Linear and Nonlinear Multi-User Detection in Coded OFDM-CDMA Systems

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ABSTRACT

This paper analyzes the performance of linear and nonlinear multi-user detection (MUD) for an asynchronous OFDM-CDMA uplink transmission. Specifically, we regard linear MUD techniques such as the decorrelator and the MMSE approach as well as the nonlinear parallel interference cancellation (PIC) and their combination. It is pointed out that OFDM-CDMA systems offer a great advantage over single carrier systems due to flat fading conditions on each subcarrier leading to much lower implementation costs of MUD techniques.

Assuming perfectly known channel impulse responses for each user and a rough synchronization it turns out that solely linear interference cancellation is not able to reach the performance of a single user system. The nonlinear parallel interference cancellation is able to totally remove the interference even for a spectral efficiency that equals that of TDMA and FDMA systems. Furthermore, the combination of linear MMSE equalization and PIC shows excellent performance even for twice as high system loads.

I. INTRODUCTION

Code Division Multiple Access (CDMA) has been chosen in various modern communication systems [1, 2, 3, 4] as multiple access technique. In this paper, the uplink of a multi-carrier CDMA (MC-CDMA) system [5, 6] is considered using OFDM (Orthogonal Frequency Division Multiplex) to combat the frequency selectivity of the mobile radio channel. Therefore, each subcarrier is affected by flat fading and a one tap equalizer suffices for eliminating channel distortion.

In contrast to a synchronous downlink transmission where orthogonal spreading sequences suppress multi-user interference (MUI) efficiently, this orthogonality is destroyed in an asynchronous uplink transmission. Therefore, pseudo-noise (PN) sequences are used and multi-user interference is the limiting factor concerning system capacity. Principally, there exist two possibilities combatting MUI.

First, MUI can be interpreted as additional white gaussian noise that is conventionally suppressed by a strong error correcting code. Due to the inherent spreading, each user occupies a large bandwidth allowing FEC coding with very low code rates and, therefore, high coding gains. It has been shown in [7, 8, 9] that powerful FEC coding is able to ensure acceptable performance for moderate system loads. However, powerful FEC coding is not the appropriate mean for very high loads.

Second, interference can be attacked by multi-user detection (MUD) techniques. In the last years, plenty of work has been spent on multi-user detection [10, 11, 12, 13, 14, 15]. Capacity bounds have been analytically derived for different MUD techniques indicating the maximum system load that should be reachable in theory [12, 16, 17]. Furthermore, a lot of simulations have been carried out for single carrier systems operating in frequency nonselective and even frequency selective environments. In the latter case MUD algorithms incorporate channel equalizer [18] resulting in high computational costs.

The aim of this paper is to analyze the performance of linear and nonlinear MUD exploiting the characteristics of coded OFDM-CDMA systems. One specific feature of OFDM-CDMA receivers is the one-tap-equalization due to flat fading on each subcarrier. This enables us to apply conventional MUD algorithms developed for frequency nonselective channels saving valuable implementation costs when compared to frequency selective fading and single carrier systems. Specifically, zero-forcing and MMSE equalization are applied as well as nonlinear parallel interference cancellation (PIC).

The paper is structured as follows: Section 2 describes the OFDM-CDMA system with FEC coding and section 3 analyzes the performance for single-user detection. Next, section 4 presents the considered MUD techniques, their application in an OFDM-CDMA environment and discusses the obtained simulation results. Finally, section 5 gives some conclusions.

II. SYSTEM DESCRIPTION

Figure 1 illustrates the structure of a typical OFDM-CDMA system for a single user j, $1 \leq j \leq J$. The remaining $J-1$ interfering users are summed up to the signal $\mathbf{MUI}$. The information bits $d^{(j)}(k)$ of duration T_d

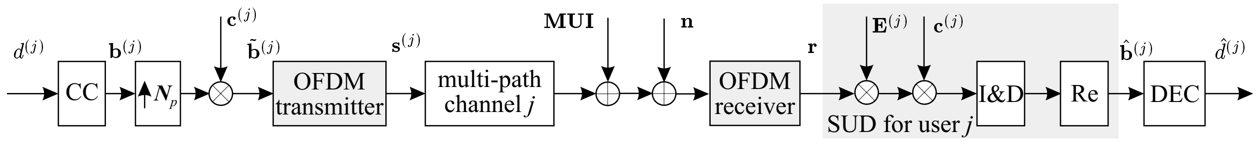

Figure 1: Typical structure of an OFDM-CDMA system with single-user detection

are encoded by a conventional convolutional code of rate $R_c = 1/n$. After encoding, the resulting vector $\mathbf{b}^{(j)}(k)$ is spread by repeating each coded bit $b_i^{(j)}(k)$, $1 \leq i \leq n$, N_p times and successive multiplication with a user-specific code $\mathbf{c}^{(j)}(k)$. Due to an asynchronous transmission in the uplink, we use simple pseudo-noise (PN) sequences for spreading. Throughout the paper, the duration of a chip $c_\mu^{(j)}(k)$ equals $T_c = T_d/G_p$ where $G_p = N_p/R_c = 64$ is the entire processing gain.

Next, the OFDM transmitter depicted in Figure 2 transforms $\tilde{\mathbf{b}}^{(j)}(k) = (\tilde{b}_0^{(j)}(k) \ \ldots \ \tilde{b}_{N_c-1}^{(j)}(k))^T$ into the time domain. In this work, the number of carriers N_c equals exactly the processing gain, i.e. one information bit $d^{(j)}(k)$ is mapped exactly onto one OFDM symbol $\mathbf{s}^{(j)}(k)$. After frequency-domain interleaving (Π_f) over N_c chips and inverse Fourier transformation (IFFT), a cyclic prefix of duration T_g called guard interval is inserted in front of each OFDM symbol.

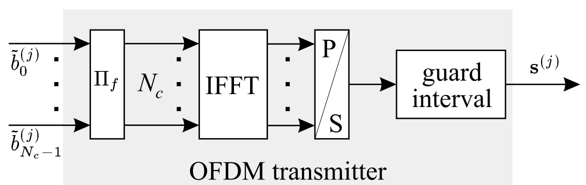

Figure 2: Typical structure of an OFDM-CDMA transmitter

The resulting signals $\mathbf{s}^{(j)}(k)$ of different users are now transmitted over J individual L-path mobile radio channels. Real and imaginary parts of the corresponding channel coefficients $h_l^{(j)}(k)$, $0 \leq l < L$, are gaussian distributed and statistically independent. Although each user is assigned to an individual channel, the number of transmission paths L is assumed to be the same for all users. The corresponding transfer functions are defined by

$$H_\mu^{(j)}(k) = \sum_{l=0}^{L-1} h_l^{(j)}(k) \cdot e^{-j2\pi\mu l/L} \ . \tag{1}$$

At the OFDM receiver, the cyclic prefix is removed first (Fig. 3). A guard time T_g larger than the delay-spread $\Delta\tau$ of the channel results in a cyclic convolution of channel impulse response and transmitted signal. This enables us to efficiently transform the received signal back into the frequency domain by the fast Fourier transform (FFT). Assuming rough synchronization, i.e. the maximum delay between different users is limited to $T_g - \Delta\tau$, one FFT window suffices for transforming all user signals back into the frequency domain. The cyclic convolution in time domain corresponds to a scalar multiplication of $H_\mu^{(j)}(k)$ with the spread signal $\tilde{b}_\mu^{(j)}(k)$ in the frequency domain. Hence, this leads to an equivalent channel model where each chip is only affected by flat fading.

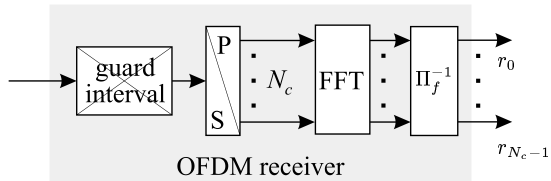

Figure 3: Typical structure of an OFDM-CDMA receiver

The received vector $\mathbf{r}(k)$ consists of N_c chips of the OFDM symbol at time instance k and can be expressed by

$$\mathbf{r}(k) = \mathbf{A}(k)\mathbf{b}(k) + \mathbf{n}(k) \tag{2}$$

where

$$\mathbf{b}(k) = \left(\mathbf{b}^{(1)}(k)^T \ \mathbf{b}^{(2)}(k)^T \ \cdots \ \mathbf{b}^{(J)}(k)^T\right)^T \tag{3}$$

contains the convolutionally encoded bits $b_i^{(j)}(k)$ of all users and $\mathbf{n}(k)$ determines the background noise. The system matrix $\mathbf{A}(k) = (\mathbf{A}^{(1)}(k) \ \cdots \ \mathbf{A}^{(J)}(k))$ comprises J user specific matrices

$$\mathbf{A}^{(j)}(k) = \begin{pmatrix} \mathbf{a}_1^{(j)}(k) & & \\ & \ddots & \\ & & \mathbf{a}_n^{(j)}(k) \end{pmatrix} \tag{4}$$

where the column vectors have the form

$$\mathbf{a}_i^{(j)}(k) = (a_{i,0}^{(j)}(k) \ \cdots \ a_{i,N_p-1}^{(j)}(k))^T \ .$$

Its elements

$$a_{i,\nu}^{(j)}(k) = c_{(i-1)N_p+\nu}^{(j)}(k) H_{(i-1)N_p+\nu}^{(j)}(k),$$

$0 \leq \nu < N_p$, are element-wise products of the signature sequences $\mathbf{c}^{(j)}(k)$ and the channel transfer function $\mathbf{H}^{(j)}(k)$. The influence of the interleaver Π_f on the indices is neglected. The special form of $\mathbf{A}$ is caused by the specific mapping of the coded bits onto the OFDM symbols. Due to the fact that generally n coded bits $b_i^{(j)}(k)$ are mapped onto one OFDM symbol, $\mathbf{A}$ is composed by nJ column vectors $\mathbf{a}_i^{(j)}(k)$. Rough synchronization mentioned before ensures that vectors with different indices i do not mutually interfere. Therefore, $\mathbf{A}$ can be split up into n different sub-matrices saving computational costs when calculating its pseudo-inverse for linear MUD.

The signal $\mathbf{r}(k)$ has now to be equalized and despread before its real part is fed to the FEC decoder. The next section describes the performance of a single-user detector.

III. SINGLE-USER DETECTION

In this section, we analyze the performance of a system with single-user detection (SUD). The optimal single-user detector is a matched filter that maximizes the signal-to-noise ratio at its output. This is done by maximum ratio combining N_p chips of $\mathbf{r}$ corresponding to one coded bit $b_i^{(j)}(k)$. Presupposing perfectly known channel impulse responses, the equalizer $\mathbf{E}^{(j)}(k)$ for user j in Figure 1 then equals the hermitian form of $\mathbf{A}^{(j)}(k)$ and the input of the FEC decoder can be described by

$$\begin{aligned}\hat{\mathbf{b}}^{(j)}(k) &= \mathrm{Re}\left\{\mathbf{E}^{(j)}(k)\cdot\mathbf{r}(k)\right\}\\ &= \mathrm{Re}\left\{\left[\mathbf{A}^{(j)}(k)\right]^H\cdot\mathbf{r}(k)\right\}\ .\end{aligned}\qquad(5)$$

The superscript $[]^H$ denotes the conjugate transpose. Vector $\hat{\mathbf{b}}^{(j)}(k)$ at the FEC decoder input of user j can be devided into three parts

$$\hat{\mathbf{b}}^{(j)}(k) = \alpha(k) + \beta(k) + \eta(k)\ .\qquad(6)$$

The first term

$$\begin{aligned}\alpha(k) &= [\mathbf{A}^{(j)}(k)]^H\mathbf{A}^{(j)}(k)\mathbf{b}^{(j)}(k)\\ &= \begin{pmatrix}\sum_{\mu=0}^{N_p-1}|H_\mu^{(j)}(k)|^2\cdot b_1^{(j)}(k)\\ \vdots\\ \sum_{\mu=(n-1)N_p}^{N_c-1}|H_\mu^{(j)}(k)|^2\cdot b_n^{(j)}(k)\end{pmatrix}\end{aligned}\qquad(7)$$

represents the desired coded information obtained by maximum ratio combining (MRC) N_p chips. The terms $\beta(k)$ and $\eta(k)$ describe the multiple access interference and the contribution of the background noise, respectively.

If the performance for $J = 1$ user is known, we can tightly approximate the results for $J > 1$ active users by the gaussian approximation [9, 19]. Figure 4 depicts the received results for convolutional codes with constraint length $L_c = 7$ of rate $R_c = 1/2$. In all investigations, a fully symbol-interleaved 4-path Rayleigh fading channel was used, i.e. the channel is assumed to remain unchanged for the duration $T_d = G_P \cdot T_c$ of one information bit. In the average, the transmitted signal's energy is equally spread over the 4 taps of the channel. Successive channel coefficients are assumed to be statistically independent.

The tremendous performance degradation due to MUI is obvious. Although the described single-user detector maximizes the SNR at its output it is not optimum at all for multi-user systems and does not achieve the single-user performance. Moreover, we can observe an error floor even in the case of $J = 16$ users. For $J = 32$, an error rate of $P_b = 10^{-4}$ cannot be reached any more. Therefore, it is advantageous to consider receivers maximizing the signal-to-interference-plus-noise ratio (SINR).

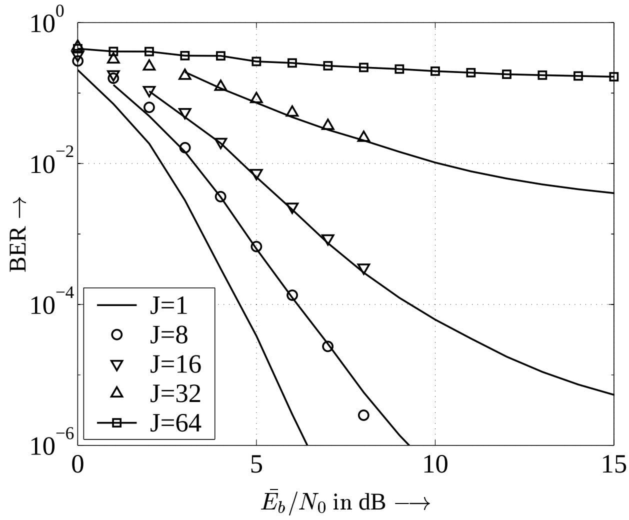

Figure 4: Simulation results for convolutional code with $R_c = 1/2$, different number of active users

IV. MULTI-USER DETECTION

A. Linear MUD techniques

Multi-user detection (MUD) schemes can be mainly devided into two groups, linear and nonlinear techniques [10]. Linear MUD schemes compute the pseudo-inverse $\mathbf{A}^\dagger(k)$ of the system matrix $\mathbf{A}(k)$ in equation (2) and thus perform a kind of equalization. It is necessary to make some comments on the calculation of the pseudo-inverse $\mathbf{A}^\dagger(k)$.

The system matrix $\mathbf{A}(k)$ consists of N_c rows and nJ columns. Therefore, it describes a system of $N_c = nN_p$ linear equations with nJ unknown variables. If the number of users J is larger than the CDMA spreading factor N_p, e.g. $J > 32$ for $N_p = 32$ and $n = 2$, there are more unknown variables than equations and the linear equation system can only be solved with additional conditions. However, the pseudo-inverse always exists and tries to find an approximation of $\mathbf{A}^\dagger(k)\mathbf{A}(k) = \mathbf{I}$ leading to an estimate $\hat{\mathbf{b}}(k)$ with minimum energy.

For the case $J < N_p$, the pseudo-inverse has the form

$$\mathbf{A}^\dagger(k) = (\mathbf{A}^H(k)\mathbf{A}(k) + \alpha\sigma_N^2\mathbf{I})^{-1}\mathbf{A}^H(k)\qquad(8)$$

where σ_N^2 represents the noise power [13]. The term $\alpha = 0$ indicates the ZF equalizer and $\alpha = 1$ the MMSE solution. For $J > N_p$,

$$\mathbf{A}^\dagger(k) = \mathbf{A}^H(k)(\mathbf{A}(k)\mathbf{A}^H(k) + \alpha\sigma_N^2\mathbf{I})^{-1}\qquad(9)$$

holds. The MMSE approach with $\alpha = 1$ realizes a compromise between sufficiently decorrelating the interfering signals and noise suppression. Generally, the linear MMSE equalizer provides a performance improvement even in the case of $J > N_p$. There also exist suboptimal reduced-rank approximations requiring less computational effort [13] but they are not considered here. The linearly filtered signal can be expressed by

$$\begin{aligned}\hat{\mathbf{b}}(k) &= \mathrm{Re}\left\{\mathbf{A}^\dagger(k)\cdot\mathbf{r}(k)\right\}\\ &= \mathrm{Re}\left\{\mathbf{A}^\dagger(k)\mathbf{A}(k)\mathbf{b}(k) + \mathbf{A}^\dagger(k)\mathbf{n}(k)\right\}.\end{aligned}\qquad(10)$$

Due to the fact that FEC decoding is carried out after linear filtering, it is necessary to supply channel state information (CSI) to the FEC decoder. As stated before,

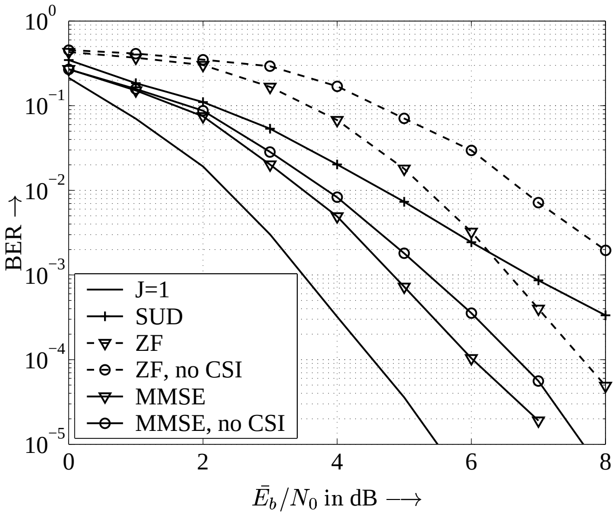

Figure 5: Performance of linear MUD for $J = 16$, $G_p = 64$

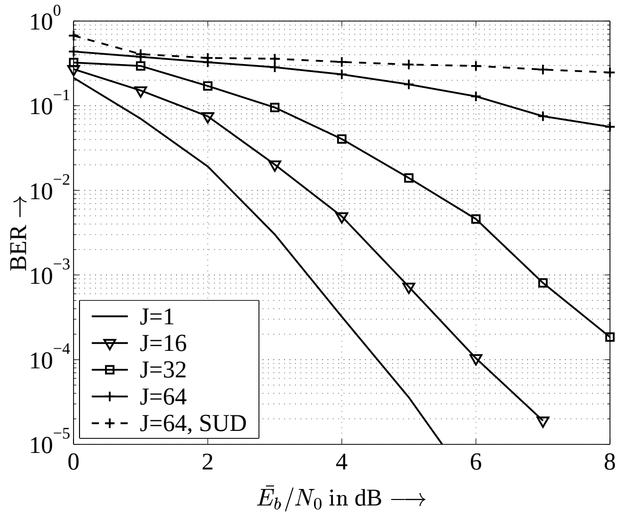

Figure 6: Performance of linear MMSE for different number of active users

OFDM offers the advantage that every chip is only affected by flat fading. Analyzing (10) for $J = 1$ it can be easily shown that a coded bit at the input of the FEC decoder can be expressed by

$$\hat{b}_i(k) = b_i(k) + \frac{\mathrm{Re}\left\{\sum_{\mu=(i-1)N_p}^{iN_p-1} n_\mu H_\mu(k)^*\right\}}{\sum_{\nu=(i-1)N_p}^{iN_p-1} |H_\nu(k)|^2 + \alpha \cdot \sigma_N^2} \tag{11}$$

with $1 \le i \le n$. Although (11) does not hold any longer for $J > 1$, an intuitive choice for channel state information is

$$CSI_i^{(j)}(k) = \frac{1}{N_p} \sum_{\mu=(i-1)N_p}^{iN_p-1} |H_\mu^{(j)}(k)|^2 + \alpha \cdot \sigma_N^2 \ , \tag{12}$$

i.e. each received coded bit $\hat{b}_i^{(j)}(k)$ is weighted with the sum of squared magnitudes of the channel coefficients associated with it.

Figure 5 shows the results for a convolutional code with $L_c = 7$ and $R_c = 1/2$ and $J = 16$ active users. It can be seen that the MMSE equalizer outperforms the ZF approach by more than 1 dB. Without CSI, the MMSE approach loses up to 0.5 dB whereas the loss amounts approximately 2 dB for the ZF solution. These results demonstrate the decoder's sensitivity concerning appropriate channel state information especially for the case of ZF-MUD where noise amplification is a serious problem.

The performance of the MMSE detector for different number of active users is shown in Figure 6. Although the MMSE approach is superior to a single-user detector (SUD) even in the case of $J = 64$ there remains a huge gap to the single-user case ($J = 1$). Therefore, the performance of linear MUD is far away from the single user case especially for high system loads.

B. Parallel Interference Cancellation

Concerning nonlinear multi-user detection we regard the parallel interference cancellation (PIC) in this paper. Whereas successive interference cancellation is suitable for systems with large power variations of the received signals, PIC is predestinated for systems with strong power control. This ensures equal receive power of all users and all signals can be detected simultaneously.

The structure of the whole PIC system is depicted in Figure 7. After individual SUD for each user, Soft-In/ Soft-Out decoders deliver estimated information bits $\hat{\mathbf{d}}^{(j)}(k)$ as well as log-likelihood ratios $L(\tilde{\mathbf{b}}^{(j)}(k))$ of the coded bits. Then, the expected values of $L(\tilde{\mathbf{b}}^{(j)}(k))$ are calculated by the $\tanh$-function. Afterwards, the received sequences are re-constructed by scrambling and weighting with the corresponding channel coefficients $H_\mu^{(j)}(k)$. Finally, the sum

$$\tilde{\mathbf{r}}^{(j)}(k) = \sum_{\substack{i=1 \\ i \neq j}}^{J} \hat{\mathbf{r}}^{(i)}(k) \tag{13}$$

over all interfering signals $\hat{\mathbf{r}}^{(i)}(k)$ regarding user j is subtracted from the received signal $\mathbf{r}(k)$. In the absence of decoding errors, this difference is an estimate of the received signal of user j without any multi-user interference. Therefore, passing this signal through the one tap equalizer and the channel decoder a second time should yield the performance of the single-user case. Due to decoding errors, the procedure described above has to be repeated several times.

The results obtained with 3 PIC iterations are depicted in Figure 8. It can be observed that, up to $J = 32$, the single user performance (SUS) is reached. Note that $J = 32$ active users lead to a system with the same spectral efficiency as half rate coded TDMA or FDMA systems. Increasing the number of users to $J = 64$ leads to a tremendous loss because the signal-to-interference ratio equals 0 dB in this case. Regarding the performance of the considered convolutional code at $E_b/N_0 = 0$ dB for $J = 1$

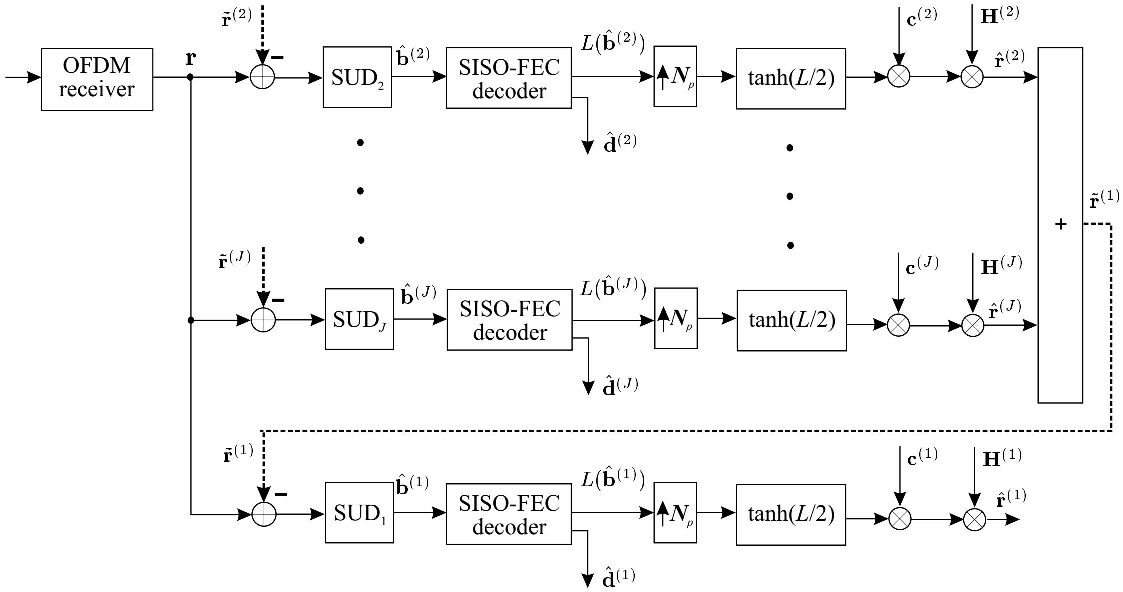

Figure 7: Principle structure of the parallel interference cancellation scheme

indicates that the error rate at the decoder output is approximately 0.2. With this high error rate the PIC scheme is not able to remove interference and the iterative process does not converge.

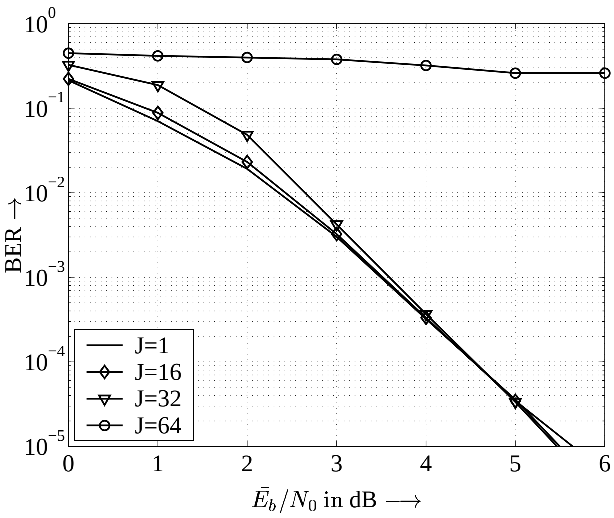

Figure 8: Performance of PIC with 3 iterations for different number of users

C. *Combined MMSE and PIC*

From Figure 8 it is obvious that solely parallel interference cancellation is not suited to remove MUI for high system loads, i.e. $J = G_p$. The initial performance of the error correcting coding scheme is not high enough to supply reliable estimates that can be used to re-construct the interfering signals accurately. Therefore, it might be advantageous to enhance the signal-to-interference ratio at the decoder input by replacing all J single-user detectors by one MMSE multi-user detector.

Figure 9 depicts the combined scheme. In a first stage, linear multi-user detection with successive single-user decoding is carried out (*switches on inner positions*). The re-constructed signals $\hat{\mathbf{r}}^{(j)}(k)$ of each user are summed according to (13) and subtracted from the received vector $\mathbf{r}(k)$. In a second stage, the switches are turned on the outer positions so that the PIC loop now operates with the original received signal $\mathbf{r}(k)$. Thus, the MMSE detector is only working in an initial phase for enhancing the SINR at the decoder inputs. The PIC scheme is then processing the original received data.

A comparison of the different MUD schemes is shown in Figure 10 for $J = 64$ active users. Obviously, the linear approach as well as the PIC scheme with 3 iterations are not able to reduce MUI significantly. However, the combination of both asymptotically reaches the performance of a single-user system. Notice that the load of the system equals $J/G_p = 1$ which is twice as high as the load of conventional TDMA or FDMA systems with half-rate FEC coding.

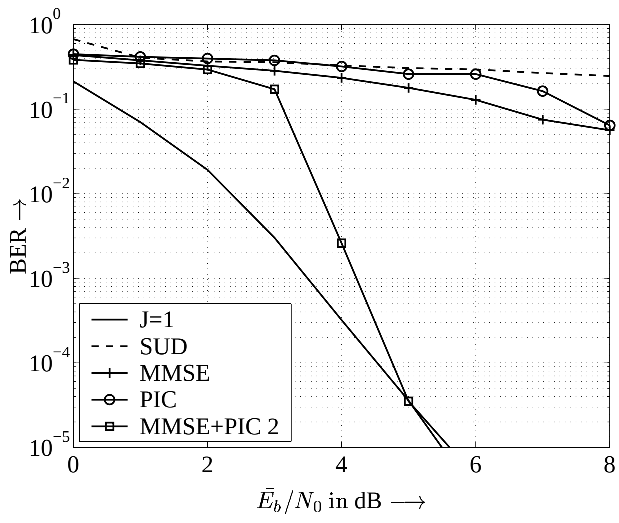

Figure 10: Comparison of different MUD techniques for $J = 64$ active users

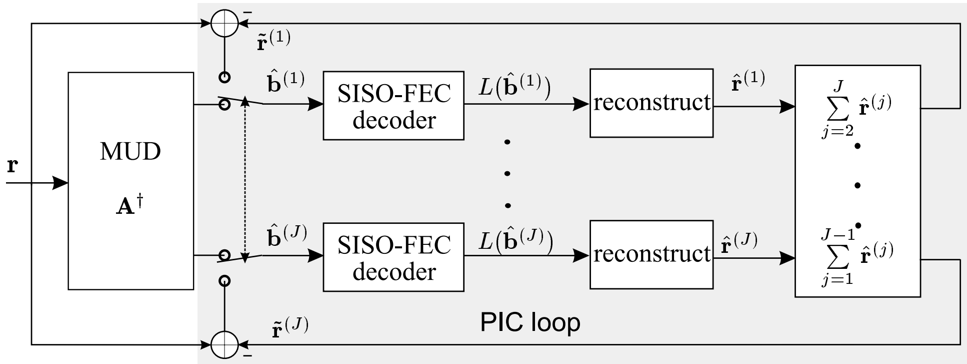

Figure 9: Combination of linear MUD and parallel interference cancellation

V. CONCLUSION

It has been shown that OFDM-CDMA offers great advantages for the application of multi-user detection in frequency selective environments. Due to flat fading on each subcarrier the computational costs for MUD are much lower than for comparable single-carrier systems. In coded systems the use of channel state information improves the performance of ZF-MUD by nearly 2 dB and by 0.5 dB for MMSE-MUD. Combining MMSE-MUD and PIC leads to a remarkable performance. In this case, the performance of a single-user system is reached for a bit error rate of $P_b = 10^{-5}$.

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