Iterative Blind and Non-blind Channel Estimation in GSM Receivers

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ABSTRACT

Channel estimation plays a leading role in wireless communication systems such as GSM (Global System for Mobile communications). Putting the main emphasis on investigating the application of non-blind and blind channel estimation approaches to the identification of a full rate data Traffic CHannel (TCH/F9.6), we present in this paper a new idea of an iterative channel estimation based upon the capabilities of channel coding. We will show that this method leads to a significantly improved performance especially for high speed propagations with Doppler frequencies up to 500 Hz. *Giving the bit error rates (BERs) before and after channel decoding in terms of the $\bar{E}_b/N_0$ ratio, we show, furthermore, that blind channel estimation schemes could be as efficient as non-blind methods.*

I INTRODUCTION

Optimum receivers in digital communication systems require the knowledge of the transmission channel. In general, this knowledge is not available so that the channel's equivalent baseband impulse response has to be estimated. Hence, state-of-the-art mobile communication systems transmit *training sequences* to assist the receiver in estimating the channel impulse response. However, their repeated transmission leaves the communication system with a considerable overhead which could be used for the transmission of additional data sequences, if the channel estimation was solved *blindly*. In recent publications [2, 6], we have shown that some blind approaches yield promising results in estimating GSM channels although there is still a significant $\bar{E}_b/N_0$ loss compared to non-blind channel estimation schemes. In order to reduce this loss, we have utilized an iterative channel estimation which employs the bits after channel decoding as a new "training" sequence. Furthermore, the separation of this sequence into different sections enables the use of GSM[1] for data or speech transmission at very high mobile unit speeds.

Since high rate data transmission over mobile radio channels is playing a growing role, we have carried out our simulations for the TCH/F9.6 transmission mode for full rate data at 12 kbit/s.

The paper is organized as follows: Section II gives an overview of the GSM communication system. Based on the so-called *burst* structure, we outline in section III the approaches of non-blind and blind channel estimation. In section IV, we describe the concept of iterative channel estimation in a GSM receiver. Finally, simulation results are presented in section V and the paper is concluded in section VI.

II GSM DATA TRANSMISSION

Fig. 1 depicts the equivalent baseband representation of the physical layer of a GSM communication system. In

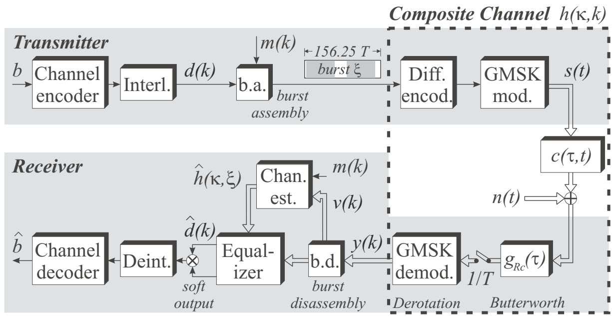

Figure 1: Physical layer of a GSM communication system

the transmitter, each *input block* of the TCH/F9.6 consists of 240 bits $b \in \{0,1\}$ containing 16 header, 200 data, and 24 FCS (frame check sequence) bits. Hence, the input rate amounts to 240 bits/20 ms = 12 kbit/s. A half-rate channel encoder applies two convolutions whose polynomials are respectively D^4+D^3+1 and D^4+D^3+D+1. Since convolutional codes are adapted to infinite sequences, 4 appended tail bits (set to "0") enable a termination of the finite input block. After puncturing, each *output block* consists of 456 coded bits. In order to scramble the coded

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[1] By GSM, we understand GSM-900 as well as its derivatives DCS-1800 or PCS-1900

bit stream, 5 consecutive *output blocks* are then interleaved unequally over 22 radio *bursts* of 156.25 bits valued $\{-1, 1\}$, where each data sequence $d(k)$ is assembled together with the reference bits $m(k)$, two tail bit sequences, and one guard interval [5]. In order to facilitate demodulation, each burst is encoded differentially. Then, it is modulated by Gaussian Minimum Shift Keying (GMSK) and transmitted over the *time-variant* (TV) channel $c(\tau, t)$. Apart from linear TV distortions, Additive White Gaussian Noise (AWGN) $n(t)$ is present. In the receiver, an anti-aliasing *Butterworth* filter $g_{Rc}(\tau)$ is applied in order to suppress adjacent channel interference. Due to differential encoding and GMSK modulation, a simple *Derotation* demodulator can be used upon symbol-rate sampling in order to obtain the sequence $y(k)$.

Maximum Likelihood Sequence Estimation (MLSE) represents a well-known procedure to remove intersymbol interference (ISI) from the received digital communication signal. Since MLSE requires the knowledge of the composite channel $h(\kappa, k)$ in order to equalize a block of the demodulated sequence $y(k)$, the following section focuses on blind and non-blind channel estimation. Therefore, let $\hat{h}(\kappa, \xi)$ denote the channel estimate which will be used to equalize the ξ-th demodulated burst.[2] By using soft-decision algorithms within the equalizer, the system performance can be improved by about 3 dB [8]. Therefore, a *symbol-by-symbol* (SS)-Max-Log-MAP soft output detector computes probabilities for each bit $\hat{d}(k) \in \{-1, 1\}$ which can be applied to the channel decoder after deinterleaving.

III BLIND AND NON-BLIND CHANNEL ESTIMATION

According to Fig. 2, each GSM "normal" burst contains a training sequence, the so-called *midamble*, $m(k)$ of 26 bits surrounded by two packets of 58 data bits emerging from the coded i.i.d. information sequence $d(k)$. With regard to Fig. 1, non-blind channel estimates can be derived by solving the relation between the received (corrupted) sequence $v(k)$ after burst disassembling (b.d.) of the ξ−th demodulated burst and the stored midamble $m(k)$ in the *least-squares* (LS) sense [2]. However, their repeated transmission leaves a GSM system with an overhead capacity of $26/116 = 22.4\%$ which can be used for other purposes if the channel is estimated blindly. The fundamental idea of blind system identification is to derive the complete channel characteristics (including phase information) from the received signal only, i.e. *without* training

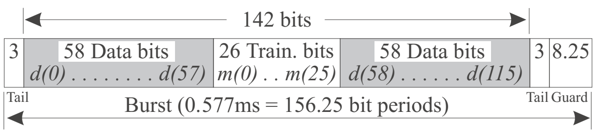

Figure 2: GSM "normal" burst structure

[2]While in $h(\kappa, k)$, the index $k = t/T$ refers to the symbol period $T \approx 3.7\ \mu$s, the index ξ used in $\hat{h}(\kappa, \xi)$ changes from burst to burst only

sequences. Fig. 3 shows the structure of a "blind" GSM burst which consists of 142 data bits.

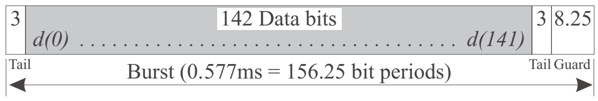

Figure 3: GSM "blind" burst structure

Depending on different ways of extracting information from the received signal, several classes of blind algorithms can be distinguished. When the received signal is sampled at the symbol-rate $1/T$, the resulting sequence is (quasi) stationary. Since, in general, *Second Order Statistics* (SOS) of a stationary signal are inadequate for the identification of the complete channel characteristics, this can be achieved by approaches based on *Higher Order Statistics* (HOS). Higher order *cumulants* contain the complete information on the channel provided that the channel input signal is non-Gaussian distributed which is true for GSM.

The application of SOS methods to blind channel estimation requires the exploitation of channel diversity which can be achieved in several ways. When the sampling period is a fraction of T, approaches based on *Second Order Cyclostationary Statistics* (SOCS) can be applied in principle provided that some excess bandwidth is available. Alternatively, the generation of channel diversity can also be achieved if the GSM derotation scheme is used in order to create two channel outputs from a single symbol-rate sampled GMSK signal [3]. Generally, algorithms demanding channel diversity are sufficient to retrieve the complete channel characteristics, but there are "singular" channel classes with common subsystems (i.e. common zeros) in all polyphase subchannels which can *not* be identified this way. Therefore, we have not considered blind SOS-based algorithms in this paper.

Besides the non-blind LS scheme, we have selected the HOS-based EIGENVECTOR APPROACH TO BLIND IDENTIFICATION (EVI) by Boss et al. [1] which is a non-linear method maximizing a fourth order cross-cumulant on a second order boundary condition. An investigation of further algorithms based on HOS and SOS can be found in [2, 6, 7].

A common drawback of *all* existing blind approaches whether they are based on HOS or SOS is the interference of the estimated channel impulse response by an unknown complex factor. In [2, 6] we have presented the so-called BLIND CHANNEL ESTIMATION CORRECTION (BECO) scheme which overcomes this fundamental problem. Based on the *a-priori* knowledge of the transmitted data symbols $d(k) \in \{-1, 1\}$, the complex factor can be calculated after filtering the received sequence $y(k)$ with a Mean Square Error (MSE) equalizer followed by time averaging over each burst.

III-A Modifications to Coding and Interleaving Schemes

Depending on the application of non-blind or blind channel estimation algorithms, the burst structure differs for both cases. This fact also influences the block structures mentioned in section II. Thus, some modifications to the coding and interleaving schemes are required in order to achieve a fair comparison between non-blind and blind channel estimation approaches. Differing from the GSM *Specifications*, in the non-blind case each input block contains 228 bits b reducing the input rate to 11.4 kbit/s. Thus, after half-rate encoding an output block consists of 464 non-punctured bits. Finally, a block interleaver scrambles the bits of 5 consecutive output blocks over 20 bursts.

If the impulse response of the transmission channel is estimated blindly, each burst consists of 142 data bits $d(k)$. In order to apply the same modified interleaver structure as in the non-blind case, an output block therefore must contain 568 encoded bits. Hence, we have to transmit 280 uncoded bits b per input block within the same period of time which results in an increased input rate of 14 kbit/s. Regarding the energy E_b of an uncoded bit, this leads to an E_b/N_0 gain of $10 \cdot \log(280/228) \approx 0.9$ dB of blind approaches relative to non-blind ones, where N_0 characterizes the power spectral density of the real valued baseband AWGN $n(t)$.

III-B Influence of High Doppler Frequencies

Both non-blind and blind channel estimation approaches assume the channel to be time-invariant (TI) over each burst. However, Fig. 4 illustrates that this assumption is violated at high speeds of the mobile unit. The dash-

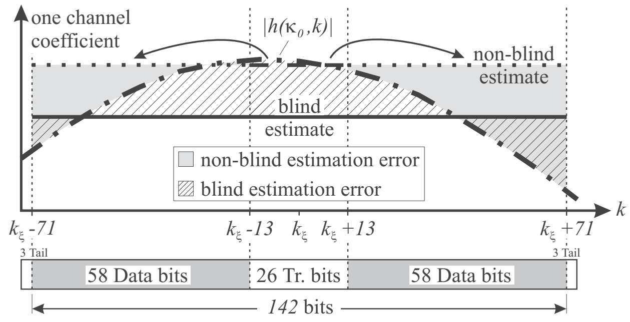

Figure 4: Estimation errors at high mobile unit speeds

dotted line indicates the TV magnitude of an arbitrary coefficient $h(\kappa_0, k)$ of a fast fading channel with a maximum Doppler frequency of $f_{\mathrm{D,max}} = 500$ Hz over one burst period.[3] Because it is based on the midamble, a non-blind estimate of $h(\kappa_0, k)$ should approximate the mean channel coefficient averaged over the training period (see the dashed horizontal line in Fig. 4). The arrows and the dotted lines indicate that this estimate is used for MLSE of the data fields adjacent to the midamble. Since $h(\kappa_0, k)$ has already changed there, the estimated coefficient is erroneous (represented by the gray areas) and consequently the distribution of bit errors might increase towards the burst boundaries.

On the other hand, a blind estimate should approach the mean channel coefficient averaged over 142 bit periods (refer to the solid horizontal line in Fig. 4), where the striped areas point out the corresponding estimation error. Thus, the decisive question is whether the bit error rate is lower if non-blind or blind channel estimates are applied to MLSE. To answer this question, we include in section V simulation results for fast fading channels.

[3]Which corresponds to mobile unit velocities of 600 km/h and 300 km/h for GSM-900 and DCS-1800, respectively

IV ITERATIVE CHANNEL ESTIMATION

Fig. 5 shows the concept of iterative channel estimation within a GSM receiver. Upon receive filtering and

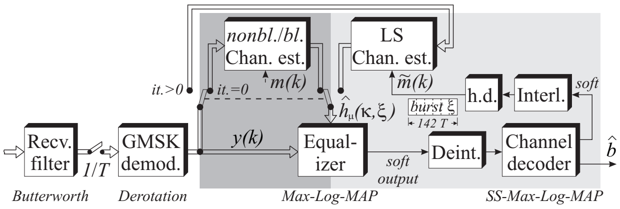

Figure 5: Iterative GSM channel estimation

symbol-rate sampling, bursts of the GMSK demodulated sequence $y(k)$ are equalized by means of a soft-output Max-Log-MAP equalizer. Let us first consider the initial step of channel estimation ($it. = 0$) characterized by the dark grey box and the two switches set to the inner position. According to section III, the equalizer can be fed with a non-blind or blind channel estimate $\hat{h}(\kappa, \xi)$, where the non-blind channel estimation is based upon the midamble $m(k)$ consisting of 26 bits while the blind one is obtained from 142 samples of $y(k)$. Then, the soft-output sequence of the equalizer is deinterleaved and channel decoded. Remember, that in the non-blind case this step is equivalent to common state-of-the-art GSM receivers. The new idea is that we apply a SS-Max-Log-MAP channel decoder delivering soft information for both the information bits $\hat{b}$ and the coded bits [9]. The process of iteration (characterized by the light grey box in Fig. 5, where both switches are set to the outer position $it. > 0$) starts with an interleaving of 5 consecutive blocks of the coded bits over 20 bursts in the same way described in section III-A. After hard-decision (h.d.), each burst can be utilized by a LS channel estimation as a new 142 bits long "training burst" $\tilde{m}(k)$ containing the midamble $m(k)$ if the initial channel estimation was based on reference data. Then, the updated channel estimate is fed into the equalizer and the next iteration starts after deinterleaving and channel decoding.

IV-A Suppressing the Disturbance of Fast Fading Channels

Especially in the non-blind case, we may reckon in accordance with section III-B that for higher Doppler frequencies the distribution of bit errors increases towards the burst boundaries. Since the iterative LS channel estimation is based on a "training" sequence $\tilde{m}(k)$ covering a whole burst period of 142 samples, it is obvious to separate it into different "training" sections $\tilde{m}_\mu(k)$ in order to suppress the disturbance caused by high mobile unit speeds. Fig. 6 depicts an example using the same channel coefficient $h(\kappa_0, k)$ as in Fig. 4, where $\tilde{m}(k)$ is divided into 5 sections $\tilde{m}_\mu(k)$, $0 \leq \mu \leq 4$, of lengths ℓ_μ.

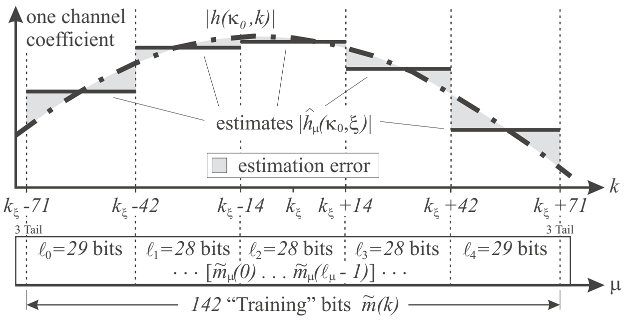

Figure 6: Iterative channel est. based on "training" sections

For a given order $\hat{q}$ of the TI(ℓ_μ)-FIR system to be estimated over ℓ_μ bit periods, the LS estimate $\hat{h}_\mu(\kappa, \xi)$ can be calculated by

$$\begin{bmatrix} \hat{h}_\mu(0,\xi) \\ \vdots \\ \hat{h}_\mu(\hat{q},\xi) \end{bmatrix} = \tilde{\mathbf{M}}_\mu^\dagger \cdot \begin{bmatrix} y_\mu(0) \\ \vdots \\ y_\mu(\ell_\mu - 1 - \hat{q}) \end{bmatrix}, \qquad (1)$$

where $\tilde{\mathbf{M}}_\mu$ is the $(\ell_\mu - \hat{q}) \times (\hat{q}+1)$ Toeplitz matrix containing the new "training" bits of the μ-th section

$$\tilde{\mathbf{M}}_\mu = \begin{bmatrix} \tilde{m}_\mu(\hat{q}) & \cdots & \tilde{m}_\mu(0) \\ \vdots & \ddots & \vdots \\ \tilde{m}_\mu(\ell_\mu - 1 - \hat{q}) & \ddots & \tilde{m}_\mu(\hat{q}) \\ \vdots & \ddots & \vdots \\ \tilde{m}_\mu(\ell_\mu - 1) & \cdots & \tilde{m}_\mu(\ell_\mu - 1 - \hat{q}) \end{bmatrix} \qquad (2)$$

and † denotes its pseudo-inverse. A comparison of Figs. 4 and 6 shows that this strategy should lead to significantly reduced estimation errors. Consequently, BERs should be reduced in the same way by feeding the equalizer with the channel estimates $\hat{h}_\mu(\kappa, \xi)$.

A very important question is how to determine the optimum lengths ℓ_μ of the "training" sections $\tilde{m}_\mu(k)$. We have to find a compromise between long sections which reduce the variance of estimation and the influence of noise but do not sufficiently suppress the disturbance of fast fading channels on the one hand and short sections behaving vice versa on the other hand. Therefore, the minimum length of each section $\tilde{m}_\mu(k)$ was set to 26 bits which is equivalent to the length of the midamble $m(k)$. Hence, the two outer sections of each burst can overlap their inner neighbours in some cases. In section V we will carry out some investigations referred to the determination of the optimum section lengths.

Finally, the question might arise why using a channel decoder which delivers soft information for the coded bits when hard decision eliminates this useful information. The answer is that we have investigated several techniques exploiting soft information, e.g. the elimination of all rows of $\tilde{\mathbf{M}}_\mu$ associated with low soft values of $\tilde{m}_\mu(k)$, but in fact none of them did improve the estimation quality. Therefore, we do not present these techniques in this paper.

V SIMULATION RESULTS

Referring to Figs. 1 and 5 and our modifications to the full rate data transmission mode TCH/F9.6 described in section III-A, a block of 232 (284) bits[4] b is channel encoded and interleaved into bursts of 116 (142) data bits $d(k)$, where the numbers in front of and within the brackets refer to the non-blind and blind case, respectively. Assembled together with the midamble[5] $m(k)$, the tail bits, and the guard time, a burst of 156 bits is differentially encoded, modulated, and then propagated through each slice $c(\tau, t_{8\xi})$ of the respective sample channel[6] derived from the stochastic GAUSSIAN STATIONARY UNCORRELATED SCATTERING (GSUS) model [4]. We use standard COST-207 *Bad Urban* (BU) profiles with maximum Doppler frequencies of $f_{\mathrm{D,max}} = 50, 200$, and 500 Hz, where we take the channel's time-variance *within each burst period* into account. Let these channel examples be denoted BU50, BU200 and BU500. According to a given $\bar{E}_b/N_0$ ratio, AWGN $n(t)$ is added to the signal. Upon receive filtering, symbol-rate sampling, demodulation, and guard time removing 148 samples of $y(k)$ are obtained per burst, where we assume perfect synchronization. In the initial iteration step ($it. = 0$), the non-blind LS approach or the blind EVI scheme is applied to $y(k)$, where the former exploits the training sequence $m(k)$ while the latter corrected by the BECO method uses the 142 data samples of each demodulated burst. Then, the resulting channel estimates $\hat{h}(\kappa, \xi)$ are passed to the equalizer delivering the hard output $\hat{d}(k) \in \{-1, 1\}$ as well as the reliability information for soft-decision decoding. After deinterleaving, the channel decoder provides the output sequence $\hat{b} \in \{0, 1\}$ and the soft information of the coded bits which are the basis for the new "reference" sequence $\tilde{m}(k)$ of the iterative channel estimation described in the previous section.

In the frame of MONTE-CARLO simulations, this procedure was executed for all channel slices $c(\tau, t_{8\xi}), 1 \leq \xi \leq 50000$, and $\bar{E}_b/N_0$ values ranging from 0 to 15 dB. Finally, BERs are calculated after channel decoding from the bits $\hat{b}$ transmitted at a given $\bar{E}_b/N_0$.

Figure 7: In order to answer the question of how to determine the optimal lengths of the "training" sections $\tilde{m}_\mu(k)$, for different numbers of iterations subplots a and c display the according BERs of the channel decoded sequence $\hat{b}$ at $\bar{E}_b/N_0 = 10$ dB after applying the non-blind LS and the blind EVI channel estimate of a BU500 channel to MLSE in the initial step, respectively. Within each burst, every "training" section has the same length $\ell = \ell_\mu$ except from the outer ones, due to the overlap technique described in section IV-A. Obviously, the optimal section length depends on the initial channel estimate $\hat{h}(\kappa, \xi)$. If it is based

[4] Including the 4 tail bits required for termination

[5] Which is only required for non-blind channel estimation

[6] Since at most each 8th burst is sent from/to the same mobile station, just each 8th slice $c(\tau, t_{8\xi})$, with $t_{8\xi} = (8 \cdot 156.25)\xi T$, is used

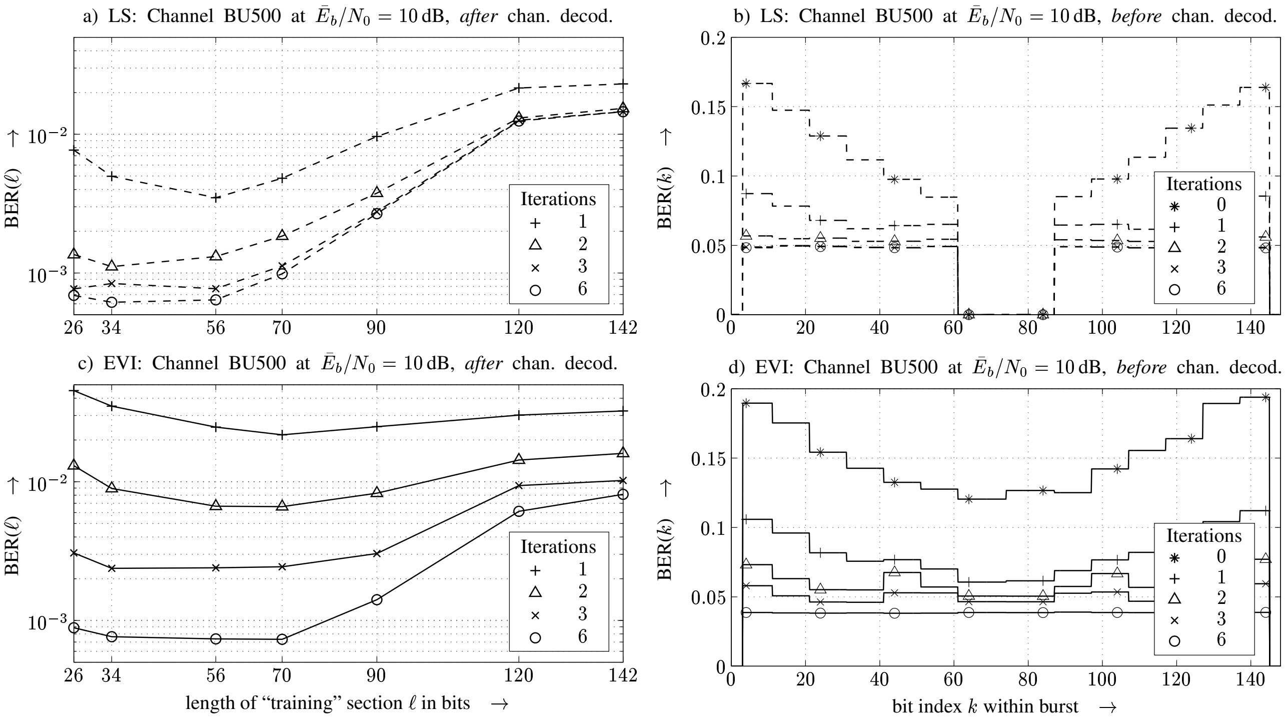

Figure 7: BERs in terms of the length ℓ of the "training" section (a, c) and the bit index k within each burst (b, d) for the non-blind LS (a, b) and the blind EVI estimates (c, d) of a COST-207 BU channel with a maximum Doppler shift $f_{\mathrm{D,max}} = 500$ Hz for different numbers of iterations at $\bar{E}_b/N_0 = 10$ dB.

Table 1: BU channels, optimal "training" section lengths

Chan.	Chan. est.	Iterations 1	2	3	4	5	6
BU50	LS	142	142	142	142	142	142
	EVI	142	142	142	142	142	142
BU200	LS	56	34	34	34	34	34
	EVI	142	70	70	56	34	34
BU500	LS	56	34	34	34	34	34
	EVI	70	56	34	34	34	34

on reference data (subplot a) the optimal length after the first iteration amounts to $\ell = 56$ bits, while it is 70 bits if the channel was estimated blindly (subplot c). According to Table 1, the following optimal section lengths can be successively approximated from iteration to iteration. However, for slow fading channels (BU50), we can see that there is no need to divide the "training burst" $\tilde{m}(k)$ into different sections.

Prior to channel decoding, subplots b and d show the distribution of bit errors over the bit index k per burst for several numbers of iterations based on the optimal "training" section lengths from Table 1 at $\bar{E}_b/N_0 = 10$ dB. As expected, after an initial LS channel estimation (subplot b, iteration 0) BER(k) rises as the burst boundaries are approached, where it is zero for $61 \leq k \leq 86$, since no information bits are transmitted in the midamble. However, according to subplot d, even for an initial EVI channel estimation, BER(k) behaves in a comparable way. This is caused by the fact that during most of the burst periods, the channel coefficients $|h(\kappa, k)|$ are either increasing or decreasing at a linear rate which implies that blind and non-blind channel estimates are very similar. Nevertheless, from both subplots (b and d) we can see that according to the flat lines the disturbance of fast fading channels is suppressed after 3 (LS) and 6 iterations (EVI).

Figure 8: Taking into account the optimal "training" section lengths of Table 1, we display the BERs in terms of $\bar{E}_b/N_0$ for an iterative channel estimation of a BU50 (a, d), BU200 (b, e), and BU500 sample channel (c, f). While the upper row of subplots (a-c) is associated with an initial non-blind LS channel estimation, the lower row (d-f) shows the performance of iterative channel estimation based on an initial blind EVI scheme. Without any iteration, we can see in general that a higher mobile unit speed causes an increased error floor. Especially for very fast fading channels with Doppler frequencies up to 500 Hz, an application of GSM is impossible, since a sufficient data transmission requires at least BERs below 10^{-3} after channel decoding. Furthermore, the performance of EVI is much worse compared to that of LS, although we have already considered the $\bar{E}_b/N_0$ gain of approx. 0.9 dB for blind approaches due to the avoided midamble (see section III-A).

Depending on the channel's maximum Doppler shift and the initial estimation approach, the utilization of the iterative channel estimation with successive approximation of the "training" section lengths leads to tremendous performance gains. Regarding the BU50 and BU200 channels, at BER $= 10^{-3}$ the $\bar{E}_b/N_0$ gain with an initial LS estimation amounts to approx. 0.9 dB (subplot a) and 1.3 dB (subplot b) after just 1 iteration, while it is approx. 4.9 dB (subplot d) and 6.2 dB (subplot e) after 6 iterations if EVI was applied to MLSE in the initial step. From subplots c

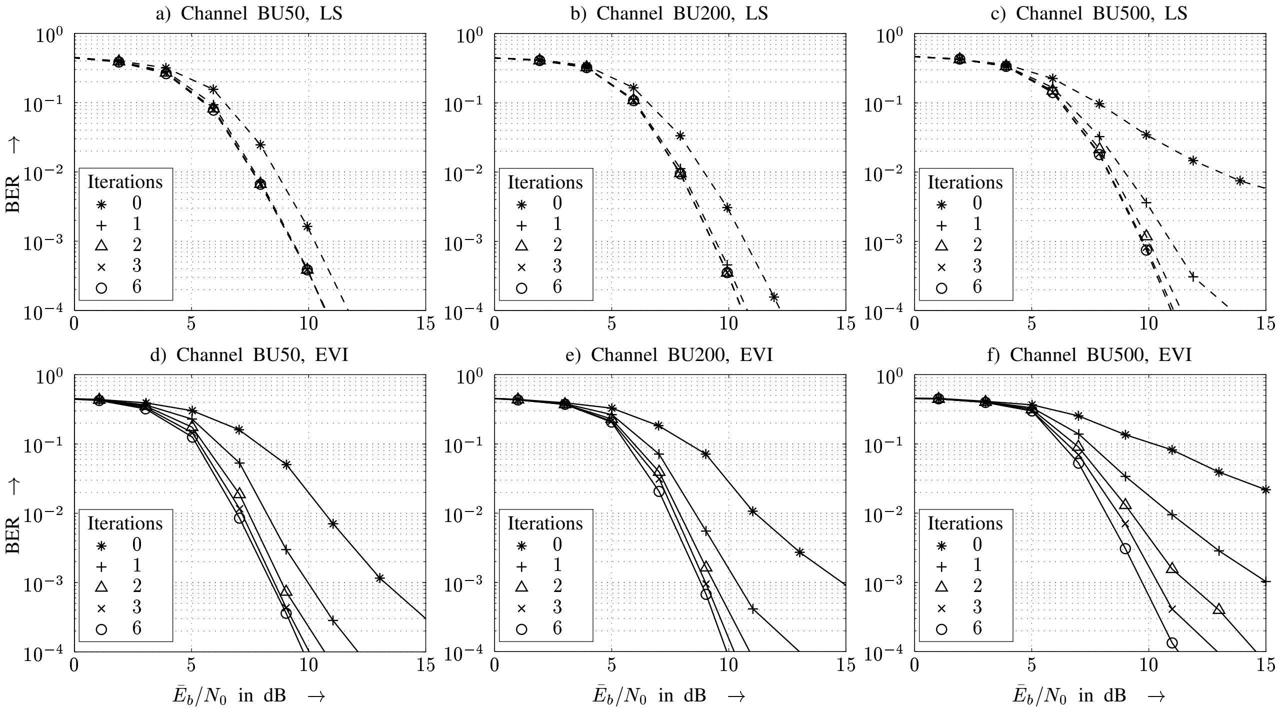

Figure 8: BERs in terms of $\bar{E}_b/N_0$ and different numbers of iterations for an initial non-blind LS (a-c) and blind EVI channel estimation (d-f) of a COST-207 BU channel with maximum Doppler shifts of $f_{\mathrm{D,max}} = 50$ Hz (a, d), 200 Hz (b, e), and 500 Hz (c, f).

and f, we see that an application of GSM to a fast fading BU500 channel can only be made possible by the iterative channel estimation scheme. In accordance with Fig. 7 b and d we see from all subplots that the iterative channel estimation scheme with successive approximated "training" section lengths is able to completely suppress the fading influences of transmission channels.

Furthermore, comparing subplots a and d as well as b and e, the considered $\bar{E}_b/N_0$ gain of 0.9 dB of blind channel estimation algorithms compared to non-blind ones is discernible after some iterations. Therefore, we have shown that blind approaches in conjunction with the iteration scheme can be as efficient as non-blind ones.

VI CONCLUSIONS

For the GSM full rate data transmission mode TCH/F9.6, we have presented in this paper a new iterative channel estimation scheme. Especially for fast fading channels, the performance after an initial blind or non-blind channel estimation could be improved significantly due to the division of the "training burst" into different sections. Remember that this technique could be applied to any state-of-the-art GSM receiver *without hurting the GSM Specifications*, if the initial channel estimation is non-blind. Although we are well aware of the fact that, on account of these specifications, the midambles are quite unlikely to be banned from the GSM "normal" burst, we have shown that after some iterations the application of blind channel estimation algorithms to MLSE indicates an $\bar{E}_b/N_0$ gain of 0.9 dB compared to non-blind ones caused by the fact that the reference sequences were replaced by data bits.

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