Estimation and Correction of transmitter-caused I/Q Imbalance in OFDM Systems

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Abstract— **In the presented paper a new method of estimating and correcting transmitter-caused I/Q imbalance in a transmission system based on the WLAN standard IEEE802.11a is proposed. The estimation process consists of an initial estimation followed by an iterative tracking of the I/Q imbalance parameters. Additionally, with the modified preamble structure presented in this paper a further improvement in estimation quality can be achieved. At the receiver the I/Q imbalance correction based on the estimated parameters leads to a significant performance gain as we will show in our simulation results.**

Index Terms— **OFDM, IEEE802.11a, I/Q imbalance, modified preamble**

I. Introduction

The application of direct conversion concepts in transmitter or receiver structures, especially for wireless LAN (WLAN) transmission systems in the 5 GHz band, has several advantages over using a heterodyne receiver with image-rejection, e.g. no image rejection filter is required and the expensive surface acoustic wave (SAW) filter of the intermediate-frequency (IF) stage is replaced with a low pass filter [1]. However, serious problems can occur when directly downconverting the signal from radio frequency (RF) to base band. For instance, tolerances in parts of the direct conversion front end can cause amplitude and phase mismatch between the inphase (I) and quadrature phase (Q) branch of the signal. This effect, called I/Q imbalance, leads to an incorrect channel estimation and equalization in the receiver reducing the overall performance of the system significantly.

This paper focuses on the estimation and correction of transmitter-caused I/Q imbalance in a WLAN system in conformity with IEEE802.11a. From the receiver's point of view, the I/Q imbalance produced by the receiver itself is assumed time invariant and can always be corrected with a constant parameter set. In contrast, the properties of the transmitter-caused I/Q imbalance change completely when communicating with a different transmitter. Thus a separate estimation of the parameters is necessary with at least every new connection.

To achieve an improved system performance it is essential that the I/Q imbalance parameters are accurately estimated as early as possible within the actual burst. Therefore we propose an estimation method consisting of two steps: The first step is an initial estimation based on the received pilot subcarriers at the transition from the preamble to the data part of a burst. The second step represents the tracking of the parameters in combination with a block-by-block channel estimation over the burst to improve the accuracy of the initially estimated parameters. In case of frequency selective transmitter-caused I/Q imbalance we propose a slight modification of the preamble structure defined in [2]. This leads to a further gain in estimation accuracy and overall system performance.

The paper is organized as follows: Section II deals with the mathematical formulation of transmitter-caused I/Q imbalance. In section III we present the new initial estimation, tracking, and correction methods and propose a modified preamble structure for improved estimation accuracy. Section IV shows some simulation results followed by a conclusion of the paper in section V.

II. Transmitter-caused I/Q imbalance

In case of a direct conversion transmitter with ideal components, the I and Q branches are perfectly matched.

The resulting bandpass signal at the carrier frequency ω_0 can be described by

$$\begin{aligned} s_{BP}(t) &= \mathrm{Re}\left\{s_T(t)e^{j\omega_0 t}\right\} \\ &= s_{I,T}(t)\cos(\omega_0 t) - s_{Q,T}(t)\sin(\omega_0 t) \end{aligned} \tag{1}$$

with $s_T(t)$ as the transmitted baseband signal, as well as $s_{I,T}(t)$ and $s_{Q,T}(t)$ as the inphase and quadrature phase components respectively. As a result of inaccuracies in parts of the transmitter, I/Q imbalance can occur, which means that a relative amplitude deviation of $2\,\varepsilon$ and/or a phase shift of $\Delta\varphi$ between the I and Q branch lead to a distorted transmit signal:

$$\begin{aligned} s'_{BP}(t) &= (1+\varepsilon)\cdot s_{I,T}(t)\cdot\cos(\omega_0 t + \Delta\varphi/2) \\ &\quad -(1-\varepsilon)\cdot s_{Q,T}(t)\cdot\sin(\omega_0 t - \Delta\varphi/2) \end{aligned} \tag{2}$$

Assuming an ideal transmission channel the received signal after quadrature amplitude demodulation can be written as

$$\begin{aligned} s'_R(t) &= s'_{BP}(t)\,e^{-j\omega_0 t} \\ &= s'_{BP}(t)\cos(\omega_0 t) - j\cdot s'_{BP}(t)\sin(\omega_0 t) \\ &= s'_{I,R}(t) + j\cdot s'_{Q,R}(t) \end{aligned} \tag{3}$$

with $s'_{I,R}(t)$ and $s'_{Q,R}(t)$ as the inphase and quadrature phase components of this signal respectively. Using some theorems for trigonometric functions it can be shown that

$$\begin{aligned} s'_{I,R}(t) &= \tfrac{1+\varepsilon}{2}\cdot\cos(\Delta\varphi/2)\cdot s_{I,T}(t) \\ &\quad +\tfrac{1-\varepsilon}{2}\cdot\sin(\Delta\varphi/2)\cdot s_{Q,T}(t) \end{aligned} \tag{4}$$

and

$$\begin{aligned} s'_{Q,R}(t) &= \tfrac{1+\varepsilon}{2}\cdot\sin(\Delta\varphi/2)\cdot s_{I,T}(t) \\ &\quad +\tfrac{1-\varepsilon}{2}\cdot\cos(\Delta\varphi/2)\cdot s_{Q,T}(t)\ , \end{aligned} \tag{5}$$

assuming that the receive filters have eliminated the parts of the received signal at the frequency $2\,\omega_0$.

To get a more compact expression, (4) and (5) can be combined to

$$\begin{aligned} \begin{bmatrix} s'_{I,R}(t) \\ s'_{Q,R}(t) \end{bmatrix} &= \begin{bmatrix} \frac{1+\varepsilon}{2}\cdot\alpha & \frac{1-\varepsilon}{2}\cdot\beta \\ \frac{1+\varepsilon}{2}\cdot\beta & \frac{1-\varepsilon}{2}\cdot\alpha \end{bmatrix}\cdot\begin{bmatrix} s_{I,T}(t) \\ s_{Q,T}(t) \end{bmatrix} \\ &= \mathbf{\Phi}\cdot\begin{bmatrix} s_{I,T}(t) \\ s_{Q,T}(t) \end{bmatrix} \end{aligned} \tag{6}$$

with $\alpha = \cos(\Delta\varphi/2)$, $\beta = \sin(\Delta\varphi/2)$, and the distortion matrix $\mathbf{\Phi}$, which represents the influence of I/Q imbalance on the original transmit signal.

In the following, an equivalent expression for (6) in frequency domain will be determined. Therefore, (3) is arranged as[1]

$$\begin{aligned} s'_R(t) &= s'_{I,R}(t) + j\cdot s'_{Q,R}(t) \\ &= \alpha\cdot(1{+}\varepsilon)\cdot s_{I,T}(t) + \beta\cdot(1{-}\varepsilon)\cdot s_{Q,T}(t) \\ &\quad +\ j\beta\cdot(1{+}\varepsilon)\cdot s_{I,T}(t) + j\alpha\cdot(1{-}\varepsilon)\cdot s_{Q,T}(t) \end{aligned} \tag{7}$$

so that after some resorting we obtain the expression

$$\begin{aligned} s'_R(t) &= \left[(\alpha + j\varepsilon\beta)\cdot(s_{I,T}(t) + js_{Q,T}(t))\right] \\ &\quad +\ \left[(\varepsilon\alpha + j\beta)\cdot(s_{I,T}(t) - js_{Q,T}(t))\right]\ . \end{aligned} \tag{8}$$

Following [4], the imbalanced receive signal $s'_R(t)$ can be expressed by a linear combination of the desired signal $s_T(t)$ and its conjugate complex counterpart $s^*_T(t)$:

$$s'_R(t) = a\cdot s_T(t) + b\cdot s^*_T(t) \tag{9}$$

where the weighted, complex coefficients

$$a = \alpha + j\,\varepsilon\,\beta \quad\text{and}\quad b = \varepsilon\,\alpha + j\,\beta \tag{10}$$

represent the influence of the transmitter-caused I/Q imbalance.

To obtain the imbalanced receive symbols $\hat{d}'_n(t)$ in frequency domain, a N-point discrete Fourier transform (DFT) is applied to the time discrete representation of (9):

$$\begin{aligned} \hat{d}'_n(t) &= \mathrm{DFT}_N\left\{s'_R(i,k)\right\} \\ &= \mathrm{DFT}_N\left\{a\cdot s_T(i,k) + b\cdot s^*_T(i,k)\right\} \\ &= a\cdot d_n(i) + b\cdot d^*_{-n}(i);\ \ n = 1,...,N_a/2 \end{aligned} \tag{11}$$

with $d_n(i)$ as the transmitted symbol on the OFDM subcarrier with index n, N_a as the number of active subcarriers, and $k = (t - i\cdot T_S)/T$ as time index within the i-th OFDM symbol of the duration T_S.

The result of (11) shows that transmitter-caused I/Q imbalance leads to interference between the symbols on opposite subcarriers. The effects on the corresponding pairs of transmitted symbols can be expressed by a matrix $\mathbf{\Psi}$ as follows:

$$\begin{bmatrix} \hat{d}'_n(i) \\ \hat{d}'^*_{-n}(i) \end{bmatrix} = \underbrace{\begin{bmatrix} a & b \\ b^* & a^* \end{bmatrix}}_{\mathbf{\Psi}}\begin{bmatrix} d_n(i) \\ d^*_{-n}(i) \end{bmatrix} \tag{12}$$

[1]The weighting factor 1/2 in (4) and (5) is only a result of the baseband transform and will be disregarded in the following.

where $\hat{d}'_n(i)$ and $\hat{d}'_{-n}(i)$ represent the imbalanced receive symbols respectively.

It can be shown, that if there's a deviation between the transmit filters in the I and Q branch, the I/Q imbalance becomes frequency selective, which means that the coefficients of $\mathbf{\Psi}$ are dependent on the particular subcarrier index n. Furthermore, in case of a non-ideal transmission channel with a channel impulse response duration shorter than that of the OFDM cyclic prefix, so that neither interchannel (ICI) nor intersymbol interference (ISI) occur, the received symbols on pairs of opposite subcarriers can be described by

$$\begin{bmatrix} \hat{d}'_n(i) \\ \hat{d}'^*_{-n}(i) \end{bmatrix} = \begin{bmatrix} C_n & 0 \\ 0 & C^*_{-n} \end{bmatrix} \cdot \begin{bmatrix} a_n & b_n \\ b^*_{-n} & a^*_{-n} \end{bmatrix} \cdot \cdot \begin{bmatrix} d_n(i) \\ d^*_{-n}(i) \end{bmatrix} + \begin{bmatrix} \tilde{n}_n(i) \\ \tilde{n}^*_{-n}(i) \end{bmatrix} \tag{13}$$

with C_n as the frequency domain channel coefficient of the n-th sub channel and $\tilde{n}_n(i)$ as additive white Gaussian noise colored by the receive filters and DFT. Due to the fact that $a_n \approx 1$ in general and the coefficient itself can be interpreted as a complex weighting factor similar to the channel coefficient for that subcarrier, we assume that channel estimation together with equalization compensate it's influence almost completely. Thus, in the following we set $a_n = 1$, which proves to be a good approximation in practice.

III. Estimation and Correction Procedures

The initial estimation procedure of transmitter-caused I/Q imbalance described in the following is partially based on an approach presented in [4]. To simplify matters, we disregard any additive noise and assume BPSK (Binary Phase Shift Keying) modulated symbols, i.e. $d_n(i) \in \{-1; 1\}$. Starting with expression (13), we get the equations

$$\begin{aligned} \hat{d}'_n(i) &= C_n \cdot d_n(i) + C_n \cdot b_n \cdot d^*_{-n}(i) \\ \hat{d}'^*_{-n}(i) &= C^*_{-n} \cdot b^*_{-n} \cdot d_n(i) + C^*_{-n} \cdot d^*_{-n}(i) \ . \end{aligned} \tag{14}$$

Provided that the transmission channel and the I/Q imbalance are supposed to be approximately time invariant for the duration of at least two OFDM symbols, we also can write

$$\begin{aligned} \hat{d}'_n(i-1) &= C_n \cdot d_n(i-1) + C_n \cdot b_n \cdot d^*_{-n}(i-1) \\ \hat{d}'^*_{-n}(i-1) &= C^*_{-n} \cdot b^*_{-n} \cdot d_n(i-1) + C^*_{-n} \cdot d^*_{-n}(i-1). \end{aligned} \tag{15}$$

In case that on one subcarrier the sign of a transmit symbol d_n changes from one OFDM symbol period to the following, and the sign for the opposite subcarrier doesn't, i.e.

$$\begin{gathered} d_n(i) = d_n(i-1) \quad \wedge \quad d_{-n}(i) = -d_{-n}(i-1) \\ \text{or} \\ d_n(i) = -d_n(i-1) \quad \wedge \quad d_{-n}(i) = d_{-n}(i-1) \ , \end{gathered} \tag{16}$$

the equations in (14) and (15) can be combined for the estimation of b_n. If for example $d_n(i) = -d_n(i-1)$ and $d_{-n}(i) = d_{-n}(i-1)$, then after some algebraic manipulations we obtain

$$\begin{aligned} \hat{d}'_n(i) + \hat{d}'_n(i-1) &= 2 \cdot C_n \cdot b_n \cdot d^*_{-n}(i) \\ \hat{d}'_n(i) - \hat{d}'_n(i-1) &= 2 \cdot C_n \cdot d_n(i) \\ \hat{d}'^*_{-n}(i) + \hat{d}'^*_{-n}(i-1) &= 2 \cdot C^*_{-n} \cdot d_{-n}(i) \\ \hat{d}'^*_{-n}(i) - \hat{d}'^*_{-n}(i-1) &= 2 \cdot C^*_{-n} \cdot b^*_{-n} \cdot d^*_n(i) \end{aligned} \tag{17}$$

which provides two independent conditional equations for the searched imbalance coefficients b_n and b_{-n}:

$$b_n = \frac{\hat{d}'_n(i) + \hat{d}'_n(i-1)}{\hat{d}'_n(i) - \hat{d}'_n(i-1)} \cdot \frac{d_n(i)}{d^*_{-n}(i)} \tag{18}$$

$$b_{-n} = \frac{\hat{d}'_{-n}(i) - \hat{d}'_{-n}(i-1)}{\hat{d}'_{-n}(i) - \hat{d}'_{-n}(i-1)} \cdot \frac{d_{-n}(i)}{d^*_n(i)} \tag{19}$$

Surveying the WLAN standard IEEE802.11a, BPSK reference symbols fulfilling condition (16) can be found on the pilot subcarriers ($n = \pm 7$ and $n = \pm 21$) at the transition from the preamble to the header and following data part. Thus, the proposed initial estimation procedure comprises the calculation of (18) and (19) for the pilot subcarriers at the beginning of each burst. In case of frequency selective I/Q imbalance, the coefficients b_n on the remaining subcarriers can be approximated by linear interpolation. To improve the estimation accuracy, it is possible to average the symbols on the pilot subcarriers in time, e.g. the two OFDM symbols of the preamble and up to four symbols after the preamble, before calculating the corresponding coefficients b_n.

After the determination of b_n for all subcarriers, the I/Q imbalance correction of the received, channel equalized symbols $\tilde{d}'_n(i)$ is performed by multiplying with the inverse of $\mathbf{\Psi}$, i.e

$$\begin{bmatrix} \tilde{d}_n(i) \\ \tilde{d}^*_{-n}(i) \end{bmatrix} = \mathbf{\Psi}^{-1} \cdot \begin{bmatrix} \tilde{d}'_n(i) \\ \tilde{d}'^*_{-n}(i) \end{bmatrix} . \tag{20}$$

If the accuracy of the initial estimation suffers from additive noise or the I/Q imbalance shows a time variant behavior, additional gain can be obtained by the tracking procedure described below.

Solving the equations in (14) for b_n and b_{-n} respectively leads to the conditional equations

$$b_n = \frac{\hat{d}'_n(i) - C_n \cdot d_n(i)}{C_n \cdot d^*_{-n}(i)} \tag{21}$$

$$b_{-n} = \frac{\hat{d}'_{-n}(i) - C_{-n} \cdot d_{-n}(i)}{C_{-n} \cdot d^*_n(i)} \, . \tag{22}$$

So, for the determination of b_n and b_{-n} the knowledge of the corresponding channel coefficients as well as the transmitted symbols is necessary. Because only the symbols of the preamble are known to the receiver, 'reference symbols' for the following symbol periods can be substituted by the decided receive symbols obtained by an iteration process which is specified below. The necessary channel coefficients are substituted by values of the preceding iteration of that process.

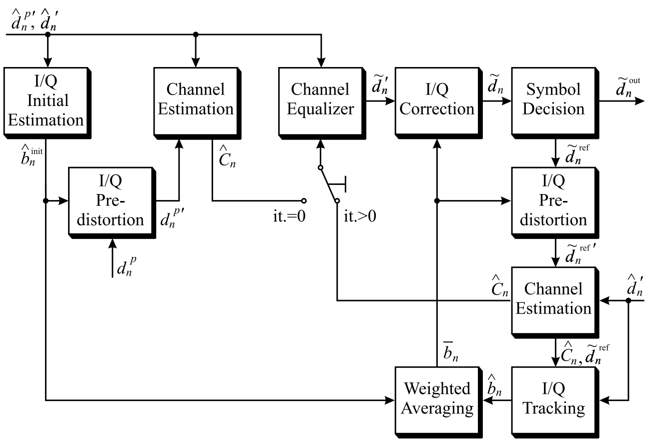

Fig. 1. Iteration process with initial estimation and tracking

An overview of the proposed iteration process can be seen in fig. 1. At the beginning of each new burst an initial estimation of the I/Q mismatch based on the received preamble and one or more of the following data symbols is performed. The result $\hat{b}_n^{init}$ is used to predistort the known preamble symbols d_n^p, so that in the subsequent channel estimation the deviation caused by I/Q imbalance can be minimized. In the next step, an iteration loop with block by block processing of the received OFDM symbols is entered. The first block of receive symbols $\hat{d}'_n$ passes through the channel equalizer and gets I/Q corrected by the initially estimated coefficients $\hat{b}_n^{init}$. Afterwards, a symbol decision is performed which provides the new 'reference symbols' $\tilde{d}_n^{ref}$ for the actual iteration block. These symbols - after an I/Q predistortion - represent the reference for the subsequent channel estimation based on the received OFDM symbols of the actual block. With the determined reference symbols $\tilde{d}_n^{ref}$ and channel coefficients $\hat{C}_n$ an estimation of $\hat{b}_n$, based on equation (21) and (22), is performed to track the I/Q imbalance in time. For the following iterations, the parameters $\bar{b}_n$ are calculated, which represent the weighted average of the recent estimated I/Q imbalance coefficients and those of the actual iteration. To further improve the performance of the iteration process, it is possible to pass several iterations with the same block of receive symbols. However, in practice this can result in undesirable time delays.

For an efficient I/Q imbalance estimation and correction it is necessary, that the initial estimation procedure provides results as accurate as possible. Only then, a fast adjustment of the tracking process is guaranteed. But especially in the case of frequency selective I/Q imbalance the four pilot carriers evaluated by the initial estimation don't provide enough information to reach a good estimation quality for all active subcarriers if the signal-to-noise ratio (SNR) is weak. Therefore we propose a slight modification of the preamble defined in IEEE802.11a.

The main idea is to raise the number of symbols that can be evaluated by the initial estimation procedure, i.e. that fulfil condition (16). The original preamble defined in IEEE802.11a consists of two sequenced OFDM symbols, where the second one is simply a repetition of the first. In contrast to that, in our proposed preamble we additionally change the sign on one half of the active subcarriers, i.e.

$$\left[d_n^{p,2}\right] = \left[+d^{p,1}_{-N_a/2\ldots-1}, -d^{p,1}_{1\ldots N_a/2}\right] \tag{23}$$

with $[d_n^{p,1}]$ and $[d_n^{p,2}]$ as the first and second OFDM symbol of the preamble respectively. According to the standard ($N_a = 52$) this leads to 26 pairs of subcarriers complying to (16) so that the coefficient $\hat{b}_n$ is estimable for every active subcarrier. Although a linear interpolation of $\hat{b}_n$ in frequency direction is no longer essential, it can suppress noise influences by smoothing.

It is obvious that the proposed change of sign doesn't directly affect the frequency synchronization and channel estimation. However, it has to be mentioned, that the modification violates the cyclic continuation, so an additional cyclic prefix between the first and second preamble OFDM symbol must be inserted to avoid ICI and ISI. This can be seen as the main drawback of the new preamble.

IV. Simulation Results

In this section we present some of our simulation results to illustrate the performance gain that can be reached with the methods described in the last section. The presented results were generated by a Matlab simulation model in accordance with the standard IEEE802.11a. Because the effects of I/Q imbalance become significant especially for high-rate modulated OFDM signals, we focused our analyses on the 54 Mbit/s mode, where the signal constellation complies with 64-QAM (Quadrature Amplitude Modulation). For each E_b/N_0 ratio in the following figures we simulated the transmission of 25000 bursts consisting of approx. 8000 random bits each, i.e. 36 OFDM symbols, over randomly generated multipath channels in accordance with the indoor channel model 'B' ($\Delta\tau = 100\text{ns}$) described in [3].

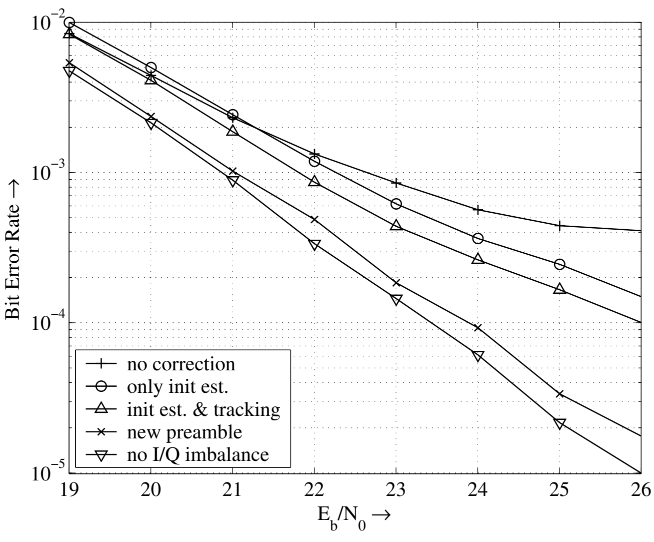

Fig. 2. Simulated bit error rates

Fig. 2 and 3 show the obtained bit (BER) and burst error rates respectively. The lower curve (no I/Q imbalance) represents transmission with ideal transmitter components whereas the bit error rates of a transmission with uncorrected I/Q imbalance are illustrated by the uppermost curve (no correction). Here, the randomly generated transmitter-caused I/Q imbalance for each burst consists of an equally distributed amplitude mismatch with a maximum of 0.5 dBr and an equally distributed phase deviation of a maximum 3 degrees. The frequency dependency of the I/Q imbalance was obtained by an equally distributed relative deviation of the transmit filter cut-off frequencies of a maximum 3%.

The second curve from above (only init est.) shows that an I/Q imbalance correction based on the initial estimation improves the system performance for E_b/N_0 ratios above 22 dB. Further gain can be reached, if the initial estimation is combined with the tracking process described before, as illustrated by the third curve (init est. & tracking). The fourth curve finally shows that an I/Q imbalance correction based on the initial estimation together with the modified preamble and the tracking process almost reaches the reference curve of no I/Q imbalance for both BER and burst error rate.

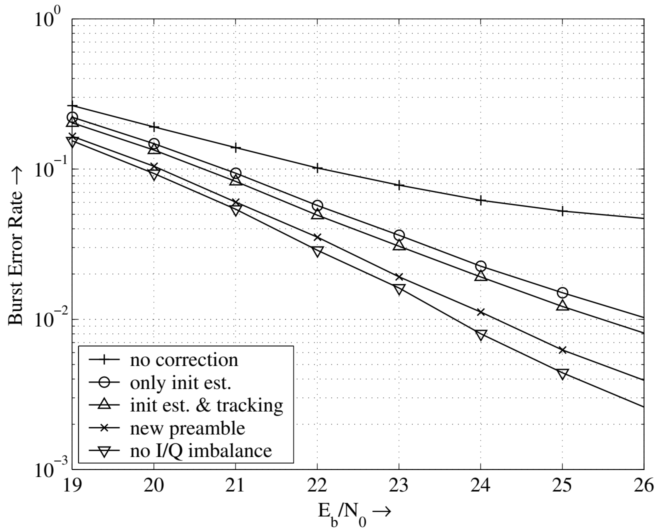

Fig. 3. Simulated burst error rates

V. Conclusion

In case of direct conversion transmission systems the overall performance can strongly suffer from I/Q imbalance caused by non-ideal transmitter components. Although a correction based on the initial estimation presented in this paper slightly reduced the effects of this distortion, the simulation results showed that a real break through regarding performance gain could only be achieved in combination with the modified preamble and iterative tracking process.

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