Combined Linear and Nonlinear Multi-User Detection for Coded OFDM-CDMA

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Abstract— ***This paper analyzes the performance of linear and nonlinear multi-user detection (MUD) for an asynchronous OFDM-CDMA uplink transmission. Specifically, we regard linear MUD techniques such as the decorrelator (zero forcing, ZF) and the MMSE approach as well as the nonlinear parallel interference cancellation (PIC) and their combination. It is pointed out that OFDM-CDMA systems offer a great advantage over single carrier systems due to flat fading conditions on each subcarrier. This leads to much lower implementation costs of MUD techniques. Furthermore, the combination of a linear MUD filter with successive PIC and FEC decoding results in a recursive receiver structure. It will be demonstrated that the linear filter should only be active in the first iteration and should be removed for further iterations of FEC decoding and PIC.***

Assuming perfectly known channel impulse responses for each user and a rough synchronization it turns out that solely linear interference cancellation is not able to reach the performance of a single user system. Nonlinear techniques like parallel interference cancellation supplies single user performance even for system loads of $J/G_p = 1/2$. Furthermore, the combination of linear MMSE equalization and PIC shows excellent performance even for $J/G_p = 1$.

Keywords— ***OFDM-CDMA, multiuser detection, interference cancellation***

I. Introduction

Code Division Multiple Access (CDMA) has been chosen in various modern communication systems [1], [2], [3], [4] as multiple access technique. In this paper, the uplink of a multi-carrier CDMA (MC-CDMA) system [5], [6] is considered using OFDM (Orthogonal Frequency Division Multiplex) to combat the frequency selectivity of the mobile radio channel. Therefore, each subcarrier is affected by flat fading and a one tap equalizer suffices for eliminating channel distortion.

In contrast to a synchronous downlink transmission where orthogonal spreading sequences suppress multi-user interference (MUI) efficiently, this orthogonality would be destroyed in the asynchronous uplink. Therefore, pseudo-noise (PN) sequences are used and multi-user interference is the limiting factor concerning system capacity.

In order to achieve high spectral efficiencies, the interference has to be attacked by multi-user detection (MUD) techniques. In the last years, plenty of work has been spent on multi-user detection [7], [8], [9], [10], [11], [12]. Capacity bounds have been analytically derived for different MUD techniques indicating the maximum system load that should be reachable in theory [9], [13], [14]. Furthermore, a lot of simulations have been carried out for single carrier systems operating in frequency nonselective and even frequency selective environments. In the latter case MUD algorithms incorporate channel equalizer [15] resulting in high computational costs.

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The aim of this paper is to analyze the performance of linear and nonlinear MUD exploiting the characteristics of coded OFDM-CDMA systems. One specific characteristic of OFDM-CDMA is the one-tap-equalization due to flat fading on each subcarrier. This enables us to apply conventional MUD algorithms developed for frequency nonselective channels saving valuable implementation costs when compared to frequency selective fading and single carrier systems. Specifically, the combination of linear MUD and nonlinear parallel interference cancellation is investigated.

The paper is structured as follows: Section 2 describes the OFDM-CDMA system with FEC coding and single-user detection (SUD). Next, section 3 presents the considered MUD techniques, their application in an OFDM-CDMA environment and discusses the obtained simulation results. Finally, section 4 gives some conclusions.

II. System Description

Figure 1 depicts the structure of the considered OFDM-CDMA transmitter. The information bits $d^{(j)}(k)$ of duration T_d for each user $1 \le j \le J$ are encoded by a conventional convolutional code (CC) of rate $R_c = 1/n$. After encoding, the resulting vector $\mathbf{b}^{(j)}(k)$ is spread by repeating each coded bit $b_i^{(j)}(k)$, $1 \le i \le n$, N_p times and successive multiplication with a user-specific code $\mathbf{c}^{(j)}$. Due to an asynchronous transmission in the uplink, we use simple pseudo-noise (PN) sequences for spreading. Throughout the paper, the duration of a chip $c_\mu^{(j)}$ equals $T_c = T_d/G_p$ where $G_p = N_p/R_c = 64$ is the entire processing gain.

Next, the OFDM transmitter transforms $\tilde{\mathbf{b}}^{(j)}(k)$ into the time domain. In this work, the number of carriers N_c equals exactly the processing gain, i.e. one information bit $d^{(j)}(k)$ is mapped exactly onto one OFDM symbol. After frequency-domain interleaving (Π_f) over N_c chip and inverse Fourier transformation (IFFT), a cyclic prefix of duration T_g called guard interval is inserted in front of each OFDM symbol.

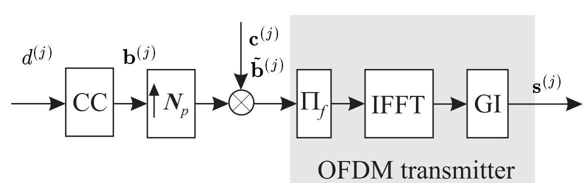

Fig. 1. Typical structure of an OFDM-CDMA transmitter

The resulting signals $\mathbf{s}^{(j)}(k)$ of different users are now transmitted over J individual L-path mobile radio channels. Real and imaginary parts of the corresponding channel coefficients $h_l^{(j)}(k)$, $0 \le l < L$, are gaussian distributed and statistically independent. Although each user is assigned to an individual channel, the number of transmission paths L is assumed to be the same for all users. The corresponding transfer function is defined by

$$H_\mu^{(j)}(k) = \sum_{l=0}^{L-1} h_l^{(j)}(k) \cdot e^{-j2\pi\mu l/L} \; . \tag{1}$$

At the OFDM receiver, the cyclic prefix is removed first (Fig. 2). A guard time T_g larger than the delay-spread $\Delta\tau$ of the channel results in a cyclic convolution of channel impulse response $\mathbf{h}^{(j)}(k)$ and transmitted signal $\mathbf{s}^{(j)}(k)$. This allows an efficient transformation of the received signal back into the frequency domain by the fast Fourier transform (FFT). Assuming rough synchronization, i.e. the maximum delay between different users is limited to $T_g - \Delta\tau$, one FFT window suffices for transforming all user signals back into the frequency domain. The cyclic convolution in time domain corresponds to a scalar multiplication of $H_\mu^{(j)}(k)$ with the spread signal $\tilde{b}_\mu^{(j)}(k)$ in the frequency domain. Hence, this leads to an equivalent channel model where each chip is only affected by flat fading.

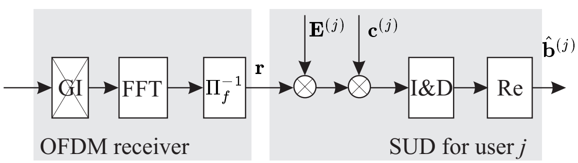

Fig. 2. Single-user receiver for OFDM-CDMA

The received vector $\mathbf{r}(k)$ at the output of the OFDM receiver at time instance k consists of N_c chips and can be expressed by

$$\mathbf{r}(k) = \mathbf{A}(k)\mathbf{b}(k) + \mathbf{n}(k) \tag{2}$$

where

$$\mathbf{b}(k) = \left(\mathbf{b}^{(1)}(k)^T,\; \mathbf{b}^{(2)}(k)^T,\; \cdots,\mathbf{b}^{(J)}(k)^T\right)^T \tag{3}$$

contains the convolutionally encoded bits $b_i^{(j)}(k)$ of all users and $\mathbf{n}(k)$ determines the background noise. The system matrix $\mathbf{A}(k) = (\mathbf{A}^{(1)}(k) \cdots \mathbf{A}^{(J)}(k))$ comprises J user specific matrices

$$\mathbf{A}^{(j)}(k) = \begin{pmatrix} \mathbf{a}_1^{(j)}(k) & & \\ & \ddots & \\ & & \mathbf{a}_n^{(j)}(k) \end{pmatrix} \tag{4}$$

where the column vectors have the form

$$\mathbf{a}_i^{(j)}(k) = (a_{i,0}^{(j)}(k), \cdots, a_{i,N_p-1}^{(j)}(k))^T \; .$$

Its elements

$$a_{i,\nu}^{(j)}(k) = c_{(i-1)N_p+\nu}^{(j)} H_{(i-1)N_p+\nu}^{(j)}(k), \quad 0 \le \nu < N_p$$

are element-wise products of the signature sequences $\mathbf{c}^{(j)}$ and the channel transfer function $\mathbf{H}^{(j)}(k)$. The influence of the interleaver Π_f on the indices is neglected. The special form of $\mathbf{A}$ is caused by the specific mapping of the coded bits onto the OFDM symbols. Due to the fact that generally n coded bits $b_i^{(j)}(k)$ are mapped onto one OFDM symbol, $\mathbf{A}$ is composed by nJ column vectors $\mathbf{a}_i^{(j)}(k)$. The rough synchronization mentioned before ensures that vectors with different indices i do not mutually interfere. Therefore, $\mathbf{A}$ can be split up into n different sub-matrices saving computational costs when calculating its pseudo-inverse for linear MUD.

The optimal single-user detection (SUD) employs matched filter that maximizes the signal-to-noise ratio at its output. Presupposing perfectly known channel impulse responses, the equalizer $\mathbf{E}^{(j)}(k)$ for user j in Figure 2 then equals the hermitian form of $\mathbf{A}^{(j)}(k)$ and the input of the FEC decoder can be described by

$$\begin{aligned} \hat{\mathbf{b}}^{(j)}(k) &= \mathrm{Re}\left\{\mathbf{E}^{(j)}(k) \cdot \mathbf{r}(k)\right\} \\ &= \mathrm{Re}\left\{\left[\mathbf{A}^{(j)}(k)\right]^H \cdot \mathbf{r}(k)\right\} \; . \end{aligned} \tag{5}$$

The superscript $[]^H$ denotes the conjugate transpose. Vector $\hat{\mathbf{b}}^{(j)}(k)$ at the FEC decoder input of user j can be devided into three parts

$$\hat{\mathbf{b}}^{(j)}(k) = \alpha(k) + \beta(k) + \eta(k) \; . \tag{6}$$

The first term

$$\begin{aligned} \alpha(k) &= [\mathbf{A}^{(j)}(k)]^H \mathbf{A}^{(j)}(k)\mathbf{b}^{(j)}(k) \\ &= \begin{pmatrix} \sum_{\mu=0}^{N_p-1} |H_\mu^{(j)}(k)|^2 \cdot b_1^{(j)}(k) \\ \vdots \\ \sum_{\mu=(n-1)N_p}^{N_c-1} |H_\mu^{(j)}(k)|^2 \cdot b_n^{(j)}(k) \end{pmatrix} \end{aligned} \tag{7}$$

represents the desired coded information obtained by maximum ratio combining (MRC) N_p chips. The terms $\beta(k)$ and $\eta(k)$ describe the multiple access interference and the contribution of the background noise, respectively. According to [16], both parts can be tightly approximated by using the gaussian approximation that calculates an equivalent E_b/N_0 and exploit the results of a single-user system. Due to OFDM we have flat fading on each subcarrier and no path cross talk occurs like in the case of a Rake receiver. Hence, the average signal-to-interference-plus-noise ratio (SINR) on the channel for a chip-synchronous transmission can be calculated by

$$\mathit{SINR} = \frac{R_c E_b/N_0}{1 + (J-1)R_c E_b/N_0} \; . \tag{8}$$

Figure 3 shows the performance of the considered coding scheme for different number of active users. A fully symbol-interleaved 4-path Rayleigh fading channel was used, i.e. the channel is assumed to remain unchanged for the duration $T_d = G_P \cdot T_c$ of one information bit. In the average, the transmitted signal's energy is spread equally over the 4 taps of the channel. Successive channel coefficients are assumed to be statistically independent. The degradation due to multiple access interference is obvious. Therefore, an adequate performance is only achievable for relative small number of users unless more sophisticated techniques for combatting MUI are employed.

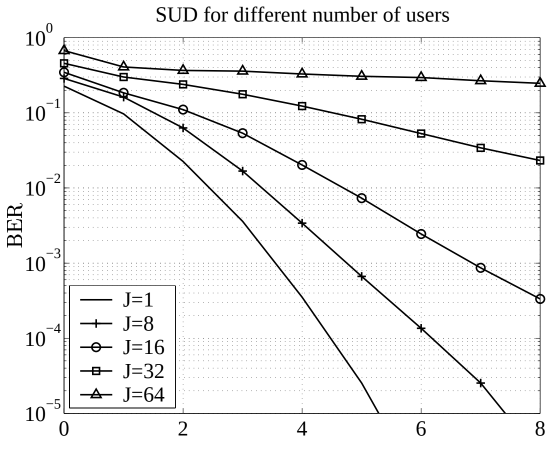

Fig. 3. Simulation results for different number of active users and Single-User Detection

III. Multi-user Detection

A. Linear MUD techniques

Multi-user detection (MUD) schemes can be mainly devided into two groups, linear and nonlinear techniques [7]. Linear MUD schemes generally compute the pseudo-inverse $\mathbf{A}^\dagger(k)$ of the system matrix $\mathbf{A}(k)$ in equation (2) and thus perform a kind of equalization. It is necessary to make some comments on the calculation of the pseudo-inverse $\mathbf{A}^\dagger(k)$.

The system matrix $\mathbf{A}(k)$ consists of N_c rows and nJ columns. Therefore, it describes a system of $N_c = nN_p$ linear equations with nJ unknown variables. If the number of users J is larger than the CDMA spreading factor N_p, e.g. $J > 32$ for $N_p = 32$ and $n = 2$, there are more unknown variables than equations and the linear equation system can only be solved with additional conditions. However, the pseudo-inverse always exists and tries to find an approximation of $\mathbf{A}^\dagger(k)\mathbf{A}(k) = \mathbf{I}$ leading to an estimate

$$\begin{aligned}\hat{\mathbf{b}}(k) &= \mathrm{Re}\left\{\mathbf{A}^\dagger(k)\cdot\mathbf{r}(k)\right\}\\ &= \mathrm{Re}\left\{\mathbf{A}^\dagger(k)\mathbf{A}(k)\mathbf{b}(k) + \mathbf{A}^\dagger(k)\mathbf{n}(k)\right\}\end{aligned} \qquad (9)$$

with minimum energy. For the case $J < N_p$, the pseudo-inverse has the form

$$\mathbf{A}^\dagger(k) = \left(\mathbf{A}^H(k)\mathbf{A}(k) + \gamma\mathbf{I}\right)^{-1}\mathbf{A}^H(k) \qquad (10)$$

where $\gamma = 0$ indicates the ZF equalizer (decorrelator) and $\gamma = \sigma_N^2$ the MMSE solution. The term σ_N^2 represents the noise power [10]. For $J > N_p$,

$$\mathbf{A}^\dagger(k) = \mathbf{A}^H(k)\left(\mathbf{A}(k)\mathbf{A}^H(k) + \gamma\mathbf{I}\right)^{-1} \qquad (11)$$

holds. The MMSE approach with $\gamma = \sigma_N^2$ realizes a compromise between sufficiently decorrelating the interfering signals and noise suppression. Generally, the linear MMSE equalizer provides a performance improvement even in the case of $J > N_p$. There also exist sub-optimal reduced-rank approximations requiring less computational effort [10] but they are not considered here.

Due to the fact that FEC decoding is carried out after linear filtering, it is necessary to supply channel state information (CSI) to the FEC decoder. As stated before, OFDM offers the advantage that every chip is only affected by flat fading. Analyzing (9) for $J = 1$ it can be easily shown that a coded bit at the input of the FEC decoder can be expressed by

$$\hat{b}_i(k) = b_i(k) + \frac{\mathrm{Re}\left\{\sum\limits_{\mu=(i-1)N_p}^{iN_p-1} n_\mu H_\mu(k)^*\right\}}{\sum\limits_{\mu=(i-1)N_p}^{iN_p-1} |H_\mu(k)|^2 + \gamma} \qquad (12)$$

with $1 \le i \le n$. Although (12) does not hold any longer for $J > 1$, an intuitive choice for channel state information is

$$CSI_i^{(j)}(k) = \frac{1}{N_p}\sum_{\mu=(i-1)N_p}^{iN_p-1} |H_\mu^{(j)}(k)|^2 + \gamma\ , \qquad (13)$$

i.e. each received coded bit $\hat{b}_i^{(j)}(k)$ is weighted with the sum of squared magnitudes of the channel coefficients associated with it.

Figure 4 shows the results for a convolutional code with $L_c = 7$ and $R_c = 1/2$ and $J = 16$ active users. It can be seen that the MMSE equalizer outperforms the ZF approach by 1.6 dB. Without CSI, the MMSE approach loses up to 0.5 dB whereas the loss amounts approximately 2 dB for the decorrelator.

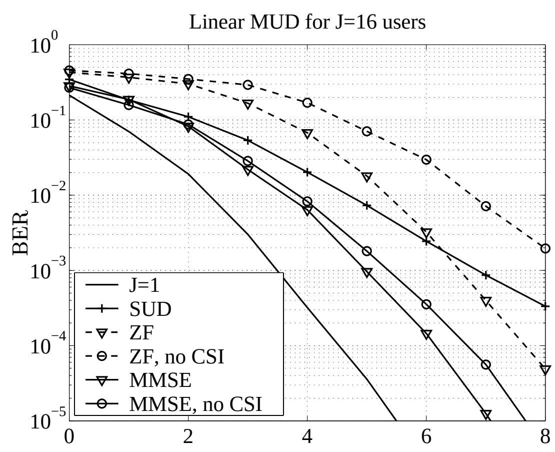

Fig. 4. BER for linear MUD for $J = 16$, $G_p = 64$

In Figure 5, the influence of the MMSE filter on the desired signal is analyzed. The performance degradation for $J = 16$ and $J = 32$ users (solid lines) in comparison with the single user case is obvious. However, the reason for this degradation is not only residual interference that has not been perfectly removed by the filter. In fact, a large portion of this impairment is caused by insufficiently equalizing the channel. In order to illuminate this effect, we carried out simulations where the interfering signals were ideally subtracted in front of the MMSE filter (dashed curves). Thus, the filter receives only the desired signal. From Fig. 5 we recognize that the performance loss for $J = 16$ due to residual interference amounts only 0.5 dB whereas the loss compared to $J = 1$ amounts 2 dB. For $J = 32$, the impairments due to residual interference is higher. However, its portion is still smaller than the degradation due to insufficient equalization.

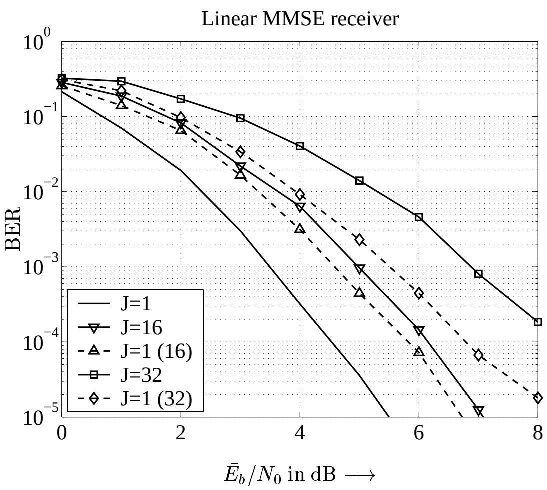

Fig. 5. BER for linear MMSE ($J = 1\,(16)$ and $J = 1\,(32)$ indicate the curves with perfect interference cancellation in front of the MMSE filter)

B. Parallel Interference Cancellation

Concerning nonlinear multi-user detection we regard the parallel interference cancellation (PIC) in this paper. Whereas successive interference cancellation is suitable for systems with large power variations of the received signals, PIC is predestinated for systems with strong power control. This ensures nearly equal receive power of all users and all signals can be detected simultaneously.

The PIC procedure is illustrated in Figure 7 and can be described in the following way. After individual SUD for each user, Soft-In/Soft-Out decoders deliver estimated information bits $\hat{\mathbf{d}}^{(j)}$ as well as log-likelihood ratios $L(\tilde{\mathbf{b}}^{(j)})$ of the coded bits [16], [17]. Then, the expected values of $L(\tilde{\mathbf{b}}^{(j)})$ are calculated by the tanh-function. Finally, the reconstructed signal $\hat{\mathbf{r}}^{(j)}(k)$ of user j is obtained by N_p-fold repetition and scalar multiplication with the coefficients $e_\mu^{(j)} = c_\mu^{(j)} \cdot H_\mu^{(j)}(k)$. The sum

$$\tilde{\mathbf{r}}^{(j)}(k) = \sum_{\substack{\nu=1 \\ \nu\neq j}}^{J} \hat{\mathbf{r}}^{(\nu)}(k) \tag{14}$$

over all interfering signals $\hat{\mathbf{r}}^{(\nu)}$ regarding user j is now subtracted from the received signal $\mathbf{r}(k)$. In the absence of decoding errors, this difference is an estimate of the received signal of user j without any multi-user interference. Therefore, passing this signal through the single-user detector and the channel decoder a second time should yield the performance of the single-user case. Due to decoding errors, the procedure described above has to be repeated several times.

The results for 3 PIC iterations are depicted in Figure 6. It can be observed that, up to $J = 32$, the single user performance is reached. Note that $J = 32$ active users lead to a system with the same spectral efficiency as half rate coded TDMA or FDMA systems. Increasing the number of users to $J = 64$ results in a tremendous loss because the signal-to-interference ratio (SIR) equals 0.07 dB in this case. Regarding the performance of the considered convolutional code at $E_b/N_0 = 0.07$ dB for $J = 1$ indicates that the error rate at the decoder output is approximately 0.2. With this high error rate the PIC scheme is not able to estimate the transmitted signals reliably. Thus, interference is not removed effectively and the iterative process does not converge.

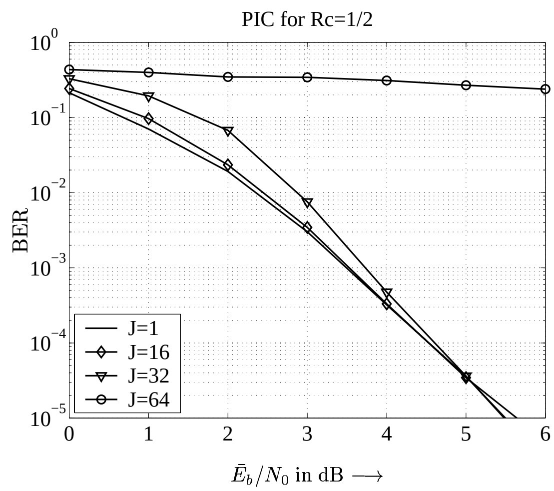

Fig. 6. BER for PIC with 3 iterations and different number of users

C. Combined MMSE and PIC

From Figure 6 it is obvious that solely parallel interference cancellation is not suited to remove MUI for high system loads, i.e. $J \geq G_p$. The initial performance of the error correcting coding scheme is not high enough to supply reliable estimates that can be used to re-construct the interfering signals accurately. Therefore, it might be advantageous to enhance the signal-to-interference ratio at the decoder input by replacing the single-user detectors by one MMSE multi-user detector.

Figure 8 depicts a possible realization of the combined scheme. The MMSE filter is placed in front of the feedback loop of the PIC scheme. As a consequence, the feedback loop is slightly different as in the case of pure PIC because channel state information has to be provided to the FEC decoders. Furthermore, the influence of the linear MUD on the received signal has to be taken into account when the transmitted signals are reconstructed. Therefore, after

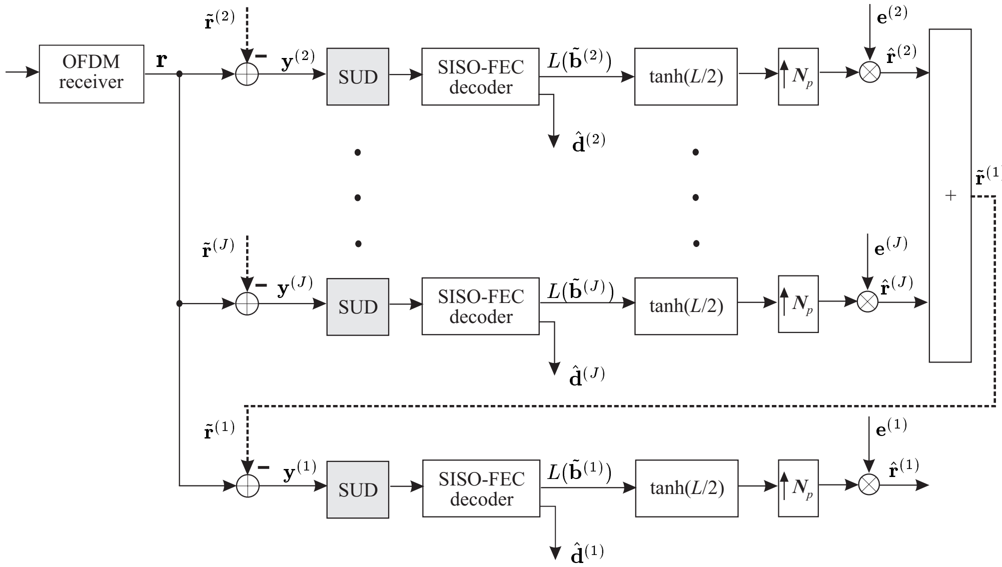

Fig. 7. Principle structure of the parallel interference cancellation scheme

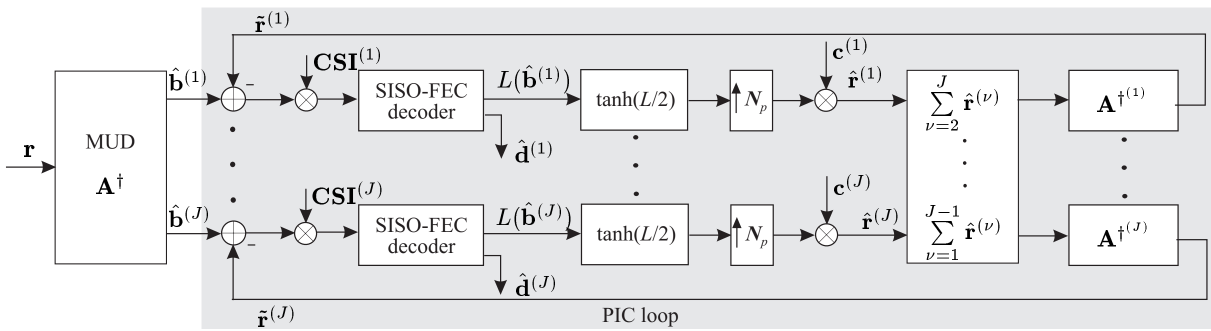

Fig. 8. Combination of linear MUD and parallel interference cancellation, PIC loop deals with linearly filtered signals (MMSE+PIC)

summing up the interfering signals, the part $\mathbf{A}^{\dagger(j)}$ of $\mathbf{A}^{\dagger}$ that concerns user j is applied to the interfering signals $\tilde{\mathbf{r}}^{(j)}(k)$. Finally, the resulting signals $\tilde{\mathbf{r}}^{(j)}(k)$, $1 \leq j \leq J$, can be subtracted from the outputs of the linear multi-user detector.

Figure 9 shows the results of this arrangement for $J = 32$ users (curve termed 'MMSE+PIC'). As can be seen, the interference is totally removed by the PIC loop because the curve termed '$J = 1\,(32)$' is reached (see section III-A). However, there remains a gap of about 2.5 dB compared to the single user performance due to an insufficient channel equalization.

This principle drawback can be avoided when the MMSE filter is only used as a catalyst and the PIC loop directly processes the received vector $\mathbf{r}(k)$. Figure 10 depicts the corresponding realization. In a first stage, linear multi-user detection is carried out increasing the SINR at the decoder inputs (switches in inner positions). After providing channel state information, single-user FEC decoding is performed, the signals $\hat{\mathbf{r}}^{(j)}(k)$ of each user are re-constructed and summed up according to (14). Then, the interference is not subtracted from the output of the MMSE filter but directly from the received vector $\mathbf{r}(k)$ and J individual single user detectors are inserted. In a second stage, the switches

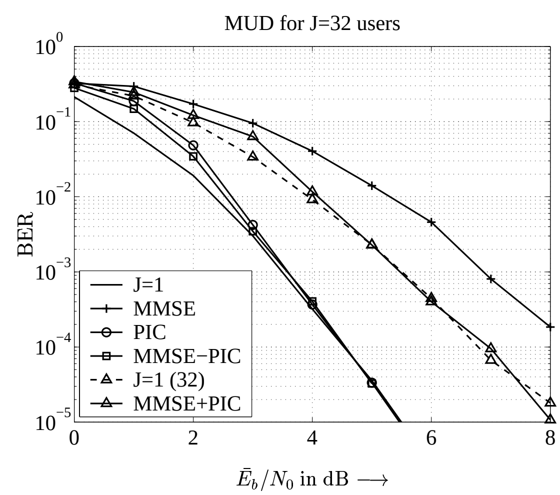

Fig. 9. Comparison of different MMSE-PIC combinations, 3 PIC iterations, $J = 32$ active users

are turned to the outer positions and the interference reduced signals are decoded again several times according to section III-B. Thus, the MMSE filter is only working during an initial phase in order to improve the SINR at the decoder inputs. In this case, the single user curve is reached after a few PIC iterations (see Figure 9, curves

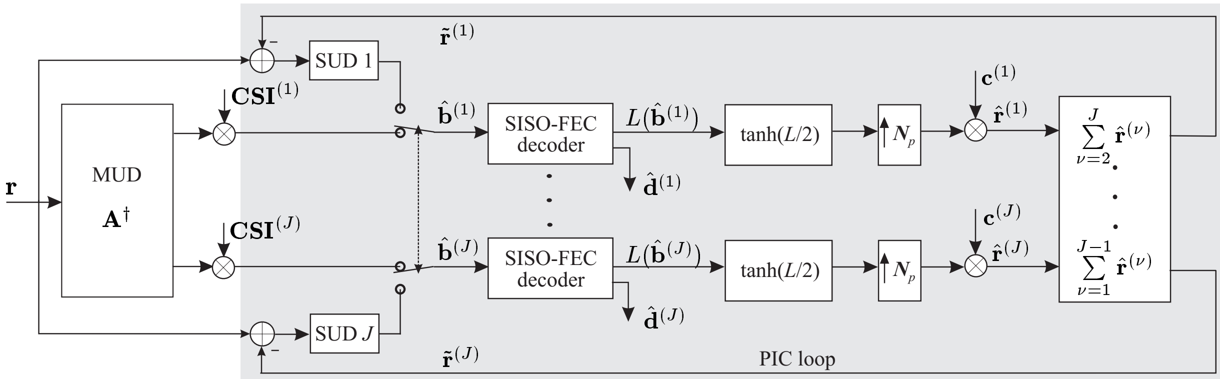

Fig. 10. Combination of linear MUD and parallel interference cancellation, PIC loop deals directly with received signals (MMSE-PIC)

termed 'MMSE-PIC').

A comparison of the different MUD schemes for $J = 64$ active users is shown in Figure 11. Obviously, the linear approach as well as the PIC scheme are not able to reduce MUI significantly. However, the combination of both asymptotically reaches the performance of a single-user system. Notice that the load of the system equals $J/G_p = 1$ and is twice as high as the load of a conventional TDMA or FDMA system with half-rate FEC coding.

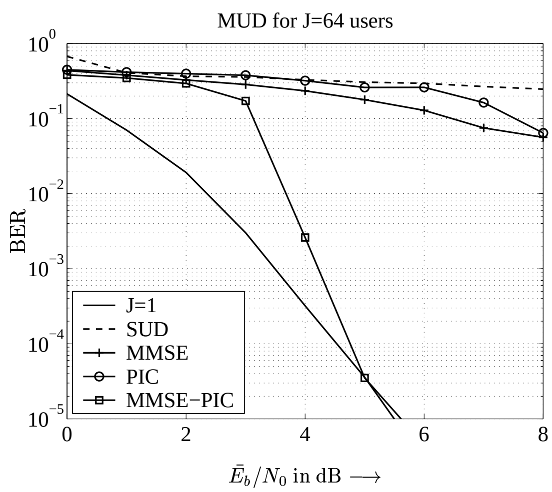

Fig. 11. Comparison of different MUD techniques for $J = 64$ active users

IV. Conclusion

It has been shown that OFDM-CDMA offers great advantages for the application of multi-user detection in frequency selective environments. Due to flat fading on each subcarrier the computational costs for MUD are much lower than for comparable single-carrier systems. In coded systems the use of channel state information improves the performance of ZF-MUD by nearly 2 dB and by 0.5 dB for MMSE-MUD. Combining MMSE-MUD and PIC leads to a remarkable performance. However, the MMSE filter should only be used in an initial run for increasing the SINR at the decoder inputs and should then be turned off. In this case, the performance of a single-user system is reached for a bit error rate of $P_b = 10^{-5}$ even for $J = G_p$.

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