On Carrier Frequency Offsets in Alamouti-coded OFDM systems similar to IEEE 802.11a

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***Abstract*— In the presented paper, the influence of Carrier Frequency Offsets (CFO) in OFDM systems with transmit diversity using the Alamouti coding scheme is investigated. The OFDM system parameters are choosen according to the WLAN standard IEEE 802.11a. Different methods for estimating the CFO based on a new, only slightly modificated IEEE 802.11a preamble are shown and the performance of these algorithms is compared to the single antenna case (SISO system). Alamouti coded OFDM systems are significantly more sensitive to carrier frequency offsets as we will show in our simulation results as well as in measurements in the 2.4 GHz ISM band.**

***Index Terms*— OFDM, IEEE 802.11a, Alamouti, transmit diversity, carrier frequency offset**

I. Introduction

Multiple Input Multiple Output (MIMO) systems are well known for increasing system capacity compared with Single Input Single Output (SISO) systems. If there is no possibility to use multiple receive antennas, one have to focus on transmit diversity schemes like space time block codes. Perhaps the most popular space time block code for two transmit antennas is the Alamouti scheme, which allows theoretically high gains in terms of bit error rate.

But in real transmission systems there are lots of effects due to nonidealities which decrease system performance, e.g. I/Q imbalances, DC offsets as well as sample and/or carrier frequency offsets. Proper estimation and correction of carrier frequency offsets is of great importance especially in OFDM systems.

For future extensions of the IEEE 802.11a standard with transmit diversity concepts, like Alamouti coding, the behaviour of these schemes under such effects has to be investigated.

Our paper focusses on the impact of the carrier frequency offset, which destroys the orthogonality of the Alamouti scheme and leads to intersymbol interference between the two alamouti coded symbols. This is shown mathematically and the results are compared with measurements in the 2.4 GHz ISM band.

Because a simple estimation of channel coefficients and carrier frequency offset based on the original IEEE 802.11a preamble is not possible in case of Alamouti coded signals, we present a new preamble structure, which is only slightly modificated compared to the standard. For estimating carrier frequency offsets, two methods are presented, one of them based on this new preamble structure. We will show in our simulation results, that even when using the new preamble for CFO estimation, which allows an estimation accuracy similar to the one in the SISO system, the loss due to estimation errors in the MISO system is higher than in the single transmit antenna system.

The paper is organized as follows: Section II deals with the mathematical formulation of carrier frequency offsets in Alamouti coded OFDM systems. Furthermore, we verify the results with measurements in the 2.4 GHz ISM band. In section III, our carrier frequency offset estimation methods are presented. Section IV shows some simulation results followed by a conclusion of the paper in section V.

II. Phase errors in Alamouti-coded OFDM systems

The transmit diversity scheme proposed by Alamouti [1] is based on systems containing only two transmit antennas. The data symbols s are divided into groups of two symbols each, s_1 and s_2. In consecutive time slots, one antenna transmits s_1 followed by s_2, while the second antenna transmits the symbols $-s_2^*$ and s_1^*. Assuming flat fading conditions, there are channel coefficients h_1 belonging to the first and h_2 to the second antenna, which we assume be constant for the duration of at least two data symbols. Defining the data symbol vector $\mathbf{s} = [s_1\ s_2^*]^T$ as well as the receive vector $\mathbf{r} = [r_1\ r_2^*]^T$, $\mathbf{r}$ becomes

$$\mathbf{r} = \mathbf{H}\mathbf{s} + \mathbf{n} \tag{1}$$

with the channel matrix

$$\mathbf{H} = \begin{bmatrix} h_1 & -h_2 \\ h_2^* & h_1^* \end{bmatrix} \tag{2}$$

and the AWGN vector $\mathbf{n}$.

Decoding of the received symbols is done by multiplying $\mathbf{r}$ with the estimated hermitian channel matrix $\hat{\mathbf{H}}^H$

$$\tilde{\mathbf{r}} = \hat{\mathbf{H}}^H\mathbf{H}\mathbf{s} + \hat{\mathbf{H}}^H\mathbf{n} \tag{3}$$

which is

$$\begin{bmatrix} \tilde{r}_1 \\ \tilde{r}_2^* \end{bmatrix} = \begin{bmatrix} |h_1|^2 + |h_2|^2 & 0 \\ 0 & |h_1|^2 + |h_2|^2 \end{bmatrix} \mathbf{s} + \mathbf{H}^H\mathbf{n}, \tag{4}$$

if the channel estimation is correct ($\hat{\mathbf{H}} = \mathbf{H}$).

In case of OFDM transmission the data symbols s become OFDM symbols in frequency domain, i. e. they consist of multiple (number of subcarriers) PSK or QAM symbols each. To be correct, all variables describing the Alamouti scheme would need a second index for the considered subcarrier, which is neglected here for simplification. Alamouti coding takes place before processing the IFFT in the transmitter, and Alamouti decoding after applying the FFT in the receiver. The block diagram of an OFDM transmitter using the Alamouti scheme is shown in Fig. 1.

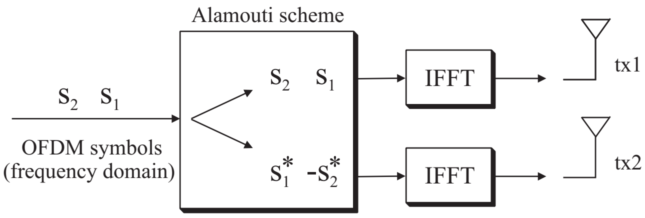

Fig. 1. OFDM transmitter with Alamouti coding

When deriving the influences of phase errors (or carrier frequency offsets, CFOs) in Alamouti coded systems, in the following we assume a noise free transmission, i. e. $\mathbf{n} = \mathbf{0}$.

The phase error is modelled by the multiplication of the receive symbols r_1 and r_2 with the phasors $e^{j\varphi_1}$ and $e^{j\varphi_2}$, respectively. Of course, φ_2 can be expressed by $\varphi_2 = \varphi_1 + \Delta\varphi$, with $\Delta\varphi$ being the phase deviation between the phase error at time of r_1 and the phase error at time of r_2, so that $\Delta\varphi$ is proportional to the frequency deviation, if the phase errors results from a CFO. The received symbols including phase errors become

$$\begin{aligned} r_1' &= (h_1 s_1 - h_2 s_2^*)e^{j\varphi_1} \\ r_2' &= (h_2 s_1^* + h_1 s_2)e^{j\varphi_2}. \end{aligned} \tag{5}$$

Assuming a correct channel estimation, i.e. $\hat{\mathbf{H}} = \mathbf{H}$, we get

$$\begin{aligned} \tilde{r}_1' &= h_1^* r_1' + h_2 r_2'^* \\ &= (|h_1|^2 e^{j\varphi_1} + |h_2|^2 e^{-j\varphi_2}) \cdot s_1 \\ &+ \underbrace{h_1^* h_2 (e^{-j\varphi_2} - e^{j\varphi_1}) \cdot s_2^*}_{\text{ISI}} \end{aligned} \tag{6}$$

and

$$\begin{aligned} \tilde{r}_2' &= -h_2 r_1'^* + h_1^* r_2' \\ &= (|h_1|^2 e^{j\varphi_2} + |h_2|^2 e^{-j\varphi_1}) \cdot s_2 \\ &+ \underbrace{h_1^* h_2 (e^{j\varphi_2} - e^{-j\varphi_1}) \cdot s_1^*}_{\text{ISI}} \end{aligned} \tag{7}$$

after Alamouti decoding. Obviously, *spatial intersymbol interference* (ISI) between the symbols s_1 and s_2 occurs in case of phase errors. We can show, that this not only leads to phase deviations, i.e. to a rotation of the signal constellation, but also to magnitude deviations. Setting all channel coefficients to one, (6) becomes

$$\tilde{r}_1' = r_1' + r_2'^* = (s_1 - s_2^*)e^{j\varphi_1} + (s_1 + s_2^*)e^{-j\varphi_2} \tag{8}$$

This is demonstrated in Fig. 2, where the signal constellation diagrams are shown for a IEEE802.11a system with Alamouti coding, carrier frequency offset (CFO) of 300 Hz (uncorrected) and QPSK modulation. Each diagram contains 54 OFDM symbols. The addition of r_1' (Fig. 2a) and $r_2'^*$ (Fig. 2b) leads to crosses, see Fig. 2c.

To verify this results, we did some measurements in the 2.4 GHz ISM band, using the MIMO transmission system built at the University of Bremen.

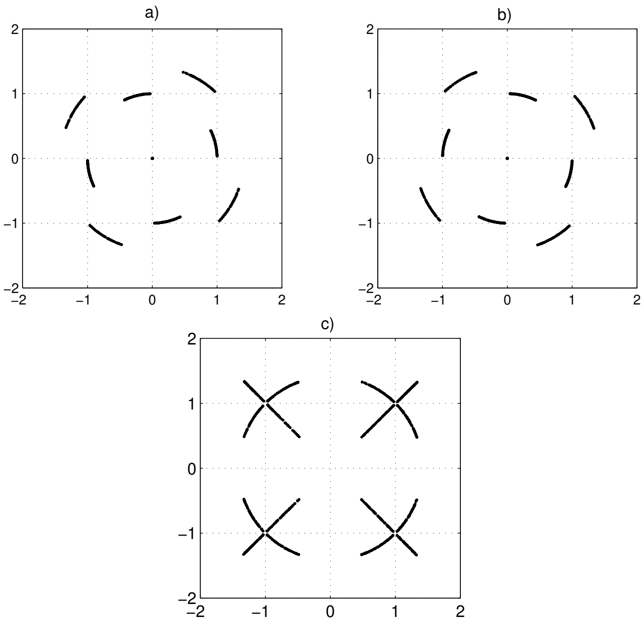

Fig. 2. Signal constellation diagrams for QPSK with CFO: a) $r'_1 = (s_1 - s_2^*)e^{j\varphi_1}$ b) $r'^*_2 = (s_1 + s_2^*)e^{-j\varphi_2}$ c) $r'_1 + r'^*_2$

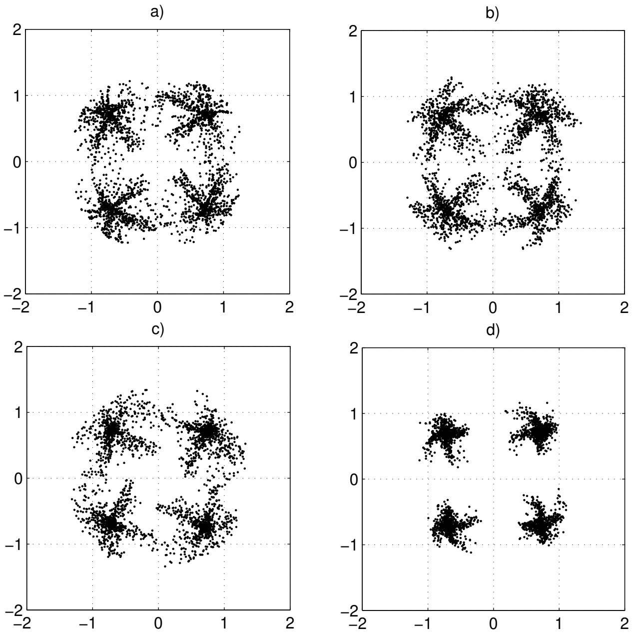

Fig. 3. Signal constellation diagrams for QPSK with CFO: a) simulated b) - d) measured

A burst of 54 QPSK modulated OFDM symbols according to IEEE 802.11a (except the carrier frequency) was sent. In Fig. 3b the signal constellation after Alamouti decoding is depicted.

The crosses known from Fig. 2c are clearly visible, deviations mainly result out of the channel influence, which is not considered in Fig. 2. For direct comparison, Fig. 3a shows an example obtained by simulation including channel influence. Fig. 3c and d are further measurement results. Due to the time varying channel the signal constellations are slightly different compared with Fig. 3b, but the crosses are also visible.

III. Estimation of Carrier Frequency Offsets in IEEE802.11a with Alamouti Coding

Since an IEEE 802.11a system with transmit diversity according to Alamouti is not conformable to the standard at all, there are lots of possibilities creating preambles for estimating the channel coefficients and the CFO. But here, we restrict ourselves to solutions, which only need relatively small changes compared with the original IEEE 802.11a preamble. This preamble contains 2 identical BPSK modulated C symbols, which normally (i.e. without transmit diversity) are used for channel and CFO estimation. For the detailed preamble structure see [5]. In case of transmit diversity, an easy solution for channel estimation is including the C symbols in the Alamouti coding scheme, so that the channel coefficients for each subcarrier can be calculated with a simple linear combination

$$\begin{aligned} \hat{h}_1 &= \pm 0.5 \cdot (r_1 + r_2) \\ \hat{h}_2 &= \pm 0.5 \cdot (r_2 - r_1). \end{aligned} \tag{9}$$

The sign depends on the sign of the known BPSK symbol on every subcarrier. Of course, there are some drawbacks. A second long guard intervall between the two C symbols is needed and no averaging of the two C symbols in the receiver to reduce noise influences can be done. An important point is, that the C symbols cannot be used for estimating the carrier frequency offset any more.

A. Coarse CFO estimation in time domain

One solution is to estimate the CFO based on the preamble B symbols. Therefore, it is important that both antennas are sending the same symbols, i.e. the B symbols are not Alamouti coded. The estimation algorithm itself is well known. For each sample of a B symbol in time domain the difference between the instantaneous phases of consecutive B symbols can be calculated and averaged. The phase difference $\Delta\varphi$ is directly proportional to the CFO Δf

$$\Delta\varphi = 2\pi \frac{\Delta f}{f_S} \cdot pL_B, \qquad p \in \mathcal{N} \tag{10}$$

where f_S denotes the FFT sampling frequency, L_B the length of one B symbol (i.e. 16 sample in IEEE

802.11a) and p is a integer factor (p=1, if consecutive symbols are considered). A different way of using B symbols for CFO estimation is described in [3].

Our second approach modifies the C part of the preamble by inserting another C symbol, which is not Alamouti coded, so that an estimation of the CFO based on the first two C symbols is possible, later on the last two symbols can be used for channel estimation. The new preamble structure (only C part) is depicted in Fig.4.

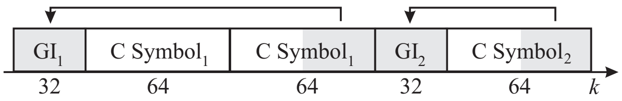

Fig. 4. Extended C preamble structure

The $\mathrm{C\ symbols}$ are all identical considering the first antenna, at the second antenna $\mathrm{C\ symbol}_2$ equals $\mathrm{C\ symbol}_1$ with negative sign. An advantage of the second approach is the possibility of averaging two C symbols, which increases the channel estimation quality due to the noise reduction. The estimation itself is unchanged compared to the single transmit antenna case. The CFO can be calculated according to (10), where L_B has to be replaced by the length of the C symbol and the factor p is set to 1.

B. Fine CFO estimation in frequency domain

The fine CFO estimation in frequency domain is based on the pilot carriers and remains almost unchanged compared with the single antenna system, if the pilot carriers are not transmitted Alamouti coded. For a detailed description, see [6]. Here, it is important to replace the channel coefficients of the single transmit antenna case with the *sum* of the two channel coefficients of the Alamouti coded system.

IV. Simulation results

Simulations with parameters according to IEEE 802.11a were carried out to evaluate the performance of the presented methods for estimating a carrier frequency offset. The results are compared with a single transmit antenna system.

In our simulations, we assume totally independent channels in case of the two transmit antenna system (Alamouti coding). It is important to note, that the overall transmit power is kept constant when increasing the system to two transmit antennas. The channel impulse responses (HIPERLAN/2, type A, see [4]) are timeinvariant within the bursts, but different for each burst. The burst length is choosen to 36 OFDM symbols in the 54 Mbit/s mode and 54 OFDM symbols in the 12 Mbit/s mode. Of course, the simulations include channel estimation as well as channel coding.

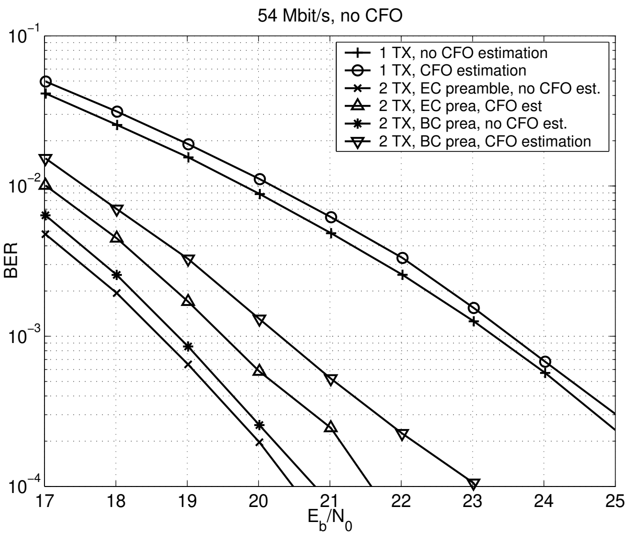

Fig. 5. Bit error rates for the 54 Mbit/s mode

Fig. 5 depicts the bit error rates (BER) in the 54 Mbit/s mode (64 QAM) for the Alamouti coded transmit diversity system (2TX) compared with the single transmit antenna system (1TX). The carrier frequency offset was set to zero, so that the loss due to the CFO estimation is visible. In the Alamouti coded system, the extended C ('EC') preamble is used when estimating the CFO based on the C symbols, the BC preamble when estimating it using only the B symbols like described before. In the latter case, 10 B symbols are included in the estimation.

There are three reference curves, denoting the BER without any CFO estimation. Because of the noise reduction in the channel estimation due to the averaging of the first two C symbol (see Fig. 4), the BER of the two transmit antenna system with EC preamble is slightly lower compared with the system using the BC preamble. Furthermore, it is visible that the CFO estimation based on the C symbols (EC prea) performs much better than that based on the B symbols: The loss due to the CFO estimation reduces from approx. 1.5 dB to approx. 1 dB at a BER of 10^{-3}.

A very interesting fact can be seen, if the loss due to the CFO estimation in the two transmit antenna system is compared with that in the single transmit antenna system.

Although the estimation algorithm is exactly the same (for the system with EC preamble), the loss is approx. 0.5 dB higher in the two antenna case. This denotes the sensitivity of Alamouti coding to phase errors caused by the loss of orthogonality, as described in section II.

To ensure this results, additional simulations were carried out (see Fig. 6). Here, the 12 Mbit/s mode of IEEE 802.11a (QPSK) is considered. The results are in principle the same as in the 54 Mbit/s mode.

While in the single transmit antenna system the loss due to the CFO estimation is approx. 0.5 dB, it is approx. 1 dB in the two transmit antenna case (at a BER of $2 \cdot 10^{-3}$).

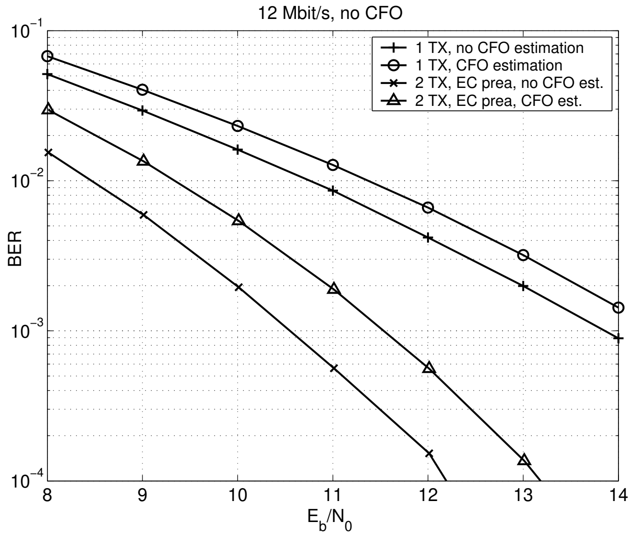

Fig. 6. Bit error rates for the 12 Mbit/s mode

V. Conclusions

In this paper, the influences of carrier frequency offsets, or, more general, phase errors including phase jitter and phase noise, on an IEEE 802.11a OFDM system extended by a transmit diversity scheme according to Alamouti were discussed.

We illustrate the effects of CFOs by means of signal constellation diagrams and verify the theoretical results with measurements in the 2.4 GHz ISM band.

The mathematical formulation reveals, that the orthogonality of the Alamouti coding scheme is lost, which makes it more sensitive to phase errors compared to a single transmit antenna system. This was shown by means of bit error rate simulations.

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