Exploiting Time and Frequency Diversity by Iterative Decoding in OFDM-CDMA Systems

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Abstract

This paper examines iterative decoding of a serial concatenated coding scheme consisting of an outer convolutional and an inner Walsh-Hadamard block code, and the scheme is applied for an OFDM-CDMA uplink transmission. New aspects arise because OFDM enables the iterative decoding structure to exploit time as well as frequency diversity of a mobile radio channel. To the authors knowledge, this is the first time that iterative decoding is performed in both time and frequency domain. Besides the derivation of basic decoding algorithms, simulation results are shown for differently interleaved indoor channels. The results confirm that the iterative decoding scheme benefits from the selectivity in time as well as in frequency direction.

1. Introduction

One interesting candidate for future mobile communication systems is the combination of *Orthogonal Frequency Division Multiplexing* (**OFDM**) with *code division multiple access* (**CDMA**) [1, 2]. In this context, OFDM is used to combat the frequency selectivity of the mobile radio channel where CDMA leads to diversity and user separation. In case of asynchronous CDMA uplink transmission inherent *multiple access interference* (**MAI**)[1] requires efficient low rate channel coding under the constraint of low decoding complexity. In order to meet these requirements *serial concatenated coding schemes* (**SCCS**) consisting of an outer *convolutional code* (**CC**) and an inner $(M, \log_2(M), M/2)$ *Walsh-Hadamard* (**WH**) block code can be applied [3, 4]. Well-known schemes are the QUALCOMM system [5] (that is based on single carrier *Direct Sequence* (**DS**)-CDMA and also uses WH block codes) or systems with one single low rate channel code [6].

In contrast to these systems, the proposed coding scheme *(i)* incorporates a serial concatenation of codes, and *(ii)* is embedded in OFDM-CDMA. Due to these differences, some important new aspects arise. First, decoding of the WH code is merely performed in frequency direction, whereas the decoder of the outer CC mainly acts in time direction. Therefore, in case of mobile radio channels, the proposed SCCS benefits from the selectivity of the channel in frequency as well as in time domain (e.g. large interleaver sizes can be avoided in case of low Doppler frequencies). Moreover, the system performance for SCCS can be improved by iterative decoding. Results for the application for the above mentioned single carrier DS-CDMA based QUALCOMM system show promising performance gains [5]. Finally, we can use the low cost *Fast Hadamard Transform* (**FHT**) for *maximum a-posteriori* (**MAP**) decoding of WH code words.

The main objective of this paper is to examine iterative decoding in both frequency and time domain in an OFDM-CDMA based uplink transmission. For low rate coding, we propose a SCCS that consists of an outer CC and an inner WH code. In the recent few years, a lot of work has been done to compare conventional *Non-Systematic Convolutional* (**NSC**) codes [7] to *Recursive Systematic Convolutional* (**RSC**) codes [8]. We will evaluate a common *log-likelihood ratio* (**LLR**) expression of the coded source bits for these two kinds of codes and will briefly discuss their *bit error rate* (**BER**). Finally, in order to show the two-dimensional exploitation of the iterative decoding structure, the examination is accomplished by Monte-Carlo simulation results for differently interleaved indoor channels. For comparison, we will also present results for an OFDM-CDMA uplink system with convolutional coding (NSC) and subsequent spreading with PN sequences (no WH code). This system will be termed as conventional throughout the paper.

The paper is structured as follows: In section 2, SCCS embedded in the OFDM-CDMA system is presented, and the iterative decoding scheme is described in section 3. We will discuss the MAP decoding algorithms in section 4. In section 5, simulation results are presented, and section 6 concludes the paper.

[1]Due to asynchronous received user signals, CDMA uplink transmission systems uses non-optimized *Pseudo-Noise* (**PN**)-sequences for user separation.

2. SCCS and OFDM-CDMA transmitter

An OFDM-CDMA transmitter with SCCS for one user is illustrated in fig. 1. The data bits $d \in \{0,1\}$, each of duration T_d, are convolutionally encoded with code rate $R_c = K/N$. The input of the encoder is a sequence $\boldsymbol{d}$ of K subsequent data bits and the output is the encoded bit sequence $\tilde{\boldsymbol{b}}$ of N bits, each with duration $T_b = R_c\,T_d$. In order to analyze the proposed system, we apply familiar NSC codes [7] or RSC codes [8].

After block interleaving (Π_t) in time domain the encoded bits are serial-parallel converted to groups of $\log_2(M)$ bits each. A $(M, \log_2(M), M/2)$ WH block code with subsequent BPSK maps the $\log_2(M)$ data bits to the vector $\boldsymbol{w}^m = [w_0^m, w^m, \ldots, w_{M-1}^m]^T$ that can be interpreted as a

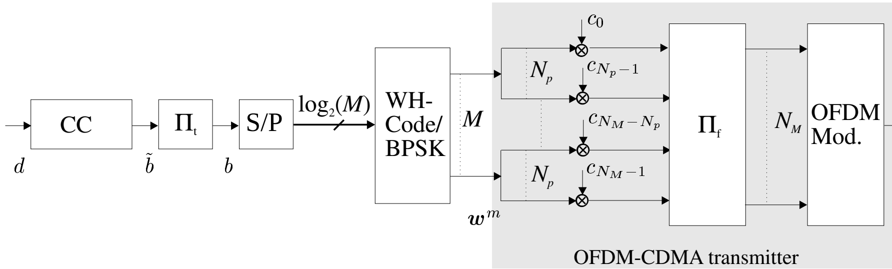

Figure 1: SCCS and OFDM-CDMA transmitter

code word in the Euclidean signal space including M code symbols $w_\mu^m \in \{\pm 1\}$, $\mu = 0, \ldots, M-1$. This mapping is also well-known as M-ary orthogonal Walsh modulation [5, 4].

In contrast to the application of WH block codes for DS-CDMA based systems [5], be aware that for the considered OFDM-CDMA system a WH code word is arranged in *frequency direction*. Furthermore, WH codes are systematic with $\log_2(M)$ systematic symbols

$$w^m_{sys(\kappa)} = (-1)^{b_\kappa}, \quad b_\kappa \in \{0, 1\} \tag{1}$$

at positions

$$sys(\kappa) = \frac{1}{2^{\kappa+1}} M, \; \kappa = 0, \ldots, \log_2(M) - 1. \tag{2}$$

After WH encoding, the code symbols are replicated into N_p copies. Each branch of the parallel stream is then multiplied with one chip of the user specific PN-code $c_i \in \{\pm 1/\sqrt{N_M}\}$, $i = 0, \ldots, N_M - 1$, and the number of carriers is denoted by N_M.

Then, the symbols are randomly frequency interleaved (Π_f) and OFDM modulated. The OFDM modulator consists of the IDFT and a parallel-serial converter, and it also includes the insertion of a guard interval between adjacent OFDM symbols.

Note that the number of carriers for OFDM transmission is chosen to be $N_M = M\, N_p$. This choice leads to a scenario where in the average $\log_2(M)$ bits b_κ are mapped to one OFDM symbol, and it will result in the advantage of less mismatching if the product $M\, N_p$, this means the number of carriers, is raised (it is assumed that the guard time and the entire bandwidth remain unchanged).

Due to the guard interval, each carrier is affected by only one channel transfer coefficient [2] that is complex valued Gaussian distributed. Since we confine on an uplink transmission scenario, the reception of the user specific signals is asynchronous. We assume perfect synchronization of the user concerned and the delay times of the other active users are equal to an integer multiple of the OFDM sampling time. Hence, the reception can also be interpreted as quasi-synchronous.

3. OFDM-CDMA receiver with iterative decoding

For one user, coherent OFDM-CDMA reception with subsequent iterative decoding is shown in fig. 2. The OFDM-CDMA receiver (shadowed block) includes the OFDM demodulation, deinterleaving (Π_f^{-1}), multiplication with the user specific code, one-tap equalization, correlation of N_p carriers, and the evaluation of the real part. For equalization, we focus on *maximum ratio combining* (**MRC**) [2] with perfectly known channel coefficients. In case of channel estimation, pilot based methods in one or two dimensions are possible [2]. Note that the first code symbol of a WH code word equals 0. This known information can be used for pilot tones[2] depending on spreading with N_p.

The applied MAP decoder for the WH block code requires LLRs as input signals. Let us assume statistically independent adjacent fading carriers ensured by perfect frequency interleaving (Π_f), statistically independent user signals, and a non-dissipative channel. Thus, we can approximate the LLR at position $\mu = 0, \ldots, M-1$ by [9]

$$L_\mu \approx 4 \; \underbrace{\frac{E_s}{N_0} \frac{1}{1 + \frac{J-1}{N_M} \frac{E_s}{N_0}}}_{L_c} \; v_\mu, \tag{3}$$

where L_c is termed reliability of the channel and v_μ is the μ-th component of vector $\boldsymbol{v}$. The WH code word energy[3] in the bandpass and the two-sided noise spectral density are denoted by E_s and $N_0/2$, respectively, and J is the number of simultaneously active users.

The iterative decoding structure is a serial decoding scheme being similar to the scheme introduced in [8], and is shown in detail in fig. 2. It consists of two *symbol-by-symbol MAP* (**SS-MAP**) decoders, where the inner one represents a special SS-MAP decoder for the systematic WH block code. This means that we can use the low cost FHT being a major advantage. The outer SS-MAP decoder is based on the MAP decoding algorithm introduced in [10] and in contrast to the modified algorithm in [8] it delivers soft information for the information bits as well as for the code bits. Both decoders

[2]The application of WH block codes also offers the possibility of channel phase estimation based on decision-directed estimation without required pilot tones. This method has been investigated in [3].

[3]with subsequent BPSK

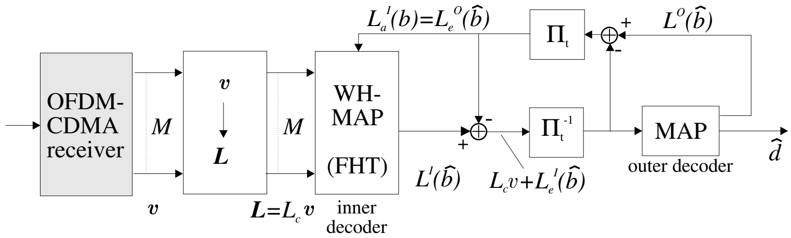

Figure 2: Coherent OFDM-CDMA receiver and iterative decoding scheme

perform decoding in the logarithmic domain and therefore require input LLRs. With (2) and (3), we can express the LLRs by three parts

$$\begin{aligned} L^I(\hat{b}_\kappa) &= L_c v_{sys(\kappa)} + L_a^I(b_\kappa) + L_e^I(\hat{b}_\kappa) \\ L^O(\hat{b}_\kappa) &= L_c v_{sys(\kappa)} + L_a^O(b_\kappa) + L_e^O(\hat{b}_\kappa). \end{aligned} \tag{4}$$

The first part includes the channel reliability, the second one is the a-priori information $L_a^I(b_\kappa)$ for the decoded bit of the inner code, and $L_a^O(b_\kappa)$ for the coded bit of the outer code. The third part describes the extrinsic LLRs, $L_e^I(\hat{b}_\kappa)$ and $L_e^O(\hat{b}_\kappa)$, gleaned from the decoding process for the decoded/coded bit, respectively. In the first iteration, no a-priori information is available, whereas for the iteration steps following the extrinsic information delivered by the inner decoder is used as a-priori information for the outer decoder, and vice versa. Thus, we obtain

$$\begin{aligned} L_a^I(b_\kappa) &\stackrel{!}{=} L_e^O(\hat{b}_\kappa) \\ L_a^O(b_\kappa) &\stackrel{!}{=} L_e^I(\hat{b}_\kappa). \end{aligned} \tag{5}$$

4. MAP decoding algorithms

4.1. SS-MAP decoding of Walsh-Hadamard block codes

Low cost SS-MAP decoding of block codes is still an open problem, and there exists different decoding algorithms or implementations. Some significant examples are the trellis implementation, the dual code method, and the direct implementation [11]. Since we use WH block codes, it is possible to meet the requirement of low cost decoding by applying the FHT embedded for the MAP decoding structure.

Furthermore, besides the foregoing mentioned general decoding implementations, we can use special WH-MAP decoding structures that were for the first time introduced in [12], enhanced in [5] for iterative decoding in DS-CDMA systems, and introduced in [13] for coherent OFDM-CDMA transmission with Viterbi decoding. All these algorithms have in common that the evaluation of the output LLR proceeds from the knowledge of the probability density functions, and uses the input signal v_μ instead of the corresponding LLR L_μ.

In this paper, we focus on the *direct implementation* of systematic block codes [5, 11], where especially for WH codes the SS-MAP decoding rule can be expressed by[4]

$$\begin{aligned} L^I(\hat{b}_\kappa) &= \ln \frac{\sum\limits_{\boldsymbol{w}^m \varepsilon C^I,\, b_\kappa=+1} P(\boldsymbol{w}|\boldsymbol{v})}{\sum\limits_{\boldsymbol{w}^m \varepsilon C^I,\, b_\kappa=-1} P(\boldsymbol{w}|\boldsymbol{v})} \\ &= \ln \frac{\sum\limits_{\boldsymbol{w}^m \varepsilon C^I,\, b_\kappa=+1} \exp\left(\frac{1}{2}\mathrm{FHT}\{L(w_\mu; v_\mu)\}\right)}{\sum\limits_{\boldsymbol{w}^m \varepsilon C^I,\, b_\kappa=-1} \exp\left(\frac{1}{2}\mathrm{FHT}\{L(w_\mu; v_\mu)\}\right)}, \end{aligned} \tag{6}$$

with

$$L(w_\mu; v_\mu) = \begin{cases} L_c v_\mu + L_a^I(b_\kappa) & \text{for} \quad \mu = sys(\kappa), \\ L_c v_\mu & \text{else.} \end{cases} \tag{7}$$

Eq. (6) shows the application of the FHT for the MAP decoding implementation where the set of code words is denoted by C^I and the *a-posteriori-probability* (**APP**) by $P(\boldsymbol{w}|\boldsymbol{v})$. With the FHT, we carry out the correlation operation.

In order to further reduce the complexity of the MAP algorithm (6), we can use the approximation $\ln(e^{x_1} + e^{x_2}) \approx \max(x_1, x_2)$. This leads to the expression [5]

$$L^I(\hat{b}_\kappa) \simeq \frac{1}{2} \max_{\substack{\boldsymbol{w}^m \varepsilon C^I, \\ b_\kappa=+1}} \mathrm{FHT}\{\cdot\} - \frac{1}{2} \max_{\substack{\boldsymbol{w}^m \varepsilon C^I, \\ b_\kappa=-1}} \mathrm{FHT}\{\cdot\}. \tag{8}$$

4.2. SS-MAP trellis based decoding of RSC/NSC codes

The SS-MAP trellis based algorithm for NSC codes was first proposed by *Bahl et. al* in [10] providing soft information for the coded and for the information bits. In [8], this algorithm was adapted to RSC codes with the restriction to deliver only soft information for the information bits. In the following, we will give a common expression for RSC and NSC codes concerning the soft information of the coded bits. The derivation chosen is based on the idea given in [10]. This means that we use the APP of a trellis transition which is directly associated with the APP to be $+1$ or -1 of the corresponding coded bits.

[4] In comparison to (1), the coded bits b_κ can have values $\{-1, +1\}$ by mapping $0 \rightarrow 1$ and $1 \rightarrow -1$.

The LLR of a code bit can be expressed by

$$L^O(\hat{b}_\kappa) = \ln\frac{P(b_\kappa=+1|R_1^N)}{P(b_\kappa=-1|R_1^N)} = \ln\frac{\sum\limits_{(m',m)\varepsilon B_{+1}^k} P(S_{k-1}=m',\, S_k=m\,|R_1^N)}{\sum\limits_{(m',m)\varepsilon B_{-1}^k} P(S_{k-1}=m',\, S_k=m\,|R_1^N)},$$

$$\kappa = 0,\dots,\log_2(M)-1, \quad k=1,\dots,N, \qquad (9)$$

where $B_i^k = \{S_{k-1}=m',\, S_k=m,\, b_\kappa^k=i\}$, $i\in\{-1,+1\}$ is the set of transitions from state $S_{k-1}=m'$ to $S_k=m$, and R_1^N is a received sequence from time 1 through some time N. Let us define the probabilities related to [8, 10]

$$\alpha_k(m) = P(S_k=m|R_1^k)$$
$$\beta_k(m) = P(R_{k+1}^N|S_k=m)/P(R_{k+1}^N|R_1^k) \quad (10)$$

and the branch transition probability

$$\gamma_i(R_k,m',m) = P(d_k=i,R_k,S_k=m|S_{k-1}=m')$$
$$= q(d_k=i|S_{k-1}=m',S_k=m)\cdot\prod_{\kappa=0}^{\log_2(M)-1}\frac{1}{1+e^{(iL_\kappa)}}. \quad (11)$$

Since the convolutional encoder is deterministic, the probability $q(\cdot)$ equals 0 or 1, and $d_k=i$ indicates the decoded data bit with $i\in\{-1,+1\}$. The second term in (11) includes the channel reliability and the a-priori-information of the data bit as LLR summed up in L_κ. Due to the assumptions that events after time k are not influenced by observations R_1^k and data bit d_k if state S_k is known, we can rewrite (9) to

$$L^O(\hat{b}_\kappa) = \frac{\sum\limits_{(m',m)\varepsilon B_{+1}^k}\sum_i \alpha_{k-1}(m')\,\gamma_i(R_k,m',m)\,\beta_k(m)}{\sum\limits_{(m',m)\varepsilon B_{-1}^k}\sum_i \alpha_{k-1}(m')\,\gamma_i(R_k,m',m)\,\beta_k(m)}. \quad (12)$$

In contrast to the realization given in [8], the summation in the numerator and denominator in (12) runs over all transitions from state $S_{k-1}=m'$ to state $S_k=m$ with regard to the code bit $b_\kappa=+1$ or $b_\kappa=-1$, resp., and not with regard to the information bit d_k.
In order to avoid numerical problems, especially in simulations using time variant channel models, we apply the suboptimal MAX-LOG-MAP approximation [11] of (12) by defining the probabilities (10), (11) in the log-domain, and additionally use the $\max(\cdot)$ approximation mentioned above.

5. Simulation Results

Detailed results concerning the trade-off between the CC, the WH code, and replication by N_p were presented in [13] when straight-forward Viterbi decoding is applied. Here, we confine the discussion on fixed $R_c=1/2$, $M=64$, and $N_p=4$ throughout the simulations. Termination of the convolutional code is performed for $18\cdot 32=576$ code bits, and, hence, the influence on the code rate of the tail bits can be neglected. Moreover, for low cost implementation, we apply the MAX approximation (8) for the decoding of the WH code words and the MAX-LOG-MAP decoding algorithm for the CC.
For all cases, $J=8$ active users are taken into account and perfect channel estimation as well as perfect synchronization is assumed for the user concerned. Results for a

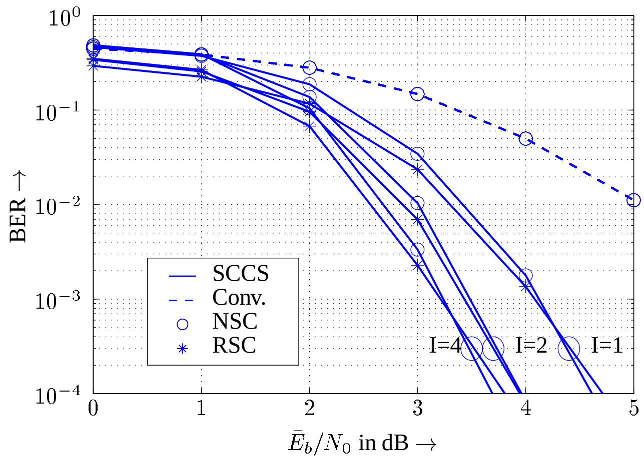

Figure 3: Perfect interleaving: SCCS with RSC or NSC code and iterative decoding $I=1,2,4$ compared to conventional OFDM-CDMA with a NSC code

RSC code as well as a NSC code are plotted in fig. 3 when perfect interleaving in time (Π_t) as well as in frequency (Π_f) direction is considered. The results indicate that RSC codes perform better for small $\bar{E}_b/N_0$, and for large $\bar{E}_b/N_0$ it is the other way around [8] (be aware that $\bar{E}_b$ denotes the bit energy at the output of the outer decoder).
Furthermore, independent of the code used, note a gain of about 0.7 dB for the second and approximately 0.9 dB for the fourth iteration at a BER of 10^{-3}. For comparison, the conventional OFDM-CDMA system with a concatenation of a NSC code $R_c=1/2$, a BPSK modulation and a simple replication ($N_p=24$) leads to an unacceptable BER[5].
Fig. 4 shows the BER for an uplink indoor channel differently interleaved. As an example, we used a RSC code and iterative decoding with four iterations ($I=4$). The indoor channel is related to the European HIPERLAN/2 standardization. In particular, we assumed a very low Doppler frequency of 9 Hz, a coherence bandwidth of approx. 1.8 MHz, and a total bandwidth of 25 MHz. Therefore, the fading on each carrier as well as the fading between carriers is highly correlated resulting in burst error structures. Four cases of interleaving are considered: (1) a random frequency (Π_f) and a 18x32 block time interleaver (Π_t), (2) merely

[5] Since no WH block code is included, N_p can be larger (24 instead of 4) in case of an unchanged entire spreading.

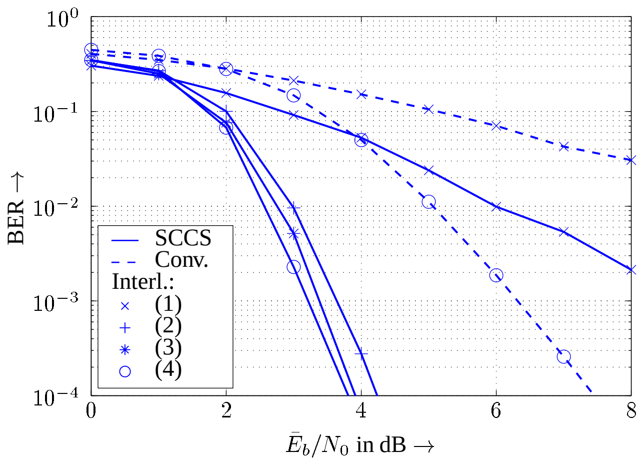

Figure 4: Differently interleaved channel: SCCS with RSC code and iterative decoding $I = 4$ compared to conventional OFDM-CDMA with a NSC code. (1) random frequency (Π_f) and a 18x32 block time interleaver (Π_t), (2) merely sufficient in time (3) merely sufficient in frequency, and (4) sufficient in time as well as frequency

sufficient in time, (3) merely sufficient in frequency, and (4) sufficient in time as well as frequency.
First, we observe that the system sufficiently interleaved in frequency direction outperforms the one sufficiently interleaved in time domain. This can be explained by the large Hamming distance ($M/2$) of the WH codes leading to a large built in degree of diversity. Thus, the inner system, and, hence, the achievable diversity gain in frequency direction mainly determines the overall system performance. There exists only a small loss for the sufficiently frequency interleaved system in comparison to the perfectly interleaved one.
The results for the random frequency and block time interleaver confirm the extreme sensitivity of the considered decoding scheme to burst error structures caused by the indoor scenario (Not shown in the figure, only less gain can be achieved by iterative decoding in this case).
Fig. 4 also shows results for conventional OFDM-CDMA transmission with a NSC code as CC, and again, they emphasize the tremendous loss compared to the SCCS with WH block codes.

6. Conclusions

In this paper, we examined iterative decoding of serial concatenating an outer CC and an inner WH block code in an OFDM-CDMA system. Since the outer code mainly acts in time direction and the WH code is exclusively arranged in frequency direction, the new aspect arises that the iterative decoding scheme exploits the time as well as the frequency domain. In this context, we discussed the MAP decoding of WH block codes, and also presented a common expression for the soft information of coded bits for a trellis based SS-MAP decoding of RSC and NSC codes.
Simulation results for RSC as well as NSC codes show a gain of about 1 dB in $\bar{E}_b/N_0$ (BER=10^{-3}) for the fourth iteration assuming a perfectly interleaved Rayleigh fading channel. The analysis for a differently interleaved indoor mobile channel indicates robustness of the proposed two dimensional coding scheme against very slow fading or non-frequency selective channels. This fact enhance the property of the system to benefit from time as well as frequency selectivity.

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