Channel Coding Aspects in an OFDM-CDMA System

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Abstract — **This paper investigates coding aspects in an OFDM-CDMA environment. The inherent bandwidth expansion in CDMA systems offers many possibilities for the application of powerful codes with low code rates. Three different coding strategies are under consideration: the combination of convolutional and repetition codes (conventional DS-CDMA system), a code-spread system consisting of one very low rate convolutional code, and a serial concatenation of convolutional, Walsh and repetition code. Analytical and simulation results turn out that the Walsh coded system is the best choice among the considered scenarios.**

I. Introduction

Mobile radio communication represents a rapidly growing market since the GSM (Global System for Mobile Communications) standard has been established. Meanwhile, third generation mobile radio systems are currently under standardization [1, 2] employing CDMA (Code Division Multiple Access) as multiple access technique. There exist several realizations of CDMA systems. The most common one is a single carrier system with Direct-Sequence (DS) spreading, i.e. the data signal is directly spread by a user specific sequence [3].

In this paper, we pursue a different approach. We consider the uplink of a multi-carrier CDMA (MC-CDMA) system [4, 5, 6, 7], where OFDM (Orthogonal Frequency Division Multiplex) is used to combat the frequency selectivity of the mobile radio channel. Therefore, each subcarrier is affected by flat fading, hence a one tap equalizer suffices for eliminating channel distortion.

In order to exploit diversity in an OFDM-CDMA system, channel coding has to be applied. Contrary to narrow band transmission where the code rate has to be as high as possible due to spectral efficiency, the inherent bandwidth expansion in a CDMA system allows very low code rates. Due to this spreading, each user occupies a very large bandwidth offering a variety of channel coding scenarios with potentially high coding gains.

Here, the question arises how to perform very low rate coding. In this paper, we pursue three different approaches: First, a conventional coding scenario (CCRPC) consisting of the serial concatenation of a simple recursive convolutional code (CC) and a repetition code (RPC) is discussed. Exchanging step by step the code rates of CC and RPC in favour to the convolutional code finally leads to the second approach termed code-spread system (CSP). This system was first proposed in [8, 9] and uses a single very low rate convolutional code constructed by a nested code search [9]. However, the obtained codes also include a repetition code but different bits of a code word are repeated unequally [9]. The idea of one powerful code was already presented by Viterbi in [10], but his super-orthogonal codes have the great disadvantage of extremely high decoding costs.

The third scenario inserts a Walsh-Hadamard code between CC and RPC yielding a serial concatenated coding scheme (SCCS) that consists of three codes. In contrast to the above mentioned approaches that can be decoded by a single Viterbi decoder, no optimal maximum likelihood decoding is applicable for the SCCS scheme. Instead, a sub-optimal iterative decoding process using soft-output MAP-decoding algorithms has to be implemented [11].

The paper is structured as follows: In section 2, the transmission system consisting of transmitter, channel and receiver is decribed. Section 3 presents an analytical derivation of the performance of coded OFDM-CDMA systems. Finally, the received simulation results are discussed in section 4, and section 5 concludes the paper.

II. Coded OFDM-CDMA System

A. Transmitter

Figure 1 shows the structure of three different transmitters. The data stream $\mathbf{d}$ consists of information bits $d_k \in \{0,1\}$ each of duration T_d that are encoded by one of the three coding schemes mentioned above. After encoding, the resulting sequence $\tilde{\mathbf{b}}$ with $T_{\tilde{b}} = T_d \cdot R_c$ is scrambled by a user-specific code $\mathbf{c}^{(i)}$ possessing the same chip duration $T_c = T_{\tilde{b}}$. Note that the 'classical' spreading is interpreted as a simple repetition code and thus the whole spreading procedure is incorporated in the channel encoder. Due to asynchronous transmission in the uplink we use simple pseudo-noise (PN) codes for scrambling.

The scrambled sequence $\mathbf{b}$ is fed to the 'OFDM-transmitter' consisting of the following parts [5]: First, $\mathbf{b}$ is mapped onto N_M subcarriers by serial-parallel conversion. Each vector with N_M elements is interleaved in frequency domain and transformed into the time domain by the IFFT. After parallel-serial conversion a guard interval is appended avoiding intersymbol interference. Therefore, a frequency-domain one-tap-equalizer with maximum ratio combining can be employed [4]. For the remainder of the paper, the number of carriers for

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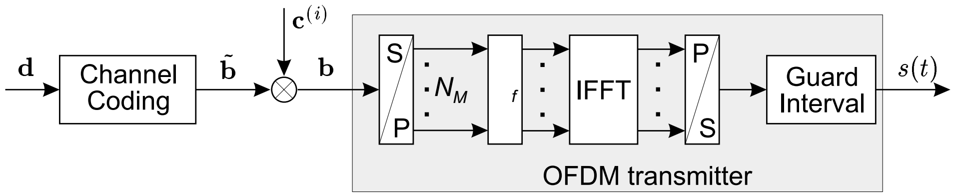

Fig. 1: Structure of coded OFDM-CDMA transmitter

OFDM transmission equals $N_M = 64$.

B. Coding scenarios

In this paper we distinguish three different coding scenarios depicted in Figure 2. In order to ensure the same processing gain for systems, the different coding schemes possess the same overall code rate

$$R_c = \frac{1}{N_M} = \frac{1}{64} \ . \tag{1}$$

This leads to a scenario, where, in the average, one information bit is mapped to one OFDM symbol.

Conventional coding scenario (CCRPC)

The conventional coding system (CCRPC) employs a recursive convolutional code of code rate $R_c^{\mathrm{cc}} = 1/n$, constraint length $L_c = 7$ and generator polynomials $(133_8,\ 171_8)$. It is followed by a repetition code with rate $R_c^{\mathrm{rpc}} = 1/N_P = n/N_M$ ensuring a constant entire code rate $R_c = R_c^{\mathrm{cc}} \cdot R_c^{\mathrm{rpc}} = 1/N_M$ that represents the reciprocal of the total processing gain. In order to shed some light on the influence of the choice of R_c^{cc} and R_c^{rpc}, we have examined three combinations in our investigations that are depicted in Table I.

Code-Spread System (CSP)

It is well known that repetition codes have very poor error correcting capabilities. Thus, the question arises if the repetition code can be replaced by a more powerful code. Reducing R_c^{cc} to the minimum value of $R_c = 1/N_M$ results in a single very low rate convolutional code, and the repetition code is discarded. This approach was first introduced in [8] and is termed code-spreading. Note that the whole bandwidth expansion is already performed by the channel encoder and the bit duration $T_{\tilde{b}}$ equals the chip rate T_c. Low rate convolutional codes have been found by computer search [9]. They possess a maximum free distance d_f and a minimum number of sequences with weight d_f. The advantage of the code-spread system employing only one single very low rate code over concatenated coding schemes is the possibility of optimal maximum likelihood decoding by the Viterbi algorithm. Table I shows the coding parameters used in our investigations.

Serially Concatenated Coding Scheme (SCCS)

The introduction of the code-spread system was motivated by replacing the RPC by a more powerful code. A different approach in reaching this goal was presented

	CC, $(133_8,\ 171_8)$	Walsh Code	RPC
CCRPC	1/2	-	1/32
	1/4	-	1/16
	1/8	-	1/8
SCCS	1/2	6/64	1/3
(N=600)	1/2	8/256	1
CSP	1/64	-	1

TABLE I
Rates of constituent codes for different coding schemes

in [5] where M-ary orthogonal Walsh modulation was introduced as an inner spreading system increasing the overall system performance. This modulation concept can be interpreted as a systematic linear block code of code rate $R_c^{\mathrm{w}} = \log_2 M/M$. Thus, the entire coding scenario describes a serial concatenated coding scheme consisting of a convolutional code, a Walsh code and a repetition code as depicted in Figure 2. Although it is known from [12] that the inner code of a concatenated coding scheme should be a recursive convolutional code, Walsh codes provides low decoding complexity by the Fast Hadamard transform.

Concerning this coding scheme, the code rate of the convolutional code was fixed to $R_c^{\mathrm{cc}} = 1/2$. Hence, introducing the Walsh code affects only the repetition code whose code rate R_c^{rpc} increases in the same way as R_c^{w} decreases (s. Table I). Simulation results show that lower rates of the outer convolutional code (e.g. $R_c^{\mathrm{cc}} = 1/6$) coming along with higher rates of the inner codes (e.g. $R_c^{\mathrm{rpc}} = 1$) leads a significant performance loss. This observation indicates that it is important to increase the performance of the inner codes, i.e. the Walsh code. The interleaver Π_t between CC and Walsh encoder is a simple block-interleaver with 20 rows and 30 columns ($N = 600$).

C. Channel Model

The considered OFDM-CDMA system is applied to an uplink transmission where the signals of different users are transmitted over different channels. As mobile radio channel, a fully symbol-interleaved 4-path Rayleigh fading channel with equal average power on each tap is used, i.e. the channel is assumed to remain unchanged during one OFDM symbol. Successive channel coefficients are assumed to be statistically independent. Due to the time varying behavior of the channel,

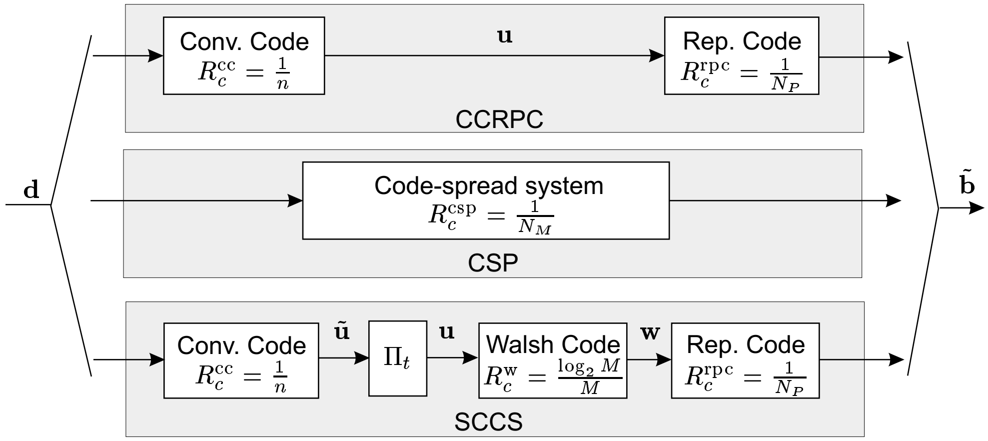

Fig. 2: Three different coding scenarios

the SNR can differ from symbol to symbol. The inherent mismatching due to the inserted guard interval equals $10\log_{10}(1-3/64) = 0.2$dB and is the same for all configurations.

Note that the number of carriers, and, hence, the mismatching is always the same regardless of the specific coding scheme. Due to this restriction, the degree of diversity is not the same for all coding scenarios. For the conventional system (CCRPC) as well as for the code-spread system (CSP) the code word of each information bit d_k is exactly mapped to one OFDM-symbol. Hence, there is a one-to-one correspondence. Introducing Walsh codes leads to a different situation. Here, e.g. for 64-ary Walsh modulation 3 information bits d_k are first half-rate convolutionally encoded and then fed to the Walsh encoder that generates 64 code bits. In order to ensure a constant overall code rate R_c, we have to append a repetition code replicating each code bit $N_P = 3$ times. These $3 \cdot 64 = 192$ bits are now distributed to three successive OFDM-symbols that are affected by different channel conditions. Hence, the code bits of each information bit are spread over three different OFDM-symbols supplying a higher diversity gain compared with the other two coding schemes. The same holds for 256-ary Walsh coding where each information bit is distributed over four Walsh symbols.

D. Receiver

The structure of the receiver is illustrated in Figure 3. First, the OFDM-receiver performs the following steps: After removing the guard interval and serial-parallel conversion, the FFT transforms the received sequence in the frequency domain. Here, an one-tap-equalization with maximum ratio combining is carried out. After subsequent de-interleaving, parallel-serial conversion, detection of the real part and descrambling, the signal $\hat{\tilde{\mathbf{b}}}$ consisting of log-likelihood-ratios enters the channel decoder. Its structure depends on the specific channel coding scenario and is depicted in Figure 4.

The conventional system (CCRPC) consists of a combiner that correlates N_P successive samples (decoding RPC by integrate&dump) followed by a Viterbi decoder. This structure is nearly the same for the code-spread system except the fact that an unequal combiner is used with respect to the specific code construction [8]. For the SCCS scheme, we need a structure as depicted at the bottom of Figure 4. First, a combiner decodes the repetition code. Then, an iterative decoding process starts consisting of an inner symbol-by-symbol Max-Log-MAP-decoder [11] for the Walsh Hadamard code and an outer symbol-by-symbol Max-Log-MAP decoder for the convolutional code. The extrinsic information of each decoder is extracted and fed to the successive decoder improving the performance compared with a single decoding iteration.

III. Performance Analysis

In order to obtain some fundamental results concerning the performance of the different coding schemes, we have analyzed their distance properties and calculated the bit error rate performance by the union bound. Regarding the mobile radio channel, it has to be mentioned that a fully bit-interleaved fading channel is assumed. This diverges from the assumptions made in our simulations (section II.C) where the 4-path Rayleigh fading channel with perfect OFDM-symbol interleaving is used. Here, four taps in the time domain lead to correlated carriers in the frequency domain contradicting the assumption of perfect bit-interleaving. Therefore, a higher diversity gain is expected for the analytical performance analysis prohibiting a quantitative comparison of analytical and simulation results. Thus, only qualitative comparisons can be made.

The *union bound* is given by [13]

$$P_b \le \sum_d c_d P_d \;, \tag{2}$$

with P_d as pairwise error probability for two code words with Hamming distance d. In case of a fully-interleaved Rayleigh fading channel P_d can be expressed by

$$P_d = \left(\frac{1-\mu}{2}\right)^d \cdot \sum_{j=0}^{d-1} \binom{d-1+j}{j} \cdot \left(\frac{1+\mu}{2}\right)^j \tag{3}$$

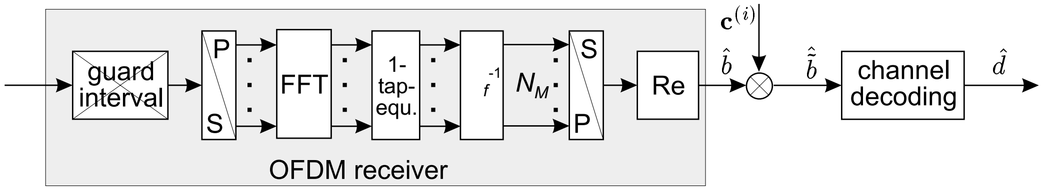

Fig. 3: Structure of OFDM-CDMA receiver

with

$$\mu = \sqrt{\frac{\gamma}{1+\gamma}} \tag{4}$$

and

$$\begin{aligned} \gamma &= \frac{E_s/N_0}{1+(J-1)E_s/N_0} \\ &= \frac{R_c E_b/N_0}{1+(J-1)R_c E_b/N_0} \ , \end{aligned} \tag{5}$$

where J indicates the number of active users. In contrast to an AWGN channel, d and R_c do not compensate each other. This is why the repetition code has to be considered explicitly. The parameter c_d has to be determined by the distance spectra of the different entire coding schemes.

A computer search delivers the the IOWEF (Input Output Weight Enumerating Function) of the convolutional codes [12]

$$A^{\mathrm{cc}}(W,D) = \sum_{w,d} A^{\mathrm{cc}}_{w,d} \cdot W^w \cdot D^d \ , \tag{6}$$

where $A^{\mathrm{cc}}_{w,d}$ denotes the number of path with input weight w and output weight d of the CC. Equation (6) is valid for all used convolutional codes including the code-spread system (CSP). Concerning the CCRPC scheme, we have to consider the repetition code by multiplying all weights d by N_P resulting in

$$\begin{aligned} A^{\mathrm{ccrpc}}(W,D) &= \sum_{w,d'} A^{\mathrm{ccrpc}}_{w,d'} \cdot W^w \cdot D^{d'} \\ &= \sum_{w,d} A^{\mathrm{cc}}_{w,d} \cdot W^w \cdot D^{d \cdot N_P} \end{aligned} \tag{7}$$

The IOWEF of the Walsh code can be determined without computer search because all code words $\mathbf{w}$ have identical Hamming weight $d = M/2$. By including the all-zero word we obtain

$$A^{\mathrm{w}}(W,D) = 1 + \sum_{w=1}^{\log_2 M} A^{\mathrm{w}}_{w,d=M/2} \cdot W^w \cdot D^{M/2} \tag{8}$$

The concatenation of Walsh and convolutional codes requires the consideration of the interleaver Π_t of length N. Assuming that m input vectors $\mathbf{u}$ of the Walsh encoder generally fits the interleaver size N, we obtain a parallel arrangement of m Walsh encoders leading to the expression [12]

$$\begin{aligned} A^{\mathrm{w}^m}(W,D) &= [A^{\mathrm{w}}(W,D)]^m \\ &= \sum_{w,d} A^{\mathrm{w}^m}_{w,d} \cdot W^w \cdot D^d \end{aligned} \tag{9}$$

Refering to [12] we now make use of the *uniform interleaver*

$$A^{\mathrm{cc,w}}(W,D) = \sum_{w,d} A^{\mathrm{cc,w}}_{w,d} \cdot W^w \cdot D^d \tag{10}$$

with

$$A^{\mathrm{cc,w}}_{w,d} = \sum_{l} \frac{A^{\mathrm{cc}}_{w,l} \cdot A^{\mathrm{w}^m}_{l,d}}{\binom{N}{l}} \ . \tag{11}$$

Finally, the repetition code is taken into account by multiplying all weights d with N_P.

$$\begin{aligned} A^{\mathrm{sccs}}(W,D) &= \sum_{w,d'} A^{\mathrm{sccs}}_{w,d'} \cdot W^w \cdot D^{d'} \\ &= \sum_{w,d} A^{\mathrm{cc,w}}_{w,d} \cdot W^w \cdot D^{d \cdot N_P} \end{aligned} \tag{12}$$

From (12), the coefficients c_d can be extracted in the following way

$$\left.\frac{\partial A^{\mathrm{sccs}}(W,D)}{\partial W}\right|_{W=1} = \sum_{d'} \underbrace{\sum_{w} w \cdot A^{\mathrm{sccs}}_{w,d'}}_{c'_d} \cdot D^{d'} \ . \tag{13}$$

With (3) and (13) all parameters are determined to calculate the bit error probability by the union bound. The results obtained are shown in Figure 5. Be aware that E_b denotes the energy per information bit d_k and not the energy of a coded bit in $\mathbf{b}$.

First, the conventional CCRPC and the CSP schemes are considered. It can be seen that a code rate lower than $R_c^{\mathrm{cc}} = 1/4$ for the CC does not lead to a significant improvement. Only the transition from $R_c^{\mathrm{cc}} = 1/2$ to $R_c^{\mathrm{cc}} = 1/4$ results in a gain of about 1 dB at $P_b = 10^{-5}$. This fact can be explained by the distance properties of the combination of CC and RPC. The free distance d_f is nearly the same for all parameter constellations, only the coefficients c_d decrease significantly from $R_c^{\mathrm{cc}} = 1/2$ to $R_c^{\mathrm{cc}} = 1/4$. For $R_c^{\mathrm{cc}} < 1/4$ only small changes of c_d can be observed leading only to minor improvements of the bit error probability. This even holds for the CSP system that incorporates an unequal repetition encoder. Thus, its distance properties are only slightly improved compared with the concatenation of CC and RPC both of rate $R_c^{\mathrm{cc}} = R_c^{\mathrm{rpc}} = 1/8$ resulting in a minor performance gain.

In contrast to the above mentioned results, the SCCS with Walsh Hadamard coding performs much better. Although the union bound is very loose at low signal to noise ratios, we found that the combination of Walsh

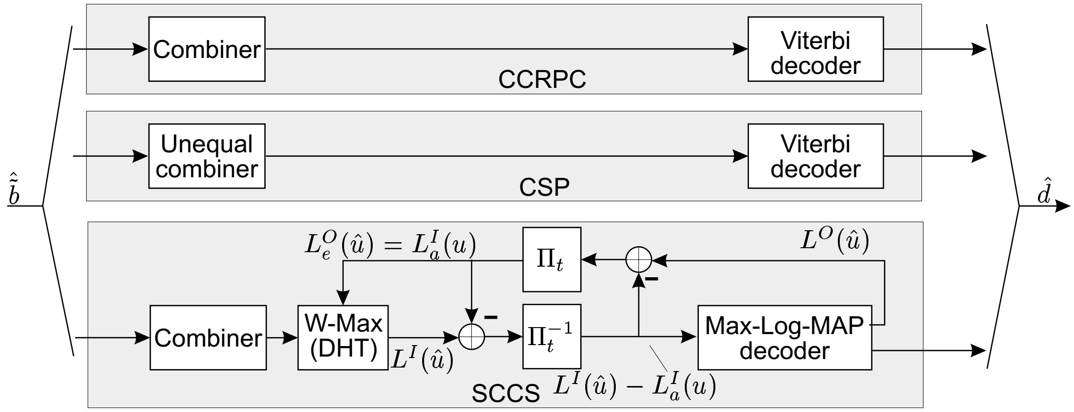

Fig. 4: Different decoding schemes

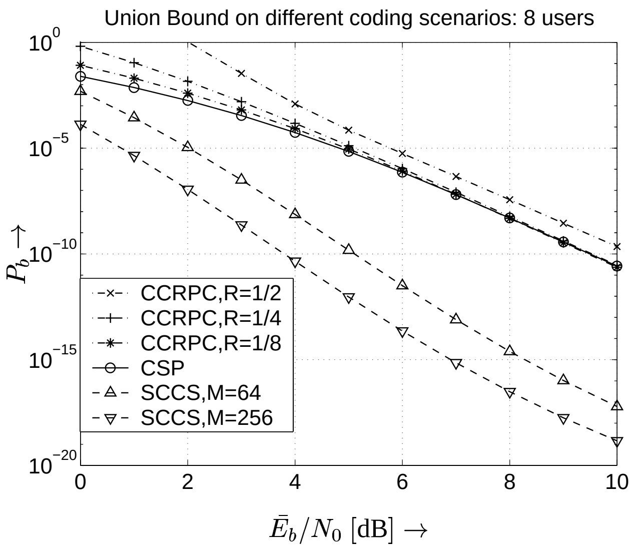

Fig. 5: Results of union bound for different coding schemes and $J = 8$ active users

Hadamard ($M = 64$) and convolutional coding yields a remarkable performance gain of nearly 3 dB at $P_b = 10^{-5}$. Additional improvements can be achieved by replacing the repetition code ($N_P = 3$) by a stronger Walsh Hadamard code ($M = 256$).

IV. Simulation Results

The received simulation results for $J = 1$ and $J = 8$ active users are shown in Figure 6 and Figure 7, respectively. For the SCCS, two decoding iterations are performed. Although the results cannot be directly compared with analytical results of section III, a good qualitative correspondence can be observed. As already mentioned, reducing R_c^{cc} do not lead to significant improvements for $R_c^{\mathrm{cc}} < 1/4$. This holds also for the code-spread system. For $P_b > 10^{-3}$, the SCCS with Walsh Hadamard code outperforms the other coding schemes.

However, for a bit error rate above $P_b > 10^{-3}$ the SCCS loses approximately 2 dB. This holds for $J = 1$ as well as $J = 8$ and contradicts the analytical results where perfect maximum likelihood decoding of the entire code is assumed. In our opinion, this difference can be explained by the sub-optimum iterative decoding procedure. It starts with the weakest code (RPC) first, then decodes the Walsh Hadamard code and finally the CC (repeating this procedure ones). Hence, if the weak inner codes produce many decoding errors, the convolutional decoder is not able to correct them. Therefore, its extrinsic information is not reliable enough to improve the performance significantly in a second iteration. This explanation is substantiated by the observation that weaking the inner code by discarding the RPC in favour to a stronger CC of rate $R_c^{\mathrm{cc}} = 1/6$ yields a significant performance loss. Furthermore, even a larger interleaver Π_t of size 6000 bits does not close the gap between CCRPC and SCCS for $P_b > 10^{-3}$.

Considering the performance of the entire coding scheme, it might be better to employ a strong convolutional code as inner code. However, the inner decoder operates at very high data rates where the low decoding complexity is necessary, e.g. the efficient Fast Hadamard Transformation for decoding Walsh codes.

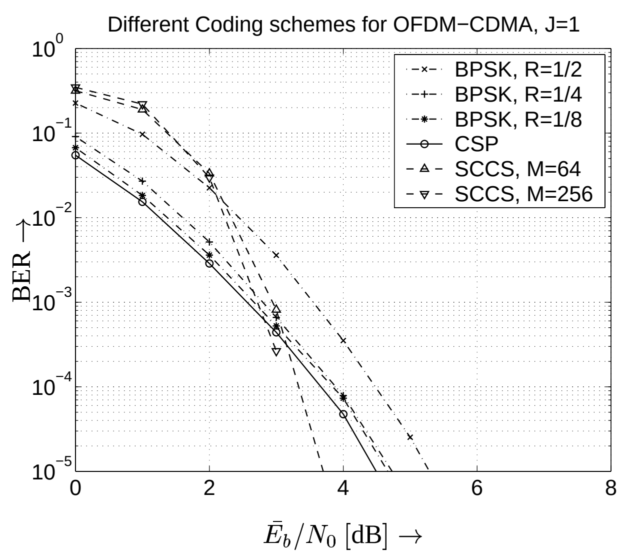

Fig. 6: Bit error rates for 4-path Rayleigh fading channel: $J = 1$

Finally, it has to be mentioned that no multi-user detection (MUD) has been applied. Especially for bit error

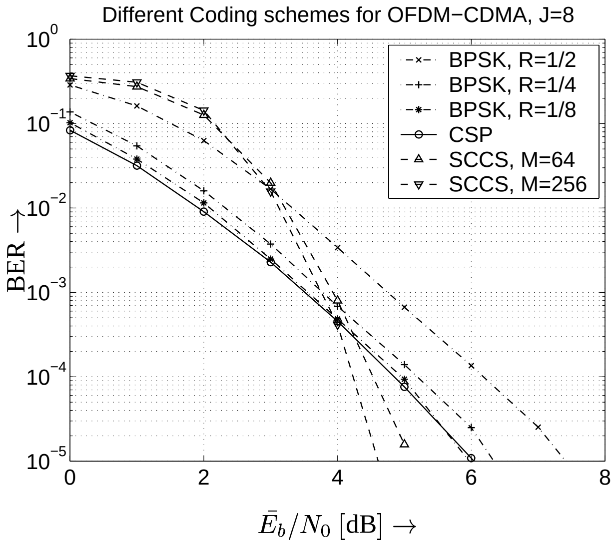

Fig. 7: Bit error rates for 4-path Rayleigh fading channel: $J = 8$ user

rates above $P_b = 10^{-3}$ where the Walsh coded system is inferior to the others there might be an advantage for CSP and CCRPC with MUD.

V. Conclusion and Outlook

In general, the paper indicates that significant performance improvements can be achieved by replacing the weak repetition code inherent in many CDMA systems by a more powerful code, e.g. a Walsh code or a convolutional code of lower rate. Otherwise, reducing the convolutional code rates below $R_c^{cc} < 1/4$ only leads to minor improvements, including also the extension to the code-spread system. Solely, the case $R_c^{cc} = 1/4$ significantly outperforms $R_c^{cc} = 1/2$. Analytical investigations by the union bound approximation as well as Monte-Carlo simulation confirm these statements. Concerning the concatenation of convolutional code, Walsh code and repetition code the SCCS outperforms CSP and CCRPC for bit error rates below $P_b > 10^{-3}$.

Further research has to be carried out concerning the influence of multi-user detection which can also be interpreted as a decoding algorithm and thus be incorporated into an iterative decoding process. Additionally, other code combinations than Walsh codes and convolutional codes should be investigated because it is well known that the inner code in a serially concatenated coding scheme should be a recursive convolutional code [12]. Another question affects the application of the discussed coding schemes in single-carrier DS-CDMA system although similar properties are expected.

VI. References

[1] E. Dahlman, B. Gudmundson, M. Nilsson, and J. Sköld. UMTS/IMT-2000 Based on Wideband CDMA. *IEEE Communications Magazine*, September 1998.

[2] T. Ojanperä and R. Prasad. An Overview of Air Interface Multiple Access for IMT-2000/UMTS. *IEEE Communications Magazine*, September 1998.

[3] K. Gilhousen, I. Jacobs, R. Padovani, A.J. Viterbi, L. Weaver, and C. Wheatley. On the Capacity of a Cellular CDMA System. *IEEE Transactions on Vehicular Technology*, 40, May 1991.

[4] Stefan Kaiser. *Multi-Carrier CDMA Mobile Radio Systems – Analysis and Optimization of Detection, Decoding and Channel Estimation*. PhD thesis, German Aerospace Center, VDI, January 1998.

[5] A. Dekorsy and K.D. Kammeyer. M-ary Orthogonal Modulation for MC-CDMA Systems in Indoor Wireless Radio Networks. In G.P. Fettweis and K. Fazel, editors, *First International Workshop on MC-SS*, volume 1, pages 69–76, Oberpfaffenhofen, Germany, April 1997. Kluwer Academic, The Netherlands.

[6] A. Dekorsy, S. Fischer, and K.D. Kammeyer. Maximum Likelihood Decoding of M-ary Modulated Signals for Multi-Carrier Spread-Spectrum Systems. In *In Proc. IEEE Int. Symp. on Personal, Indoor and Mobile Radio Communications (PIMRC)*, September 1998.

[7] A. Dekorsy and K.D. Kammeyer. A new OFDM-CDMA Uplink Concept with M-ary Orthogonal Modulation. *Accepted for European Transactions on Telecommunications (ETT).*

[8] P. Frenger, P. Orten, and T. Ottosson. Code-Spread CDMA using Maximum Free Distance Low-Rate Convolutional Codes. In *In Proc. IEEE Int. Symp. on Spread Spectrum Techniques and Applications (ISSSTA)*, pages 374–378, September 1998.

[9] P. Frenger, P. Orten, T. Ottosson, and A. Svensson. *Multi-rate Convolutional Codes*. Technical Report No. 21, Göteborg, 1998.

[10] A.J. Viterbi. Very low rate convolutional codes for maximum theoretical performance of spread-spectrum multiple-access channels. *IEEE Journal on Selected Areas in Communications*, 8(4):641–649, May 1990.

[11] R. Herzog, A. Schmidbauer, and J. Hagenauer. Iterative Decoding and Despreading improves CDMA-Systems using M-ary Orthogonal Modulation and FEC. In *In Proc. IEEE International Conference on Communications (ICC)*, volume 2, pages 909–913, June 1997.

[12] S. Benedetto, G. Montorsi, D. Divsalar, and F. Pollara. Serial Concatenation of Interleaved Codes: Performance Analysis, Design, and Iterative Decoding. *TDA Report*, 42(126), August 1996.

[13] J. G. Proakis. *Digital Communications*. McGraw-Hill, third edition, 1995.