variables. In order to keep the notation as simple as possible, we comprise all information bits of user $j$ associated with one OFDM symbol into the vector $d^{(j)}(k) = (d_1^{(j)}(k), \ldots, d_{w_j}^{(j)}(k))$ where $k$ indicates the OFDM symbol rate $1/T_w$.

Thus, vector $b^{(j)}(k) = (b_1^{(j)}(k), \ldots, b_{w_j \cdot n}^{(j)}(k))$ consists of $w_j$ successive $n$-bit code words of the convolutional code (CC) with fixed rate $R_c = 1/n$.

After encoding and BPSK modulation, the resulting vector $b^{(j)}(k)$ is individually spread by repeating each of the $w_j \cdot n$ coded bits $N_p^{(j)}$ times and successive multiplication with a user-specific signature code $c^{(j)}(k)$. Considering an asynchronous transmission in the uplink, we use simple pseudo-noise (PN) sequences for scrambling where the period of one sequence is much longer than $T_s$ (long codes). Due to the constraint of a fixed duration $T_s$ of a chip $c^{(j)}(k)$ the user specific processing gains are

$$G_p^{(j)} = n \cdot N_p^{(j)} = N_c \cdot \frac{T_s}{T_d^{(j)}}$$

(1)

Table I depicts the processing gains and the corresponding number of information bits mapped per OFDM symbol used in this paper.

<table>
<thead>
<tr>
<th>$w_j$</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>8</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_p^{(j)}$</td>
<td>64</td>
<td>32</td>
<td>16</td>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td>$N_p^{(j)}$</td>
<td>32</td>
<td>16</td>
<td>8</td>
<td>4</td>
<td>2</td>
</tr>
</tbody>
</table>

At the OFDM transmitter the spread signal $b^{(j)}(k)$ is first interleaved in the frequency domain by $\Pi_j$ and transformed into the time domain with the inverse Fourier transform (IFFT). Finally, a cyclic prefix of duration $T_{cy}$ called guard interval is inserted in front of each OFDM symbol.

The resulting signals $s^{(j)}(k)$ of different users are now transmitted over $J$ individual $L$-path mobile radio channels. Real and imaginary parts of the corresponding channel coefficients $h_{i,j}(k)$, $0 \leq l < L$, are Gaussian distributed and statistically independent. Although each user is assigned to an individual channel, the number of transmission paths $L$ is assumed to be the same for all users. The corresponding transfer function is defined by

$$H_{i,j}^{(j)}(k) = \sum_{l=0}^{L-1} h_l^{(j)}(k) \cdot e^{-j2\pi l \mu^j / L}, \quad 0 \leq \mu < N_c.$$  

(2)

At the OFDM receiver, the cyclic prefix is removed first (figure 2). A guard time $T_g$ larger than the delay-spread $\Delta t$ of the channel results in a cyclic convolution of channel impulse response $h^{(j)}(k)$ and transmitted signal $s^{(j)}(k)$. This allows an efficient transformation of the received signal back into the frequency domain by the fast Fourier transform (FFT). Assuming coarse synchronization, i.e., the maximum delay between different users is limited to $T_s - \Delta t$, one FFT window suffices for simultaneously transforming all user signals back into the frequency domain. The cyclic convolution in time domain corresponds to a scalar multiplication of $H_{i,j}^{(j)}(k)$ with the chips $b_{i,j}^{(j)}(k)$ in the frequency domain. Hence, this leads to an equivalent channel model where each chip is only affected by flat fading.

![OFDM receiver diagram](image)

Fig. 2. Single-user receiver for OFDM-CDMA

The received vector $r(k)$ at the output of the OFDM receiver at time instance $k$ consists of $N_c$ chips and can be expressed by

$$r(k) = A(k)b(k) + n(k)$$

(3)

where

$$b(k) = \left(b^{(1)}(k)^T, b^{(2)}(k)^T, \ldots, b^{(J)}(k)^T\right)^T$$

(4)

contains the convolutionally encoded bits of all users and $n(k)$ determines the background noise. The system matrix $A(k) = (A^{(1)}(k) \ldots A^{(J)}(k))$ comprises $J$ user specific matrices

$$A^{(j)}(k) = \begin{bmatrix} a_1^{(j)}(k) \\ \vdots \\ a_{w_j \cdot n}^{(j)}(k) \end{bmatrix}$$

(5)

where the column vectors have the form

$$a_i^{(j)}(k) = (a_{i,0}^{(j)}(k), \ldots, a_{i,N_p^{(j)}-1}^{(j)}(k))^T, \quad 1 \leq i \leq w_j \cdot n.$$  

Its elements

$$a_{i,v}^{(j)}(k) = \frac{e^{j2\pi v \mu^j / L}}{(i-1)N_p^{(j)}+v} H_{i,v}^{(j)} (i-1)N_p^{(j)}+v (k), \quad 0 \leq v < N_p^{(j)}$$
are element-wise products of the signature sequences \( e^{(j)}(k) \) and the channel transfer function \( H^{(j)}(k) \). The influence of the interleaver \( \Pi \) on the indices is neglected. The special form of \( A \) is caused by the specific mapping of the coded bits onto the OFDM symbols. Due to the fact that \( w_j \cdot n \) coded bits \( b^{(j)}(k) \) are mapped to one OFDM symbol, \( A \) is composed by \( n \sum_j w_j \) column vectors \( a^{(j)}(k) \).

The optimal single-user detection (SUD) employs a matched filter that maximizes the signal-to-noise ratio at its output. Presupposing perfectly known channel impulse responses, the equalizer \( E^{(j)}(k) \) for user \( j \) in Figure 2 then equals the hermitian form of \( A^{(j)}(k) \) and the input of the FEC decoder can be described by

\[
\hat{b}^{(j)}(k) = \text{Re} \left\{ E^{(j)}(k) \cdot r(k) \right\} = \text{Re} \left\{ \left[ A^{(j)}(k) \right]^H \cdot r(k) \right\}.
\]

Vector \( \hat{b}^{(j)}(k) \) at the FEC decoder input of user \( j \) consists of three parts: the desired coded information obtained by maximum ratio combining (MRC) \( N_p^{(j)} \) chips, the multiple access interference, and the contribution of the background noise. Assuming a chip-synchronous transmission, the signal-to-interference-plus-noise ratio (SINR) at the decoder input of a user with spreading factor \( N_p^{(j)} \) can be calculated by

\[
\text{SINR} = \frac{N_p^{(j)} E_s/N_0}{1 + (J - 1) E_s/N_0} = \frac{N_p^{(j)} (J - 1) + N_0/E_s}{N_0}. \tag{7}
\]

Equation (7) demonstrates that users with low processing gain and therefore small \( N_p^{(j)} \) suffer from severe MUI whereas users with large \( G_p^{(j)} \) get a better SINR at the decoder input. This has deep impact on possible multi-user detection strategy described in the next section.

### III. Multi-user Detection

#### A. Linear MUD techniques

Multi-user detection (MUD) schemes can be mainly divided into two groups, linear and nonlinear techniques [6]. Linear MUD schemes generally compute the pseudo-inverse \( A^H(k) \) of the system matrix \( A(k) \) in Equation (3) and thus perform a kind of equalization. It is necessary to make some comments on the calculation of the pseudo-inverse \( A^H(k) \).

The system matrix \( A(k) \) consists of \( N_c \) rows and \( n \sum_j w_j \) columns. Therefore, it describes a system of \( N_c \) linear equations with \( n \sum_j w_j \) unknown variables. If \( n \sum_j w_j \) is larger than \( N_c \) there are more unknown variables than equations and the linear equation system can only be solved with additional conditions. However, the pseudo-inverse always exists and tries to find an approximation of \( A^H(k) A(k) = I \) leading to an estimate

\[
\hat{b}(k) = \text{Re} \left\{ A^H(k) \cdot r(k) \right\} = \text{Re} \left\{ A^H(k) A(k) b(k) + A^H(k) n(k) \right\} \tag{8}
\]

with minimum energy. For the case \( n \sum_j w_j < N_c \), the pseudo-inverse has the form

\[
A^H(k) = (A^H(k) A(k) + \gamma I)^{-1} A^H(k) \tag{9}
\]

where \( \gamma = 0 \) indicates the ZF equalizer (decorrelator) and \( \gamma = \sigma_N^2 \) the MMSE solution. The term \( \sigma_N^2 \) represents the noise power [9]. For \( n \sum_j w_j > N_c \),

\[
A^H(k) = A^H(k) (A(k) A^H(k) + \gamma I)^{-1} \tag{10}
\]

holds. The MMSE approach with \( \gamma = \sigma_N^2 \) realizes a compromise between equalization, sufficiently decorrelating the interfering signals, and noise suppression. Generally, the linear MMSE equalizer provides a performance improvement even in the case of \( n \sum_j w_j > N_c \). However, with increasing \( \sum_j w_j \) the average processing gain decreases and the maximum system load will decrease.

Due to the fact that FEC decoding is carried out after linear filtering, channel state information (CSI)

\[
\text{CST}^{(j)}(k) = \frac{1}{N_p^{(j)}} \sum_{\mu=(i-1)N_p^{(j)}}^{iN_p^{(j)}-1} |H^{(j)}(k)|^2 \tag{11}
\]

with \( 1 \leq i \leq w_j n \) is provided to the FEC decoders. According to (11), each received coded bit \( \hat{b}^{(j)}(k) \) is weighted with the sum of squared magnitudes of the channel coefficients associated with its replicas [15].

Figure 3 shows the results for a convolutional code with \( L_c = 7 \), \( R_c = 1/2 \) and a MMSE receiver for different system loads. Obviously, the performance degrades for higher loads. However, even for \( J = G_p = 64 \) a slight improvement can be observed. This improvement is important for the combination of the MMSE receiver and interference cancellation schemes described below.

Concerning the application of the MMSE filter in multirate systems, we first analyze the effect of changing the overall processing gain. Figure 4 illustrates the average BER for the case that all users have the same processing gain \( G_p^{(j)} = G_p \). The load is assumed to be constant \( (J = G_p/2) \) and \( G_p \) is varying in the range between 8 and 64. As can be seen, the performance degrades for decreased processing gain and increased \( w_j \). This observation can be explained by different diversity degrees for different processing gains. We have to emphasize
that 4 channel taps in the time domain correspond to correlated coefficients in the frequency domain. For $G_p = 64$, $N_p = 32$ replicas are associated with each coded bit leading to a good averaging between good and bad coefficients. For $G_p = 8$, instead, only $N_p = 4$ replicas are distributed over correlated subcarriers resulting in much less diversity.

As an intermediate conclusion, we can state that adapting the processing gains in an OFDM-CDMA system for the purpose of multirate transmission causes performance degradations due to changing diversity degrees. Therefore, it might be advantageous to realize a multirate opportunity by assigning several spreading codes to a certain user and not to lower its processing gain. However, in order to illuminate the effects and the overall capacity for variable processing gains, we present further analysis below.

We consider two different transmit scenarios where different users obtain different processing gains. Scenario 1 includes $J = 8$ users with the configuration shown in Table II.

<table>
<thead>
<tr>
<th>Number of users</th>
<th>2</th>
<th>1</th>
<th>3</th>
<th>2</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_j$</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td>8</td>
<td>16</td>
</tr>
<tr>
<td>$G_p$</td>
<td>64</td>
<td>32</td>
<td>16</td>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td>$N_p$</td>
<td>32</td>
<td>16</td>
<td>8</td>
<td>4</td>
<td>2</td>
</tr>
</tbody>
</table>

Scenario 2 includes $J = 6$ users mapped to the different processing gains as shown in Table III.

<table>
<thead>
<tr>
<th>Number of users</th>
<th>2</th>
<th>1</th>
<th>1</th>
<th>1</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_j$</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td>8</td>
<td>16</td>
</tr>
<tr>
<td>$G_p$</td>
<td>64</td>
<td>32</td>
<td>16</td>
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<td>4</td>
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<tr>
<td>$N_p$</td>
<td>32</td>
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<td>8</td>
<td>4</td>
<td>2</td>
</tr>
</tbody>
</table>

Note that both scenarios possess the same spectral efficiency, i.e. a total of 32 information bits are mapped onto $J$ OFDM symbols. This implies that the performance of $J = 32$ and $G_p = 64$ from Figure 4 can be referred to as a reference system since it represents the same spectral efficiency. Figure 5 shows the performance of both multirate scenarios compared to the reference system (marked 'ref'). Users with the same number of information bits per OFDM symbol (i.e. bit rate) are grouped together. Those users with $w_j = 1$ perform better than the reference system because in the multirate systems there are less interferences. The performance loss of users with $w_j > 1$ is caused by a loss of diversity (as can be noticed by the slopes of the BER curves), a smaller $E_b/N_0$ and worse interference suppression. In this plot the abscissa represents the SNR on the channel.

All BERs of scenario 2 show a better performance compared to those of scenario 1 since the impact of MUI is less as it consists of only six users compared to eight in scenario 1.

BERs for $E_b/N_0$ for each user group are shown in Figure 6.

The performance of users with small processing gains is worse despite of applying a MMSE filter. The spread between the best and the worst users is larger than $6$ dB at a BER of $10^{-3}$.

These results imply that it seems to be better to have less users with higher data rates.

B. Parallel Interference Cancellation

Concerning nonlinear multi-user detection we consider the parallel interference cancellation (PIC) in this paper.
The PIC procedure is illustrated in figure 9 and can be described in the following way. After individual SUD for each user, Soft-In/Soft-Out decoders deliver estimated information bits \( \hat{d}^{(j)} \) as well as log-likelihood ratios \( L(\hat{b}^{(j)}) \) of the coded bits [16], [17]. Then, the expected values of \( L(\hat{b}^{(j)}) \) are calculated by the tank function. Finally, the reconstructed signal \( \tilde{r}^{(j)}(k) \) of user \( j \) is obtained by \( N_p \) fold repetition and scalar multiplication with the coefficients \( \epsilon^{(j)}_\mu = \epsilon^{(j)}_\mu : H^{(j)}_\mu(k) \). The sum

$$
\tilde{r}^{(j)}(k) = \sum_{\nu \neq j} \tilde{r}^{(\nu)}(k)
$$

over all interfering signals \( \tilde{r}^{(\nu)} \) regarding user \( j \) is now subtracted from the received signal \( r(k) \). In the absence of decoding errors, this difference is an estimate of the received signal of user \( j \) without any multi-user interference. Therefore, passing this signal through the single-user detector and the channel decoder a second time should yield the performance of the single-user case. Due to decoding errors, the procedure described above has to be repeated several times.

C. Combined MMSE and PIC

In [15], the advantage of enhancing the signal-to-interference ratio at the decoder input of the parallel interference cancellation by replacing the single-user detectors by one MMSE multi-user detector is described. However, the MMSE filter suffers from insufficient channel equalization and therefore the subsequent PIC loop would converge to a bad performance.

The principle drawback can be avoided when the MMSE filter is only used as a catalyst and the PIC loop directly processes the received vector \( r(k) \). Figure 10 depicts the corresponding realization. In a first stage, linear multi-user detection is carried out increasing the SNIR at the decoder inputs (switches in inner positions). After providing channel state information, single-user FEC decoding is performed, the signals \( \tilde{r}^{(j)}(k) \) of each user are reconstructed and summed up according to (12). Then, the interference is not subtracted from the output of the MMSE filter but directly from the received vector \( r(k) \) and \( J \) individual single user detectors are inserted. In a second stage, the switches are turned to the outer positions and the interference reduced signals are decoded again several times according to section III-B. Thus, the MMSE filter is only working during an initial phase in order to improve the SINR at the decoder inputs.

In this case we can achieve further performance improvement for both scenarios compared to sole linear MUD. The BERs for the SNR on the channel (refer to figure 5) are shown in figure 7. Figure 8 provides BERs over \( E_b/N_0 \). The reference system now shows the best performance. Differences between the usergroups with same \( w_j \) in system 1 and 2 are smaller due to the PIC iterations. The spread of \( E_b/N_0 \) required to achieve \( P_b = 10^{-3} \) is decreased from more than 6 dB down to 3 dB.

Considering the different \( E_b \) for each usergroup a successive interference cancellation could lead to even better performance of multirate systems.

IV. Conclusion

It has been shown that it is advantageous to realize multirate OFDM-CDMA systems by assigning several spreading codes to a user requesting higher bit rates rather than lowering the processing gain of the user. This reverse results obtained for multirate systems considering an AWGN channel[18]. The opposing conclusions can be explained by the performance degradation caused by a lowered diversity degree when reducing the processing gain and thus producing less replica of coded bits on correlated subcarriers.
By applying the MMSE-PIC multi-user detection performance of a multirate system can be improved. It might be advantageous to design a detection process that also includes a successive interference cancellation step. This could further mitigate the effects caused by multi-user interference and lead to a performance improvement.

References


Fig. 9. Principle structure of the parallel interference cancellation scheme

Fig. 10. Combination of linear MUD and parallel interference cancellation, PIC loop deals directly with received signals (MMSE-PIC)